To O.G.
with sincere good wishes
Christmas 1937
G.W.L.

THE THEORY AND PRACTICE

OF

MODERN FRAMED STRUCTURES.

DESIGNED FOR THE USE OF SCHOOLS,

AND FOR

ENGINEERS IN PROFESSIONAL PRACTICE.

BY

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PREFACE TO THE SEVENTH EDITION.

WHILE many minor changes and corrections have been made in each new edition of this work, these have not been sufficient to warrant calling special attention to them. The following important additions were made in the sixth edition:

(1) Moment Tables for Cooper's conventional method of treating wheel loads (Art. 112*b*).

(2) Chapter IX, on Column Formulæ, has been supplemented by new working formulæ which give correct working loads for all lengths of column, and at the same time give greater factors of safety on short than on long columns, and the reasons therefor. This is an innovation in column formulæ, but the authors feel that the arguments fully justify the change. Diagrams are also given for these new formulæ which enable the designer to take off his working stress as soon as his ratio $\frac{l}{r}$ is approximately known. These are drawn for all grades of material from wrought iron to hard steel, or for all "apparent elastic limits" from 30,000 to 50,000 lbs. per square inch.

(3) A new discussion of swing-bridges (Arts. 175 and 178*a*), which proves that the ordinary formulæ are practically correct, since the neglecting of the web system usually compensates the errors made in assuming the moment of inertia of the chord sections constant. The method given in the last edition of this work, therefore, and since adopted in other recent publications on drawbridges, of considering the moment of inertia of the chords as variable and of neglecting the deflections due to the web system, is here shown to give very erroneous results. These methods, therefore, are not only very tedious in application, but quite misleading in practical designing.

In this edition the following additions have been made:

(1) A Review of Fundamental Principles in Art. 48*a* of Chapter II.

(2) The Derivation of the Fundamental Formula of the Continuous Girder, p. 142.

(3) The experimental strength of Cast-Iron Columns, p. 151.

(4) An approximate analysis of stresses in Full-Spandrel, Two-Hinged, Steel Arches, p. 218*a*.

(5) Twelve full-page plates of illustrations of Bridge Erection Methods in Appendix C.

September, 1898.

PREFACE TO THE FIRST EDITION.

It is now less than fifty years since the first successful attempt was made to correctly analyze the stresses in a framed structure and to proportion the members to resist the given external forces.* In this comparatively short period the rational designing of framed structures has ripened into practical perfection, and the best current practice leaves little to be desired in the way of further development. The only material uncertainties remaining are the dynamic effects of moving loads, and these will probably never submit themselves to any very accurate determination or prediction. This would seem to be a fitting time, therefore, for the presentation to the engineering profession of a general treatise on *Modern Framed Structures.*

The evolution of methods of analysis and of construction has been so rapid during this generating period that no sooner has a work on structures appeared than it has been found to be behind the current practice and no longer representative. It is believed that this rapid evolution of new methods has about run its course, and that we have now settled upon a line of practice, both in analysis and in construction, which will be reasonably fixed so long as the materials employed remain as they are to-day. It was this conviction that led the authors of this work to undertake the task of presenting the subject in as concise and inclusive a form as possible, to serve at once the needs of the student and of the practitioner.

This work is something of a compendium, a text-book, and a designer's hand-book, all in one. As a compendium it is intended to cover a great deal of ground without going too much into details which are found in standard works on mathematics and mechanics. As a text-book it is intended to serve as the student's manual in framed structures, after he has had a course in mathematics and mechanics. As a designer's hand-book it is intended to contain such ready information as any competent designer must constantly use, but which he does not care to burden his mind with.

As a text-book the discreet teacher will not undertake to go over it all with equal care. According to the amount of time he can spare to this subject he will use more or less of it. Chapters I, II, III, IV, V, VIII, and IX are essential as a ground-work for any intelligent designing. After these are mastered any portion of the remainder may be taken at pleasure. It would hardly be wise, in any case, to teach all of Part I before taking something in Part II. After studying a portion or all of the seven chapters named above, it might be

* By Mr. Squire Whipple, of Albany, N. Y. See foot-note, p. 8.

well to assign to each student some simple design, as of a roof truss (each one taking a different style of truss, but all of the same span, loads, spacing, etc.), the teacher leading the class in the problem, and assigning such parts only of the various chapters in Part II as bear on the several elements of the design as they arise for solution. This would indicate at once how Part II is to be used in actual designing, and it would maintain the student's interest in the theoretical portion of Part I by the practical application of it.

If the course is a fairly thorough one nearly all of Part I should be studied sooner or later, and as much of Part II as there is time for. Probably in no case would it all be taught, but the student, in the various problems in designing which are assigned to him, should have occasion to consult nearly all parts of the book.

The work may be criticised on the one hand for being too concise, and on the other for being too inclusive. The authors have tried to avoid all unnecessary verbiage and such mathematical developments as are given in works necessarily preparatory to this, to keep the book from becoming too bulky; and they have intended to fairly cover the field of structural designing in which the engineer of to-day is called upon to practise.*

This work has been written by so many persons that it is only in a limited sense that those whose names appear on the title-page may be considered its authors. These latter, however, have had the direction of the work and have written much the larger portion of it. It has been their controlling motive to have the book represent correctly the latest and best practice, and in many ways to even point out some improvements in both the analysis and the designing of structures.

In order that the reader may always know the particular author he may be reading, the following scheme is given as a key to such information. While Prof. Johnson has had general charge of the entire work, in an editorial capacity, and has written portions of various chapters not ascribed to him, the work has been divided as follows:

Prof. J. B. Johnson, Chapters I, VI, VIII, IX, X, XI, XV, XXIII, XXV (Parts I and III), and XXVII.

Prof. F. E. Turneaure, Chapters II, III, IV, V, VII, XII (in part), XIII, and XIV.

Mr. C. W. Bryan, C.E., Chapters XVI, XVII, XVIII, XIX, XX, XXI, XXII, and XXV (Part II).

Mr. J. W. Schaub, M. Am. Soc. C. E., Chapters XII (in part) and XXIV.

Mr. David A. Molitor, C.E., Chapter XXVI.

Mr. C. T. Purdy, C.E., Chapter XXVIII.

Mr. Geo. H. Hutchinson, C.E., Chapter XXIX.

Mr. F. H. Lewis, M. Am. Soc. C. E., Appendix A.

Mr. A. L. Johnson, C.E., Appendix B.

Mr. Frank W. Skinner, M. Am. Soc. C. E., Appendix C.

It is only due to Washington University to say that at the time he did the work Prof. Turneaure was Instructor in Civil Engineering in that institution (C.E. Cornell University), while Messrs. Schaub, Bryan, Molitor, and A. L. Johnson are graduates from its civil engineering course.

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* The chapter on Lock Gates which was in the original scheme was made unnecessary by the excellent monograph on this subject by Lieut. Hodges, published in 1892 by the Corps of Engineers, U.S.A., as *Professional Papers*, No. 26.

Mr. Schaub has been Chief Engineer of two of the largest bridge works of America, namely, the Dominion Bridge Company of Montreal and the Detroit Bridge Company. He was for many years an assistant to Mr. C. Shaler Smith, one of the great bridge engineers this country has produced. He is now General Manager of the Pottsville Bridge Works, Pottsville, Pa. His chapters on draw bridges can therefore be regarded as authoritative.

Mr. Bryan speaks also with authority, as he has for many years been the Designing Engineer of the Edge Moor Bridge Works, which are the largest structural works in the world.

Mr. Purdy has designed many of the tall steel-skeleton buildings of Chicago, and Messrs. Hutchinson, Lewis, A. L. Johnson and Skinner are also fully qualified, both theoretically and practically, to speak on the subjects treated by them.

Mr. Molitor has written a chapter in an entirely new field, so far as the English literature is concerned. He has spent a number of years in Europe in engineering practice, and has had an opportunity to cultivate a naturally strong æsthetic sense. His private library and his collections of photographs are the main sources from which he obtained his material. It is to be hoped that this chapter may give an impetus to the growing sense of dislike for the innate ugliness which now characterizes many of the largest bridges of this country.

The authors wish also to acknowledge their indebtedness to Prof. Green of the University of Michigan, to Prof. Crandall of Cornell University, to Prof. Swain of the Massachusetts Institute of Technology, to Dr. Eddy, President of Rose Polytechnic Institute, for many ideas and methods which have been incorporated in the body of the work, and to Mr. Wolcott C. Foster, for the use of some plates from his "Wooden Trestle Bridges." Other acknowledgments will be found in foot-notes scattered through the book.

The authors have spared no expense in the matter of cuts and plates, nearly all of which have been specially drawn for this work, and engraved by the American Bank Note Company of New York. Only a few of the plates have been reproduced by photographic processes. The publisher of Hutton's monograph of the Washington Bridge has kindly granted the use of several plates from that excellent work.

That this work should fairly and adequately exemplify the principles and practice of structural designing in America, and meet with the approval of their fellow teachers and practitioners, has been the constant hope and aim of

THE AUTHORS.

July, 1893.

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THEORY AND PRACTICE

IN THE DESIGNING OF

MODERN FRAMED STRUCTURES.

PART I.

THEORY OF FRAMED STRUCTURES.

CHAPTER I.

DEFINITIONS AND HISTORICAL DEVELOPMENT.

1. **A Simple Framed or Articulated Structure** is one composed of straight members so attached at their extremities as to cause the structure to act as one rigid body. It may be contrasted with masonry structures on the one hand and with solid beams, plate girders, arches, wire suspension-bridges, and the like on the other.

2. **The External Forces** include all the loads and foundation reactions, including the weight of the structure itself, which act upon and which tend to distort it. These forces are always replaced by their equivalent forces applied at the joints before the direct stresses in the members can be computed.

3. **Strain** is the distortion of a body caused by the application of one or more external forces.* It is measured in units of length, as inches, and not in pounds. The proportional, or relative, strain is usually meant, this being the distortion per unit of original length, or in other words, the actual distortion divided by the original length of the member.

4. **Stress** is the resistance of a body to distortion, and can only exist in unconfined bodies when these are solid or plastic. It is measured in pounds or tons the same as the external forces. The stresses resist, or hold in equilibrium, the external forces, but the immediate cause of the stress is the distortion of the body.† The external forces upon a framed structure distort the members until the resisting stresses developed in them are sufficient to hold in equilibrium these external or distorting forces. For bodies in equilibrium the external forces and the internal stresses stand in the relation to each other of action and reaction in mechanics. Furthermore, when the stress in one member is resisted by or transmitted to another member or part of a structure it acts upon the latter as an external force. Thus the reaction of the foundation is a stress in the masonry support, but is to be treated as an external force acting upon the superposed structure.

5. **Relation between Stress and Strain.**—In all solid bodies there is a definite relation between the intensity of the stress and the amount of the accompanying strain. No body is so rigid as to remain unstrained, or undistorted, under the application of any finite external

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* In popular language "strain" and "stress" are often confused and used indiscriminately. Some authors of repute have also followed the popular usage, but the definitions here given conform to the practice of the leading authorities.

† It is common to say the distortion is caused by the stress. But a resistance cannot be the cause of the thing resisted. Though coincident in time and place, the distortion is really the cause of the stress.

force, however small.* Within a certain limit for any particular material a given increase in the external force is always accompanied by a proportionate increase in the strain, or distortion, and this develops a like increase in the stress, or resistance. Thus if any bar of rolled iron or steel be distorted by an external force (pull or thrust) of 28 lbs. per square inch, it will stretch or shorten, as the case may be, an amount equal to one one-millionth part of its length, the internal stress, or resistance to distortion, then coming to be just equal to the external force of 28 lbs. per square inch. An external force of 28,000 lbs. per square inch distorts or strains the bar one one-thousandth part of its length and then develops in the bar a resistance or stress of 28,000 lbs. per square inch. It is evident, however, that this resistance cannot continue to increase indefinitely in proportion to the distortion. There always comes a time, if the external force continues to increase, when a greater increment of distortion is requisite to develop a given increment of resistance. This point is called

6. **The Elastic Limit.**—Below this limit the stress and the strain are proportional, equal increments of one always producing equal increments of the other.† Also, below this limit, when the distorting force ceases to act the body returns to its original shape and dimensions and the stress is relieved. If the body be distorted beyond the elastic limit, the strain increases more rapidly than the stress, or than the external force, these two always of necessity being equal to each other, and some of the distortion becomes permanent. That is, when the external force is removed the body does not fully return to its original dimensions, but remains permanently distorted somewhat, or it is said to have "taken a set."

7. **The Modulus of Elasticity** is the ratio of the stress per unit of area to the relative strain, or distortion, which accompanies it. In other words, it is *unit stress divided by unit strain*, or

$$E = \frac{\text{unit stress}}{\text{unit strain}} = \frac{f}{\frac{\alpha}{l}} = \frac{fl}{\alpha}; \qquad \text{(1)}$$

where α = distortion or strain (either elongation or compression);
l = original length of part under stress;
f = stress per unit area (pounds per square inch in English units).

Since pounds and inches are the standards used in English, the modulus of elasticity as given and used in all English works must be understood to represent pounds per square inch, the denominator of our fraction in eq. 1 being an abstract number.‡

This modulus or ratio is constant within the elastic limit. Beyond that it steadily decreases until it reduces to zero in the case of solid metals where the material becomes plastic and draws out or compresses under a constant load. All working stresses are, or should be, well within the elastic limit, and hence for all such stresses this ratio is constant for any given material. When it is known the resisting stresses can be found for a known distortion, or the distortion may be computed for a known stress. The determination of this ratio requires very delicate measuring apparatus with the most careful and expert handling. Tabular values given for these moduli for different materials in standard works are not very reliable. Thus, for all the rolled irons and steels this modulus is remarkably constant, being perhaps always between 26,000,000 and 31,000,000 lbs. per square inch for the ordinary

* It may be found helpful to think of all engineering materials as composed of india-rubber in order to free our minds from the notions of absolute rigidity which are apt to be associated with the harder kinds of structural materials.

† This is known as Hooke's Law and was originally expressed by the Latin phrase "*Ut tensio sic vis.*"

‡ If this denominator could become unity, which it never can in solids, then the fraction, or E, would represent the number of pounds per square inch required to stretch a body to twice its original length, and the modulus of elasticity is sometimes so defined.

temperatures, while the tensile strength of these metals will vary from 45,000 lbs. per square inch for wrought-iron and soft steel to over 200,000 lbs. per square inch for hard-drawn steel wire. It is an extremely valuable property of engineering materials, and is used to great advantage by the scientific designer.

8. **Examples.**—The following examples are given to illustrate some of the uses to be made of the modulus of elasticity. In solving these problems, take the modulus of wrought-iron as 27,000,000, and of steel as 28,500,000; of cast-iron as 12,000,000; and of timber as 1,500,000 lbs. per square inch.

1. A steel-wire cable 5 miles long and one square inch in solid section is pulled with an average force of 15,000 lbs. What is the strain, or stretch?

2. The rim of a cast-iron fly wheel 10 feet in diameter is subjected to a tensile stress of 5000 lbs. per square inch from the centrifugal force. How much is its diameter increased?

3. If an iron or steel rail 30 feet long is prevented from expanding, what will be the stress in it per square inch resulting from a rise of temperature of 80° F., the coefficient of expansion being taken at 0.0000065?

4. A series of wooden posts superposed upon each other in a building to a total height of 60 feet are subjected to an average compressive stress of 1000 lbs. per square inch. How much will be the settlement at the top from this cause?

THE TRUSS AND ITS ELEMENTS.

9. **A Truss** is a framed or jointed structure designed to act as a beam while each member is usually subjected to longitudinal stress only, either tension or compression.

10. **The Struts** are those members which are compressed endwise, and which therefore have developed in them compressive resistances or stresses. Struts are sometimes called Posts, or Columns.

11. **The Ties** are those members which are extended, and which thus have developed in them tensile resistances or stresses.

12. **The Upper and Lower Chords** are composed of the upper and lower longitudinal members respectively. When the loads are downward and the truss is supported at its ends, the upper chord is always in compression and the lower chord always in tension. The spaces between the chord joints are called *panels*.

13. **The Web Members** are those which join the two chords. They are alternately in tension and compression, or the struts and ties alternate in the web system.

14. **A Counterbrace** is a member which is designed to resist both tensile and compressive strains. That is, for one position of the load the member may be elongated, while for another it may be compressed, and hence at different times it must resist both extension and compression. When two or more external forces act upon it, some of which tend to compress the member and others to extend it, it is evident that only the algebraic sum of these forces really acts upon the member. It is subjected to a stress, therefore, equal to and of the same kind as the algebraic sum of all the external forces acting upon it.

Hence, also, a tension member or tie may resist a compressive external force without becoming a counterbrace, so long as this compressive force is smaller than another extending force which also acts. Similarly with struts, they can resist tensile external forces without becoming ties, so long as these are less than other compressive forces which continue active. The residual stress is tension in the one case and compression in the other, for which only the member is designed, and therefore we may say that both struts and ties may resist the contrary external forces without becoming counterbraces, the stresses in the member always being of one sign * or kind.

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* In this book tension is called *minus* and compression *plus*. There is no objection to following the contrary rule. It is only important that both stresses and forces of opposite kinds should enter with opposite signs.

15. Mains and Counters.—A main member, whether strut or tie, is one which acts when the entire structure is loaded. A counter is one which acts only for particular partial loads.

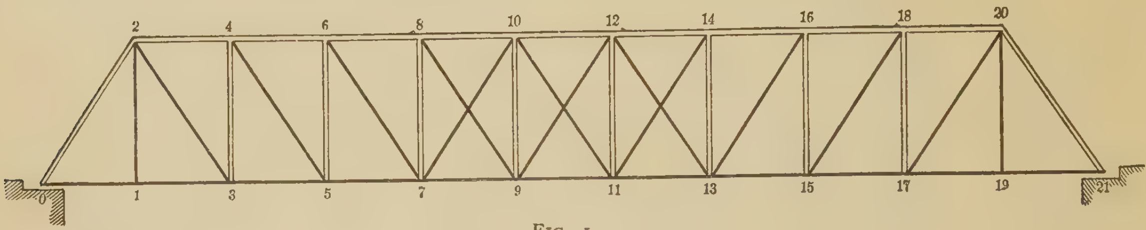

FIG. 1.

16. Illustration.—In Fig. 1, which represents a Pratt Truss, the compression members, or struts, are shown by double lines, and the tension members, or ties, by single lines. When the end posts are inclined as in the figure, they would seem to belong about as much to the upper chord as to the web system. They are usually spoken of separately as the "inclined end-posts" or the "batter-braces."

The members 7–10, 9–12, 10–11, and 12–13 are counters, or counter-ties. There are no counter-braces in this truss; that is, no members have to resist both tension and compression.

The tension members 2–3, 4–5, 6–7, 8–9, 11–14, 13–16, 15–18, and 17–20 are main tie-rods. The members 1–2 and 19–20 are not elements of the truss proper, since they serve only to carry the loads at 1 and at 19 to the hip-joints.

17. The Action of a Truss.—Since a truss is a jointed structure composed of rigid but elastic members so arranged as to form an unyielding combination, it must be composed of an assemblage of rigid polygons. But the only rigid polygonal figure is a triangle. A truss must therefore be composed of an assemblage of triangles. Any assemblage of triangles fastened together at their apices, consecutive figures having sides in common, is a truss, and will act as a beam. In Fig. 1 the triangles are all right-angled. A load placed at joint 7, for instance, is carried by the truss as a beam to the abutments at 0 and 21. The part of this load which goes to the left abutment may be conceived as being carried up to 6, down to 5, up to 4, down to 3, up to 2, and then down to 0, where it passes to the ground. The part which goes to the right passes over the path 7, 10, 9, 12, 11, 14, 13, 16, 15, 18, 17, 20, and 21. Thus for such a load the ties 7–10 and 9–12 are put under stress, while 8–9, 10–11, and 12–13 are idle, as well as the post 7–8 and the hangers 1–2 and 19–20. If all these idle members were removed, the truss would stand under this particular loading, since it would still remain an assemblage of triangles, properly joined. If the load were placed at 9, 9–8 and 9–12 would be under stress, while 7–10 and 10–11 would be idle. If the two middle joints 9 and 11 were loaded equally, the part of the load at 9 going to the right is just balanced by the part of the load at 11 going to the left, and hence there is no stress in the intermediate web members 9–12, 10–11, 9–10, and 11–12. The counter-ties 7–10 and 12–13 are also idle.

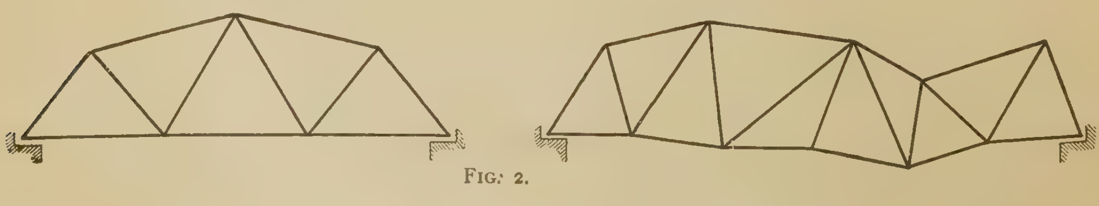

FIG. 2.

In Fig. 2 we have generalized conceptions of a truss. They are assemblages of triangles, adjacent figures having a common side, and exemplify the generic idea of a truss.

A truss is not weakened from its want of symmetry or from its sagging in the middle, provided all the members are properly proportioned to carry their loads.

A Through-bridge is one in which the roadway is carried directly at the bottom-chord joints, with lateral bracing overhead between the top-chord joints, thus enclosing a space through which the load passes.

A Deck-bridge is one in which the roadway is carried directly at the top-chord joints, or on the upper chords themselves. The trusses are usually placed closer together than on through-bridges, the roadway extending over them.

A Pony Truss is a low truss of short span, with the roadway carried at the bottom joints, but not of sufficient height to allow of the upper lateral bracing. The trusses are stayed, or held to place, by bracing, connected with the floor system.

HISTORICAL DEVELOPMENT OF THE TRUSS IDEA.

18. Primitive Systems.—The earliest forms of truss were built of timber, the progressive development of the forms used for bridge purposes being shown in the following figures.

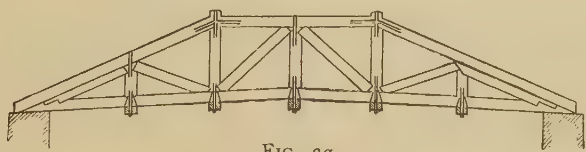

FIG. 3*a*.

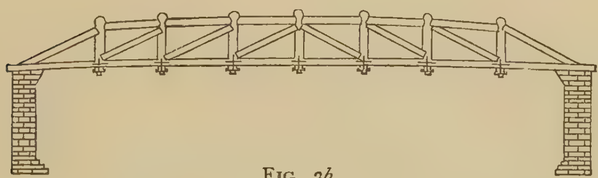

FIG. 3*b*.

FIG. 3*c*.

Figs. 3*a*, 3*b*, 3*c* are three forms of truss construction employed by Palladio, a famous Italian architect, 1560–80. These trusses were built entirely of timber, and are believed to be the earliest examples of a scientific use of the truss element, the rigid triangle. Palladio wrote an elaborate illustrated treatise on architecture in which these and other forms of truss have been preserved.

The mastery of the principles of truss construction did not follow the practice of Palladio, and so fine an example of the use of the truss element is not found again for nearly three hundred years.

Fig. 4 represents one span, 170 feet long, of a bridge over the Rhine at Schaffhausen, built in 1758 by Ulric Grubenmann, a carpenter by trade but really a great engineer. He

afterwards built a wooden bridge of similar design, 366 feet long, near Baden. Both these bridges served their purpose till destroyed by Napoleon in 1799.

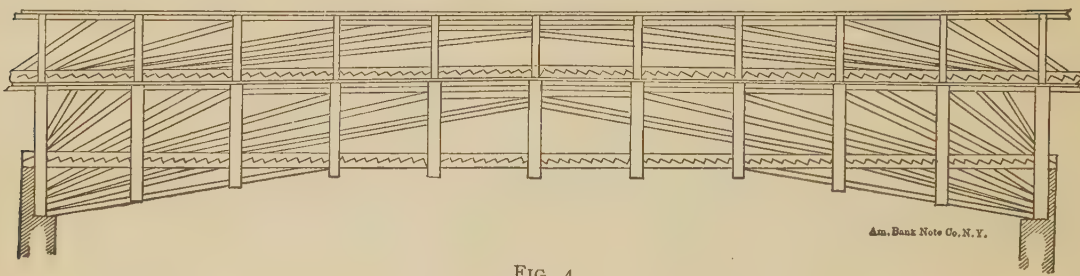

FIG. 4.

Fig. 5 represents the "Permanent Bridge" over the Schuylkill River at Philadelphia, built by Mr. Timothy Palmer of Newburyport, Mass., in 1804.* The middle span was 195 feet and the side spans were 150 feet each. It was covered in and continued in service till 1850, when it was replaced by a bridge for railroad purposes.

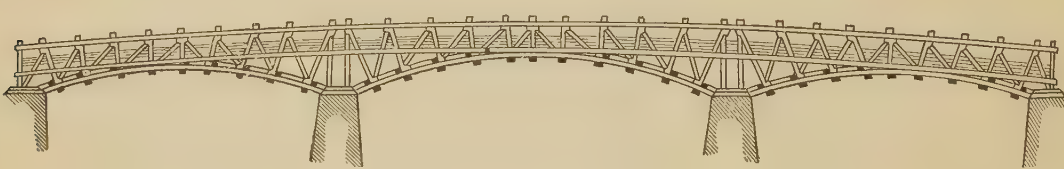

FIG. 5.

In 1804 Mr. Theodore Burr built a bridge over the Hudson River at Waterford, in four spans of 154, 161, 176, and 180 feet clear span, respectively, after the pattern shown in Fig. 6.

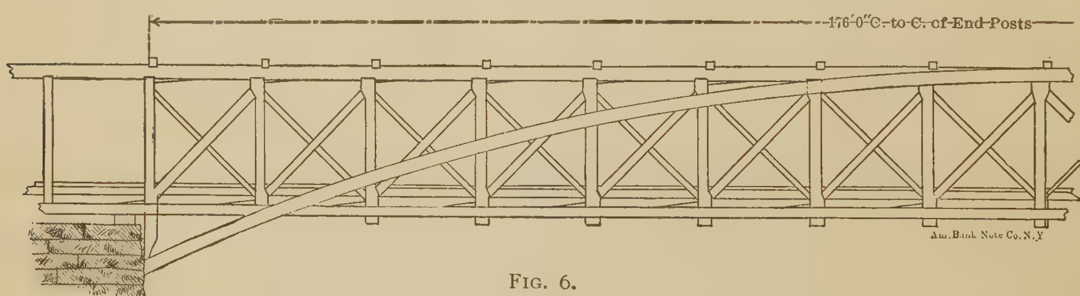

FIG. 6.

All members were of timber, and counter-struts were inserted the entire length, thus giving great rigidity. This is probably the most scientific design for an all-wooden bridge ever invented, and for a half-century it stood unrivalled for cheapness and efficiency for highway purposes in this country. These bridges were always covered in, the covering extending many feet beyond the end of the truss proper.

In Fig. 7 is shown a view of the Colossus Bridge over the Schuylkill River at Philadelphia, built in 1812 by Mr. Lewis Wernway. It was 340 feet clear span, and marked a great advance on previous practice in America in the length of span. It was destroyed by fire in 1838. These three gentlemen, Palmer, Burr, and Wernway, were, up to 1840, the leading bridge engineers of America.

All the above types of bridges, Figs. 4–7, are composite forms and not simple trusses.

* See Paper on *American R. R. Bridges*, by Theodore Cooper, *Trans. Am. Soc. Civ. Engrs.* Vol. XXI, p. 1.

on the Rensselaer and Saratoga Railway (Fig. 11). The ties in the web system extend over two panels, and it is therefore called a "double-intersection" truss. The lower chord was composed of wrought-iron links passing over wrought-iron trunnions in the bottoms of the posts. All the compression members were of cast-iron, and it was pin-connected in both upper and lower chords. This form of arrangement of members is still known as the Whipple truss.

In 1863 Mr. John W. Murphy first used wrought-iron for all the compression members in truss construction, but still used cast-iron in joint-blocks and pedestals. On account of this improvement by Mr. Murphy in the Whipple truss, the modern wrought-iron or steel double-intersection, horizontal chord, truss is sometimes called the Murphy-Whipple truss.

In 1861 Mr. J. H. Linville first used wide forged eye-bars and wrought-iron posts in the web system. He still retained the cast-iron upper chord.

To Messrs. Whipple, Murphy, and Linville, therefore, is largely due the credit for establishing in this country the distinctive practice of eye-bar and pin connections which are still used here on all long-span iron bridges.

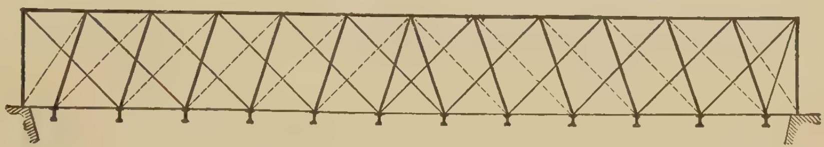

FIG. 12.

From 1865 to 1880 a great many railroad and highway bridges were built under patents granted to Mr. S. S. Post, all being of the style shown in Fig. 12. This truss is known as the Post truss, its distinctive feature being that the web struts, instead of standing vertically, have a horizontal run of one half a panel length, while the ties have a horizontal run of one and one-half panel lengths. The theoretical economy from this arrangement is now thought to be offset by corresponding practical disadvantages and the truss is no longer built.

Since this historical account treats only of truss forms, no mention is made of the early cast-iron arch-bridges, and of iron-link and wire suspension-bridges, many kinds of which, of long spans, preceded the introduction of the truss proper. In fact, in England and on the Continent the truss developed out of a combination of the arch and suspension systems, cast-iron being used in the arched upper chord, and wrought-iron links in the curved lower chord, the two being rigidly held with vertical struts and diagonal tie-rods.

Since about 1870 cast-iron has been entirely abandoned in America in the construction of railroad bridges, and since about 1880 in highway bridges as well.

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were at first made of the same size from end to end. Mr. Whipple's work is preserved in a small book of 120 pp. entitled "A Work on Bridge Building, consisting of Two Essays, the one elementary and general, the other giving Original Plans and Practical Details for Iron and Wooden Bridges. By S. Whipple, C.E. Utica, N. Y., 1847."

This is a remarkable work. The author not only has correctly analyzed bridges for both static and moving loads, correctly dimensioning all members, including the counters, but he computes the total "strain-lengths" of various styles of bridges and compares their relative weights in this manner. He also gives a very good column formula, finds the best ratio of panel length to height of truss, and of height of truss to length, and compares the relative cost of wood and iron bridges. There are ten plates of details, including his first designs for the double-intersection iron truss since called by his name. His methods of analysis were graphical but strictly correct. He was the first to use pin-connections in the bottom chord, and his designs for this wrought-iron pin-joint are extremely interesting.

This book had been published three or four years when Hermann Haupt wrote his work on bridges. Apparently, Mr. Haupt had never seen a copy of it, since he claims his work as original also, and there is no internal evidence that he had seen Whipple's book. His methods of analysis are much cruder than Mr. Whipple's and far less complete.

The theory of the stone arch, and of arch and suspension bridges under fixed or uniform loads, was early developed, but the true theory of truss action seems to have originated with Mr. Whipple. His manuscript was written in 1846. See *Development of the Iron Bridge*, by S. Whipple in *R. R. Gazette*, Apr. 19, 1889; and Discussion by A. P. Boller in *Transactions Am. Soc. Civ. Engrs.* Vol. XXV, p. 362. Also *American R. R. Bridges*, by Theodore Cooper, *Trans. Am. Soc. Civ. Engrs.* Vol. XXI, p. 1.

The favorite style of truss now for moderate spans for all purposes is the Pratt truss (Fig. 1). It was patented in 1844 by Thomas W. and Caleb Pratt as a combination wood and iron bridge. It was a variation from the Howe truss in that the diagonals were of iron and used in tension, while the verticals were struts and were made of timber. It never became a popular style until wrought-iron came to be used exclusively in bridge construction, when it was found to have advantages over all other forms. It will be fully developed in the body of this work.*

In closing this short account of the development of the idea of the simple truss, it should be said that only within the last twenty-five or thirty years have the mathematical principles governing the distribution of stresses in a truss been generally understood, and for a still shorter period has the actual strength of full-sized members and joints been even approximately known. All the earlier examples of bridge construction were designed and executed by carpenters and mechanics wholly ignorant either of the values of the stresses or of the strength of the parts, except as experience had educated their judgments of what would probably serve the purpose. They are deserving, therefore, of high honor for the great works they were able to build without any of those scientific aids now offered to every student in our numerous engineering schools.

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* No historical account of American iron bridges would be complete without some notice of the efforts of Thomas Paine to introduce long cast-iron arch-bridges of low rise. As early as 1786 he advocated the use of cast-iron for long arch-bridges, with a rise of about one twentieth of the span; and he had such arches made and tested at his own expense 90 feet in length, as models for a 400-foot span which he urged Congress to build as an example to educate the public. The Academy of Sciences of Paris reported favorably on his design for this length of span, but his model was sold for debt and afterwards used in England. He never took out a patent, his object being purely benevolent.

CHAPTER II.

APPLICATION OF THE LAWS OF EQUILIBRIUM TO FRAMED STRUCTURES.

DEFINITIONS.

20. **Forces** are *concurrent* when their lines of action meet in a point; *non-concurrent* when their lines of action do not so meet.

Forces may also be *coplanar*, that is, lying in the same plane; or *non-coplanar*, lying in different planes. Coplanar forces only will be here considered.

A force is fully defined when its *amount*, its *direction*, and its *position* are known.

21. **The Moment of a Force** about a point is the product of the force into the perpendicular distance from the point to the line along which the force acts; it is a measure of the rotative action of the force about the point.

22. **A Couple** is a pair of equal and opposite forces having different lines of action.

23. **Equilibrium.**—A system of forces acting upon a body is in equilibrium, or balanced, when the *state of motion* of the body is not thereby changed; e.g., a body at rest or moving at a uniform velocity is being acted upon by a balanced system of forces. The body also is said to be in equilibrium.

As we distinguish two kinds of motion, translation and rotation, so we may distinguish two kinds of equilibrium, equilibrium of translation and equilibrium of rotation. A body to be in complete equilibrium must be so in both these senses, and one does not imply the other.

24. **The Resultant** of a system of forces is a single force which will replace that system as regards its effect upon the state of motion of the body acted upon. A force equal and opposite to the resultant will balance the resultant and therefore the original system. In the single case where the system reduces to a couple no one force will replace the system.

For equilibrium of translation the resultant must equal zero. For equilibrium of rotation the sum of the moments of the forces about any point must equal zero.

RESULTANT AND EQUILIBRIUM OF CONCURRENT FORCES.

25. **Graphically.**—Let $P_1 \ldots P_5$, Fig. 13 (*a*), be a system of concurrent forces applied at *A*. Their resultant may be found as follows:

Lay off $O1$, Fig. 13 (*b*), equal by scale to P_1 and having the same direction, then from 1 lay off 1–2 equal and parallel to P_2. By the principle of the parallelogram of forces, $O2$ is the resultant of P_1 and P_2 in amount and direction. Similarly, by laying off 2–3, 3–4, and 4–5 equal and parallel to P_3, P_4, and P_5, respectively, we have $O3$ equal to the resultant of P_1, P_2, and P_3; $O4$ the resultant of P_1, P_2, P_3, and P_4; and finally $O5$ equal to the resultant of our given system in amount and direction. Its point of application is *A*.

The order in which the forces are laid off in (*b*) is immaterial, as each force will evidently have the same effect upon the final position of a point following around the figure in whatever part of the path the force occurs. Thus the order P_4, P_3, P_1, P_2, and P_5 gives the figure $O1'2'3'4\,5$. We must, however, be careful to draw each force in its true *direction;* e.g., P_3 must be drawn from 2 towards 3, and not from 2 towards $3''$.

A figure such as Fig. 13 (*b*) is called a *Force Polygon.*

Since R is the resultant of $P_1 \ldots P_5$, if we apply a force R' at A, equal and opposite to R, the forces $R', P_1 \ldots P_5$ will form a balanced system. This is seen to be true from the

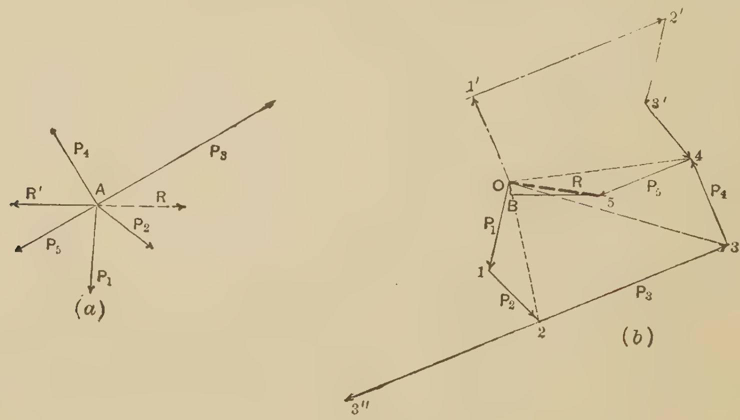


FIG. 13.

force polygon also, for since $5O$ is equal and parallel to R', the resultant of the system $P_1 \ldots P_5$, R' is zero, and therefore we have equilibrium of translation. Equilibrium of rotation is not in question, the forces being concurrent. Expressed graphically, the only condition necessary for equilibrium of a system of concurrent forces is that their *force polygon must close.*

To sum up, we may state that the resultant of a system of concurrent forces is given in amount and direction by the closing line of the force-polygon, this closing line being drawn from the origin to the end of the last force; and the force necessary to balance the given system is given by this same closing line drawn in the opposite direction. The resultant is simply the shortest way of passing from the origin of our roundabout system to its end. Any system of concurrent forces with its force polygon beginning at O and ending at 5 would be equivalent to the given system, and if beginning at 5 and ending at O would balance the system. Moreover, each of the forces in a closed polygon is equal and opposite to the resultant of all the other forces.

26. Algebraically.—Take the same system of forces as before, Fig. 14. Resolve each force into horizontal and vertical components (any two directions at right angles will do as well), or along the axis of X and the axis of Y respectively. These components form a system equivalent to the given system. Let Σ hor. comp. be the algebraic sum of the horizontal components, $= H$ in the figure, and Σ vert. comp. that of the vertical components, $= V$ in figure. The resultant, R, in amount is evidently equal to

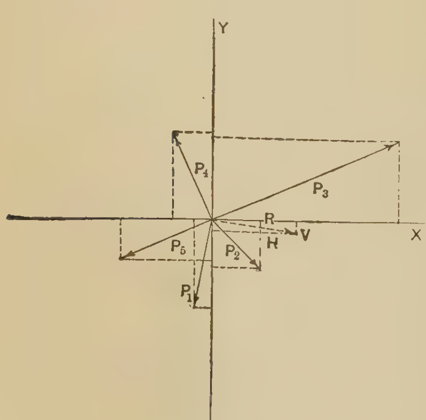


FIG. 14.

$$\sqrt{(\Sigma \text{ hor. comp.})^2 + (\Sigma \text{ vert. comp.})^2}.$$

The tangent of the angle between R and the axis of Y is given by $\dfrac{\Sigma \text{ hor. comp.}}{\Sigma \text{ vert. comp.}}$, and thus R is determined in amount and direction. Its point of application is A as before, whence it becomes fully known.

For equilibrium, R must be zero, or $\sqrt{(\Sigma \text{ hor. comp.})^2 + (\Sigma \text{ vert. comp.})^2} = 0$, which requires that

$$\Sigma \text{ hor. comp.} = 0, \quad \ldots \ldots \quad (1)$$

and

$$\Sigma \text{ vert. comp.} = 0. \quad \ldots \ldots \quad (2)$$

These two equations express algebraically the condition we have already expressed graphically by requiring that the force polygon must close. Referring to Fig. 13 (b), we see that Σ hor. comp. is the net horizontal displacement in passing from O to $5, = B5$, and that Σ vert. comp. is the net vertical displacement, $= OB$; also that $R = \sqrt{(\Sigma \text{ hor. comp.})^2 + (\Sigma \text{ vert. comp.})^2}$. For equilibrium, $R = 0$.

27. Remarks.—The foregoing conditions of equilibrium are precisely similar to the conditions necessary to a balanced land-survey. As there, the plot must close, or the latitudes and departures must each sum up zero, so here, our force polygon must close, or Σ hor. comp. $= 0$ and Σ vert. comp. $= 0$. And further, as we can supply two unknown quantities in the former case, so here, having a system in equilibrium, we can, either graphically or by the use of the above equations, compute two unknowns. They may be either the amounts or directions of two forces, the amount of one and the direction of another, or the amount and direction of one.

RESULTANT AND EQUILIBRIUM OF NON-CONCURRENT FORCES.

28. Graphically.—Let $P_1 \ldots P_4$, Fig. 15, be a system of non-concurrent forces. Required their resultant and conditions of equilibrium.

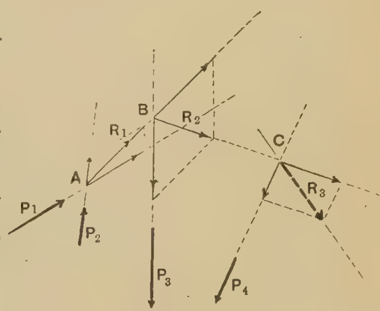


FIG. 15.

We can combine P_1 with P_2, getting their resultant R_1, then this resultant with P_3, getting R_2, and finally R_2 with P_4, getting R_3, the resultant of the entire system in amount, direction, and position. To save drawing the separate parallelograms we may construct a force polygon as in Fig. 16 (b), and draw the lines $O2$, $O3$, and $O4$, these being the resultants, R_1, R_2, and R_3, in amount and direction. Then from A, the intersection of P_1 and P_2, draw AB parallel to R_1, and from the intersection of AB with P_3 draw BC parallel to R_2, and lastly CD parallel to R_3. The line CD will be the line of action of R_3 whose amount and direction are given in the force polygon.

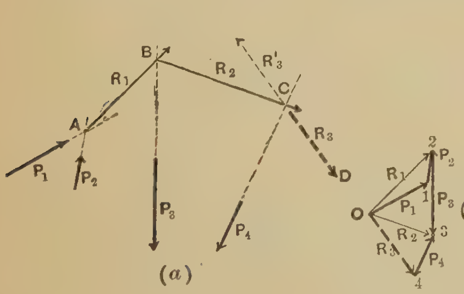


FIG. 16.

The figure $ABCD$ is called an *equilibrium polygon*, and AB, BC, and CD are its *segments*. The point O is called the *pole*, and R_1, R_2, and R_3 are *rays*, of the force polygon; $O1\ 2\ 3\ 4$ is called the *load line*.

To cause equilibrium there must be a force R_3', equal and opposite to R_3 and applied in the line CD. Graphically then, for equilibrium of non-concurrent forces, the *force polygon must close*, giving equilibrium of translation, and the *last force must coincide with the last segment of the equilibrium polygon*, giving equilibrium of rotation. It is important to remember that each segment of an equilibrium polygon is the line of action of the resultant of all the forces to the left of that

segment; and if the system is a balanced one, it is as well the line of action of the resultant of all the forces upon *either* side of that segment. When the forces are parallel or nearly so, a special expedient is necessary to enable us to draw an equilibrium polygon. This case is treated of in Arts. 37–44.

29. Algebraically.—Resolve the forces into horizontal and vertical components, as in the case of concurrent forces. The amount of the resultant, R, is given by $\sqrt{(\Sigma \text{ hor. comp.})^2 + (\Sigma \text{ vert. comp.})^2}$; its direction by $\frac{\Sigma \text{ hor. comp.}}{\Sigma \text{ vert. comp.}}$ Its line of action is found by putting its moment about A, equal to the sum of the moments of the given forces, or $Ra = \Sigma$ mom.

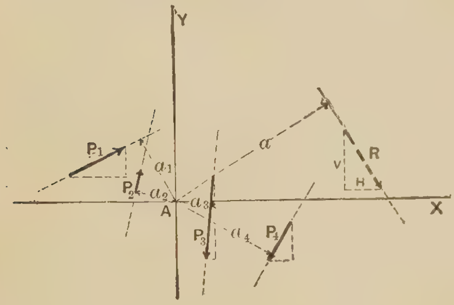


FIG. 17.

To cause equilibrium we must have R, or

$$\sqrt{(\Sigma \text{ hor. comp.})^2 + (\Sigma \text{ vert. comp.})^2}, = 0,$$

giving equilibrium of translation, and the sum of the moments, or Σ mom., $= 0$, giving equilibrium of rotation. This gives us three equations of equilibrium:

$$\Sigma \text{ hor. comp.} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\Sigma \text{ vert. comp.} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\Sigma \text{ mom.} \qquad = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which hold true for any system of non-concurrent forces in equilibrium, and hence, having such a system, we can in general determine three unknowns. As the forces are as a rule known in position and direction from other considerations, the unknowns are usually the amounts of three forces. If, however, the three unknown forces themselves meet in a point, they are indeterminate; for, since the system is by supposition in equilibrium, the resultant of the known forces must pass through the same point, and the system is for our purposes a concurrent one, and we have but *two* independent equations. Instead of the first two equations above, it is often more convenient to write two other moment equations, taking a new centre of moments each time. Equations (3) and (4) are then not independent.

APPLICATION OF THE EQUATIONS OF EQUILIBRIUM.

30. Methods.—In determining the stresses in framed structures there are three general methods of applying the equations of equilibrium.

1st. To the structure as a whole.

2d. To any single joint.

3d. By passing a section through the structure, removing one portion and applying the equations of equilibrium to the remaining portion, the stresses in the members cut being replaced by equal external forces. This is known as the *method of sections.*

I. *Algebraical Application.*

31. First. To the Structure as a Whole.—The external forces acting upon a structure in equilibrium form a balanced system, to which may be applied the three equations of equilibrium of Art. 29. We can therefore in general fully determine these external forces, provided there are not more than three unknown.

EXAMPLE 1. Suppose the roof-truss in Fig. 18 to be acted upon by the wind-pressure, W, acting normally to the roof; the weight, G, of the roof and truss applied at their centre of gravity and acting downwards; and the abutment reactions as yet unknown in direction or amount. These comprise all the external forces.

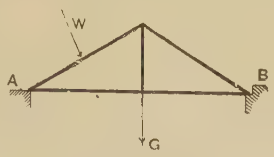


FIG. 18.

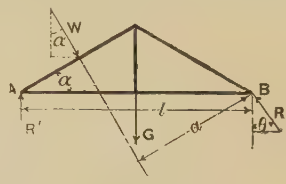


FIG. 19.

Fig. 19 shows the truss free from the abutments, with the abutment reactions, R' and R'', put in. The left end of the truss is supposed to rest upon rollers, the abutment at B taking all the horizontal thrust due to wind. This being the case, R' will be vertical and R'' inclined at some angle θ with the vertical. The unknowns are R', R'', and θ. Applying our three equations of equilibrium: Σ hor. comp. = o gives

$$W \sin \alpha - R'' \sin \theta = 0; \quad \text{. . . . .} \quad (a)$$

Σ vert. comp. = o gives

$$R + R'' \cos \theta - W \cos \alpha - G = 0; \quad \text{. . . . .} \quad (b)$$

Σ mom. about B = o gives

$$R'l - G \times \tfrac{1}{2}l - Wa = 0. \quad \text{. . . . .} \quad (c)$$

From (c)

$$R' = \frac{Wa + \frac{1}{2}Gl}{l}. \quad \text{. . . . .} \quad (d)$$

From (a)

$$R'' \sin \theta = W \sin \alpha. \quad \text{. . . . .} \quad (e)$$

From (b) and (d)

$$R'' \cos \theta = W \cos \alpha + G - \frac{Wa + \frac{1}{2}Gl}{l},$$

and since $R'' = \sqrt{R''^2 \sin^2 \theta + R''^2 \cos^2 \theta}$, and $\tan \theta = \dfrac{R'' \sin \theta}{R'' \cos \theta}$, therefore R', R'', and θ are readily found.

In the above example we have for convenience called forces to the right, and upwards, and moments tending to produce rotation with the hands of a watch, *positive*; and those in the opposite directions, *negative*. The opposite convention would do as well, the only thing necessary being to introduce opposite forces and opposite moments with unlike signs.

EXAMPLE 2. Bridge-truss (Fig. 20) with three loads, each = 10,000 lbs. Required the abutment reactions. Rollers at A. Weight of truss, G, = 30,000 lbs.

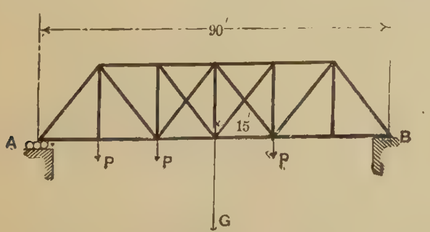


FIG. 20.

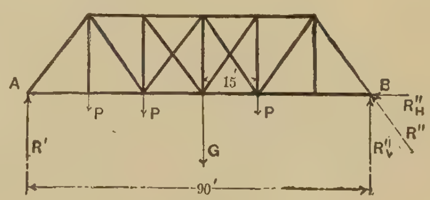


FIG. 21.

Fig. 21 shows the truss free with all external forces put in. There being rollers at A, R' will be vertical; R'' will make some angle θ with the vertical. It will usually be more

convenient to deal with the hor. and vert. components of forces whose directions are unknown, the unknowns then being these components in *amount*. Our unknowns are thus R', R_H'', and R_V''. Applying now our equations of equilibrium: Σ hor. comp. $= 0$ gives

$$-R_H'' = 0, \quad \text{or} \quad R_H'' = 0.$$

This result might readily have been foreseen by inspection, there being no other horizontal force. In arriving at conclusions by inspection we must, however, be very careful to see that they are based upon some one of the three conditions of equilibrium, and where the result is doubtful we should always return to the rigid method,—consider the structure by itself, put in all forces, and write out in detail the equations of equilibrium.

Returning to the example:—An equation of moments about B as a centre will give R' directly, after which R_V'' can be found by a second moment equation with A as centre, or by Σ vert. comp. $= 0$.

EXAMPLE 3 (Fig. 22). Hinges at A, B, and C. Loads P_1 and P_2; weight of structure neglected. What are the reactions at A and B, also the hinge reaction at C? In neglecting

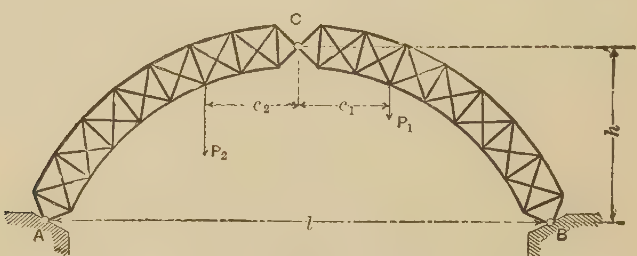


FIG. 22.

the weight of the structure the problem is to find that portion of the reactions due to the loads P_1 and P_2. The portion due to the weight may be found separately and combined with the other, giving the total reactions.

There being a hinge at C, allowing one part of the structure to turn freely upon the other, we have two separate framed structures, AC and CB, to which may be applied the conditions of equilibrium. Fig. 23 shows each structure separated and the external forces

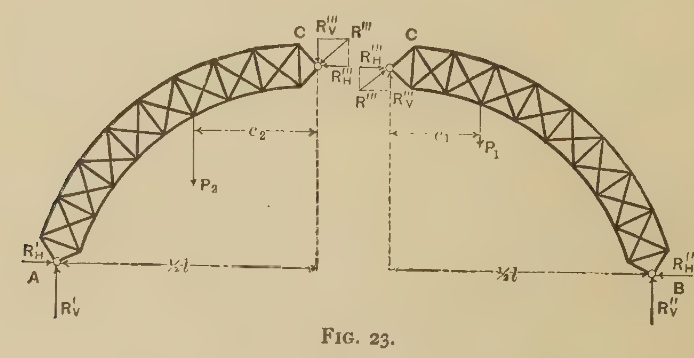


FIG. 23.

put in, the vert. and hor. components of the unknown forces being indicated. At C we have a simple case of action and reaction,—the forces acting upon the two portions are equal

and opposite. We have now six unknown quantities and can write three equations for each structure. For the left-hand structure, Σ vert. comp. $= 0$ gives us

$$R_V' - P_2 - R_V''' = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Σ hor. comp. $= 0$ gives

$$R_H' - R_H''' = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Σ mom. about $C = 0$ gives

$$R_V' \times \tfrac{1}{2}l - P_2 \times c_2 - R_H' \times h = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

For the right-hand structure, Σ vert. comp. $= 0$ gives

$$R_V'' + R_V''' - P_1 = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

Σ hor. comp. $= 0$ gives

$$R_H''' - R_H'' = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

Σ mom. about $C = 0$ gives

$$P_1 \times c_1 + R_H'' \times h - R_V'' \times \tfrac{1}{2}l = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

These six equations are readily solved for the six unknowns. From (*b*) and (*e*) we have at once, $R_H' = R_H''' = R_H''$, a result evident from inspection.

In this example let $P_1 = 1000$ lbs., $P_2 = 2000$ lbs., $l = 100$ ft., $c_1 = 10$ ft., $c_2 = 15$ ft., and $h = 40$ ft. What are the numerical values of the reactions?

Note.—The value of R_V''' should come out $= -300$ lbs. The minus sign merely indicates that the direction of R_V''' has been wrongly assumed; it should act *upwards* on the portion AC.

The above problem may be solved more directly as follows: Since the entire structure, ACB, is in equilibrium, our equations will apply as well to it. The external forces are R_H', R_V', P_1, P_2, R_H'', and R_V''. Σ mom. about $A = 0$ gives

$$P_2 \times (\tfrac{1}{2}l - c_2) + P_1 \times (\tfrac{1}{2}l + c_1) - R_V'' \times l = 0,$$

from which R_V'' is found. Σ vert. comp. $= 0$ gives

$$R_V' + R_V'' - P_1 - P_2 = 0,$$

from which we get R_V' at once. To get R_H', R_H''', R_V''', and R_H''. we must pass to the single structure, AC, or CB, whence these unknowns are readily found.

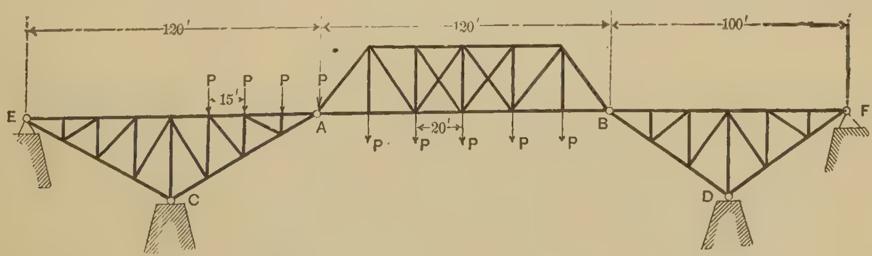


FIG. 24.

EXAMPLE 4. Cantilever bridge. Joints at A, B, C, and D. Loads as shown; weight of bridge neglected. $P = 10{,}000$ lbs. Find reactions at C, D, E, and F. Notice that here we have *three* independent structures.

32. **Second. To Single Joints to Find Stresses.**—Since all parts of a structure at rest are in equilibrium, we may evidently apply the laws of equilibrium to the forces acting upon any portion of that structure. That portion may be a single joint, a single member or part of a member, or it may include several joints and members. The forces acting upon the portion may be part external forces and part internal forces or stresses, or they may be wholly stresses. Fig. 25 illustrates five different portions of the roof-truss, to which may be applied the equations of equilibrium. The letters S_1, S_2, etc., denote forces due to stresses in the members cut; the direction in which they act along the members, whether towards or away from the portion considered, is known only after solution.

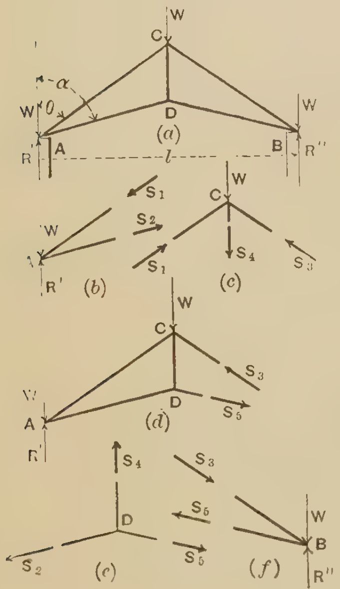


FIG. 25.

EXAMPLE I. Let it be required to find the stresses in all the members of the above truss; loads as shown.

We must first find the abutment reaction at A or B, treating the structure as a whole. Σ mom. about $B = 0$ gives $R'l - Wl - W \times \frac{1}{2}l = 0$, from which $R' = \frac{3}{2}W$.

In treating now of single joints we are dealing with concurrent forces, hence we have but two independent equations at our disposal and can therefore find but two unknown forces. After finding R' there are but two unknown forces acting at A, viz., the stresses in AC and AD. Separating this joint, Fig. 25 (b), and replacing these stresses by the forces S_1 and S_2, assuming them to act as shown, we are ready to apply our equations of condition. Σ vert. comp. $= 0$ gives

$$R' - W - S_1 \cos\theta + S_2 \cos\alpha = 0.$$

Σ hor. comp. $= 0$ gives

$$S_2 \sin\alpha - S_1 \sin\theta = 0.$$

From these two equations S_1 and S_2 are easily found. If the result in either case is negative, then we have assumed our force in the wrong direction. Now S_1 being the force exerted *upon* the lower end of AC *by* the upper end, the direction of the arrow indicates compression in AC. Similarly, AD is in tension by the amount S_2, assuming the direction indicated as correct.

Having found the stress in AC we may pass to the joint C. The unknowns are the stresses in CD and CB. In Fig. 25 (c) the joint is shown free with all forces put in; S_1 will point towards C, AC being now known to be in compression. Assume either direction for S_3 and S_4. As before, Σ vert. comp. $= 0$ gives $S_1 \cos\theta + S_3 \cos\theta - W - S_4 = 0$ and Σ hor. comp. $= 0$ gives $S_1 \sin\theta - S_3 \sin\theta = 0$, from which S_3 and S_4 may be found.

We may then pass to D or B, there being one unknown at D and two at B, one of which is the abutment reaction.

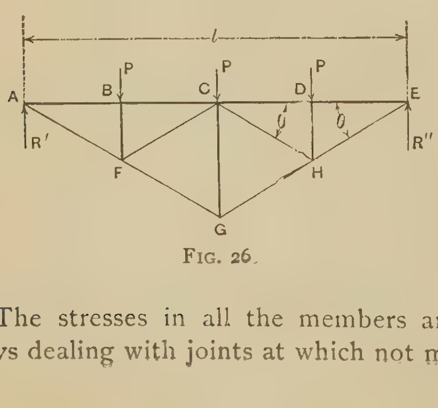


FIG. 26.

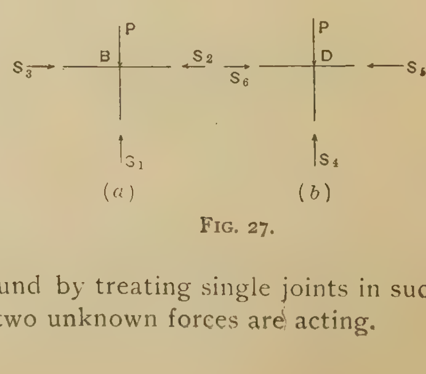


FIG. 27.

The stresses in all the members are thus found by treating single joints in succession, always dealing with joints at which not more than two unknown forces are acting.

EXAMPLE 2. Let it be required to find the stresses in all the members of the truss in Fig. 26; $P = 5000$ lbs., $l = 50$ ft., $CG = 10$ ft.

If the foregoing problem has been carefully followed, the student will have no difficulty in solving this or any similar problem.

In example 2 we may find, if desired, the stresses in BF and DH at once, thus: Fig. 27 (*a*) represents the joint B free; S_1, S_2, and S_3 are the stresses in the members cut. Σ vert. comp. $= 0$ gives at once $S_1 - P = 0$, or $S_1 = P$; likewise in Fig. (*b*), $S_4 = P$.

In general we see that if two of three unknown forces have the same line of action, the third may always be determined by putting Σ comp. perpendicular to this line of action $= 0$. This principle is a very useful one. In the example above, after finding the stress in BF, we can find that in FC by equating components perpendicular to AG at joint $F = 0$. In like manner CH can be found, and finally CG.

EXAMPLE 3. Find the stresses in all the members of the first two panels in Fig. 20, Art. 31.

EXAMPLE 4. Roof-truss (Fig. 28). $P = 2000$ lbs., $\theta = 45°$, $\alpha = 60°$, $l = 60$ ft. Find the stresses in all the members.

Note.—Begin at D and find stress in DF, then pass to F, A, etc.

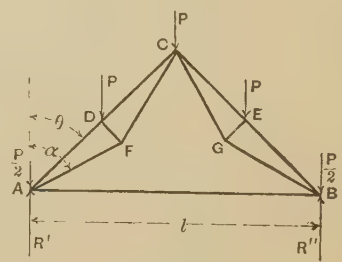


FIG. 28.

33. Third. Method of Sections.—In this method, instead of taking single joints, a section is passed through the structure cutting the members whose stresses are desired, and the equations of equilibrium applied to one of the portions into which the structure is thus divided. A part of the forces are thus external forces, and a part are due to stresses in the members cut. The portion of the structure considered usually includes several joints, and thus the forces are in general non-concurrent. This gives us three equations of equilibrium, and hence if we cut but three members (not meeting in a point) whose stresses are unknown, these stresses may be found.

Another way of stating the equilibrium existing between the forces acting upon either portion of the structure, is to say that the stresses in any section hold in equilibrium the external forces acting upon either side of that section. Or, more in detail,

Σ vert. comp. external forces $= \Sigma$ vert. comp. internal forces;
Σ hor. comp. external forces $= \Sigma$ hor. comp. internal forces;
Σ mom. external forces $= \Sigma$ mom. internal forces.

The equality is, however, one of numerical value but not of sign, as is seen by comparison with the fundamental equations of equilibrium.

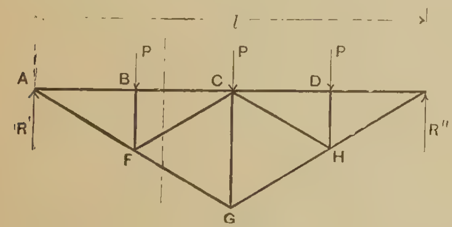


FIG. 29.

EXAMPLE 1 (same truss as in Fig. 26). Required the stresses in BC, FC, and FG. We first find the abutment reaction, R', by methods already given. Passing a section through the above members, separating the portion to the left, and replacing the stresses in the members cut by the forces S_1, S_2, and S_3, Fig 30, we may now apply our three equations. Σ vert. comp. $= 0$ gives $R' - P + S_2 \sin\theta - S_3 \sin\theta = 0$; Σ hor. comp. $= 0$ gives $S_2 \cos\theta + S_3 \cos\theta - S_1 = 0$; Σ mom. about $F = 0$ gives $R' \times AB - S_1 \times BF = 0$. These three equations enable us to find the unknown stresses. The student may substitute numerical values.

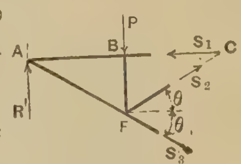


FIG. 30.

Notice that the last equation gives at once $S_1 = \dfrac{R' \times AB}{BF}$, the centre of moments being at F, the intersection of S_2 and S_3. Similarly, an equation of moments with centre at C will give S_3 directly, and one with centre at A will give

S_2: thus we may make use of three equations of moments. The lever-arms of the forces when not directly known can easily be computed from the given dimensions.

EXAMPLE 2. Fig. 31 (same truss as in Fig. 28). Find the stresses in all the members by the method of sections.

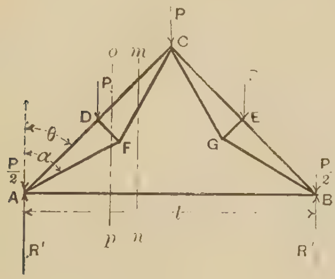


FIG. 31.

We first get R' by treating the structure as a whole. Then passing a section mn, cutting but three members not meeting in a point, we separate the left-hand portion, put in all forces, and proceed to apply our three equations, Fig. 32 (a).

We may use three moment equations, taking for centres of moments the points A, K, and C successively; or, as some of the lever-arms are tedious to compute, a better method would be to compute S_1 by a moment equation, centre moments at C, then use the other equations, the angles θ and α being given. In any case that equation should be used which gives the result sought in the simplest way.

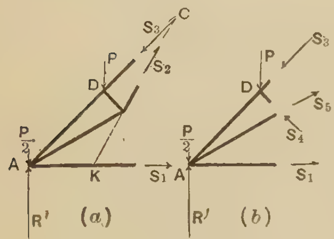


FIG. 32.

Now S_1, S_2, and S_3 being known, a section op through DC, DF, AF, and AB cuts but two pieces whose stresses are unknown, Fig. (b). Moment equations with D and A as centers then give S_4 and S_5.

The stresses in AF and AB having been found, the stress in AD is readily found by passing a section through these three pieces, or what is the same thing, treating the single joint A. The stresses in the remaining members are found by methods similar to the preceding.

It is often expedient to combine the method of sections with the preceding method; thus in the present example the stress in AB may be found by passing the section mn, after which we may pass to the joint A, and thence by single joints. A single application of the method of sections to this problem to get the stress in AB thus enables us to apply the other method in a regular way, beginning at A and passing to other joints in turn.

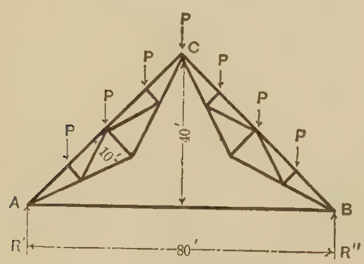


FIG. 33.

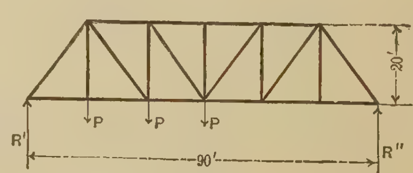


FIG. 34.

EXAMPLE 3. Roof-truss (Fig. 33) $P = 3000$ lbs. Find R' and R'', and the stresses in all the members.

EXAMPLE 4. Bridge-truss (Fig. 34) $P = 5000$ lbs. Find stresses in all the members by the method of sections.

II. *Graphical Application of the Equations of Equilibrium.* (Art. 30.)

The Equilibrium Polygon.

34. Conditions of Equilibrium of Non-concurrent Forces Restated.—Reproducing Fig. 16 (see Fig. 35), R_3 was found to be the resultant of P_1, P_2, P_3, and P_4; and the force R_3', such a force as would balance these four forces, thus forming a system in equilibrium. The conditions of equilibrium were found to be, first, that the force polygon must close; second, that the position of the last force (R_3') must coincide with the last segment (CD) of the equilibrium polygon.

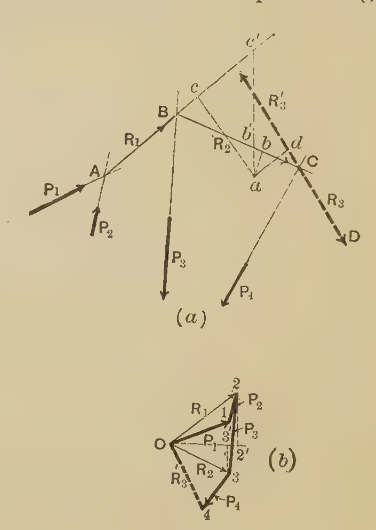


FIG. 35.

It was also noted that each segment of the equilibrium polygon is the line of action of the resultant of all the forces upon either side of that segment. This resultant is given in amount by that ray in the force polygon to which this segment is parallel. Its direction is *from O* when it is the resultant of the forces on the left of the segment, and *towards O* when it is the resultant of the forces to the right.

35. Resolution of the Forces.—Since R_1 is the resultant of P_1 and P_2, the sum of the horizontal components of P_1 and P_2 is evidently equal to the horizontal projection of R_1, and the sum of their vertical components is equal to the vertical projection of R_1. Similarly, R_2 with respect to P_1, P_2, and P_3; and in general, *the sum of the horizontal components and the sum of the vertical components of all the forces to the left of any segment are equal respectively to the horizontal and the vertical projections of the ray in the force polygon to which this segment is parallel.*

36. Moments of the Forces.—It follows from what has already been shown, that the sum of the moments of all the forces to the left of any segment, as BC, about any point a, is equal to the product of the parallel ray in the force polygon, $O3$ (their resultant), multiplied by the perpendicular distance, ab, from the point to the segment (the line of action of the resultant). Thus the sum of the moments of P_1 and P_2 about $a = R_1 \times ac$; of P_1, P_2, P_3, and P_4 about $a = R_3 \times ad$, etc.

Let ac' be drawn vertically through a; also project R_1 and R_2 upon a horizontal line. Then from the similar triangles abb' and $O33'$, we have: $O3 \times ab = O3' \times ab'$, or the sum of the moments of P_1, P_2, and P_3 about $a = O3' \times ab'$; and similarly the sum of the moments of P_1 and P_2 about $a = O2' \times ac'$. We have then this useful principle, that *the sum of the moments of the forces to the left of any segment about any point is equal to the vertical ordinate from the point to the segment, multiplied by the horizontal projection of the corresponding ray in the force polygon.* It is evident that instead of vertical ordinates and horizontal projections we may use *any* two directions at right angles.

The foregoing two articles apply equally well to the forces on the *right* of any segment if the system is a balanced one.

The sign of the moment, or the direction in which it tends to turn about the point, is at once seen when we remember that the segment is the line of action of the resultant of the forces in question. Thus the moment of P_1 and P_2 about a is right-handed or positive, that of P_1, P_2, P_3, and P_4 is positive, that of R_3' and P_4 is negative, etc.

37. **Forces Parallel or Nearly so.**—Given the forces P_1, P_2, P_3, and P_4 (Fig. 36); required their resultant.

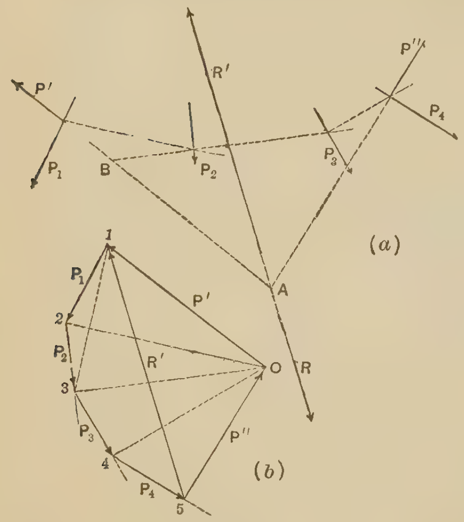


FIG. 36.

The force polygon (*b*) gives their resultant, 1–5, in amount and direction; 5–1 is a force which will balance the given system. Resolve this force into any two components, P'' and P', by drawing the triangle $5O1$, O being chosen so as to give fair angles at 1 and 5. These components together with the given system will form a system in equilibrium, provided they are inserted in Fig. (*a*) in the proper position. The position of one of them, as P', may be chosen at will, and the other will be given in position by the last segment of the equilibrium polygon constructed for the forces P', P_1 . . . P_4, with O as the pole of the force polygon. The last force thus coinciding with the last segment, and our force polygon closing, equilibrium is assured. The intersection, A, of P' and P'' is a point on the resultant of these two forces, and since 5–1 gives this resultant in amount and direction, it is therefore fully known. It is represented by R' in Fig. (*a*), and since it balances P_1 . . . P_4, an equal and opposite force, R, is the required resultant of our given system in amount, direction and position.

Corollary.—Since the resultant of all the forces up to any segment applied along that segment would balance the remaining forces, it follows that the intersection of *any* two segments of an equilibrium polygon is a point on the resultant of the intermediate forces. Thus the point B is a point on the resultant of P_1 and P_2; the line 1–3 in the force polygon gives the amount and direction of this resultant.

38. **Abutment Reactions.**—When the given system of forces acts upon a beam or framework supported at two points, the abutment reactions are themselves components of R', Fig. 36, and it is these two components that are usually desired. One point in each, the abutment, is given, and also usually the direction of one; the direction of the other and the magnitudes of both, follow from the conditions of equilibrium. It will now be shown how these reactions may be fully determined.

Let the forces P_1 . . . P_4 act upon any rigid structure ACB, Fig. 37, resting upon abutments at A and B (not shown in the figure). Suppose the end B to rest upon rollers and the end A to be fixed; the reaction at B will be vertical, that at A unknown in direction.

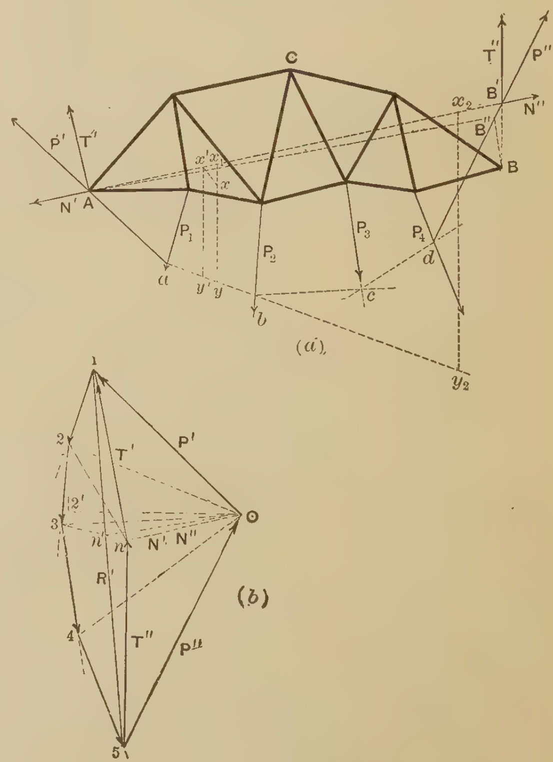


FIG. 37.

Construct the force polygon, Fig. (*b*), as in the preceding article, choosing any point O as pole. Draw the corresponding equilibrium polygon, $AabcdB'$, inserting the force P' at A; B' is the intersection of P'' with the vertical reaction at B. Draw the line AB'

called the *closing line* of the equilibrium polygon, and On parallel to it, meeting a vertical through 5, at n. Join $n1$. We have now resolved P'' into the components $5n$ and nO, and P' into On and $n1$. Replacing P'' and P' by their components, inserted at B' and A respectively, we have the opposite and equal forces N' and N'' acting along the same line AB'; hence they balance each other and the forces T' and T'' must alone hold $P_1 \ldots P_4$ in equilibrium. They are therefore the required reactions, as they fulfil all conditions. That they are components of 5–1, or R', is seen from the force polygon.

If the direction of T'' had been unknown and that of T' known, then we should have begun our equilibrium polygon at B instead of at A, and worked towards the left.

If both ends of the structure are fixed, T' and T'' are parallel, which can be true only when they are both parallel to R'. In that case we know the direction of both reactions and our equilibrium polygon need not pass through either A or B as the extremities of the closing line are then the intersections of P' and P'' with lines through A and B parallel to R'. In the above problem, under those conditions, AB'' would be the closing line, and $5n'$ and $n'1$ the abutment reactions, BB'' being parallel to 5–1, and On' parallel to AB''.

39. Resolution of the Forces.—In Fig. 37, the equal and opposite forces N' and N'' do not really act upon the structure, they being virtually inserted at the time we assume a convenient pole. They do not, therefore, enter among the external forces in finding stresses in the structure. It must be borne in mind, however, that these forces are always included when we speak of the segments of the equilibrium polygon as being the lines of action of certain resultants.

The sum of the vertical and the sum of the horizontal components of all forces, actually acting upon the structure to the left of any segment, as bc, are evidently equal to the vertical and the horizontal projections, respectively, of their resultant, $n3$. If the force N' be included, the resultant of all forces to the left of bc is $O3$, the vertical and horizontal projections of which are equal to the sums of the corresponding components of the forces.

40. Moments of the Forces (Fig. 37).—The sum of the moments of N', T', and P_1 about any point x is, as in Art. 36, equal to the vertical ordinate xy multiplied by $O2'$, $O2'$ being the horizontal component of the resultant, $O2$, of the three forces.

If now we wish to eliminate the moment of the imaginary force N', we can take our centre of moments upon AB', its line of action, thereby reducing its moment to zero. Thus the sum of the moments of T' and P_1 about $x_1 = x_1y \times O2'$; the sum of their moments

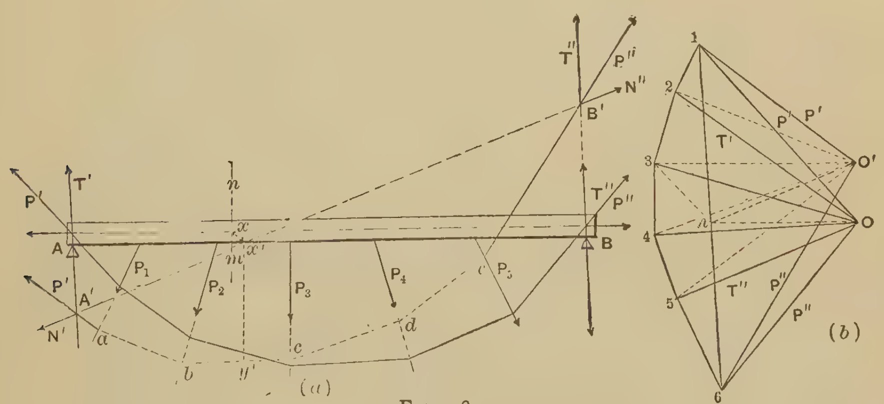


FIG. 38.

about $x_2 = x_2y_2 \times O2'$, etc. To get their moments about x, we draw xx' parallel to their resultant, $n2$. The sum of their moments about all points in xx' is the same, hence their moment about $x = x'y' \times O2'$. The sign of this moment is determined by applying the force $O2$ along ab; as this force acts right-handed about x', the above moment is positive.

41. Application to a Beam.—Let it be required to find the abutment reactions, T' and T'', of the beam AB, Fig. 38, supporting the loads $P_1 \ldots P_5$. Beam fixed at each end against horizontal motion.

Fig. (b) shows the force polygon; 1 2, . . . 6 being the load line, and O' the pole. The reactions will be parallel to 6–1; A' and B' are their intersections with P' and P''. We get for their values, $T' = n1$ and $T'' = 6n$. The student may follow out the details of the construction.

If we wish the sum of the moments of the real forces to the left of any section, mn, about the neutral axis, x, of the section*, we may, as in Art. 40, draw xx' parallel to the resultant $n3$ of these forces (T', P_1, and P_2), whence the required moment $= x'y' \times O'3'$.

Where many moments are required it will be more convenient to draw a new equilibrium polygon whose closing line shall pass through the centres of moments, or coincide with the neutral axis of the beam. This requires a new pole for our force polygon, so chosen that the equilibrium polygon, if made to pass through A, will also pass through B. The abutment reactions depending only upon the given system of forces and the positions of A and B, the point n in our force polygon will not be changed; and as the closing line is to be AB, the required pole must lie somewhere on the line nO drawn parallel to AB. With any point O on this line as pole, draw the force polygon, and beginning at A construct the corresponding equilibrium polygon; it will pass through B if accurately drawn. The vertical ordinate from the neutral axis of the beam to the polygon, multiplied by the horizontal projection of the proper ray in the force polygon, then gives the sum of the moments of all the forces actually acting upon the beam to the left or right of the centre of moments. This moment is positive for the forces on the left and negative for those on the right.

42. Problem. To Pass an Equilibrium Polygon through Three Given Points.—Let P_1, P_2, and P_3 be the given forces (Fig. 39) and A, B, and C the given points.

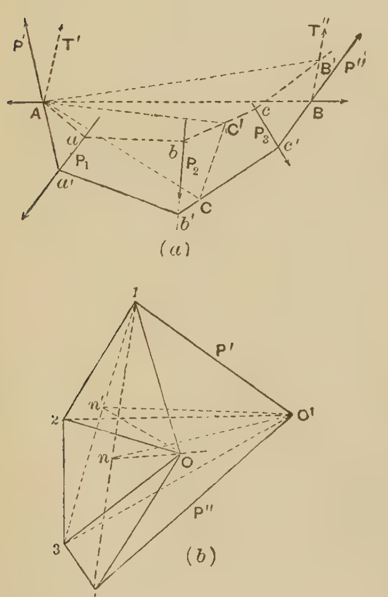


FIG. 39.

Draw a force polygon, (b), with any pole O', and the corresponding equilibrium polygon, $AabcB'$, passing through the point A, the reaction line, BB', being drawn parallel to 4–1. Draw the closing line AB' and the parallel line $O'n$; $4n$ and $n1$ would be the abutment reactions were the given forces acting upon a beam with fixed abutments at A and B. By the preceding article, the required pole lies somewhere on the line nO, drawn parallel to AB.

In like manner treat the two forces P_1 and P_2 and the points A and C. The trial force polygon, O' 1 2 3 and the equilibrium polygon $Aabc$ are already drawn. The resultant of P_1 and P_2 is 1–3, and a line through C parallel to 1–3 intersects bc at C'. The closing line is AC', and the line $O'n'$ drawn parallel to it meeting 1–3 at n' determines the reactions at A and C for loads P_1 and P_2. In order that the required equilibrium polygon may pass through C, the pole of the force polygon must lie somewhere on the line $n'O$ drawn parallel to AC. Hence the required pole O lies at the intersection of $n'O$ and nO.

* The student is assumed to be familiar with the theory of flexure. If he has not studied this subject, he is referred to Chap. VIII.

43. **Parallel Forces.**—Let P_1, P_2, and P_3, Fig. 40, be a system of parallel, and in this case vertical, forces acting upon any structure with abutments A and B.

The abutment reactions, which are both vertical, are found in the usual way. The load line is the vertical line 1–4. The reactions T'' and T' $= 4n$ and $n1$ respectively.

The sum of the vertical components of the forces, or of the forces themselves, actually acting upon the structure to the left of any segment, is equal to the distance from n to the extremity of the ray parallel to the segment. Thus the sum of T' and $P_1 = n2$; of T', P_1, and $P_2 = n3$, etc.

The sum of the horizontal components $=$ zero.

The sum of the moments of T' and P_1 about any point $x =$, as before, $x'y \times Om$, where xx' is drawn parallel to the resultant of T' and P_1 (vertical in this case), and Om is the horizontal projection of $O2$. Likewise the sum of the moments of T', P_1, and P_2 about $x_1 = x_1'y_1 \times Om$, etc. The distance Om is here called the *pole distance* of the force polygon. We have then in general: *For vertical forces, the sum of the moments of all the external forces to the left, or right, of any segment, about any point, is equal to the intercept on the vertical ordinate through the point included between the closing line and that segment, multiplied by the pole distance of the force polygon.*

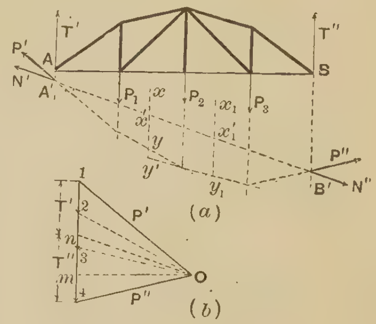


FIG. 40.

If $x'y \times Om =$ sum of the moments of T' and P_1 about x, and $x'y' \times Om =$ sum of the moments of T', P_1, and P_2 about the same point, then must $yy' \times Om =$ moment of P_2 about x. That is, *the moment of any force about any point is equal to the intercept on the vertical ordinate through the point included between the adjacent segments of the equilibrium polygon, multiplied by the pole distance.* This is known as *Culmann's Principle.*

The above discussion applies equally well to parallel forces in *any* direction, provided the abutment reactions are also in the same direction. When that is not the case, as when inclined forces act upon a structure where one end reaction is vertical, we must follow the general method of Arts. 38–40.

44. **Forces Taken in Any Order.**—Suppose the forces P_1, P_2, P_3, and P_4, Fig. 41, to act upon the beam supported at A and B.

Lay off the load line 1–5, taking, for variety, the forces in the order P_1, P_3, P_2, and P_4. Draw the force polygon with pole O and the corresponding equilibrium polygon, beginning at a point in the vertical through A. The equilibrium polygon is $A'abcdB'$, and the closing line $A'B'$. The line On drawn parallel to $A'B'$ determines the abutment reactions, T'' and T', these being equal respectively to $5n$ and $n1$ both in amount and direction; T' therefore acts downwards.

The sum of the moments of T' and P_1 about x $= x'y \times Om$ and is negative. The sum of the moments of T', P_1, and P_2 about a point x_1 is not given by a single ordinate multiplied by Om, since the force P_2 does not come next in the construction. The sum of the moments of T' and P_1 about $x_1 = x_1'y_1 \times Om$, and the moment of $P_2 = zz' \times Om$; hence, these moments being of the same sign, the required sum $= (x_1'y_1 + zz') \times Om$. Similarly, the sum of the moments of T', P_1, P_2, T'', and P_3 about $x_2 = (x_2'y_2 - x_2'z_2) \times Om$. Thus by a little inspection any desired moment may be found.

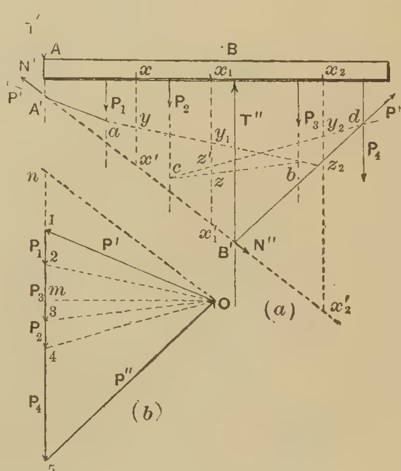


FIG. 41.

Application to Framed Structures.

45. **First. To the Structure as a Whole** to determine a portion of the external forces by means of the equilibrium polygon.

EXAMPLE 1. Three equal cylinders, each weighing 1000 lbs. (Fig. 42), B and C just touching at k; surfaces smooth so that pressures are normal. Required the reactions at c, d, e, and f and the pressures at g and h.

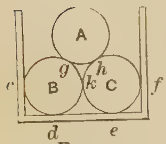


FIG. 42.

Fig. 43 shows each cylinder with all external forces indicated. We have here three systems of concurrent forces in equilibrium, hence the force polygon of each system must close.

First, the system acting upon A, P_1 and P_2 being unknown. In Fig. 44 (*a*) lay off $O1 = 1000$ lbs., to scale; then draw 1–2 parallel to P_2, and $O2$ parallel to P_1, cutting 1–2 at 2 (P_1 and P_2 will be inclined 30° from the vertical). Then 1–2 and $2O$ are the amounts of the forces necessary to close the polygon, when acting in the directions given by P_2 and P_1. Therefore P_1 and P_2 are respectively equal to $2O$ and 1–2.

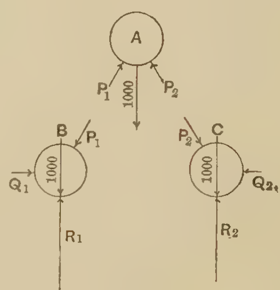


FIG. 43.

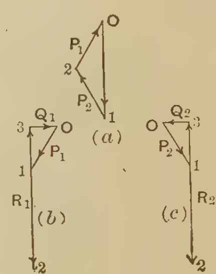


FIG. 44.

Second, the system acting upon B. The forces Q_1 and R_1 are unknown. In Fig. 44 (*b*) draw $O1$ equal and parallel to P_1 as found from (*a*) but in the opposite direction; then 1–2 vertically and equal to 1000 lbs. The lines 2–3 and 3O drawn parallel to R_1 and Q_1 close the polygon, and give these forces in amount. They are thus fully determined.

Third, the system acting upon C. A figure (*c*) similar to (*b*) gives $R_2 = 2\text{–}3$ and $Q_2 = 3O$.

The numerical values are found by scale to be as follows: $P_1 = P_2 = 580$ lbs.; $Q_1 = Q_2 = 270$ lbs.; $R_1 = R_2 = 1500$ lbs.

We see from the above that two unknown forces in any concurrent system are easily found by closing the force polygon by lines parallel to the directions of these two forces.

A few examples will now be given as a further illustration of the application of the equilibrium polygon in finding abutment reactions.

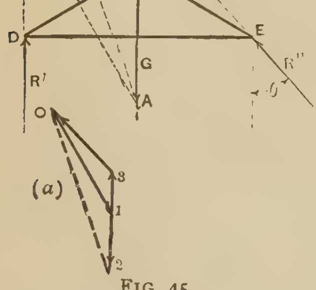


FIG. 45.

EXAMPLE 2 (same truss as in Fig. 18). The unknowns are R', R'', and θ.

We have here a system of non-concurrent forces in equilibrium, and hence the two conditions: their force polygon must close and the last force must coincide with the last segment of the equilibrium polygon. Beginning with the known forces, W and G, draw the corresponding portion of the force polygon, O 1 2, and the ray $O2$; also the segment AB, parallel to this ray. The direction of R' being known we take this force next in order; B is its intersection with AB. The amount of R' being unknown, we cannot at once draw the next ray in the force polygon and so get the direction of the next segment of the equilibrium polygon. We know, however, that this next segment must coincide with the last force R'', and hence must pass through E; BE is therefore this segment. The ray $O3$ is then drawn parallel to BE to its intersection with the line 2–3 drawn parallel to R'. The figure O 1 2 3 O is a closed polygon, and R' and R'' are respectively equal to 2–3 and $3O$; BE is the line of action of R''.

EXAMPLE 3. Find graphically the abutment reactions of the truss in Fig. 20, a case of parallel forces.

EXAMPLE 4. Given the roof-truss, Fig. 46, with loads $P_1 \ldots P_5$ and wind-pressures W_1, W_2, W_3. Required the abutment reactions, R' and R'': first, when the horizontal thrust due to wind is taken up by each abutment, the truss being fixed at both ends; second, when one end is on rollers, the other end taking all the thrust.

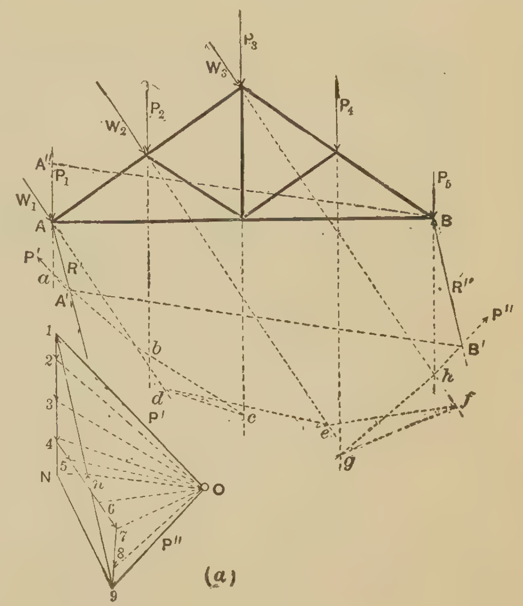


FIG. 46.

Construct the force polygon (a), laying off the forces in any convenient order. The order chosen is P_1, P_2, P_3, W_1, W_2, etc.; 1–9 is their resultant. Since many of the forces are parallel, it will be necessary to resolve 9–1 into the components P' and P'' by selecting a pole O. Draw the rays $O2$, $O3$, etc., and construct the corresponding equilibrium polygon, beginning at any point A', on a line through A parallel to 1–9. In constructing this polygon we must be careful to take the forces in the same order in which they occur in the load line. The polygon is $A'abcdefghB'$, $A'B'$ being the closing line. The line On drawn parallel to $A'B'$ gives the required reactions; $R' = n1$ and $R'' = 9n$.

If, for example, the left end be on rollers, the only part of R' acting upon the truss at A is its vertical component, $= N1$. Its horizontal component can be applied only at B, another point in the line of action of this component, and a point where the truss is fastened to the abutment. This horizontal component, nN, combined with R'', or $9n$, gives $9N$ as the resulting reaction at B.

For this last case the reactions might have been found directly by following the method of Art. 38; that is, by beginning our polygon at B, the fixed end, and ending it in a vertical through A. The closing line would have been some line BA'', and the line in the force polygon parallel to it would be ON, giving the same reactions as found above.

46. Second. To Single Joints to Find Stresses: Maxwell Diagrams.—In this method we first find the abutment reactions either algebraically or graphically, then, commencing at one abutment, find the stresses in the members at successive joints by means of force polygons only. We must always, as in the analytical method, select joints at which there are but two unknown stresses; or if there are three, we can determine one if the other two have the same line of action.

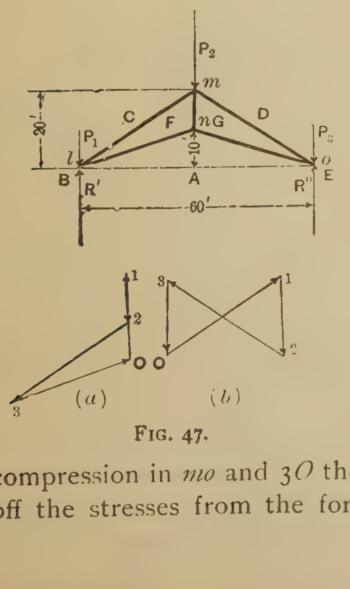


FIG. 47.

EXAMPLE 1 (Fig. 47). P_1 and P_3 each $= 500$ lbs.; $P_2 = 1000$ lbs.; dimensions as given. Required the stresses in the members.

We see at once that $R' = R'' = 1000$ lbs. For joint l we lay off in (a), $O1 = 1000$ lbs. by scale, and parallel to R'; 1–2 $= 500$ lbs. downwards; and 2–3 and $O3$ parallel to the pieces lm and ln respectively. Then 2–3 = stress in lm, and, pointing in the direction ml, it indicates compression; $3O$ = tensile stress in ln. Fig. (b) is the force polygon for joint m, 2–3 being the compression in mo and $3O$ the tension in mn. The other joints are treated similarly. Scaling off the stresses from the force polygons we have the following results: stress in lm = stress

in mo = 1800 lbs. compression. Stress in ln = stress in no = 1580 lbs. tension. Stress in mn = 1000 lbs. tension.

Instead of drawing a separate figure for each joint, we may combine the force polygons into a single figure, using each line twice as being common to two polygons. The combined figure may be called a *stress diagram*. A convenient notation to accompany this method is to letter each triangle of the truss, and also each space between the external forces, as in Fig. 47. Each piece and each external force is then known by the two letters in the adjacent spaces, as the piece *CF*, the load *CD*, etc.

Let us apply this method to the truss in the figure. We will find it convenient to lay off the loads and reactions at once, forming the load line *BE*, Fig. 48, the loads being lettered to correspond with Fig. 47. The abutment reactions are *EA* and *AB*. (If these are found graphically by means of an equilibrium polygon, we will have our load line already laid off.) Beginning as before at joint l, the force polygon for that joint will be *ABCFA*; *CF* the stress in piece *CF*, and *FA* that in piece *FA*. The nature of these stresses is easily determined by following around the polygon. Passing to joint m, the force polygon will be *FCDGF*, in which *FC* and *CD* are already drawn. The force *FC* of course acts in the opposite direction from what it did upon joint l. The polygon for joint n is *AFGA*, and for o is *AGDEA*, *EA* being the abutment reaction: its coming out so is a check upon the work. In treating a joint always begin with the piece farthest to the left whose stress is known, and then pass around right-handed. Thus, at joint m the forces *FC* and *CD* are known; begin then with *FC* and pass to *D*, *G*, and *F*; etc. If o had been the starting point, the opposite mode of procedure would have been the most convenient.

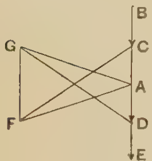


FIG. 48.

EXAMPLE 2. Required the stresses in the roof-truss of Fig. 49. Loads *NO* and *VW* each = 1000 lbs.; all others = 2000 lbs. Span = 100 ft., rise = 35 ft.; distance from apex to horizontal tie, *XM*, = 30 ft.; all struts, *AB*, *CD*, *EF*, etc., are normal to roof.

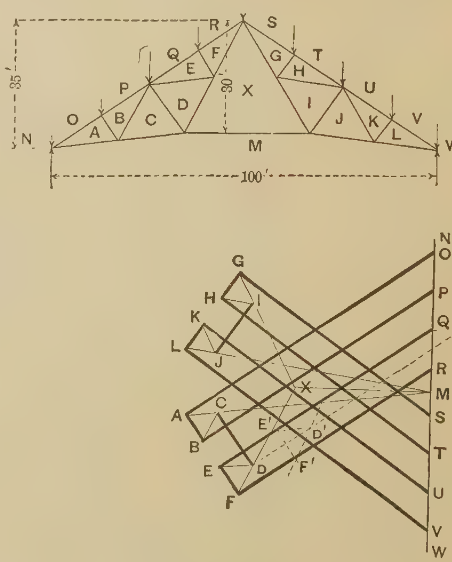


FIG. 49.

The load line is *NW*; abutment reactions, *WM* and *MN*. The stress polygons for the first three joints, beginning at the left, are readily drawn. At the third upper joint (*BPQ*, etc.), however, the stresses in the *three* pieces *CD*, *DE*, and *EQ* are unknown. As in Art. 32, Ex. 4, we may pass to the next upper joint, find the stress in *EF*, then in *ED*, and finally the stresses in *CD* and *QE*. As to the stress in *EF*:—From *Q* and *R* draw the indefinite lines *QE′* and *RF′* parallel to the pieces *QE* and *RF*. The stresses in the pieces *QE* and *RF* are unknown, but whatever they are, the stress in *EF* must be such as to close the force polygon *E′QRF′* when drawn parallel to the piece *EF*. The line *F′E′* then gives this stress, a compressive one. To get the stress in *ED* we may in a similar manner draw the indefinite line *F′D′* parallel to piece *FX*, then the line *E′D′* parallel to piece *ED*; *D′E′* will be the required tensile stress. Knowing *ED*, we may now pass to the third upper joint. The portion *CBPQ* of the force polygon is already drawn. Taking the piece *ED* next in order, its stress being known, draw *QD″* equal and parallel to *E′D′*; then *D″D* parallel to *QE*, meeting *CD* at *D*. We have then *D″D* equal to the stress in *QE*, and *DC* that in piece *DC*. The forces may now be arranged in proper order, *Q*, *E*, *D*, *C*, in the force polygon. The remaining stresses are easily found.

An easier solution of the above example is now seen from the diagram. It depends upon the fact that E, F, A and B are in the same straight line, and to find E and F we have only to produce AB to cut the lines QE and RF. We may then draw FX parallel to the piece FX, and CD likewise, thus determining the point D. The half diagram is completed by drawing XM and DE. This method is applicable only when the struts are normal to the roof, and when the secondary truss NS is symmetrical about CD.

To aid in distinguishing the nature of the stresses, lines representing compressive stresses may be made heavier than those representing tensile stresses, or different-colored inks may be used. Where the loads are symmetrical it is necessary to draw but one half the stress diagram, the stresses in corresponding members of the two halves of the structure being equal. Where this is done, however, the work should be checked by computing a few of the stresses by analytical methods.

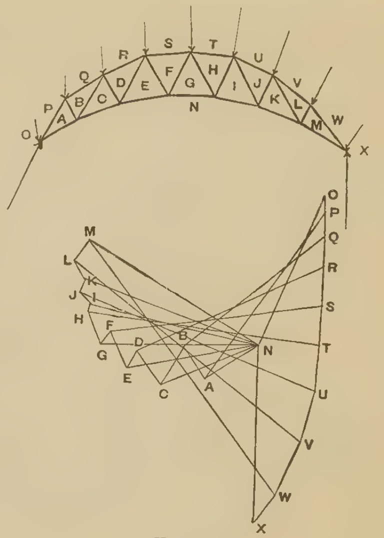


FIG. 50.

Fig. 50 shows a truss with accompanying stress diagram. A portion of the loads are inclined, due to wind-pressure from the right. Rollers under right end; $XN =$ right abutment reaction, NO the left.

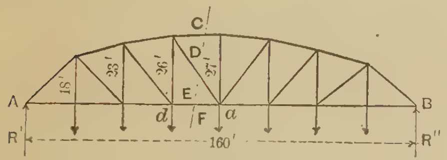


FIG. 51.

EXAMPLE 3. Find all stresses in the bridge-truss shown in Fig. 51. Each load = 30,000 lbs.

47. Special Application.—In case we desire the stresses in but one or two members of the structure when under a given loading, the preceding method necessitates the finding of all the stresses up to the ones in question, and thus is a much longer process than the analytical method of sections. Where there are no loads between one of the abutments and these members, as is frequently the case, we may find the desired stresses very quickly by graphics as follows:

Suppose we wish to find the stresses in the pieces CD, DE, and EF (Fig. 51), there being no loads to the left of a.

Pass a section through these pieces. Fig. 52 shows the left-hand portion free, with the forces put in; S_1, S_2, and S_3 are the unknown stresses. (The load P is not at present supposed to exist.) Now these stresses with R' form a balanced system whose equilibrium is independent of the shape of the structure acted upon. We may therefore replace the portion $Aecd$ by a single triangle, Acd, the structure still being a rigid one. If we then begin at A and find the stresses in the members of this new structure, the force polygons for the joints c and d will give us the required stresses. The complete diagram is given in

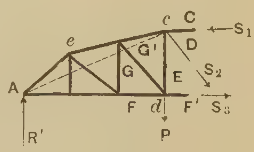


FIG. 52.

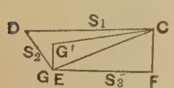


FIG. 53.

Fig. 53. Beginning at F, FC is the abutment reaction R' (R' is best found analytically); $FCGF$, the force polygon for joint A; $FGEF$, that for joint d, there being no stress in piece cd, as it exists in our new structure; $EGCDE$ is the polygon for joint c. We have then $EF = S_3$, $CD = S_1$, and $DE = S_2$. If we desire the stress in cd as a part of the original truss, we may pass to joint c of that truss. There are but two unknown stresses. The force polygon $EDCG'E$ gives these stresses; $G'E =$ stress in cd.

If there be also a load at d, our triangle, Acd, will still give us a rigid structure. The

stress diagram will then be as in Fig. 54, where $F'F =$ load P. Let the student find the numerical values of the stresses in both cases, each load being equal to 30,000 lbs., and compare with those found in Ex. 3, above.

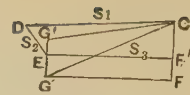


FIG. 54.

48. Third. **To a Part of the Structure, using Successive Sections commencing at One End.**—If we pass a section through a structure, cutting but two members whose stresses are unknown, the single condition that the force polygon, drawn for the forces acting upon one portion of the structure, must close, will enable us to find the stresses in these pieces. Commencing at one end of a structure and passing a section cutting but two pieces, we can determine their stresses; then passing another section cutting three members, one of which has already been treated, we can find the stresses in the other two, and finally by successive sections we can determine all the stresses by simple force polygons.

Take, for example, the truss in Fig. 55. Lay off at once the load line, MR (Fig. 56); RA and AM are the reactions at right and left abutments, found by any method. Pass a section through ABM, cutting two pieces. A simple force polygon, AMB (Fig. 56), gives the stresses in MB and BA.

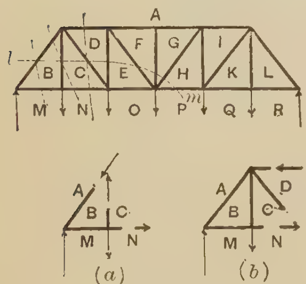


FIG. 55.

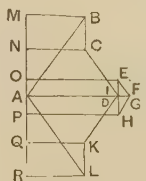


FIG. 56.

(In this case it is more convenient to work left-handed, the forces in the load line being laid off in left-handed order in the sense of rotation about the truss.)

Now pass a section through $ABCN$. The stresses in BC and CN are unknown. Fig. 55 (a) shows the portion to the left of the section free. The forces acting are AM, MN, NC, CB, and BA, and since they are in equilibrium their force polygon must close. The portion $BAMN$ (Fig. 56) of the polygon is already drawn; NC and CB drawn parallel to their respective pieces closes the polygon and determines the stresses in these pieces. Next pass a section through $ADCN$; CD and DA are the unknown stresses. Fig. (b) shows the portion of the structure considered. Of the force polygon for the forces there acting, the portion $AMNC$ is drawn. The lines CD and DA close the polygon and determine the unknown stresses. In like manner proceed throughout the structure. The complete stress diagram is given in Fig. 56.

While the above is a different method than that given in Art. 46, yet the resulting diagram is precisely the same as would have been obtained by that method, the force polygon for any joint being given directly in the figure. Moreover, if we pass any section whatever through the structure, the polygon of the forces acting upon either portion will be given by the diagram. Thus, passing a section lm, the forces acting upon the left-hand portion are the loads, abutment reaction, and the stresses in the members cut. Their force polygon is $AMNOPHGFEDCBA$, a closed figure. Likewise with the portion on the right.

48a.* **Review of Fundamental Principles.**—We have seen that there are three fundamental equations of equilibrium which may be applied (a) to the structure as a whole; (b) to any flexible joint of the structure; (c) to any part of the structure when cut by a section and one portion removed. Since these are all the fundamental equations, or propositions, which can be made use of in solving statically determinate structures, we will here restate them, and the student should fix them firmly in his mind, and *constantly recur to them whenever a new problem in structural analysis is presented to him:*

I. FUNDAMENTAL PRINCIPLES APPLIED TO THE STRUCTURE AS A WHOLE.

If a structure is in equilibrium under the action of external forces,

1. *The sum of the vertical components of these forces is equal to zero;*
2. *The sum of the horizontal components of these forces is equal to zero; and*

* This article added by J. B. J. in the seventh edition to replace a graphical method of moments given in previous editions.

3. *The sum of the moments of these forces, taken about any point in the plane of their action, is equal to zero.*

II. FUNDAMENTAL PRINCIPLES APPLIED TO ANY FLEXIBLE JOINT IN THE STRUCTURE.

If a jointed structure is in equilibrium under the action of external forces,

4. *The sum of the vertical components of the external forces and internal stresses meeting at any joint is equal to zero;*

5. *The sum of the horizontal components of the external forces and internal stresses meeting at any joint is equal to zero; and*

6. *The sum of the moments of the external forces and internal stresses meeting at any joint is of necessity zero, since they are concurrent.* (No use can be made of this proposition in structural analysis, and it is given here only for completeness.)

III. FUNDAMENTAL PRINCIPLES APPLIED TO ANY PORTION OF A STRUCTURE ON ONE SIDE OF A SECTION.

If a structure (or a beam), in equilibrium under the action of external forces, be cut by a section,

7. *The sum of the vertical components of the stresses in the members (or fibres) cut is numerically equal to the sum of the vertical components of the external forces acting on either side of this section;*

8. *The sum of the horizontal components of the stresses in the members (or fibres) cut is numerically equal to the sum of the horizontal components of the external forces acting on either side of the section; and*

9. *The sum of the moments of the stresses in the members (or fibres) cut is numerically equal to the sum of the moments of the external forces acting on either side of the section when both are taken about the same point.*

Of these nine propositions only five are commonly used, these being the *third*, for finding algebraically the *supporting forces*, Art. 31; the *fourth and fifth*, which give rise to the *Maxwell system of diagrams*, Art. 46; the *seventh*, the summation in which is called the *shear* on the section; and the *ninth*, which is used most of all, and gives rise to the *algebraic method of moments*, Art. 33.

Although all of these propositions may lead to both algebraic and graphical solutions, the only graphical solutions which are in very common use are those of the Maxwell diagrams, founded on propositions four and five, and explained in Art. 46. While the number of applications here given of this method are few, if the student has mastered the *principles* on which this system is based he can safely be trusted to apply it to other cases. This method is commonly employed in the analysis of stresses in roof-trusses, and sometimes in trussed arches. The common analysis of bridge trusses is based on propositions seven and nine, using the method of sections. The vertical component of the external forces on one side of the section is now the algebraic summation of the loads and the supporting force on one side, and this is called the *shear* on the section. If the two chord members cut by the section are both horizontal, then the shear is wholly resisted by the web member cut by the section, and hence the shear is the vertical component of the stress in that member. This, multiplied by the secant of its angle with the vertical, gives the stress in the member.* The stresses in the chord members are found by taking the centres of moments at the intersections of the web member with the opposite chord. The moment of the external forces on one side of the section about this point, divided by the height of the truss in this case, gives the stress in the chord member. If the chords are not parallel, then all the stresses may be found by proposition nine, as the centre of moments can be taken, for any one of the three members cut, at the intersection of the other two, and this is the common method employed when moving loads are to be provided for. When the loads are fixed (that is to say, not moving), as in roof-trusses, the Maxwell diagrams are commonly employed. When solving for moving loads these loads have a different position for every pair of web and chord members, and hence as many diagrams would have to be drawn as there are load systems, if graphical methods were employed. By using the method of sections, however, with shears and moments treated algebraically, the stress in any particular member can be found at once for any set of loads as soon as one of the supporting forces has been computed.

49. Beam with Uniform Load.—Law of Variation of Moment (Fig. 58).—Beam with uniform load $= p$ lbs. per foot. Required the moment of all external forces upon either side

* When resolving stresses about the sides of a right-angled triangle, instead of using the trigonometrical functions it is often more convenient to use the rule: *Divide the stress by the parallel dimension and multiply by the desired dimension.* Thus, knowing the vertical component, to find the stress in the diagonal member divide by the height of the truss and multiply by the length of the diagonal.

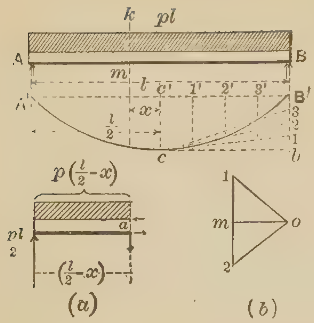


FIG. 58.

of any section, about the neutral axis of the section (called the *bending moment* in the beam).

The equilibrium polygon $A'cB'$ will be a curve whose ordinates, measured from the closing line, will be proportional to the required moments; and if we make our pole distance, Om, unity, they will be equal to them. To derive the equation of this curve we will therefore find the law of variation of these moments. Fig. 58 (a) shows the portion to the left of a section, km, taken at a distance x from the centre. The external forces are the load, $p\left(\frac{l}{2} - x\right)$, and the abutment reaction, $\frac{pl}{2}$. Their moment about a is

$$M = \frac{pl}{2}\left(\frac{l}{2} - x\right) - \frac{p}{2}\left(\frac{l}{2} - x\right)^2 = \frac{1}{8}pl^2 - \frac{p}{2}x^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is the equation of a parabola with axis vertical and whose vertex is at c, a distance $= \frac{1}{8}pl^2$ below the origin c'. This parabola being drawn, the ordinates from the line $A'B'$ to the curve will give the required moments.

A convenient way of drawing the parabola is to lay off $B'b = c'c = \frac{1}{8}pl^2$; then divide $B'b$ into any number of equal parts and $B'c'$ into the same number. Draw the lines $c1$, $c2$, etc., and the verticals through $1'$, $2'$, etc. The intersections of corresponding lines will be points on the curve.

50. Truss with Uniform Load.—Law of Variation of Moment.—In framed structures, uniform loads are carried to joints usually by means of secondary members, and it is these joint loads only which act upon the structure as a whole. For example, in Fig. 59 a uniform load of p lbs. per foot will be given over to the truss in five concentrations at C, D, E, F, and G, each $= pd$, where d is the panel length; and two loads at A and B, each $= \frac{pd}{2}$. The total load $= pl$ and the abutment reactions each $= \frac{pl}{2}$. The uniform load to the left of any joint, as E, is concentrated at the joints A, C, D, and E, $\frac{pd}{2}$ being the loads at A and E due to this portion of the uniform load. The moment of these joint loads about E is evidently the same as the moment of the uniform load to which they are equivalent, and the abutment reaction R' being equal to $\frac{pl}{2}$, we see that the moment of all the external forces to the left of E is given by the ordinate aa' to the moment parabola constructed for the uniform load of p lbs. per foot. The vertices of the equilibrium polygon drawn for the joint loads, therefore, lie on this parabola.

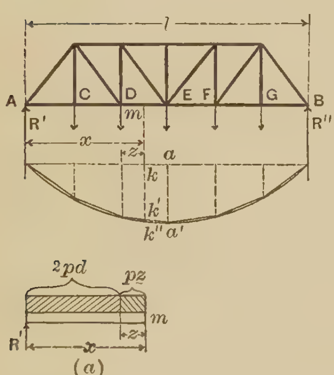


FIG. 59.

If we take our centre of moments at any other point than a point in a load line, as at m, the moment about m of all the external forces (joint loads and abutment reaction) to the left of m is given by the ordinate kk', drawn to the proper segment of the equilibrium polygon, and *not* by the ordinate kk'' to the parabola, as would be the case were the uniform load acting upon a beam. We can see this more clearly by actually computing the moments in the two cases. For the truss, the moment can be written in the form: $R' \times x - [\frac{1}{2}pdx + pd \times (x - d) + \frac{1}{2}pd \times (x - 2d)] - \frac{1}{2}pdz$. For the beam, Fig. ($a$), the moment is $R' \times x - [2pd \times (x - d)] - \frac{1}{2}pz^2$. The quantities within the brackets in the two expressions are equal; they are the moment about m of the portion of the load upon the length AD. The moment upon the beam is greater, therefore, than the moment upon the truss by $\frac{1}{2}pdz - \frac{1}{2}pz^2$. This is seen to be the bending moment at m in the secondary member, DE, produced by the uniform load in the panel.

CHAPTER III.

ANALYSIS OF ROOF-TRUSSES.

LOADS AND REACTIONS.

51. Dead Load.—The dead or fixed load supported by a roof-truss is made up of: the weight of the truss itself; the roof, including roof-covering, sheeting, rafters, and purlins; and sometimes the weight of ceilings and floors suspended from the truss. The roof being designed first, its weight can be directly computed, as can be also the weight of ceilings and floors. The total weight of roof will vary from 5 to 30 lbs. per square foot of roof-surface.

The weight of the truss can only be approximated. From actual calculations it is found to be equal to about $\frac{1}{24}l$ lbs. per square foot of area covered, where $l =$ span length in feet; or, if $b =$ distance between trusses, then the total weight of one truss =

$$T = \tfrac{1}{24}bl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For short spans the weight of the truss is small compared with the total load, and an error in its assumption is correspondingly unimportant. For long spans, however, the error becomes larger, and if, after a preliminary design, it is found to be excessive, the weight must be reassumed in accordance with the design and the computations revised.

52. Live Load.—The live or variable load consists of the wind load, snow load, and floor loads, if any. The maximum wind pressure on a surface normal to its direction is variously estimated at from 30 lbs. to 56 lbs. per square foot. Some experiments made by Sir Benjamin Baker during the erection of the Forth bridge* indicate that the pressure per unit area upon large surfaces is considerably less than upon small surfaces. The ratio of the unit pressure upon an area of $1\frac{1}{2}$ square feet to that upon an area of 300 square feet varied from 1.3 to 2.5, being on the average about 1.5. The highest pressure recorded during the seven years over which the observations extended was 41 lbs. per square foot upon the smaller surface and 27 lbs. upon the larger. As the gales experienced in that vicinity are very severe, it seems reasonable to assume, for ordinary cases at least, a wind pressure of 45 lbs. per square foot upon small surfaces and 30 lbs. upon large ones.

According to experiments recently made upon Mt. Washington by Asst. Prof. C. F. Marvin, U. S. Sig. Service,† the relation between wind pressure and velocity is given very accurately by the formula $u = .004V^2$; where $u =$ pressure per square foot and $V =$ velocity of the wind in miles per hour. These experiments were made upon surfaces of 4 and 9 square feet, the unit pressures on each being practically the same. The difference in area was, however, probably too small to detect any slight difference in unit pressure which may have occurred. Our assumed value of 45 lbs. corresponds by the above formula to a velocity of 105 miles per hour, and thus would seem to cover any case short of a tornado which would destroy almost any building supporting a roof.

The longitudinal component of the pressure of the wind upon a roof is zero for smooth roofs and nearly so for any. The normal component is usually computed by the empirical formula established by Hutton's experiments, i.e.,

$$u' = u \sin \alpha^{\,1.84 \cos \alpha - 1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* See Lon. *Engineering*, Feb. 28, 1890.

† See *Eng. News*, Dec. 13, 1890.

where u' = normal component, u = pressure per square foot on a vertical surface, and α = angle of inclination of the roof with the horizontal. The following values of u', for u = 30 lbs., are computed for various values of α:

α	u'	α	u'	α	u'
5°	3.9	25°	16.9	45°	27.1
10°	7.2	30°	19.9	50°	28.6
15°	10.5	35°	22.6	55°	29.7
20°	13.7	40°	25.1	60°	30.0

For α greater than 60°, u' is taken at 30 lbs.

The snow load is estimated, according to the locality, at from 10 lbs. to 30 lbs. per square foot of horizontal projection, the weight of new snow per cubic foot being from 5 lbs. to 12 lbs. according to its dryness. Snow load need not be considered where α is greater than 45° to 60°, depending on the smoothness of the roof.

The loads upon floors vary from 50 lbs. per square foot to 200 lbs. or more, according to the use to be made of the building. In any case the load anticipated should be approximated as nearly as possible.

53. **Apex Loads.**—The weight of the roof, and the wind and snow loads, are transferred to the truss by means of the *purlins*. In large roofs the purlins should, if possible, be placed upon the trusses at the joints; but if it is necessary to place them between joints, the members of the upper chord supporting them must be designed to resist as a beam as well as a compression member of the truss.

The snow and roof loads being vertical and uniformly distributed over each panel, the joint loads are each equal to one half the sum of the adjacent panel loads. Thus the load at b, Fig. 60, is equal to one half the panel load on bc plus one half the panel load on ab. The snow load on the panel ab is of course less *per square foot of roof* than on bc. The wind load at b is equal to one half the wind load on bc combined with one half the wind load on ab, the load on each panel being normal to the surface. If all panels in one half the truss lie in the same plane and are equal, then all joint loads are equal.

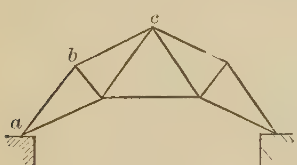


FIG. 60

The above applies only when the purlins are placed at joints. If placed at intermediate points, the loads on the purlins are found as above and divided between adjacent joints in the inverse ratio of the distance of these joints from the purlins.

In roofs of ordinary span it is usual to attach the shingles directly to small angle-iron purlins. In this case the roof load may be treated as a uniform load upon the truss, both in getting apex loads and in computing the bending moment in the upper chord.

The weight of the truss may be considered as applied equally at each of the upper joints.

54. **Reactions.**—For snow and dead loads both reactions are vertical. For wind load the reactions depend upon the manner of supporting the truss. If both ends are fixed, the wind reactions are parallel to the resultant wind load; if one end is free to move, i.e., on rollers or supported on a rocker, the reaction at this end is vertical and that at the fixed end follows from the analysis. If one end be fixed and the other merely supported upon a smooth iron plate, the reaction at the free end may have a horizontal component equal to the vertical component multiplied by the coefficient of friction, which is about $\frac{1}{6}$.

ANALYSIS.

55. **Forms of Trusses.**—A few of the standard forms of trusses are shown in the adjoining figures.

Fig. 61 shows a French or, as it is sometimes called, a Fink roof-truss. It is a very common and economical form for trusses up to about 150 feet span. The struts *bc*, *de*, etc., are placed normal to the roof. These with the upper chord are made either of wood or iron. The other members are ties and are made of iron.

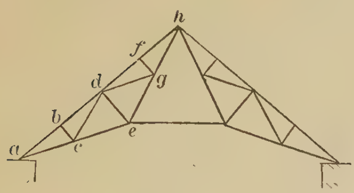


Fig. 61.

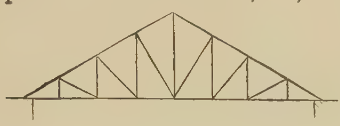


Fig. 62.

Fig. 62 is a common form for wooden trusses. The verticals are iron tie-rods. A special diagram for this form of truss is described in Art. 81, p. 66.

Fig. 63 is a quadrangular truss adapted especially to roofs of small rise. It is constructed of iron, riveted joints being used for short spans and pin connections for long spans.

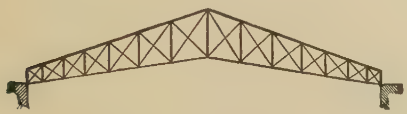


Fig. 63.

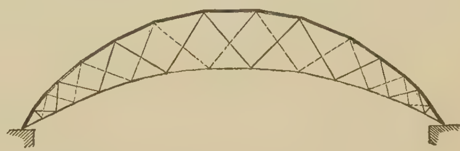


Fig. 64.

The crescent or sickle truss, Fig. 64, with either a single or double system of web members, is a good form for comparatively large spans. Riveted iron-work is used throughout.

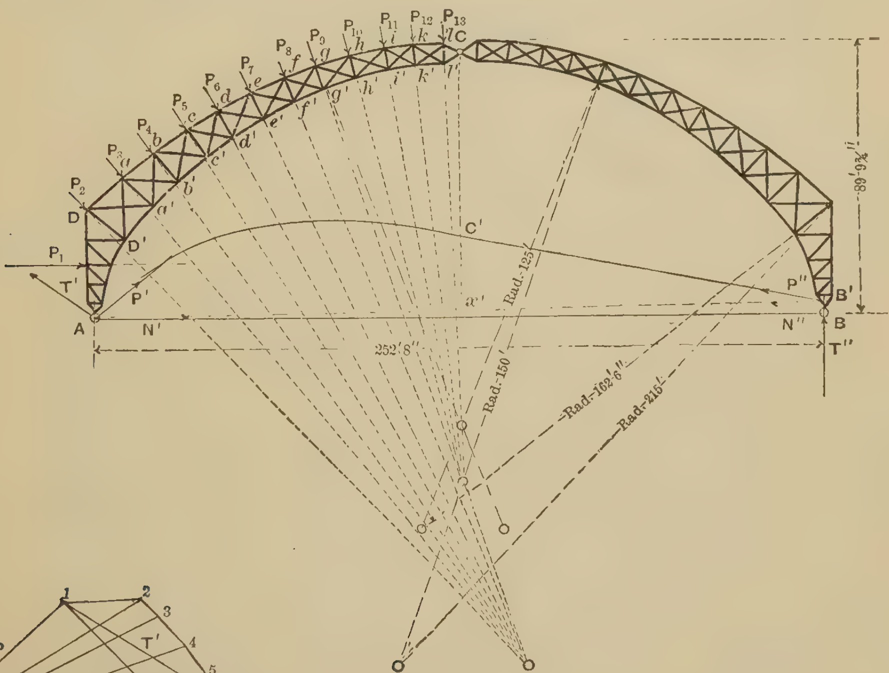


Fig. 65.

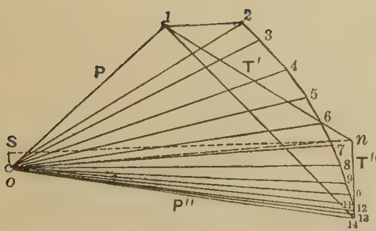


Fig. 65*a*.

For very long spans, as for train-sheds and the like, some form of the arch-truss is often used. Fig. 65 is an outline of the arch-truss of the train-shed of the Pennsylvania Railroad station at Jersey City. The arch is hinged at *A*, *B*, and *C*. The horizontal thrust at the abutments is resisted by means of a tie-rod, *AB*, placed beneath the floor.

If an arch has no hinges, or has but the two hinges at the abutments, the stresses depend upon distortion as well as upon the static load. These forms are treated in Chapter XIV.

56. Analysis of a French Truss.—In the analysis of a roof-truss we must find the stresses due to dead load and combine them with the stresses due to live load so as to get the greatest possible tension and compression in each member. As there are but three or four different possible loadings for roof-trusses, the graphical method is well adapted to their calculation, it being necessary to draw but one diagram for each loading. Where the pieces of a truss have many different inclinations the algebraical method is exceedingly tedious; but if that method is preferred, the truss should be drawn to a large scale and all lever-arms scaled from the diagram. The application of the principles of Arts. 32, 33, will then enable the stresses to be readily computed.

A complete graphical analysis of the truss of Fig. 49, p. 28, will now be made. Span = 100 ft.; rise = 35 ft.; distance apart of trusses = 20 ft.; roof divided into eight equal panels: rollers at A. Length of one side of roof $= \sqrt{50^2 + 35^2} = 61.0$ ft. Angle $\alpha = 35°$.

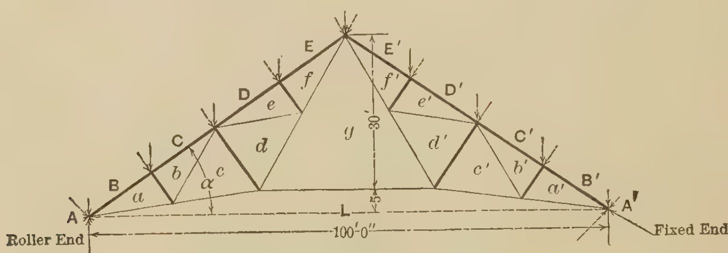


FIG. 66.

Let D = total dead load, S = total snow load, W = total wind load upon either side, P_d = panel dead load, P_s = panel snow load, and P_w = panel wind load.

From Art. 51, eq. (1), the weight of the truss may be taken at $\frac{1}{24}bl^2$, $= \frac{1}{24} \times 20 \times (100)^2 =$ 8333 lbs. Assuming weight of roof at 15 lbs. per square foot, the total weight $= 15 \times 2 \times 61.0 \times 20 = 36600$ lbs. Total dead load, or D, $= 44933$ lbs., and $P_d = 5620$ lbs.

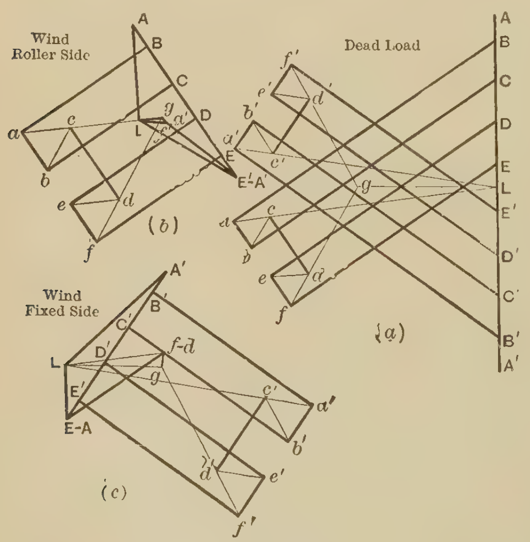


FIG. 67.

Taking snow load at 20 lbs. per square foot of hor. proj., we have $S = 40000$ lbs. and $P_s = 5000$ lbs.

The normal component of the wind pressure, according to Art. 52, with $\alpha = 35°$, is 22.6 lbs. per square foot. Whence, $W = 22.6 \times 20 \times 61.0 = 27570$ lbs., and $P_w = \frac{1}{4}W = 6890$ lbs.

After drawing the truss carefully to scale (the scale used should be from 10 to 20 feet to an inch), we proceed to draw the diagram for dead load, Fig. 67 (a). Each joint load $= P_d = 5620$ lbs., except the loads at the end joints, each of which $= \frac{1}{2}P_d = 2810$ lbs. These loads are laid off to form the load line AA'. The abutment reactions are $A'L$ and LA, each $= \frac{1}{2}A'A$. Beginning at A, the diagram is drawn exactly as in Art. 46, Fig. 49. The scale actually used in the calculations was 5000 lbs. to one inch. In the diagram, heavy lines denote compression, light lines

tension. The stresses in the members due to dead load are given in the second column of the following table; they were readily scaled off to the nearest hundred pounds. The stress in *Lg* is given as 18700 lbs., while by actual calculation it is 18733 lbs.

The diagram for snow load will be a figure similar to the one for dead load, and the stresses in the two cases will be proportional to the corresponding loads. If we multiply each dead-load stress, therefore, by $\frac{40000}{44933}$, we will have the corresponding snow load stress. These are best found with the slide-rule; they are given in the third column of the table.

For wind stresses we must consider the wind blowing first from one side, then from the other, since the abutment reaction at the roller end must in both cases be vertical, and the stresses produced in the two cases will therefore not be symmetrical. Fig. 67 (*b*) is the diagram for wind from the left. The load line is AE'. The abutment reaction, LA, at A, is easily found by putting Σ mom. about $A' = 0$; the force polygon $AE'L$ then gives the other reaction, $A'L$. Beginning at A, the diagram is readily constructed. It will be found that there are no stresses in $f'e'$, $e'd'$, etc. Column 4 gives the stresses found from the diagram. It is seen that the stresses in the pieces *Lg*, *gf'*, *gd'*, *La'*, and *Lc'* are compressive, whereas they were tensile for dead load; the *resultant* stresses are, however, all tensile. If the roof had a greater rise, the compressive stresses due to wind load would be increased; and if the rise were great enough, they would be greater than those due to dead load and the resultant stresses would be compressive. These members would then need to be counter-braced.

The diagram for wind pressure on the right is given in Fig. 67 (*c*). The load line is $A'E$. The reactions are found as before and the diagram then drawn. The stresses thus found are given in column 5.

The maximum stress of each kind in each piece is now obtained by combining with the dead-load stress, whatever possible combination of the snow and wind load stresses will give the greatest total tension and the greatest total compression. Column 6 gives these maximum stresses. It is seen that no piece is ever subjected to counter-stresses.

TABLE OF STRESSES.

Member.	Dead Load.	Snow-load.	Wind from Left.	Wind from Right.	Maximum.
Ba	+ 43400	+ 39100	+ 24500	+ 18700	+ 107000
Cb	+ 40150	+ 36150	+ 24500	+ 18700	+ 100800
De	+ 36950	+ 33250	+ 24500	+ 18700	+ 94700
Ef	+ 33700	+ 30300	+ 24500	+ 18700	+ 89500
ab	+ 4600	+ 4100	+ 6900		+ 15600
cd	+ 9200	+ 8300	+ 13800		+ 31300
ef	+ 4600	+ 4100	+ 6900		+ 15600
bc	− 5150	− 4650	− 7700		− 17500
de	− 5150	− 4650	− 7700		− 17500
La	− 35850	− 32200	− 18300	− 15500	− 86400
Lc	− 30700	− 27600	− 10600	− 15500	− 73800
Lg	− 18700	− 16800	+ 4400	− 14100	− 49600
gd	− 13600	− 12300	− 14600	− 2700	− 40500
gf	− 18800	− 16900	− 22300	− 2700	− 58000
gf'	− 18800	− 16900	+ 700	− 25800	− 61500
gd'	− 13600	− 12300	+ 700	− 18100	− 44000
Lc'	− 30700	− 27600	+ 4800	− 31000	− 89300
La'	− 35850	− 32250	+ 4800	− 38700	− 106800
d'e'	− 5150	− 4600		− 7700	− 17500
b'c'	− 5150	− 4600		− 7700	− 17500
e'f'	+ 4600	+ 4100		+ 3900	+ 15600
c'd'	+ 9200	+ 8300		+ 13800	+ 31300
a'b'	+ 4600	+ 4100		+ 6900	+ 15600
E'f'	+ 33700	+ 30300	+ 13600	+ 29800	+ 93800
D'e'	+ 36950	+ 33250	+ 13600	+ 29800	+ 100000
C'b'	+ 40150	+ 36150	+ 13600	+ 29800	+ 106100
B'a'	+ 43400	+ 39000	+ 13600	+ 29800	+ 112200

57. The Quadrangular Truss.—Fig. 68 is a half-diagram of the roof-truss of the Jersey City station of the Central Railroad of New Jersey. Distance between trusses = 32.5 ft. All upper panels = 13 ft. $1\frac{9}{16}$ in. Other dimensions as shown. Left end on rollers.

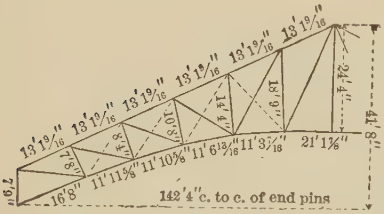


FIG. 68.

In this truss the lower chord and diagonals are eye-bars capable of resisting tension only (except the end panels of the lower chord, which members are counter-braced). Wherever, therefore, these members would be subjected to compression from wind loads, it is necessary to insert the counters (shown by dotted lines) which would then be in tension and would prevent the distortion of the quadrilaterals.

The diagram for dead load is drawn as before, omitting the counters. Snow load stresses are also found as before.

For wind load, some of the counters will come into action, and as the dead load always acts with the wind load, it will be more convenient to combine the dead and wind load for each joint and draw a diagram for this combined loading. Where there are counters, only the diagonal which is in tension should be considered. This can easily be determined by a trial diagram. If any diagonal be found in compression where there is no counter, a counter must be inserted. The wind must be considered as acting first on one side, then on the other.

For maximum stresses of both kinds we have simply to remember that the dead load may act alone, or combined with either of the other three, or combined with snow and either of the wind loads.

The student should assume loads according to Arts. 51, 52, and make the complete analysis.

58. Analysis of a Crescent Truss.—Fig. 69 is a half diagram of the roof-truss of the

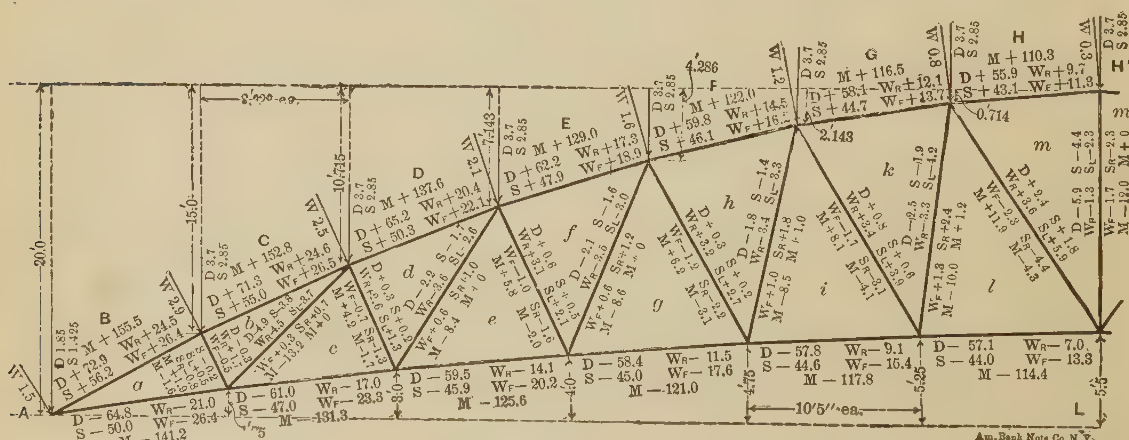


FIG. 69.

St. Louis Exposition Building. The complete analysis is given below, according to the following data and assumptions:

Distance between trusses $= b = 16$ ft.

Total weight of one truss $= T = \frac{1}{24}bl^2 = 10400$ lbs.

Weight of roof = 20 lbs. per square foot of roof-surface.

Snow load = 20 lbs. per square foot of horizontal projection, to be treated as acting: first, all over; and second, on one half only.

Wind pressure according to the table of Art. 52.

Assuming an average panel length of the upper chord of 9.3 ft., the total dead load per panel =

$$P_d = 20 \times 9.3 \times 16 + \tfrac{10400}{14} = 2976 + 743 = 3719. \quad \text{Call it 3700 lbs.}$$

The snow load per panel =

$$P_s = 20 \times 8.9 \times 16 = 2848. \quad \text{Call it 2850 lbs.}$$

The wind load per panel varies with the inclination. The different panel loads are given in Fig. 69, in thousands of pounds.

The complete dead load diagram is given in Fig. 70 (a); the stresses for uniform snow

FIG. 70a.

load are found by multiplying the dead load stresses by $\frac{2850}{3700}$. These dead and snow load stresses are marked D and S, respectively, in Fig. 69 and are given in thousands of pounds.

Fig. 70 (b) is the full diagram for snow load on the left side only. The corresponding stresses are marked S_L in Fig. 69. The stresses in the members of the left half of the truss due to snow load on the right only, are the same as those in the right half due to snow on the left. These stresses are already given in Fig. 70 (b), and are scaled off and placed along the pieces of Fig. 69 and marked S_R. Only web stresses are desired for unsymmetrical loading, as the chord stresses are greater with full snow-load.

The diagrams for wind load are given in Figs. 70 (c) and (d). Fig. (c) is for wind from left, left end fixed. In drawing the diagram for wind from the right, we may, for convenience, consider the truss turned end for end. The left end will now be on rollers and the right end fixed. The vertical components of the reactions remain the same, but the horizontal component now acts at the right end. The stresses found for the right end are thus really

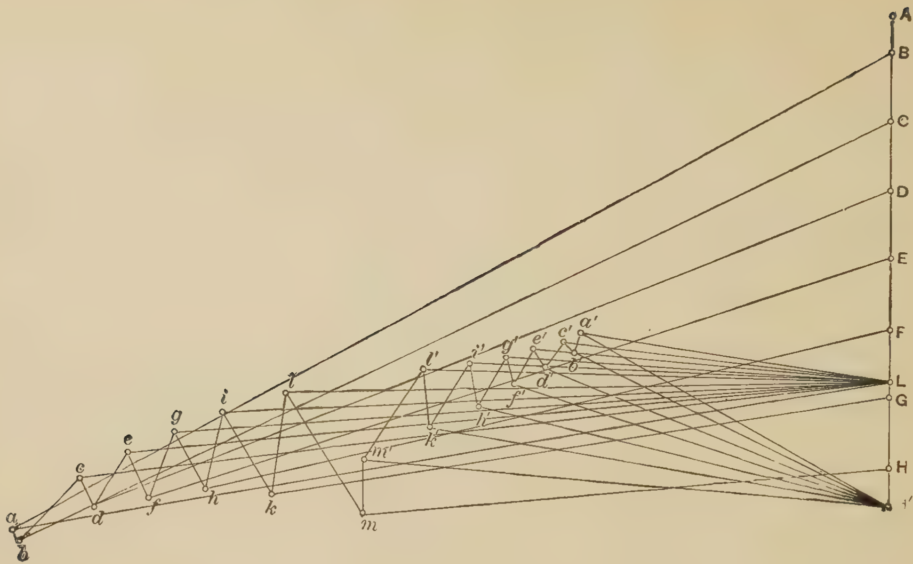


FIG. 70*b*.

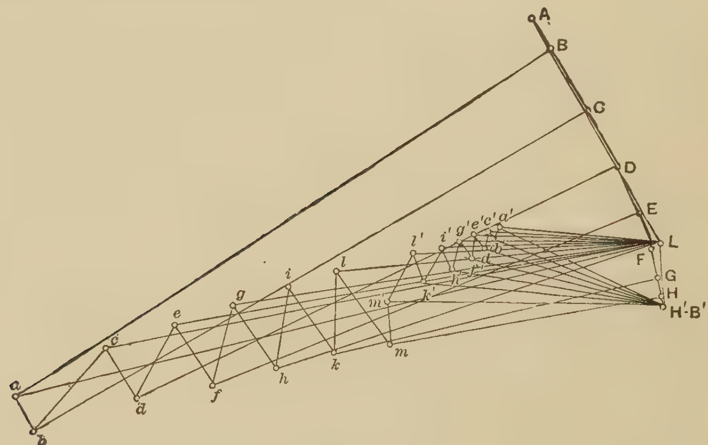


FIG. 70*c*.

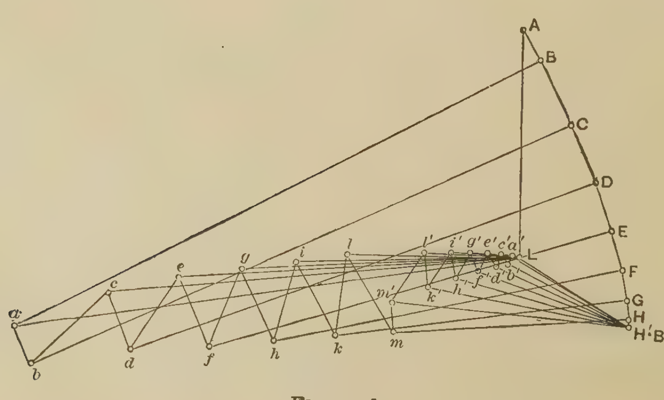


FIG. 70*d*.

those for the left end. It is to be noted that the stresses in the right half due to wind pressure are not quite the same as those in the left half. The stresses marked in Fig. 69 are the maximum of each kind which occur in either of the two symmetrical members due to wind from fixed end (W_F) and wind from roller end (W_R). Thus, with wind from either direction, the greatest chord stresses are in the half truss towards the wind, and these are the stresses given. In practice the truss would be built symmetrically. The total maximum stresses are marked M.

All loads and stresses are given in thousands of pounds.

59. The Arch-truss.—The stresses in a three-hinged arch, as in Fig. 65, p. 35, are readily found by diagram, the reactions at A, B, and C having been once obtained. Considering B as the roller end, we can get the abutment reactions at A and B as in any other truss, either analytically or graphically, by treating the structure as a whole. The stress in the tie AB is found by passing a section through C and the tie, treating the structure to the left and putting Σ mom. about $C = 0$. Knowing the stress in AB and the abutment reaction at A, we have at joint A but two unknown forces. Beginning then at this point we can construct the diagram for the truss AC. Above the point D a double system of web members is inserted which is to be treated as explained in the next article.

The loads assumed in the actual computation of the stresses in this truss were:* a dead load of about 30 lbs. per square foot (whether of roof or of horizontal projection is not stated); a snow load of 17 lbs. per square foot; and a wind pressure of 35 lbs. per square foot of elevation. The snow load was assumed to exist: first, all over; second, on the twelve centre panels only; and third, on one side only.

Since the two trusses, AC and CB, are supported alike, it is necessary to consider the wind pressure from one side only, the stresses in CB for wind from the left being the same as in AC for wind from the right. For the tie AB, however, wind pressure on the left increases its tension, while wind from the right decreases it and in fact produces a slight compression.

If the tie AB is omitted, the horizontal thrust must be resisted by the abutments themselves, the structure then being a true arch. In that case the abutment reactions are found as in Art. 31, Ex. 3. The process there given amounts to precisely the same thing as that above for finding the resultant reaction of tie and abutment.

For wind loads, which are so variously inclined, it will be simpler to find the reactions and also the stress in tie AB by means of an equilibrium polygon. Fig. 65 (a) shows a force diagram for wind load on the left. The load line is 1, 2 . . . 14; the pole O is chosen at any convenient point. To draw the equilibrium polygon, begin at A, the fixed end; the closing line is drawn from A to the intersection of the last segment with the vertical reaction line BB'. The line On drawn parallel to AB', meeting the vertical $14\text{–}n$ at n, determines the reactions at B and A; they are equal to $14\text{–}n$ and $n\text{–}1$, respectively. Knowing the reaction at B, the stress in AB is found by a simple equation of moments, centre at C, of the forces acting on the right-hand portion; or, the equilibrium polygon may be utilized as in Art. 40, thus:—Treating the forces (T', P_1, P_2, etc.) to the left of a section through C and AB, their moment about C may be found by drawing Cx' parallel to their resultant, $n\text{–}14$, and $x'C'$ vertically. Their moment about C is then equal to their moment about x', which equals $x'C' \times sn$, the horizontal projection of the ray $O\text{–}14$. This moment divided by the lever-arm of AB gives the required stress. This method is especially useful where we combine the dead with the wind loads, as in that case the portion CB is loaded and the analytical method necessitates the computation of the moments of all these loads.

60. Trusses with Double Systems of Web Members, as in Figs. 64 and 65, may be analyzed by treating each system separately, together with the loads acting at the vertices of

* See *Engineering News*, Sept. 26, Oct. 3, 1891.

that system. Thus in Fig. 64, the chords with the system shown by full lines constitute one system, while the chords with the dotted diagonals constitute the other.

The stresses in the diagonals result directly, while those in the chords are found by adding the stresses in each member due to each system. In drawing the diagrams it is sufficiently accurate to consider the chord as a straight line between consecutive joints of the same system.

In Fig. 65 one system may be taken as $Da'bc'de'fg'hi'kl'C$ with the radial struts at a, c, e, g, i, and l; and the other as $DD'ab'cd'ef'gh'ik'lC$ with the struts at b, d, f, h, and k. In treating each system, one half of a panel load is to be placed at each apex and the stresses determined as for a simple truss, each piece being designed to take either tension or compression. One diagram must be constructed for each system and for each kind of loading, and the results combined for the maximum stresses. The maximum chord stresses result by adding those due to each system.

CHAPTER IV.

BRIDGE-TRUSSES—ANALYSIS FOR UNIFORM LOADS.

GENERAL CONSIDERATIONS.

61. Preliminary Statement.—The particular method of analysis to be selected for a bridge-truss depends upon the kind of loading and upon the form of truss. The analysis of bridge-trusses will therefore be treated under three general heads corresponding to the three methods of loading, viz., Uniform Loads, Actual Wheel Loads, and Conventional Methods of Loading, no distinction being made between highway bridges and railroad bridges, except as to loads assumed. Under each of these heads will be treated the various forms of trusses for which such loads are commonly specified.

62. The Dead Load consists of the entire weight of the bridge, including floor system. For *highway bridges* the total dead load per foot may be taken from the diagram on the following page, prepared by plotting the weights per foot for bridges 18 feet in width given in Tables I, II, and III in Waddell's "Designing of Ordinary Iron Highway Bridges."* These weights are from actual computations of the bills of material for sixty bridges, and are for the ordinary type of single and double intersection trusses. "Class A" indicates bridges frequently subjected to heavy loads; "Class B" indicates city bridges occasionally subjected to heavy loads; and "Class C," country bridges. The average change in weight per foot for a change in width of two feet is given by the curves in the lower part of the diagram. Thus, for a 200-ft. span, Class A, width of roadway 22 ft., the total weight per foot = weight per foot for 18-ft. roadway $+ 2 \times$ increase in weight for a change in width of two feet, $= 960 + 2 \times 100 = 1160$ lbs. per foot.

For *railroad bridges* the dead load per foot is given very closely by the following formulæ, where $l =$ length of span in feet and $w =$ dead load per foot in pounds, not including the track system (rails, ties, guard-rails, and safety-stringers, if any).

For deck plate girders,

$$w = 9l + 120. \quad \ldots \quad (1)$$

For lattice girders,

$$w = 7l + 200. \quad \ldots \quad (2)$$

For through pin-connected iron bridges with steel eye-bars,

$$w = 5l + 350. \quad \ldots \quad (3)$$

For Howe trusses,

$$w = 6.5l + 275. \quad \ldots \quad (4)$$

---

* These curves were first plotted by Prof. C. L. Crandall in his "Notes on Bridge Stresses." They are here redrawn and slightly changed.

These formulæ are for single-track bridges; for double-track add ninety per cent. The weight per foot of a single track may be taken at 400 lbs. For the load on each truss take one half the above values.

Formulæ (1), (2), and (3) are of the same form as those in use by a number of the leading bridge companies, and are based on Cooper's loading class, "Extra Heavy A" (see page 85).

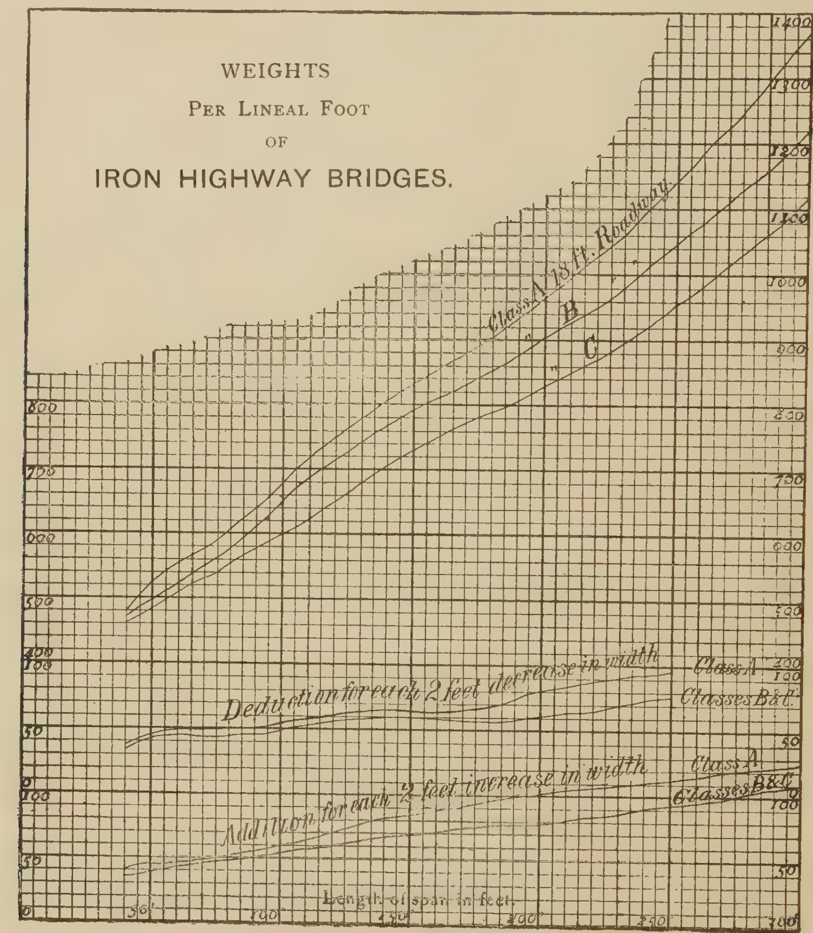


FIG. 71.

Formula (4) is from assumed weights of Howe trusses on the Oregon Pacific Railroad, A. A. Schenck, Chief Engineer.*

63. The Live Load for highway bridges is usually taken as a uniform load of from 50 to 100 lbs. per square foot of roadway, or the heaviest concentrated load, due to a road-roller or the like, which is likely to come upon the structure. The uniform load generally gives the maximum stresses in the main truss members, while for stringers, floor-beams, beam-hangers, etc., the concentrated load usually gives the greater stresses. The loads specified by Waddell are given on the next page. Classes A, B, and C have the same signification as above.

For railroad bridges the load usually specified is that due to two of the heaviest locomotives on the road in question when coupled in direct position, and followed by a uniform load due to the heaviest possible train. An example of such loading is given in the next chapter, and various standard loadings are given in the chapter on Specifications.

* See *Engineering News*, April 26, 1890.

Span in Feet.	Moving Load per Square Foot of Floor.	
	Classes A and B.	Class C.
0 to 50	100 lbs.	80 lbs.
50 to 150	90 "	80 "
150 to 200	80 "	70 "
200 to 300	70 "	60 "
300 to 400	60 "	50 "

64. **The Wind Pressure** upon bridges is carried by horizontal truss systems placed between the chords of the main trusses. The loads assumed and the corresponding stresses in these trusses are discussed in Chap. VII.

The stresses in the main trusses and in these lateral systems due to *vibration* are discussed in Part II.

65. **Apex Loads.**—The dead load for short spans is usually considered as applied at the panel points of the loaded chord. For long spans one third may be taken at the unloaded chord and two thirds at the loaded chord, or the actual concentrations may be computed.

Since the live load is given over to the truss at the panel points it can thus affect the truss only at these points. The portion of each end-panel load carried by the abutment does not affect the truss and need not be taken into account in finding either loads or reactions. Thus, for a bridge of 200 ft. span and eight equal panels having a uniform load of 2000 lbs. per foot, each of the seven apex loads = 25 × 2000 = 50000 lbs., and each abutment reaction = 7 × 50000 ÷ 2 = 175000 lbs.

Stresses in Simple Beams.

66. **Bending Moment in a Beam.**—If a beam, *AB*, Fig. 72, be loaded in any manner and any section taken, the sum of the moments of the external forces upon either side of the section about the neutral axis, *N*, is called the *bending moment*, or simply the *moment*, at *N*. From Σ mom. = 0 we see that the bending moment is equal but of opposite sign to the moment of the stress couple at *N*, Fig. (*a*). Whence we say that the moment of the external forces is balanced by the moment of the internal stresses. For convenience we call bending moment positive when it causes convexity downwards or produces tension in the lower fibres, and negative when the reverse. Its sign is thus seen to agree with the sign of the moment of the external forces to the left of the section, and to be the opposite of the sign of the moment of the forces to the right.

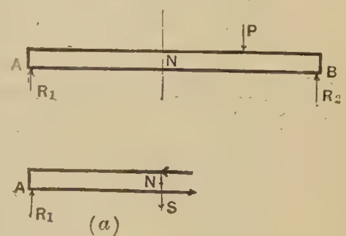


FIG. 72.

The bending moment at any point *N* is always positive, the beam being supported at the ends; for a load to the right of *N* affects the forces on the left only by increasing the abutment reaction and consequently the positive moment, while a load to the left of *N* affects the forces on the right only by increasing the right abutment reaction and consequently the positive bending moment. For a uniform load, therefore, the maximum bending moment at every point occurs when the load extends the whole length of the beam. In Chap. II, p. 31, we have shown that the bending moment in a beam under a uniform load varies along the beam as the ordinates to a parabola, the middle ordinate being $= \frac{1}{8}pl^2$, where p = load per foot and l = span. Also that the moment at a point any distance x from the centre is given by the equation

$$M = \tfrac{1}{8}pl^2 - \tfrac{1}{2}px^2. \quad \ldots \quad (5)$$

The above equation may be written in the form

$$M = \tfrac{1}{2}p\left(\frac{l^2}{4} - x^2\right) = \tfrac{1}{2}p\left(\frac{l}{2} - x\right)\left(\frac{l}{2} + x\right). \quad \ldots \quad (5a)$$

That is, *the bending moment at any point in a beam under a uniform load equals one half the load per foot multiplied by the product of the two segments into which the beam is divided.*

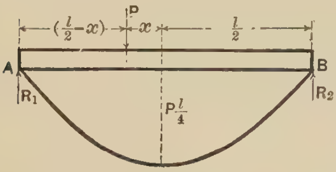


FIG. 73.

Equation (5*a*) will enable the moment to be computed at any point in a beam or plate girder under uniform loading.

For a single concentrated load, Fig. 73, the maximum moment at any point occurs when the load is at that point, for a movement to either side reduces the opposite abutment reaction and hence the moment. This maximum moment is given by the equation

$$M = P\left(\frac{\frac{l}{2}+x}{l}\right)\left(\frac{l}{2}-x\right) = P\left(\frac{l}{4}-\frac{x^2}{l}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is seen to be the equation of a parabola whose ordinate is a maximum for $x = 0$, the value of this ordinate being equal to $P\frac{l}{4}$.

If we substitute $\frac{2P}{l}$ for p in eq. (5) we shall get eq. (6), thus showing that the maximum bending moments due to a single moving load are the same as for twice that load when uniformly distributed over the beam.

For two equal loads a fixed distance apart, Fig. 74, the bending moment under the left hand load, P_1, for example, is found by adding the moments due to each load. The moment due to P_1 is given by eq. (6). The curve representing the variation in this moment as the loads move over the beam is AcB. The moment at N due to P_2 is

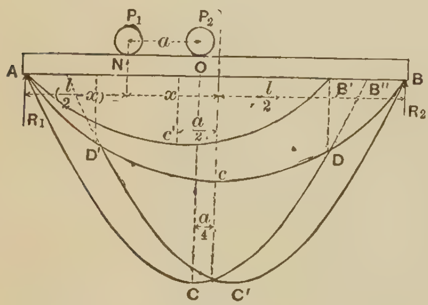


FIG. 74.

$$M_2 = P_2\left(\frac{\frac{l}{2}+x-a}{l}\right)\left(\frac{l}{2}-x\right) = \frac{P_2}{l}\left(\frac{l^2}{4}-x^2-\frac{al}{2}+ax\right). \quad (7)$$

This can readily be shown to be the equation of a parabola with maximum ordinate $= \frac{P_2}{l}\left(\frac{l}{2}-\frac{a}{2}\right)^2$ when $x = \frac{a}{2}$. The curve $Ac'B'$ is this parabola, with vertex at c'. The curve representing the sum of these moments is evidently obtained by adding the ordinates of the curves AcB and $Ac'B'$, and is $ACDB$. The equation of the portion ACD is obtained by adding eqs. (6) and (7), remembering that the loads are equal. This gives the total moment

$$M = \frac{P}{l}\left(\frac{l^2}{2} - 2x^2 - \frac{al}{2} + ax\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For a maximum, by putting $\frac{dM}{dx} = 0$, we find $x = \frac{a}{4}$. Changing our origin to this point by putting $\left(x' + \frac{a}{4}\right)$ for x in eq. (8), we have

$$M = \frac{P}{l}\left(\frac{l^2}{2} - \frac{al}{2} + \frac{a^2}{8} - 2x'^2\right) = \frac{P}{2l}\left[\left(l - \frac{a}{2}\right)^2 - 4x'^2\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This is the equation of a parabola, with axis vertical and passing through the origin. The maximum ordinate is for $x' = 0$ and equals $\frac{P}{2l}\left(l - \frac{a}{2}\right)^2$, the ordinate OC in the figure. For moment under the right wheel the curve is $AD'C'B$ and is symmetrical to $ACDB$. The greatest moments for the left half occur therefore under the left wheel, and for the right half under the right wheel. The maximum moment in the beam is at a distance from the centre equal to one-fourth the distance between the loads.*

The curve of maximum moments for any form of loading may be found by a method similar to the above, but the subject will not be treated further here. The succeeding chapter discusses the location of the point of maximum moment in a beam for any number of loads.

67. Shear in a Beam.—If a section be taken at any point N, Fig. 75, in a loaded beam, the sum of the vertical components of the external forces upon either side of the section is called the *shear* on the section. The sign of this resultant vertical force is evidently plus on one side and minus on the other; but for convenience we shall give to the shear the same sign as that of the resultant force on the left. *Positive* shear, then, is when the left-hand portion tends to move *upwards* on the right, and *vice versa*. From Σ vert. comp. $= 0$ we know that the shear must be balanced by the internal force or stress in the section, that is, by the action of the portion removed upon the portion considered. This stress is called a *shearing* stress and in Fig. (a) it is replaced by the force S.

The shear at any point N in a beam, Fig. 76, for a fixed uniform load of p lbs. per foot, is equal to the left abutment reaction minus the load between A and N, or

$$S = R_1 - px = p\left(\frac{l}{2} - x\right). \qquad (10)$$

This is the equation of a straight line having a maximum positive ordinate of $p\frac{l}{2}$ when $x = 0$, and an equal negative ordinate when $x = l$. When $x = \frac{l}{2}$, $S = 0$.

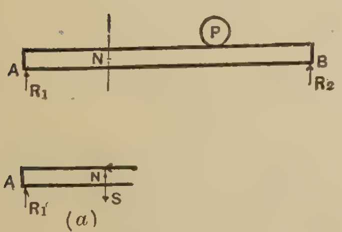


FIG. 75.

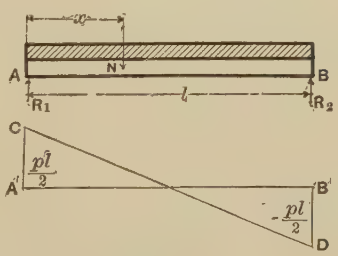


FIG. 76.

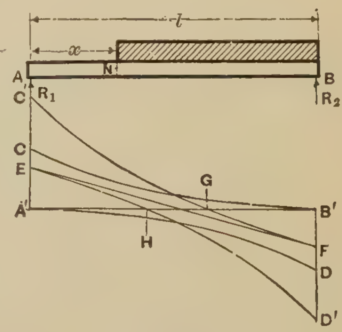


FIG. 77.

For a moving uniform load the maximum positive shear at any point N, Fig. 77, occurs when all possible loads are added to the right and when there are no loads on the left; for adding a load to the right increases the left reaction and therefore the positive shear, while adding loads to the left increases the right reaction without affecting the other forces to the right, and hence decreases the positive shear. The maximum shear at N is therefore

$$S = R_1 = p(l - x)\frac{l - x}{2l} = \frac{p}{2l}(l - x)^2, \qquad (11)$$

* By equating the maximum bending moment, as above obtained, with the moment at the centre when one wheel is at that point, it may be shown that the latter will be the greater when a is greater than $0.586l$.

the equation of a parabola with vertex at the right end. This parabola is $B'C$, BB' being the axis; the ordinate $A'C$ is equal to $\frac{pl}{2}$. The maximum negative shear is found by loading to the *left* of the point, and is

$$S = -R_2 = -\frac{p}{2l}x^2, \qquad (12)$$

the equation of the parabola $A'D$.

Where a beam is subjected to both a fixed and a movable uniform load, as from dead and live loads, the maximum positive and negative shears are found by combining the shears due to each system of loading. In Fig. 77, EF represents dead load shears; whence the maximum positive shears are found graphically by adding the ordinates of this line to those of CB', giving the curve $C'F$.* This curve crosses the axis at G, the dead load negative shears to the right of this point being greater than the live load positive shears. From G to B', therefore, positive shear cannot occur. The curve ED' gives the maximum negative shears from H to B', none being possible from A' to H. Between H and G both kinds of shear are possible. The actual values of the shears are best found by the use of eq. (10) with eqs. (11) and (12). In practice we need find only the maximum positive shears, since the negative shears are equal and symmetrical to them.

For a single load P_1, Fig. 78, the positive shear at any point N is greatest when the load is just to the right of the point, for the left reaction is then a maximum. This maximum shear is

$$S_1 = P_1\left(\frac{l-x}{l}\right), \qquad (13)$$

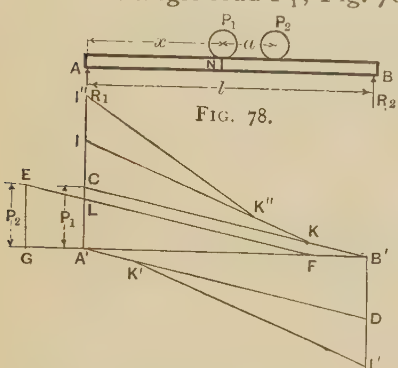


FIG. 78.

FIG. 79.

the equation of the straight line CB', Fig. 79. The ordinate $A'C = P_1$. In like manner the maximum negative shear occurs with the load just to the left of the point and is

$$S_1 = -P_1\frac{x}{l}. \qquad (14)$$

This negative shear is represented by the line $A'D$.

For two equal loads, P_1 and P_2, the maximum positive shear is when P_1 is just to the right of the point. The shear due to P_1 is given by eq. (13) and the line CB'. The shear due to P_2 when P_1 is at N, is equal to left abutment reaction for P_2, or

$$S_2 = P_2\frac{l-(x+a)}{l}. \qquad (15)$$

This is zero for $x = l - a$, and is equal to P_2 for $x = -a$ if that were possible; it is represented by the line FE, where FB' and $GA' = a$ and $GE = P_2$. The total shear is the sum of the second members of eqs. (13) and (15). It is found graphically by making $CI = LA'$ and drawing IK to meet CB' in the vertical through F. The total negative shears are found likewise. They are equal to the ordinates to the line $A'K'I'$.

For three loads the shear diagram would be $I''K''KB'$, and so on, the curve approaching to a parabola as the limiting case when the load is uniform per unit length.

In this article and the preceding, but two of the equations of equilibrium have been used, viz., Σ mom. $= 0$ and Σ vert. comp. $= 0$. These two equations are the only ones that involve the external forces, since these forces have been taken as vertical, the usual condition for bridges and beams. In the arch, however, the third condition is involved and we have moment, shear, and *thrust*.

* This curve is also a parabola and may be constructed by laying off vertically from EF the distances $\left(\frac{x}{A'B'}\right)A'C$, where x = horizontal distance of the point from F.

BRIDGE-TRUSSES WITH PARALLEL CHORDS.

68. Chord Stresses.—If AB, Fig. 80, be any truss subjected to the vertical loads P_1 and P_2, the stress in any member 3–4 of the lower chord may be found by passing the section lm cutting 3–4 and but two other pieces, treating the portion to the left and putting Σ mom. about $2 = 0$. The abutment reaction R_1 is supposed to have been found already by treating the structure as a whole. The moment of the external forces to the left, about 2, is called the bending moment in the truss at 2, and may be computed or be scaled off from the equilibrium polygon drawn for the given loads. The equilibrium polygon is $A'abB'$, and $ee' \times Om =$ bending moment. Its sign is plus. The moment of the stress in 3–4 must balance this moment and is therefore negative, or left-handed about 2. The stress in 3–4 is then tensile and equal to the bending moment divided by its lever-arm. Likewise with the same section and centre of moments at 3 we may find the stress in 1–2; it will be compression.

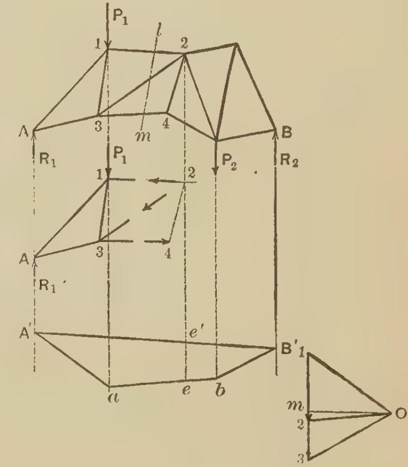


FIG. 80.

The bending moment at *any* point between A and B is seen by the equilibrium polygon to be positive, as in a beam. The stresses in all members of the lower chord are therefore tensile, and in the upper chord compressive.

Again, for maximum bending moment at any point for a uniform load, Fig. 81, the truss must be fully loaded, but in the case of a truss the uniform load can act only as joint loads. We have shown in Chap. II, p. 32, that the vertices of the equilibrium polygon for these loads lie on the parabola of moments drawn for a beam with the same uniform load. The middle ordinate $= \frac{1}{8}pl^2$. For the upper or unloaded chord, the centres of moments are at the loaded joints, and the bending moments are therefore given by the ordinates to the equilibrium polygon or to the parabola. For the lower chord the centres of moments are at the upper joints and the bending moments are given by ordinates to the equilibrium polygon only. Thus for piece 5–7 the bending moment at 6 is given by cc', the ordinate to the parabola (pole distance = unity); but for piece 6–B, with centre of moments at 7, the bending moment is given by dd'. Where the upper joints are over the centres of the lower panels the moments at the upper joints are means between the moments at adjacent lower joints.

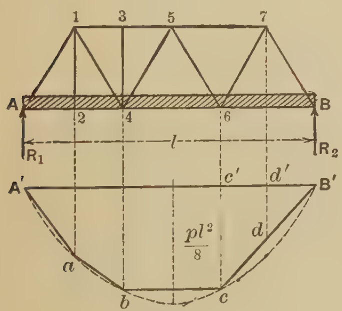


FIG. 81.

The computation of the moments at joints in the loaded chord is best made by means of eq. ($5a$), p. 45, since these moments are the same as in a beam with the same loading. Thus the moment at $6 = \frac{p}{2}(A6 \times 6B)$; moment at $4 = \frac{p}{2}(A4 \times 4B)$; etc. These moments when divided by the lever-arms of the respective pieces give stresses.

If the panel lengths are all equal to d, and m be the number of panels to the left and m' the number to the right of the centre of moments, then we have from eq. ($5a$) above referred to,

$$M = \frac{p}{2}(md \times m'd) = \frac{pd^2}{2}(mm'). \qquad (16)$$

Equation (16) applies equally well where m and m' are fractional, provided the joint loads are those due to a uniform load.

The moments at the panel points of the unloaded chord, when these points are not in the same verticals as those of the loaded chord, are best found by proportion, from the moments at the two adjacent joints of the loaded chord.

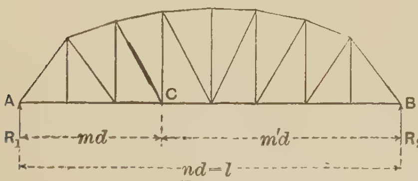


FIG. 82.

Equation (16) may be derived directly as follows: Let AB, Fig. 82, be any truss having equal panels in the lower or loaded chord. Let m be the number of panels to the left of any panel point C, and m' the number to the right. Let p = load per foot. Panel load $= pd$. There are $n - 1$ joint loads and therefore $R_1 = \frac{pd(n-1)}{2}$. To the left of C are $m - 1$ loads whose average lever-arm about $C = \frac{md}{2}$. The bending moment at C is therefore

$$M = R_1 \times md - (m-1)pd \times \frac{md}{2} = \frac{pd(n-1)}{2}md - \frac{pd^2}{2}(m-1)m = \frac{pd^2}{2}(mm'). \quad \text{Q. E. D.}$$

69. Web Stresses, Parallel Chords.—If AB, Fig. 83, be a truss with parallel chords, the stress in any web member, as 2–3, may be found by passing the section cd and putting

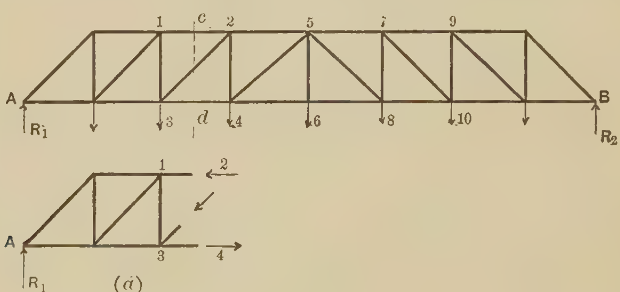


FIG. 83.

Σ vert. comp. of the forces acting upon the portion to the left of the section, equal to zero. The sum of the vertical components of the external forces is, as in the case of a beam, called the *shear*, and must be balanced by the vertical components of the stresses in the members cut; whence the vert. comp. of stress in 2–3, Fig. (a), is equal and opposite to the shear. If the shear is positive or upwards on the left, then 2–3 is in compression; and if negative, then it is in tension. The vertical component in 2–4 is the same as in 2–3, since the shear on the section cutting 2–4 is the same as that on the section cd, no load being at 2. Likewise the vertical components in 7–10 and 7–8 are equal, etc. Since the shear is constant between two adjacent loaded joints, we usually speak of the shear in the *panel*, as shear in panel 3–4, etc.

For maximum positive live load shear in panel 3–4, and hence maximum compression in 2–3 and maximum tension in 2–4, all joints to the right of the panel should be fully loaded and all joints to the left unloaded. This follows for the same reason that was given in the case of the beam, Art. 67. For maximum negative shear in 3–4 and hence maximum stresses of the opposite kinds in 2–3 and 2–4, the reverse should be the case.

For a uniform live load the method generally used in computing maximum shear, as in panel 3–4 for example, is to consider joint 4 fully loaded and joint 3 unloaded. This is evidently an impossible condition, for in order that 4 may be fully loaded the load must extend to 3, which would then give a half panel load to 3. The shears computed by this method are too great, but it will be the one generally adopted in the following analysis.

A general expression for the value of this shear may be easily derived.

Let m, Fig. 84, = the number of the panel in question (3–4) counting from the left end, = the number of the last loaded joint, calling A zero; and n = the total number of panels. The shear in the panel = the left abutment reaction = R_1. Taking moments about B, we have

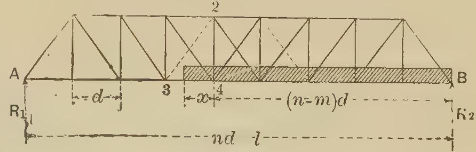


FIG. 84.

$$R_1 \times nd - pd[1 + 2 + 3 + \ldots + (n-m)]d = 0,$$

whence the maximum positive shear =

$$S_1 = R_1 = \frac{pd}{n}[1 + 2 + \ldots + (n-m)] = \frac{pd}{2n}(n-m)(n-m+1). \quad . \quad . \quad . \quad (17)$$

True Maximum Shear for Uniform Loads.

The *exact* position for maximum shear will now be derived, and the shears for this position will be found subsequently for a few cases and compared with those found by the approximate method.

Take the same figure and notation as in the above case. Let x = distance from 4 to the head of the load. As loads are added to the left of 4, the shear in the panel is increased so long as the increment to the abutment reaction at A is greater than the corresponding increment to the panel reaction at 3, since shear = abutment reaction minus load at 3. This is true until $\frac{x}{d} = \frac{(n-m)d + x}{l}$, or until $x = \frac{(n-m)d^2}{l-d} = \frac{n-m}{n-1}d$, at which point the increments of the reactions are equal. The moving load should therefore extend to the left of 4 a distance $x = \frac{n-m}{n-1}d$. The shear for this position in panel 3–4 is

$$S_2 = R_1 - \frac{px^2}{2d} = \frac{p[(n-m)d + x]^2}{2l} - \frac{px^2}{2d} = \frac{pd}{2(n-1)} \cdot (n-m)^2. \quad . \quad . \quad . \quad (17a)$$

This differs from (17) in having the factor $\frac{n-m}{n-1}$ in place of $\frac{n-m+1}{n}$, and is thus seen to give somewhat the smaller value. The actual difference is a maximum for $m = \frac{n+1}{2}$, or at the centre, and at that point has a value of $\frac{pd}{8} \cdot \frac{n-1}{n}$, or nearly $\frac{pd}{8}$, the same for all spans.

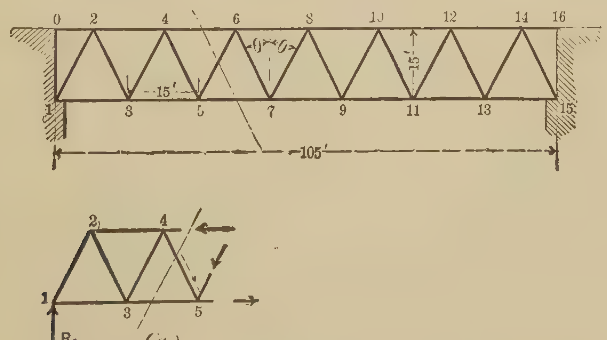


FIG. 85.

Notice that although the loading by this method is not so simple as by the approximate method, yet the expression for actual shear is quite as easy of application as formula (17).

70. The Warren Girder.—Fig. 85 shows a triangular truss or Warren girder as a deck

railroad bridge. As such it is usually a riveted structure constructed of angles and plates, and is used for comparatively short spans. The analysis of this truss is as follows:

Let the span = 105 ft.; the height, h, = 15 ft., and the panel length, d, = 15 ft. The number of panels, n, = 7. The dead load may be taken from formula (2), p. 43. We have then, dead load per foot per truss $= w = \dfrac{7l + 200 + 400}{2} = 667$ lbs. It will be considered as uniformly distributed along the upper chord joints; 2 and 14 will thus receive three fourths of a panel load each. The live load we will take at 1500 lbs. per foot per truss. The dead and live load stresses will be found separately.

Chord Stresses; Dead Load.—For the lower-chord members the centres of moments are at the upper panel points, and the stresses in these pieces result by dividing the bending moments at these points by the height of the truss. The moment at any point is, by eq. (16), $\dfrac{wd^2}{2}(mm')$. The stress in any lower chord member is then equal to $\dfrac{wd^2}{2h}(mm')$, where m and m' are the number of panels to the left and right of the centre of moments, calling 0–2 and 14–16 half panels.

We have, $\dfrac{wd^2}{2h} = \dfrac{667 \times 15^2}{2 \times 15} = 5002$, whence the following chord stresses:

$$\begin{aligned}
\text{Stress in } 1\text{–}3 &= 5002\left(\tfrac{1}{2} \times \tfrac{13}{2}\right) = 1250\,(1 \times 13) = 16250 \text{ lbs.}\\
\text{Stress in } 3\text{–}5 &= 1250(3 \times 11) = 41250 \text{ lbs.}\\
\text{Stress in } 5\text{–}7 &= 1250(5 \times 9) \;\;= 56250 \text{ lbs.}\\
\text{Stress in } 7\text{–}9 &= 1250(7 \times 7) \;\;= 61250 \text{ lbs.}
\end{aligned}$$

These stresses are all taken out with one setting of the slide-rule.

The stresses in 9–11, 11–13, and 13–15 are equal to those in 5–7, 3–5, and 1–3, respectively. These are all tensile stresses.

The centres of moments for the upper chord members are at the lower chord points. These moments are means between those at adjacent upper joints, and the lever-arms all being equal to h, the actual stresses are like means of the stresses in the lower chord pieces. We have then the following stresses:

$$\text{Stress in } 2\text{–}4 = \frac{16250}{2} + \frac{41250}{2} = 28750 \text{ lbs.}$$

$$\text{Stress in } 4\text{–}6 = \frac{41250}{2} + \frac{56250}{2} = 48750 \text{ lbs.}$$

$$\text{Stress in } 6\text{–}8 = \frac{56250}{2} + \frac{61250}{2} = 58750 \text{ lbs.}$$

etc. etc.

These are all compressive stresses.

Chord Stresses; Live Load.—The maximum live load chord stresses occur when the bridge is fully loaded. They are found in precisely the same way as the dead load stresses above; or they may be obtained by multiplying the above dead load stresses by the ratio of live to dead load, $= \frac{1500}{667}$ in this case. By the latter method a single setting of the slide-rule gives all the stresses. They are as follows:

1–3 = 36580.	2–4 = 64710.
3–5 = 92870.	4–6 = 109710.
5–7 = 126580.	6–8 = 132210.
7–9 = 137840.	

Web Stresses.—The vertical component of the stress in any web member is equal to the shear on the section which cuts that member and two horizontal chord pieces. The stress is therefore a maximum when the shear is a maximum. The dead panel load $= 667 \times 15 = 10000$ lbs. Left reaction, R_1, $= 10000 \times (5 + 2 \times \frac{3}{4}) \div 2 = 32500$ lbs. This is also the shear in panel 0–2, and being upwards on the left it is positive. The dead load shear in each of the other panels is found by subtracting from the abutment reaction the loads on the left of the panel. We have then the following shears:

$$
\begin{aligned}
0\text{–}2 &= +\,32500.\\
2\text{–}4 &= 32500 - \tfrac{3}{4} \times 10000 = +\,25000.\\
4\text{–}6 &= 25000 - 10000 = +\,15000.\\
6\text{–}8 &= 15000 - 10000 = +\,5000.
\end{aligned}
$$

The maximum positive live load shear in any panel occurs when all joints on the right are loaded. For maximum shear in 0–2 the bridge is fully loaded. The panel live load $= 1500 \times 15 = 22500$ lbs.

$$\text{Shear in } 0\text{–}2 = 22500 \times (5 + 2 \times \tfrac{3}{4}) \div 2 = 73120 \text{ lbs.}$$

For maximum shear in 2–4 all joints except 2 are loaded. Taking moments about right end, we have, as in formula (17):

$$
\begin{aligned}
\text{Shear in } 2\text{–}4 &= \frac{22500}{7}\left[\left(\frac{3}{4} \times \frac{1}{2}\right) + \frac{3}{2} + \frac{5}{2} + \frac{7}{2} + \frac{9}{2} + \frac{11}{2}\right]\\
&= 3214 \times \frac{143}{8} = 57450.
\end{aligned}
$$

Likewise:

$$
\begin{aligned}
\text{Shear in } 4\text{–}6 &= 3214[(\tfrac{3}{4} \times \tfrac{1}{2}) + \tfrac{3}{2} + \tfrac{5}{2} + \tfrac{7}{2} + \tfrac{9}{2}]\\
&= 57450 - 3214 \times \tfrac{11}{2} = 39770.
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
\text{Shear in } 6\text{–}8 &= 39770 - 3214 \times \tfrac{9}{2} = 25310.\\
\text{Shear in } 8\text{–}10 &= 25310 - 3214 \times \tfrac{7}{2} = 14060.\\
\text{Shear in } 10\text{–}12 &= 14060 - 3214 \times \tfrac{5}{2} = 6030.\\
\text{Shear in } 12\text{–}14 &= 6030 - 3214 \times \tfrac{3}{2} = 1210.
\end{aligned}
$$

The shear in 12–14 should be equal to $3214(\frac{3}{4} \times \frac{1}{2}) = 1205$, which it is very nearly, thus checking the work.

Adding the above shears to those due to dead load, we will have the greatest possible positive shears for all panels. They are as follows:

$$
\begin{aligned}
0\text{–}2 &= 32500 + 73120 = 105620. & 6\text{–}8 &= 5000 + 25310 = 30310.\\
2\text{–}4 &= 25000 + 57450 = 82450. & 8\text{–}10 &= -\,5000 + 14060 = 9060.\\
4\text{–}6 &= 15000 + 39770 = 54770. & 10\text{–}12 &= -\,15000 + 6030 = -\,8970.
\end{aligned}
$$

We see from this that positive shear cannot occur to the right of joint 10.

The above shears multiplied by sec θ give the maximum stresses in the web members due to positive shear. For members inclining downwards toward the left, as 1–2, 3–4, etc., the stresses are the same sign as the shear, that is, positive or compressive. Conversely for those inclining in the other direction. Thus in Fig. 85 (*a*), the shear in panel 4–6 being upwards on the left, the force exerted on 5–6 by the right-hand portion of the truss must be downwards, or 5–6 must be in compression. For the same reason 4–5 is in tension.

Sec $\theta = \sqrt{15^2 + 7.5^2} \div 15 = 1.118$. Multiplying the shears by this factor, we have the corresponding stresses:

$$
\begin{array}{ll}
1\text{–}2 = +118080. & 6\text{–}7 = -33890. \\
2\text{–}3 = -\ \ 92180. & 7\text{–}8 = +33890. \\
3\text{–}4 = +\ \ 92180. & 8\text{–}9 = -10130. \\
4\text{–}5 = -\ \ 61230. & 9\text{–}10 = +10130. \\
5\text{–}6 = +\ \ 61230. &
\end{array}
$$

The maximum negative shears and resulting stresses are equal and symmetrical to the positive shears and stresses. Just as positive shear cannot occur on the right of joint 10, so negative shear cannot occur on the left of 6, while between these points both kinds are possible. Members to the right of 10 and to the left of 6 are therefore subjected to but one kind of stress, while between 6 and 10 they may be subjected to either kind and hence must be counter-braced. The stresses in the latter members due to negative shear are:

$$
\begin{array}{ll}
6\text{–}7 = +10130. & 8\text{–}9 = +33890. \\
7\text{–}8 = -10130. & 9\text{–}10 = -33890.
\end{array}
$$

On the left of the centre the positive shear is the greater, and on the right the negative shear, these being of the same sign as dead load shear; the opposite kind in either case is called *counter-shear*, and the corresponding stresses *counter-stresses*.

71. The Howe Truss is shown in Fig. 86. The chords and diagonal web members are of wood, and the verticals of iron.

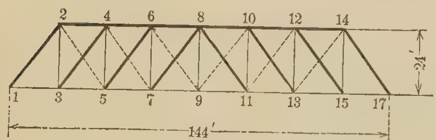


FIG. 86

This form of truss is largely used where timber is cheap, for both highway and railway bridges, and under that condition is very economical. Let us take a railway through-bridge of 144 ft. span with $n = 8$ and $h = 24$ ft.; $d = 18$ ft. From formula (4), p. 43, we find the dead load per ft. per truss, $= w$, to be $(6.5 \times 144 + 275 + 400) \div 2 = 806$ lbs. Assume the live load at 1500 lbs. per ft. per truss as before.

The dotted diagonals are not in action for uniform load, as explained subsequently. The upper chord stresses are found by dividing the moments about lower chord points by h; also we have: stress in 1–3 = stress in 2–4, stress in 3–5 = stress in 4–6, etc., because the moments at 3 and 2 are equal and at 4 and 5; or, from Σ hor. comp. = 0 for the portion to the left of a section cutting two chords and a vertical.

Making use of the four simple formulæ,

$$\text{dead load chord stress} = \frac{wd^2}{2h} mm'; \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. (16)]}$$

$$\text{live load chord stress} = \frac{pd^2}{2h} mm'$$

$$= \text{dead load chord stress} \times \frac{p}{w};$$

$$\text{dead load web stress} = \frac{wd}{2}(n + 1 - 2m) \sec \theta; \text{ and}$$

$$\text{live load web stress} = \frac{pd}{2n}(n - m)(n - m + 1) \sec \theta, \quad \ldots \quad \text{[Eq. (17)]}$$

the computations can conveniently be put into the following tabular form, which is adapted to either the Howe, Pratt, or Warren type:

$l = 144$ ft.; $d = 18$ ft.; $h = 24$ ft.; $n = 8$; sec θ for the diagonals $= 1.25$.

$w = 810$ lbs. per foot; $\frac{wd}{2} = 7290$ lbs.;

$p = 1500$ lbs. per foot;

$\frac{wd^2}{2h} = 5470$ lbs.;

$\frac{p}{w} = 1.85$;

$\frac{pd}{2n} = 1687$ lbs.

TABLE OF STRESSES IN ONE TRUSS.

Chord Stresses.					Web Stresses.						
Members.	Dead Load.		Live-load Stress = (3) × $\frac{p}{w}$.	Total = (3)+(4).	Panel.	Dead Load.		Live Load.		Total Shear = (8)+(10).	Stress in Diagonal = (11) × sec θ.
	mm'.	Stress = (2) × $\frac{wd^2}{2h}$.				$n+1-2m$	Shear = (7) × $\frac{wd}{2}$.	$(n-m) \times (n-m+1)$	Shear = (9) × $\frac{pd}{2n}$.		
1	2	3	4	5	6	7	8	9	10	11	12
1–3, 2–4	7	38300	70900	109200	1– 3	7	51000	56	94500	145500	181900
3–5, 4–6	12	65000	121500	187100	3– 5	5	36400	42	70900	107300	134100
5–7, 6–8	15	82000	151900	233900	5– 7	3	21900	30	50600	72500	90600
7–9	16	87500	162000	249500	7– 9	1	7290	20	33750	41040	51300
					9–11	− 1	− 7290	12	20250	12960	16200

Column (2) can be written out at once, and column (3) is found by multiplying the numbers in (2) by $\frac{wd^2}{2h}$, which can be done with a single setting of the slide-rule. Another setting gives column (4), and (5) is the sum of (3) and (4). Column (8) may be found as indicated, or by subtracting wd four times in succession from 51000, the left abutment reaction. Column (10) is obtained by multiplying (9) by the constant factor $\frac{pd}{2n}$ as given in eq. (17). The *positive* shears have here been found. Adding the dead load shears to these we have the maximum positive shears, column (11), in all the panels. On the right of the centre they are the counter-shears. Only one panel, 9–11, has such shear, no positive shear being possible to the right of 11.

Multiplying the total shears by sec θ, $= 1.25$, we have the stresses in the diagonals. These members are designed to resist compression only, hence the full diagonals on the left of the centre and the dotted one in panel 9–11 will be in action. For negative shears the full diagonals on the right and the dotted one in panel 7–9 are in action. The diagonals 6–9 and 9–10 are *counters*.

The stress in any vertical is equal to the shear on the section cutting it and two chord members, or is equal to the shear in the panel towards the abutment from the vertical.

The maximum stress in 8–9 is equal to the maximum positive shear in panel 7–9, or to a full panel load, whichever is the greater. For if this maximum positive shear is greater than a panel load, then the shear in 9–11 under the same loading is also positive, and 8–11 will not be in action, thus throwing all the shear in 7–9 upon 8–9; and again, the stress in 8–9 is always

equal to the load at 9 whenever the counters are *not* in action. The maximum positive shear in 7–9 = 7290 + 33750 = 41040 lbs., and a full panel load = 14580 + 27000 = 41580 lbs. The latter value is therefore the required stress.

The live load shears computed by the exact formula, $S = \frac{pd}{2(n-1)}(n-m)^2$, p. 51, are as follows, beginning at the left: 94500, 69430, 48210, 30860, 17360, 7715. The differences between these and those in the table are: 0, 1450, 2420, 2890, 2890, 2410. The error by the approximate method is on the safe side and is not objectionable, for, being greatest at the centre, it takes partial account of the effect of impact, which is very considerable on the counters. Being a constant error for all spans (in terms of panel load), it also has a greater relative influence in short spans, where again impact has a maximum effect. The effect of impact is discussed in Part II.

In the above bridge, if one-third the dead load be transferred to the upper panel points, the chord stresses and diagonal web stresses will not be changed. The stresses in the verticals will, however, each be reduced by the amount of the load transferred.

In practice all the dotted diagonals are employed to hold the cast-iron angle-blocks in place. These are not required in metallic structures.

72. **The Pratt Truss** is shown in Fig. 87 as a deck bridge. All parts are of iron or steel, the verticals being compression members and the diagonals tension members.

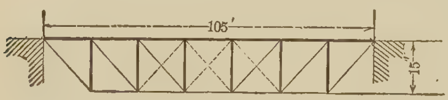


FIG. 87.

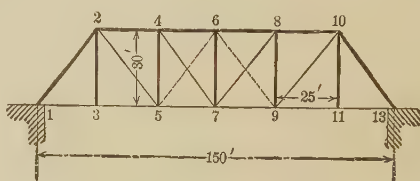


FIG. 88.

EXAMPLE 1. Highway bridge as shown in Fig. 87; class C, roadway 14 ft. wide; $l = 105$ ft.; $n = 7$; $h = 15$ ft. Find all the stresses.

Fig. 88 shows the usual form for through or pony Pratt trusses. The through or deck Pratt truss is the standard form of truss for both highway and railway bridges of moderate spans. However it is not generally used for railway bridges where the span is much less than 100 ft.

The stresses in the truss of Fig. 88 result readily by methods similar to those already illustrated. The pieces 2–3 and 10–11, called *hip verticals*, carry only the loads at their bases The end pieces 1–2 and 10–13 are compression members. The maximum stress in 6–7 is from the stress in the counter 6–9 or 5–6.

EXAMPLE 2. Find the stresses in all members of the truss in Fig. 88, a railway bridge of 150 ft. span; $h = 30$ ft.; $n = 6$; live load = 1700 lbs. per foot per truss.

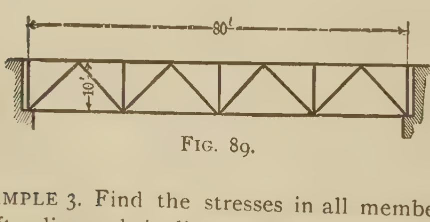


FIG. 89.

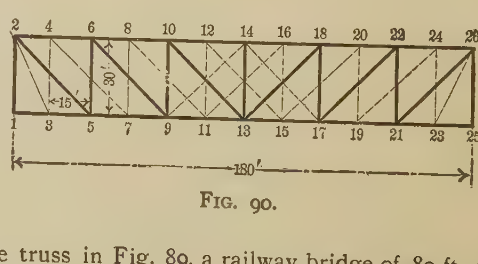


FIG. 90.

EXAMPLE 3. Find the stresses in all members of the truss in Fig. 89, a railway bridge of 80 ft. span; $h = 10'$ ft.; diagonals inclined 45°; live load = 2000 lbs. per foot per truss. Take dead load from formula (2), p. 43.

73. The Whipple Truss, shown in Fig. 90, consists of two simple Pratt trusses combined. It is in fact often called a "double intersection" Pratt truss. The advantage over the Pratt for long spans is in having short panels, and yet an economical inclination for the diagonals (about 45°). It is used extensively for highway bridges, but for railway bridges it has been as a rule discarded in favor of the Baltimore truss, or for the form shown in Fig. 114, p. 69.

The two systems of web members distinguished by full and dotted lines are commonly assumed to act independently.

This is not, however, strictly true, for the chords connecting the systems, while allowing independent vertical deflection, do not allow independent horizontal displacements, so that the loads on one system affect to some extent the form of, and hence the distribution of stress in, the other system. The exact analysis can only be made by the theory of redundant members given in Chap. XV., and even then it depends upon the adjustment of the counters. The assumption of independent systems is probably very nearly correct and enables the stresses to be statically determined. In any case the question affects the web stresses only, as the chord stresses are a maximum for a full load, and in that case the counters may, with practical exactness, be assumed as fully relieved.

Let us take a highway bridge with dimensions as in Fig. 90; class A, roadway 20 ft. wide.

From the diagram p. 44, dead load, $w_{,} = 1000$ lbs. per foot, $= 500$ lbs. per foot per truss. Live load for class A $= 80 \times 20 = 1600$ lbs. per foot, $= 800$ lbs. per foot per truss.

Chord Stresses.—For uniform loads the web members meeting the upper chord at 12, 14, and 16 are not in action, there being no shear in panel 11–15 of the dotted system. The stress in 10–18 is therefore equal to (mom. at 13) $\div h, = \dfrac{wd^2}{2h}(6 \times 6), = 67500$ lbs. Stress in 8–10 $=$ stress in 10–12 minus hor. comp. of stress in 10–13; that in 6–8 $=$ 8–10 minus hor. comp. in 8–11; etc.

The horizontal component in a diagonal $=$ vertical component multiplied by $\dfrac{2d}{h}$, or, in the case of the diagonals 2–3 and 23–26, by $\dfrac{d}{h}$. The vertical component $=$ shear in the panel of the system to which the diagonal belongs. We have then for the full system, since $2d = h$,

$$\begin{aligned} \text{hor. comp. } 10\text{–}13 &= \tfrac{1}{2}wd = 3750; \\ \text{"} \quad \text{"} \quad 6\text{–}\ 9 &= \tfrac{3}{2}wd = 11250; \\ \text{"} \quad \text{"} \quad 2\text{–}\ 5 &= \tfrac{5}{2}wd = 18750. \end{aligned}$$

For the dotted system,

$$\begin{aligned} \text{hor. comp. } 8\text{–}11 &= wd = 7500; \\ \text{"} \quad \text{"} \quad 4\text{–}\ 7 &= 2wd = 15000; \\ \text{"} \quad \text{"} \quad 2\text{–}\ 3 &= \tfrac{1}{2} \times 3wd = 11250. \end{aligned}$$

We have then by subtraction the chord stresses as follows:

$$\begin{aligned} 10\text{–}12 &= 67500. \\ 8\text{–}10 = 11\text{–}13 &= 67500 - 3750 = 63750; \\ 6\text{–}\ 8 = 9\text{–}11 &= 63750 - 7500 = 56250; \\ 4\text{–}\ 6 = 7\text{–}\ 9 &= 56250 - 11250 = 45000; \\ 2\text{–}\ 4 = 5\text{–}\ 7 &= 45000 - 15000 = 30000. \end{aligned}$$

The sum of the hor. comps. of 2–5 and 2–3 should be equal to the stress in 2–4, thus giving a check upon the work.

The live load chord stresses are obtained as before by proportion.

Web Stresses.—The dead load vertical components, or shears in the separate systems, are given above.

For live load shears, $\frac{pd}{6} = 2000$, and we have for the full system,

$$\begin{aligned}
&\text{shear in } 1\text{–}\ 5 = (pd \times 5) \div 2 = 30000; \\
&\text{“ \quad “ } 5\text{–}\ 9 = 2000\,(1 + 2 + 3 + 4) = 20000; \\
&\text{“ \quad “ } 9\text{–}13 = 2000\,(1 + 2 + 3) = 12000; \\
&\text{“ \quad “ } 13\text{–}17 = 2000\,(1 + 2) = 6000.
\end{aligned}$$

For dotted system, $\frac{pd}{12} = 1000$, and

$$\begin{aligned}
&\text{shear in } 1\text{–}\ 3 = (pd \times 6) \div 2 = 36000; \\
&\text{“ \quad “ } 3\text{–}\ 7 = 1000\,(1 + 3 + 5 + 7 + 9) = 25000; \\
&\text{“ \quad “ } 7\text{–}11 = 1000\,(1 + 3 + 5 + 7) = 16000; \\
&\text{“ \quad “ } 11\text{–}15 = 1000\,(1 + 3 + 5) = 9000; \\
&\text{“ \quad “ } 15\text{–}19 = 1000\,(1 + 3) = 4000.
\end{aligned}$$

Adding to the above the dead load shears, we have the stresses in the verticals, and the vertical components of the stresses in the diagonals. Vertical component of stress in the counter 14–17 = shear in 13–17 = 6000 − 3750 = 2250 lbs. This is also the maximum compression in 13–14. The vertical component of stress in 12–15 or 11–16 = 9000 lbs. = stress in 11–12 and 15–16. There is no positive shear in 15–19 and hence no counter is needed. The stresses in 1–2 and 25–26 are equal to the sum of the shears in 1–3 and 1–5.

For an odd number of panels the arrangement of diagonals of Fig. 92 is to be preferred to

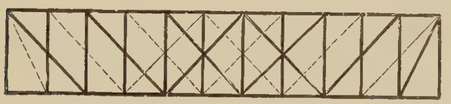


FIG. 91.

FIG. 92.

that in Fig. 91, as it gives two web systems, each of which is symmetrical about the centre. The assumption of independent systems then gives a more even and probably a more nearly correct distribution of stress over the two systems. In either case, granting this assumption, the stresses are found as in the above example.

Where the arrangement of the end posts is as in Fig. 93, there is a further ambiguity

FIG. 93.

from not knowing to which system the loads at *A* and *B* belong. Assuming them equally divided between the systems, is nearly correct and enables the stresses to be readily found. The uncertainty of computations of stresses by the usual methods, in double systems, constitutes a somewhat serious defect for such systems, and is one cause that has led to the adoption of the forms referred to at the beginning of this article.

74. The Triple Intersection Truss has been built to some extent. It is similar to the Whipple truss, but has three instead of two sets of web members. The stresses are found in the same way as in the Whipple truss.

75. The Double Triangular Truss shown in Fig. 94 has two systems of triangular bracing.

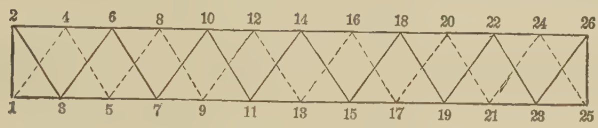


FIG. 94.

This truss is used both as a short-span riveted structure and as a long-span pin-connected bridge; the Memphis bridge and the Kentucky and Indiana bridge are of this form, though modified as shown in Fig. 101, p. 61.

For chord stresses under a full uniform load the pieces 11–14 and 14–15 are not in action. The moment at 13 divided by h gives the stress in 12–16. The upper chord stresses are then found by subtracting successively the sums of the horizontal components of the stresses in the two web members which meet at each panel point. These horizontal components are found as in Art. 73, assuming each web system as independent. For the lower chord stresses, that in 11–13 is equal to 12–16 minus hor. comp. of 12–13; 9–11 = 11–13 minus hor. comp. of 11–14, minus hor. comp. 10–11; etc.

The web stresses are readily found as in a simple triangular truss, assuming each system as independent.

76. The Lattice Truss, Fig. 95, contains four web systems. It is built only as a short-span

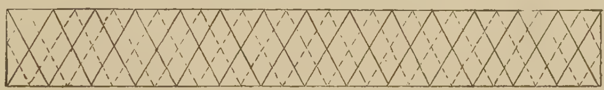


FIG. 95.

riveted structure. The web members being riveted together at each intersection, the different systems cannot act independently, and in finding stresses it is usual to treat the structure as a beam. The maximum moment and shear is found at several different sections; the stresses in the chords or flanges are found by assuming them to take all the moment, and those in the web members by assuming the shear as equally divided among the members cut.

EXAMPLE.—Find the chord and web stresses at sections 15 ft. apart in a lattice-truss of 90 ft. span; live load = 1800 lbs. per foot; dead load from formula (2), p. 43; $h = 12$ ft.; panel length between chord points of the same system = 15 ft.

77. The Post Truss shown in Fig. 96 is a special form of the double triangular truss.

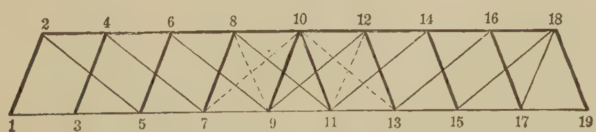


FIG. 96.

The lower chord is divided into an odd number of panels, and the upper chord contains one less panel than the lower. The posts are inclined by one-half of a panel length, and the ties by one and one-half panel lengths. The chord stresscs are readily found as before by

considering the web members meeting at 8, 10, and 12 as not acting under a full load if it is a through-bridge, or the members 8–11 and 9–12 if a deck-bridge. For web stresses, it is impossible to separate the systems, as they are connected at the centre. The best that can be done is to assume the systems to be 1, 2, 5, 6, 9, 12, 13, 16, 17, 18, 19, and 1, 2, 3, 4, 7, 8, 11, 14, 15, 18, 19. For the counters 9–10 and 10–13 the first system becomes 9, 10, 13, instead of 9, 12, 13; and for the counters 11–12 and 12–15, if required, the second system becomes 11, 12, 15, in the place of 11, 14, 15.

78. The Baltimore Truss and the Subdivided Triangular Truss.—The Baltimore truss, Fig. 97, or a modified form of it, Fig. 114, p. 69, is used very generally for long spans The stresses are all easily determined; the panel lengths are short, while the diagonals have an economical inclination.

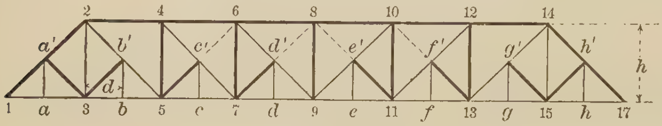
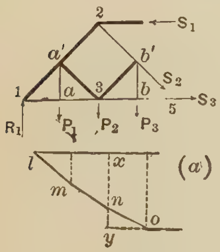


FIG. 97

Chord Stresses. — The upper chord stresses are found as usual by taking centres of moments at lower-chord points, the dotted diagonals not being in action for full load.



The stresses in the lower chord members are found by taking moments about upper chord points. Thus for piece $b5$ we pass a section cutting 2–4, $b'5$, and $b5$, separate the portion to the left, Fig. (*a*), and put Σ mom. about $2 = 0$. We have, therefore, $R_1 \times 2d - P_1 \times d + P_3 \times d - S_3 \times h = 0$, whence S_3 is determined. It is to be noted that the moment of the external forces acting on the portion considered, about the point 2, is not the ordinary "bending moment" in the truss at 2, since here we have included the force P_3 which is on the *right* of the centre of moments. In the equilibrium polygon this moment is represented by the ordinate xy.

The stress in $3b$ is evidently equal to $b5$. The other chord stresses are found similarly to the above.

Web Stresses.—The stress in each sub-vertical, aa', bb', etc., is equal to the load at its base $= (w + p)d$. The vertical component of the compressive stress in each of the pieces, $a'3$, $b'3$, $c'5$, etc., is found, by a diagram of joints a', b', etc., to be equal to one half the stress in the sub-vertical, $= \frac{1}{2}(w + p)d$. Thus for joint b', Fig. 98, draw 1–2 vertical and equal to the stress in bb', then 2–3 parallel to $3b'$, and 3–4 and 4–1 parallel to 2–5, closing the polygon, the point 4, however, being unknown. The vertical component of $2\text{–}3 = \frac{1}{2}(1\text{–}2) = \frac{1}{2}(w + p)d$.

FIG. 98

The vertical components in $1a'$, $b'5$, $c'7$, $d'9$, $8e'$, and $10f'$ are equal respectively to the maximum positive shears in the panels to which these pieces belong, as they are the only inclined members in the panels, which carry positive shear. The stresses in the verticals 4–5, 6–7, and 8–9 are equal to the vertical components in $4c'$, $6d'$, and $8e'$, respectively, plus whatever load may be applied at the upper panel point. The tension in $2\text{–}3 =$ vert. comp. in $a'3$ + vert. comp. in $b'3$ + load at 3, $= 2(w + p)d$.

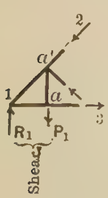


FIG. 99.

The vertical component of the stress in $2b' =$ shear in $3b$ — vert. comp. in $3b'$. To determine for what position of the loads this stress is a maximum we note that the addition of a live panel load at b increases the shear in $3b$ by $\frac{13}{16}pd$, and increases the vertical component in $3b'$ by $\frac{1}{2}pd$. The vertical component in $2b'$ is then increased by $(\frac{13}{16} - \frac{1}{2})pd = \frac{5}{16}pd$, thus showing that the loads should extend up to b from the right for a maximum in $2b'$. With such loading, then, the shear minus $\frac{1}{2}(w + p)d$ is the vertical component of the required stress. Similarly for $4c'$ and $6d'$. The vertical component in

$a'2$ = shear in $a3$ + vert. comp. $a'3$, as is seen from Fig. 99 by putting Σ vert. comp. = 0. Adding pd to a decreases the positive shear in $a3$ by $\frac{1}{16}pd$, but increases the vertical component in $a'3$ by $\frac{8}{16}pd$, thus increasing the vertical component in $a'2$ by $\frac{7}{16}pd$. Hence for a maximum stress in $a'2$ the bridge should be fully loaded.

When the pieces $e'11$ and $f'13$, and similar members on the left, act with the counters, they are in tension. The vertical component of the tension in $e'11$ = positive shear in $e11$ + vert. comp. in $e'10$, the piece $9e'$ not being in action and the vert. comp in $e'10$ being equal to one half the load at e. A process of reasoning similar to that above shows that for a maximum in $e'11$ the load should extend to 11. The stress in $f'13$ is found in like manner. If the shear in $f13$ plus $\frac{1}{2}wd$ (= vert. comp. in $f'12$) should be negative, then there would be no tensile stress in $f'13$.

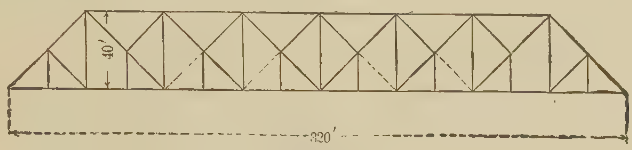


FIG. 100.

EXAMPLE.—Find the stresses in all members of the truss in Fig. 100. Span = 320 ft.; h = 40 ft.; live load = 1500 lbs. per foot; dead load by formula (3), p. 43. Consider one third the dead load as applied at the upper chord.

Fig. 101 shows a method of subdividing the panels in the double triangular truss. The computation of stresses is but slightly altered.

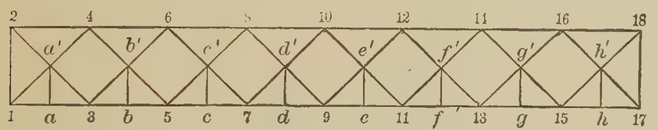


FIG. 101.

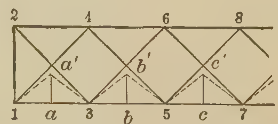


FIG. 102.

The intermediate panel loads at a, b, c, etc., may be considered as transferred to the main panel points 1, 3, 5, etc., by means of separate small trusses or trussed stringers, $1a'3$, $3b'5$, etc., as shown in Fig. 102. Whatever stresses there may be in the inclined and horizontal members of these small trusses must of course be added to the stresses for the same loading in those members of the main truss with which they in reality coincide.

The upper-chord stresses are then found as in Art. 75, considering all loads as applied at the main panel points. The stresses in the lower chord as part of the main truss are found in the same way; to these must be added the stresses in the chord as belonging to the trussed stringers.

The dead-load web stresses are found in a similar way to the chord stresses; that is, by treating the members that are common to the two trusses as belonging first to one and then the other, and adding the results.

For live-load web stresses the member $6c'$, for example, receives its maximum tension when the main joints 7, 11, and 15 are fully loaded, which requires loads at c, d, e, f, g, and h. The load at c affects the stress in $6c'$ only by adding to the load at 7, since $6c'$ belongs only to the main truss. For the maximum tension in $c'7$, however, c should not be loaded, since the addition of this load causes a compression in $c'7$, as part of the small truss, which is greater than the additional tension produced in this piece as part of the main truss. Similarly, for the maximum compression in $8c'$, joints d to h are loaded, while for $5c'$ joint c is also loaded.

EXAMPLE.—Find the stresses in a *deck*-bridge similar to the above but with the sub-verticals extending upwards from the centre. Span = 360 ft.; h = 45 ft.; loading as in previous example.

BRIDGE TRUSSES WITH INCLINED CHORDS.

79. Chord Stresses.—Since the chord stresses are all a maximum for full live load they are most readily found graphically, only one diagram being necessary.

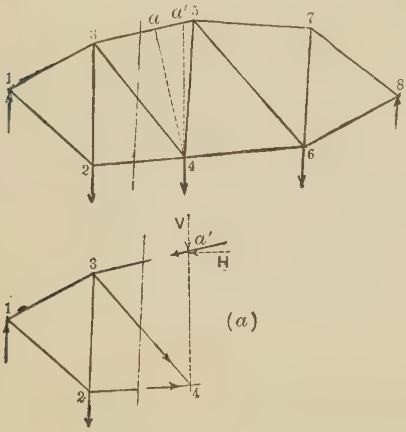


FIG. 103.

The analytical method has been sufficiently indicated in Art. 68.

Instead of the actual chord stresses, it is often desired to get only their horizontal components.

In Fig. 103 the stress in 3–5 = mom. at 4 ÷ $4a$. Draw the vertical, $4a'$. Then in Fig. (a) substitute the vertical and horizontal components for the stress in 3–5, applying them at a'. Then, since the moment of V about 4 is zero, we have

$$H = \text{mom. at } 4 \div 4a'; \quad . \quad . \quad . \quad . \quad (18)$$

and in general, the horizontal component of the stress in any chord member is equal to the bending moment at the opposite joint, divided by the vertical ordinate from the joint to the chord.

80. Web Stresses.—With inclined chords the shear in any panel is not taken by the web member alone, since the chord stress has a vertical component.

For the maximum stress in any web member each joint up to the section cutting the member should be fully loaded. If loaded on the longer segment of the bridge, this will give the main stress in the member or the stress in the main diagonal; and if on the shorter segment, it will give the counter-stress or stress in the counter. This is true, however, only when the chord members which are cut meet beyond the abutment, the usual condition. Thus for a maximum tension in 2–5, Fig. 104, joints 5, 7, and 9 should be loaded; for adding a load to the right of pq increases R_1, and hence the negative moment about I and stress in 2–5, while adding a load to the left of pq, increases R_1 less than the load, hence increases the negative moment about I less than the positive moment and therefore decreases the stress in 2–5. Similarly for any other web member.

Analytically the stresses are best found in the verticals, as 4–5, Fig. 104, by putting Σ mom. about $I = 0$, the point I being the intersection of the two chord members cut by the section pq through the vertical. Those in the inclined members, as 2–5, may be found in a similar way, t being the lever-arm of 2–5; or since t is awkward to compute, we may find the stress in 2–5 by first finding the horizontal components in 2–4 and 3–5. Then

hor. comp. 2–5 = hor. comp. 2–4 — hor. comp. 3–5.

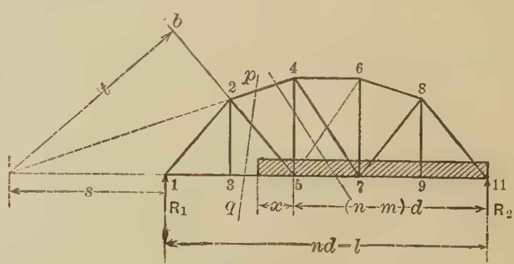


FIG. 104.

If hor. comp. 2–4 > hor. comp. 3–5, then 2–5 must be in tension, and *vice versa.* The horizontal components of the chords are very readily computed by eq. (18), especially where there are verticals.

A formula for the horizontal component of the stress in a web member could easily be written out, but it would be too complicated for ready use. Each case can easily be worked out for itself.

Graphically, the dead load web stresses will be found by diagram at the same time as the chord stresses. The counters are at first to be considered as not acting, and the stresses in

the main diagonals found. Then, in order to get the combined live and dead load stresses in the counters, we must find the dead load stresses by assuming the main diagonals as not acting. The resulting dead load stresses in the counters will be of opposite sign to the live load stresses and will subtract from them.

For live load web stresses a separate diagram must be drawn for each position of the load, a different position being required for each pair of diagonals meeting at the unloaded chord. This diagram need be drawn only up to the pieces whose stresses are desired. The abbreviated diagram of Chap. II, p. 29, will be found to apply well here. The reactions for the several positions of the loads should be first computed, then laid off on the same vertical and all the diagrams drawn. Or the live-load web stresses may be found by a single diagram as follows: Assume a left abutment reaction of some convenient amount, as 100000 lbs.. and, with *no loads* on the truss, begin at the left and draw a stress diagram for a little more than one half the truss, or as far as counters are required. The main diagonals are to be considered as acting on the left of the centre, and the counters on the right. Scale off and tabulate the stresses thus found. Compute the actual reactions for the various positions of the loads required. Then to get the maximum stress in any diagonal, multiply the stress found from the diagram by the true reaction corresponding to the position of loads for a maximum stress in this piece, and divide by 100000. With a slide rule this method is very rapid and easy. It is based on the fact that the diagram as drawn above and those drawn for each position of the loads are similar figures.

Position of Load for Maximum Shear.

The assumption of full joint loads on one side of the panel only is often made as in parallel-chord trusses. The *exact* position of the end of a uniform moving load for maximum web stress may be found as follows:

Let x, Fig. 104, = the distance from the panel point on the right of the section to the head of the load; m = the number of panels to the left of this panel point, and n = the whole number of panels. Let s = distance $I\ 1$. Then the stress in 2–5 will be increased by adding loads to the left of 5 until we reach a distance x from 5, at which point the addition of a load will produce no additional stress. The stress in 2–5 due to a load dP a distance x from 5 is

$$dS = \frac{\dfrac{dP[x + (n - m)d]}{l} \times s - dP\dfrac{x}{d}[s + (m - 1)d]}{t}.$$

Putting this equal to zero, we have

$$\frac{x + (n - m)d}{l}s - \frac{x}{d}[s + (m - 1)d] = 0,$$

whence

$$x = \frac{n - m}{n - 1 + n(m - 1)\dfrac{d}{s}}d. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Comparing this with the value of x given on p. 51 for parallel chords, we notice that in this last expression we have the additional term $n(m - 1)\dfrac{d}{s}$ in the denominator. This term becomes zero when $s = \infty$, or for parallel chords, as should be the case.

The corresponding stress in 2–5 is found by taking moments about I. If R_1 is the left abutment reaction, P' the panel load at 3, and p the load per foot, we have stress in 2–5 =

$$S = \frac{R_1 \times s - P' \times [s + (m-1)d]}{t}.$$

Now

$$R_1 = \frac{p[(n-m)d + x]^2}{2l} \quad \text{and} \quad P' = \frac{px^2}{2d}.$$

Substituting the above value of x and reducing, we have

$$S = \frac{pd(n-m)^2}{2n}\left(1 + \frac{1}{n - 1 + n(m-1)\frac{d}{s}}\right)\frac{s}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

81. The Parabolic Bowstring Truss.—In this truss, Fig. 105, the lower chord is horizontal and the upper chord joints lie in the arc of a parabola. The bracing may be as in Fig. 105, or as shown in Fig. 107. Formerly this truss was quite extensively used, but poor details and the difficulty of making it rigid against wind-pressure have caused it to be generally abandoned. However, as the analysis has several interesting features it will be given.

Chord Stresses (Fig. 105).—The horizontal component in any chord member is equal to the moment at the opposite joint divided by the length of the vertical at that joint. For full

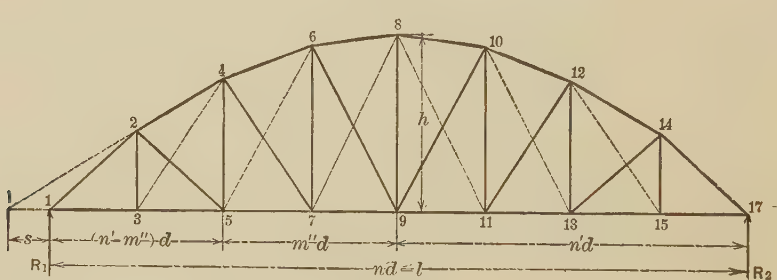


FIG. 105.

load the moments vary as the ordinates to a parabola, and likewise these verticals or lever-arms; hence their quotient is the same for any chord member, and may be written as the moment at the centre divided by the centre height of the truss. If n = number of panels, p = load per foot, h = height at centre, and d = panel length, we have

$$\text{hor. comp. chord stress} = \frac{pd^2n^2}{8h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

This is of course the actual stress throughout the lower chord.

Web Stresses.—For full load, the horizontal components in the chords being equal, there is no stress in any diagonal. The corresponding stress in the verticals is $(w + p)d$.

For moving load, assuming full joint loads up to the panel in question, the horizontal component of the maximum stress in any diagonal is constant and equal to $\frac{pd^2n}{8h}$, as will now be proved.

Let m'', Fig. 105, be the number of panels from the centre to the joint on the right of the diagonal whose stress is required. For convenience, let $n' = \frac{n}{2}$, and $P = pd$ = live panel load.

Abutment reaction $= R_1 = P\dfrac{[1+2+3+\ldots(n'+m'')]}{2n'} = P\dfrac{(n'+m''+1)(n'+m'')}{4n'}$. (a)

Hor. comp. 2–4 $= R_1 \times (n'-m'')d \div$ 4–5

$$= R_1 \frac{(n'-m'')d}{h\left(1-\dfrac{m''^2}{n'^2}\right)} = R_1 \frac{n'^2 d}{h(n'+m'')}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Hor. comp. 3–5 $= R_1 \times (n'-m''-1)d \div$ 2–3

$$= R_1 \frac{(n'-m''-1)d}{h\left(1-\dfrac{(m''+1)^2}{n'^2}\right)} = R_1 \frac{n'^2 d}{h(n'+m''+1)}. \quad . \quad . \quad . \quad . \quad . \quad (c)$$

Subtracting (*c*) from (*b*) and substituting the value of R_1 from (*a*), we have

$$\text{hor. comp. 2–5} = P\frac{n'd}{4h} = \frac{pd^2n}{8h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Q. E. D.

The stress in any vertical, as 4–5, is found by taking moments about *I*. The abutment reaction is equal to

$$R_1 = P\frac{(n'+m'')(n'+m''+1)}{4n'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

By proportion we find

$$\text{The distance } s = \frac{(n'-m'')(2\text{–}3) - (n'-m''-1)(4\text{–}5)}{(4\text{–}5)-(2\text{–}3)}d. \quad . \quad . \quad . \quad (e)$$

$$\text{The length of 2–3} = h\left(1-\frac{(m''+1)^2}{n'^2}\right) = h\frac{n'^2-(m''+1)^2}{n'^2}. \quad . \quad . \quad . \quad . \quad (f)$$

$$\text{The length of 4–5} = h\left(1-\frac{m''^2}{n'^2}\right) = h\frac{n'^2-m''^2}{n'^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$

$$\text{Stress in 4–5} = R_1\frac{s}{s+(n'-m'')d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h)$$

By substituting from (*d*), (*e*), (*f*), and (*g*) in (*h*), we have, after reduction,

$$\text{Stress in 4–5} = P\frac{(n'-1)^2-m''^2}{4n'} = P\left(\frac{n'^2-m''^2}{4n'} - \frac{2n'-1}{4n'}\right). \quad . \quad . \quad (23)$$

Now the length of 4–5 $= h\dfrac{n'^2-m''^2}{n'^2}$, whence

$$\text{Stress in 4–5} = \text{length of 4–5} \times \frac{pdn}{8h}, \text{ minus the constant, } pd\frac{n-1}{2n}. \quad . \quad . \quad (24)$$

To find the maximum stresses in all the members of a parabolic bowstring truss we have only to draw the truss to a scale such that the length of span $= \dfrac{pd^2n^2}{8h}$, or equal to the horizontal component of the chord stress. The length of each upper chord member multiplied by n will be the stress in that member. The length of each diagonal will be the

stress in that diagonal, for by construction the horizontal component is equal to $\frac{pd^2n}{8h}$. And finally, the length of each vertical minus the constant, $pd(n-1) \div 2n$, is the stress in that vertical.

Counters are evidently required in each panel.

Another example in which the stresses may be measured directly from the structure itself when drawn to a proper scale is the roof-truss in Fig. 106 when under uniform load. The diagonals may easily be proved to have a constant horizontal component equal to $\frac{pd^2n}{4h}$, which equals the hor. comp. of the stress in 6–8–10, divided by $\frac{n}{2}$.

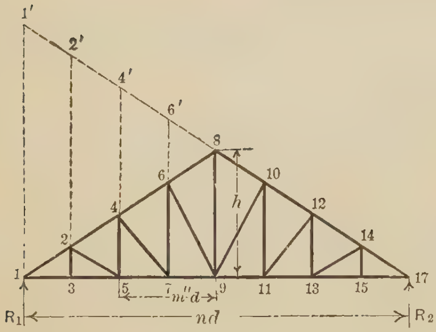


FIG. 106.

If the span 1–17 is then made equal to $\frac{pd^2n^2}{4h}$, the truss drawn to scale, and the construction made as in the figure, the following is true:

Each diagonal is equal to its stress.

Each vertical is equal to the stress in the next vertical toward the centre of the truss, assuming all loads to be applied at the upper panel points.

The hor. comp. of the stress in 6–8–10 = length of 9–17, and the stress itself = 8–17; the stress in 4–6 = 6′–17, that in 2–4 = 4′–17, and in 1–2 = 2′–17.

The stress in 7–9 = 7–17, that in 5–7 = 5–17, that in 3–5 = that in 1–3 = 3–17.

The proof of the above is left to the student.

In the parabolic bowstring with triangular bracing, Fig. 107, the ordinates from the lower

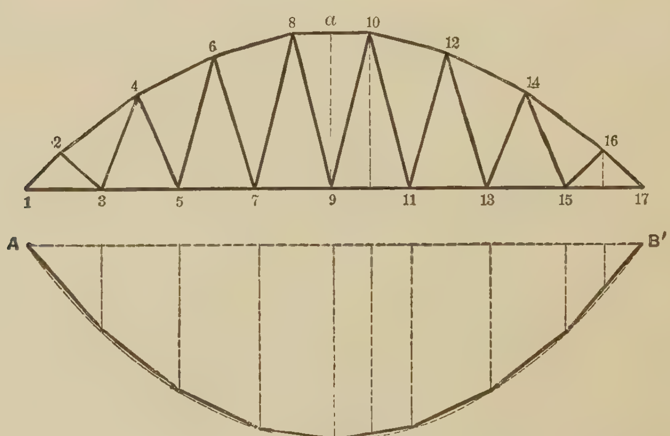


FIG. 107.

joints to the upper chord members are less than the ordinates to the parabola, since each chord member is straight between joints. The horizontal components in the upper chord are therefore not quite constant. For the lower chord, with centres of moments at upper chord points, the moments are proportional to ordinates from the closing line, $A'B'$, to the segments of the equilibrium polygon and not to the parabola; hence the lower chord stress is not quite constant. The actual horizontal components are readily computed by the method already explained.

The web stresses for moving load are computed by taking the difference between the horizontal components of the stresses in the chord members.

EXAMPLE.—Find the stresses in the truss of Fig. 107. Span = 80 ft.; ordinate $9a$ to the parabola (not to the chord member) = 16 ft. A highway bridge, class C. The web members form isosceles triangles with bases along the lower chord.

82. The Double Bowstring or Lenticular Truss, Fig. 108, has both chords in the

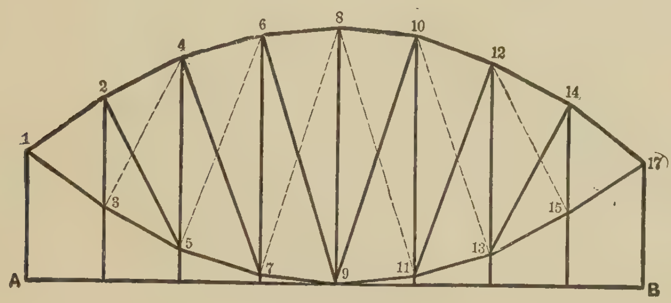


FIG. 108.

form of a parabola. The floor may be supported along the centre line 1–17, or may be hung below along the line AB. In the latter case the horizontal wind-truss in the plane AB prevents the swaying of the main truss longitudinally. With verticals and diagonals as in the figure, the horizontal component of the chord stress is constant, for the sums of the ordinates to two parabolas give the ordinates to a third parabola. The stresses in the members bear the same relation to their lengths as in the single parabolic truss.

83. The Pegram Truss, Fig. 109, has several claims to economy and general excellence of design. Each chord consists of panels of equal length, the upper chord panels being shorter than the lower. The upper chord points lie in the arc of a circle, the chord of which, 2–16, is made about one and one-third to one and one-half panel lengths shorter than the span. The versed sine may be so taken that with an economical centre height the lengths of the posts will be nearly equal, or it may be so taken that they will decrease in length toward the ends where the shear is great. In a deck-bridge the upper chord is made straight and the lower chord curved. Having assumed the chord and versed sine of the circular arc, the coördinates of the joints are readily computed, each chord section subtending the same angle at the centre of the circle.

Let us take as an example a 200-ft. through-span, Fig. 109, with seven panels, each equal to 28.57 ft. The coördinates of the upper panel points are given in the figure. These points lie in a circular arc with a chord of 160 ft. and versed sine of 15 ft. Each top chord member between pins is 23.55 ft. long except the centre one, which is $\frac{1}{20}$ less or 22.37 ft. long. This is made shorter to enable the chord sections between splices to be of uniform length, the splices being towards the end of the truss from the pin-points.

The dead load by formula (3), p. 43, will be equal to $\dfrac{5l + 350 + 400}{2} = 875$ lbs. per foot per truss. The live load we will take at 1800 lbs. per foot per truss. The dead panel load $= 875 \times 28.57 = 25000$ lbs. Live panel load $= 1800 \times 28.57 = 51430$ lbs. The stresses will be found by diagram.

Dead Load Stresses.—The abutment reaction, R_1, $= 3 \times 25000 = 75000$ lbs. Laying off BA, Fig. 111, equal to this, and AP, PQ, and QR each equal to 25000 lbs., we draw the diagram for one half the truss as in Chap. II. The dotted diagonals are considered as not in

action for uniform load, but in order to get the stress in the counter 5–8 or 10–11 due to dead and live load we must here draw the diagram first with 6–7 in action and then with 5–8 in action. The resulting compressive stress in 5–8, $G'H'$ in the diagram, is afterwards combined with the maximum tension due to live load. The resulting stresses as scaled off from the diagram are written along each member in Fig. 110, and are marked "*D*." For a check the stress in 8–10 is, by moments, equal to $\frac{875 \times (28.57)^2 \times 12}{2 \times 38.72} = 110700$ lbs.

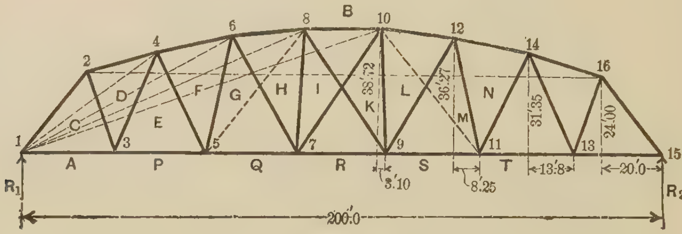


FIG. 109.

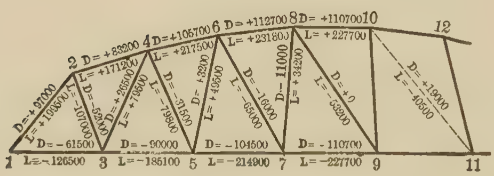


FIG. 110.

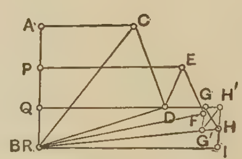


FIG. 111.

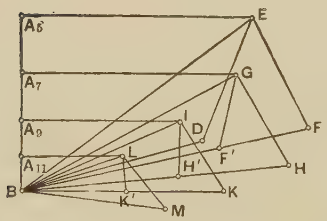


FIG. 112.

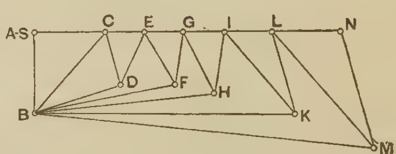


FIG. 113.

Live Load Stresses.—The stresses in the chords and in 1–2 and 2–3 are a maximum for full load, and may therefore be obtained by multiplying the corresponding dead load stresses by $\frac{1800}{875}$.

For a maximum in 3–4 and 4–5, all joints up to 5 should be loaded. The reaction R_1 is then equal to $\frac{pd}{7}(1+2+3+4+5) = 110200$ lbs. Laying this off as BA_6, Fig. 112, we proceed to draw the diagram as far as piece 4–5 by the method explained in Art. 47, p. 29. Substituting the triangle 1–4–5 for the original framework we find the stress in 4–5, or EF, by drawing A_6E parallel to 1–5 and BE parallel to 4–1; then EF parallel to 4–5 and BF parallel to 4–6; whence EF is the required stress in 4–5. To find the stress in 3–4, draw the diagram for joint 4 of the original truss. This diagram is $BFEDB$, the portion BFE being already drawn; ED is the stress in 3–4.

For a maximum in 5–6 and 6–7 all joints but 3 and 5 should be loaded. The reaction $R_1 = \frac{pd}{7}(1+2+3+4) = 73470$ lbs. Laying off BA_7 equal to this, we proceed as for 3–4 and

4–5. Substituting the triangle 1–6–7 for the portion of the truss to the left of 7, the diagram BA_7G and thence GBH determines the stress in 6–7, or GH. The diagram for joint 6 is all drawn except the line GF'. This drawn gives the stress in the post 5–6. The stresses in the other web members are found in like manner. The last loaded panel in each case is indicated by the subscript to the letter A in the diagram. The stresses are given in Fig. 110, marked "L."

The live load web stresses may be otherwise found by diagram as explained in Art. 80, p. 62. That is, by assuming a reaction of 100000 lbs., drawing the corresponding diagram, and finding the actual stresses from this diagram by proportion. Fig. 113 is such a diagram, with BA = 100000 lbs. by scale. The few computations may be tabulated thus:

LIVE LOAD WEB STRESSES.

Member.	Stress from Diagram. R_1 = 100000 lbs.	Actual Reaction.	Actual Stress.
1	2	3	4
3–4	72,100	110,200	79,500
4–5	72,500	110,200	79,900
5–6	67,000	73,500	49,200
6–7	88,700	73,500	65,200
7–8	77,500	44,100	34,200
8–9	121,000	44,100	53,200
10–11	185,000	22,000	40,700

Column (3) contains the actual reactions when the truss is loaded so as to produce the maximum stresses in the corresponding members of column (1). Column (4) is obtained by multiplying the quantities in column (2) by those in (3) and dividing by 100000. The resulting stresses should be the same as those found from Fig. 112.

If analytical methods are preferred, the same general methods are to be used as given in Art. 79, i.e., the chord stresses found by moments and the web stresses by subtracting horizontal components of chord stresses. In panel 5–7 the compression in 5–8 is to be found for dead load by assuming 6–7 as not acting, for the same reason as given in the above analysis.*

84. **The Petit Truss** shown in Fig. 114 is the standard form for very long spans. It is

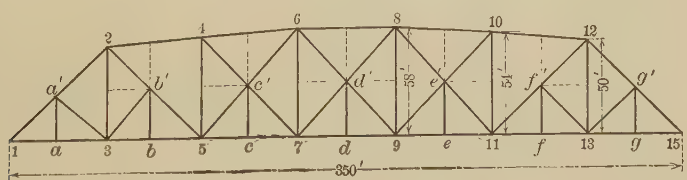


FIG. 114.

very similar to the Baltimore truss, the only difference being in the inclined upper chord, which is a more economical arrangement for long spans. The pieces shown by dotted lines serve merely to support the chords and posts at intermediate points, and form no part of the

* For a full description of the Pegram truss, see *Engineering News*, Dec. 10 and 17, 1887. For illustrations of details of three such trusses, including the one above analyzed, see *Engineering News*, Feb. 14, 1891.

truss proper. The vertical ones may be designed to carry the weight of the upper chord, the horizontal ones have no definite load and are made of uniform size, sufficiently strong to resist in either direction the buckling of the posts. Omitting these members, the analysis offers no special difficulties, as the variation from the Baltimore truss due to inclined chords is easily taken into account. If a diagram is used, the fact that the vertical components of the stresses in $a'3$, $b'3$, $c'5$, etc., are each equal to one half of a panel load, enables the diagrams for joints 3, 5, 7, etc., as these points are reached, to be readily constructed.

EXAMPLE.—Find the stresses in the truss of Fig. 114, a double-track railroad-bridge with assumed live load of 3000 lbs. per foot per truss.

85. Double Systems.—In double-intersection trusses, with curved upper chords, each system is affected by loads on the other owing to the rising tendency of each angle of the upper chord whenever there is any stress in the chord.

Fig. 115 shows a double-intersection Pegram truss, which will serve as a general example of the forms under discussion. The span is 336 ft.; length of panel 24 ft.; height at centre 45 ft., and at ends 32 ft. The pieces 2–3 and 27–30 are vertical, thus making the distance 2–30 (the chord of the circular arc) equal to 288 ft.; the versed sine = 45 − 32 = 13 ft. All upper chord panels are equal. For dead load or full live load, the diagonals meeting the upper chord at 14, 16, and 18 are assumed as not acting. The stresses for such loading are

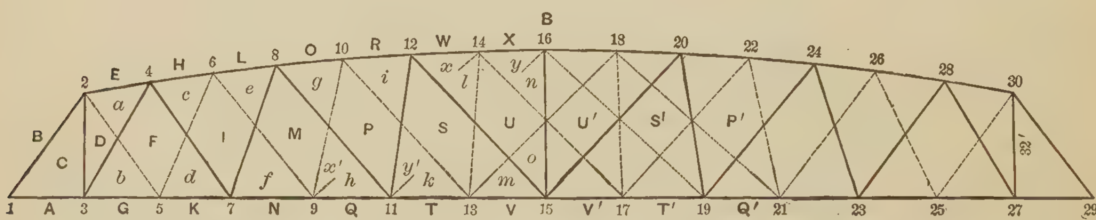


FIG. 115.

then readily found, either analytically or graphically, by commencing at the centre, finding 12–14–16–18–20 by moments, and then passing towards the end. The graphical method is

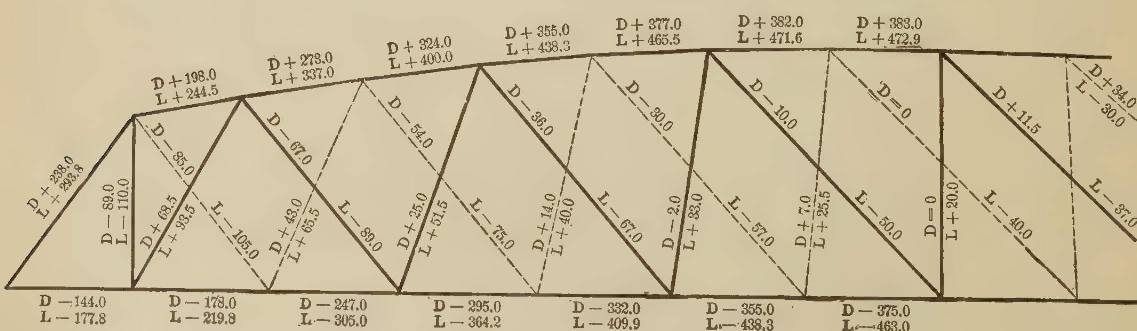
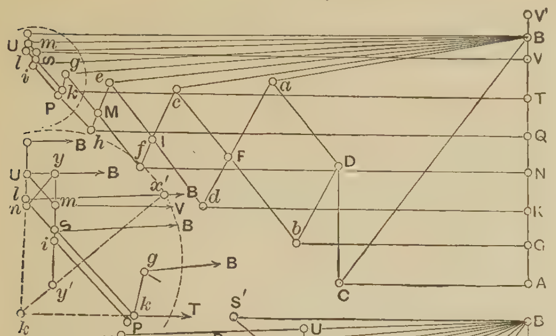


FIG. 116.

much the better here. The diagram for dead load, taken at $\frac{5 \times 336 + 350 + 400}{2} = 1215$ lbs. per foot, is given in Fig. 117. After having found 12–20 by moments the diagrams for joints 16 and 14 were drawn, thus getting the tensions in 15–16 and 13–14. Then assuming

12–15 and 15–20 equally stressed the diagram for 15 was drawn; then for joints 13, 12, 11, 10 9, etc. The diagram was also drawn for the counters, 11–16 and 9–14, acting, as was done in the previous examples. This part of the diagram is shown to a larger scale in the lower left-hand corner of Fig. 117. The resulting stresses, marked "*D*," are written along the corresponding members in Fig. 116 The live load chord stresses were found by proportion, making a single setting of the slide-rule, assuming a live load of 1500 lbs. **per foot per truss.**

FIG. 117.



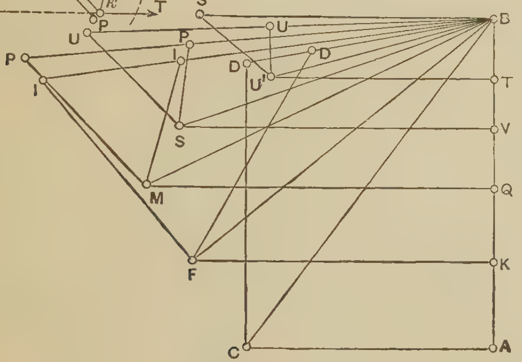


FIG. 118*a*.

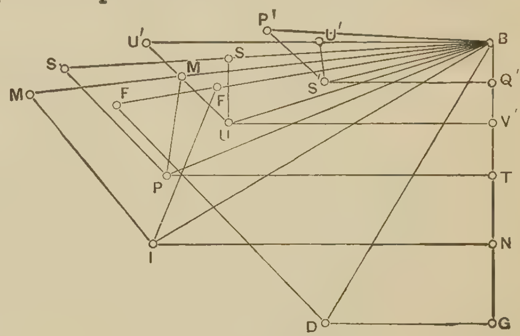


FIG. 118*b*.

For maximum live load web stresses it is sufficiently accurate to treat the systems as independent and assume the chord members straight between joints of the same system. The maximum stresses are then found as in a single-intersection truss, a set of diagrams being constructed for each system.

Fig. 118*a* is the complete diagram for the full system and Fig. 118*b* that for the dotted system. The stresses are marked "*L*" in Fig. 116.

SKEW-BRIDGES.

86. Skew-bridges are those in which one or both end-supports of one truss are not directly opposite to those of the other. Fig. 120 is a plan and Figs. 119 and 121 are elevations of the two trusses of such a bridge. The intermediate panel points are usually placed opposite, in the two trusses, so that all floor-beams are at right angles to the line of the truss. Where the skew is not exactly one panel, as at the left end, it is necessary to move the point *K* backward and *K'* forward in order that *AKK'A'* may be a plane figure.

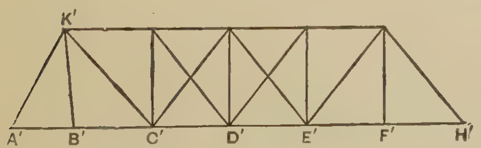


FIG. 119.

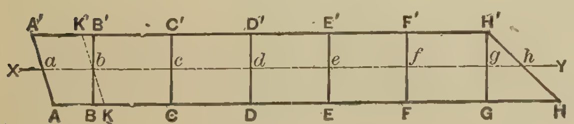


FIG. 120.

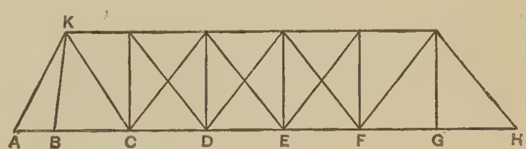


FIG. 121.

The hip verticals are thus slightly inclined, and loads at their bases will affect the lower chords directly.

In the analysis, each truss must be treated separately unless the skew is the same at each end and the trusses therefore symmetrical. As far as the load on the trusses is concerned, it may be assumed as applied along the centre line XY. A full floor-beam load is equal to one panel load except for GH' and BB'. In the former case it is $pd \times \frac{fh}{2}$ and in the latter $pd \times \frac{ac}{2}$. All floor-beam loads are divided equally between the trusses. For any particular loading it will be necessary to compute actual joint loads according to the above principles, since in no case are they the same as would be the case in a square bridge. With the joint loads computed the analysis is simple.

86a. The Ferris Wheel.—PROBLEM: To find the stresses in the rim and spokes of a wheel supported at the centre and loaded with equal loads, W, placed at each of the joints of the rim.

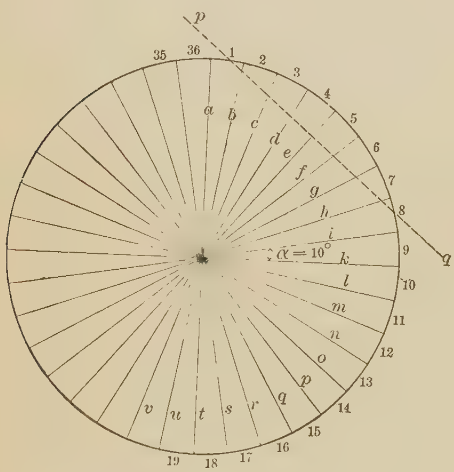


FIG. 121a.—THE FERRIS WHEEL.

Let there be 36 segments as in the well-known Ferris wheel; let r = radius and α = angle between consecutive spokes, = 10°. We will consider two cases:

1st. *When the spokes are rods capable of resisting tension only.*—In this case it will be assumed that the spokes have such initial tension in them that when the loads are applied all will still be in tension except the spoke a, whose stress will be reduced just to zero.

Let S_1, S_2, etc., be the stresses in segments 1, 2, etc., of the rim, and S_a, S_b, etc., be the stresses in spokes a, b, etc. Treating the joint at the top of the wheel as free and remembering that $S_a = 0$, we have $(S_1 + S_{36}) \sin \frac{\alpha}{2} - W = 0$, or since $S_1 = S_{36}$, $S_1 = \frac{W}{2} \operatorname{cosec} \frac{\alpha}{2}$. The stress in any other segment, as S_8, may be found by passing the section pq, treating the portion to the right and taking moments about the centre of the wheel. This gives

$$S_8 r = S_1 r + Wr \sin \alpha + Wr \sin 2\alpha + \ldots + Wr \sin 7\alpha,^*$$

hence $$S_8 = W\left(\tfrac{1}{2} \operatorname{cosec} \frac{\alpha}{2} + \sin \alpha + \sin 2\alpha + \ldots + \sin 7\alpha\right).$$

The segment having the greatest stress is No. 18, and the value of this stress is

$$S_{18} = W\left(\tfrac{1}{2} \operatorname{cosec} \frac{\alpha}{2} + \sin \alpha + \sin 2\alpha + \ldots + \sin 17\alpha\right) = 17.16\,W.$$

The tension in any spoke, as h, is $S_h = (S_7 + S_8) \sin \frac{\alpha}{2} - W \cos \beta$, where β is the inclination of spoke h to the vertical. The spoke having the maximum tension is t and its stress is $S_t = (S_{18} + S_{19}) \sin \frac{\alpha}{2} + W = 4W$.

The initial tension required to produce the conditions assumed above is $2W$ in each spoke, as may be proved thus: By symmetry, if we apply loads of W *upwards*, the tension in a will be $4W$ and the stress in t will be zero; hence if we apply loads of W both upwards and downwards (equivalent to removing all loads) the stresses in a and t will be means between these caused by the extremes of load, or the stress in both a and t will be $2W$, and hence $2W$ in all other spokes.

2d. *When the spokes are stiff members and put in without initial stress.*—The tension in any spoke will in this case evidently be the same as in the first case, minus the initial tension of $2W$; that is, the stress in a will be $2W$ compression and that in t will be $2W$ tension. The stress in any segment of the rim is the same as in the first case, minus the stress caused by the initial tension of $2W$ in the spokes, or minus $W \operatorname{cosec} \frac{\alpha}{2}$. The stress in segment 1 will then be $\frac{W}{2} \operatorname{cosec} \frac{\alpha}{2} = 11.48\,W$ tension, and the stress in segment 18 will be $S_{18} = W\left(-\tfrac{1}{2} \operatorname{cosec} \frac{\alpha}{2} + \sin \alpha + \sin 2\alpha + \ldots + \sin 17\alpha\right) = (17.16 - 11.48)\,W = 5.68\,W$ compression. The above are the maximum and minimum stresses occurring in the spokes and rim for this case. The wind stresses are readily obtained, since each joint is supported separately by diagonals from the axle.

* Since there are 36 segments of this wheel, or $\alpha = 10°$ and $\frac{\alpha}{2} = 5°$, it is here assumed that the length of the radius is the distance from the centre to the joint and also to the middle of the rim-segment. In other words, it is assumed $\cos 5° = 1$, which involves an error of $\frac{3}{8}$ of one per cent.

CHAPTER V.

ANALYSIS OF BRIDGE-TRUSSES FOR WHEEL-LOADS.

87. THE preceding chapter has treated all live loads as uniformly distributed While this method of treatment is in general use for highway bridges and to some extent for railway bridges, it has become the general practice in the latter case to deal with actual specified wheel-loads and to find the maximum stress in each member due to these loads. In highway bridges also, the concentrated load specified usually determines the maximum stresses in the floor-stringers and sometimes in the floor-beams. It is proposed in the following discussion to show the method of finding the position of any given system of wheel-loads which will give the maximum stress in any member, and also how to find such stress.

DERIVATION OF FORMULÆ.

PLATE GIRDERS. TRUSSES WITH PARALLEL CHORDS AND VERTICAL WEB-BRACING.

88. **Influence Lines; Definition.***—A curve representing the variation of moment, shear, panel load, stress, or any similar function, at a particular point in a structure or in any particular member, due to a load unity moving over the structure, is called an *influence line* The difference between an influence line and an ordinary moment or shear curve is that the former represents the variation in the function for a particular point, due to a moving load, while the latter represents the variation in the function along the structure due to some fixed load.

The equation of the influence line for any function is derived by writing out the value of the function for a load unity when placed at a variable distance x from one end of the structure taken as the origin. In all the cases here treated, the equation is of the first degree and the influence lines are therefore all straight lines.

The chief use of influence lines is in determining that position of a given set of loads which will produce the maximum value of any function; and in representing to the eye the influence exerted upon the value of this function by the various elements of the load, and also the effect of shifting the loads in either direction. A subordinate use, however, of these lines is in getting the actual value of these functions.

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* For a more detailed discussion of influence lines than here given, see a paper on *Stresses in Bridges for Concentrated Loads* by Prof. G. F. Swain, *Trans. Am. Soc. C. E.*, July, 1887, from which much has been drawn in the following discussion.

Bending Moment.

89. Influence Line for Bending Moment in a Beam, or at any Joint of the Loaded Chord of a Truss.—Let C, Fig. 122, be the point at a fixed distance a from the left end; and let $x =$ the distance from the load unity to the right end. Then we have, when P is to the right of C, moment at $C = P\frac{x}{l} \times a = \frac{ax}{l}(P, \text{ or } 1)$. This is the equation of the straight line $B'D$, where the ordinate $C'D = \frac{a(l-a)}{l} \times 1$. Likewise when P is to the left of C the moment at C is represented by ordinates to the line $A'D$. Then $A'DB'$ is the influence line for moment at C.

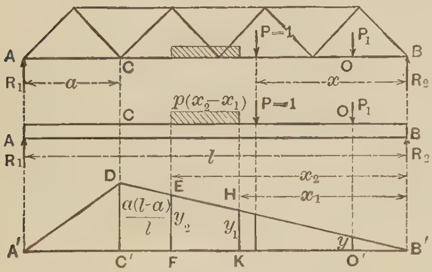

FIG. 122.

The moment at C due to a load P_1, at any point O, is equal to the ordinate y, under the load, multiplied by the load; for the ordinate y is equal to the moment due to unity load. The moment at C due to any number of loads may thus be found by multiplying each load by its corresponding ordinate and adding the products.

The moment at C due to any length of uniform load of p per unit length is equal to the area between the extreme ordinates, multiplied by p. For the moment due to an element, pdx, of load, $= pdxy$, where y is the ordinate under the load; and integrating between the limiting values of x, x_1 and x_2, we have: total moment $= p\int_{x_1}^{x_2} ydx = p \times$ area $EFHK$. The moment due to a full uniform load $=$ area $A'B'D \times p$.

90. Position of Moving Loads for a Maximum Bending Moment in a Beam or at any Joint of the Loaded Chord of a Truss.—Let $A'DB'$, Fig. 123, be the influence line for moment at C.

The maximum moment due to a uniform load is when the load extends from A to B, and is equal to the area $A'DB'$ multiplied by p, where $p =$ load per unit length.

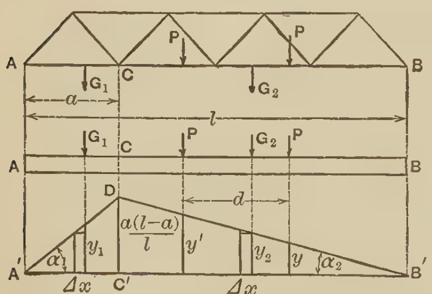

FIG. 123.

$$\text{Area } A'DB' = \tfrac{1}{2}l \times \frac{a(l-a)}{l} = \tfrac{1}{2}a(l-a).$$

$$\text{Whence the maximum moment} = \frac{p}{2}a(l-a). \quad . \quad (1)$$

Since the moment at C due to any load $P, = Py$, this moment is a maximum when the load is at C and is equal to $P\frac{a(l-a)}{l}$.

For two equal loads, P, a fixed distance, d, apart, the moment is evidently a maximum when one load is at the point and the other is on the longer segment of the beam, for then $y + y'$ is a maximum. This maximum moment is equal to

$$P\left(\frac{a(l-a)}{l} + \frac{a(l-a)}{l} \times \frac{l-a-d}{l-a}\right) = P\frac{2a(l-a)-ad}{l}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The above results for these three special loadings have been derived in Chap. IV, though in a different manner.

The position of *any* given system of loads to produce a maximum moment at C will now be derived. This general case includes also the above special cases.

Let G_1, Fig. 123, represent the sum of all the loads between A and C for any particular

position of the loading, and let this single force be applied at the centre of gravity of these loads. In a similar manner let G_2 replace the loads on the right of C. Now since G_1 has the same effect upon the abutment reaction at B as do the loads which it replaces, it follows that it has the same effect upon the moment at C as do these loads. Likewise for G_2. Hence the moment at C due to our given system is

$$M = G_1 y_1 + G_2 y_2 .$$

Now let the entire system move a small distance δx to the left, no load passing A, C, or B The moment will become

$$M + \delta M = G_1(y_1 - \delta x \tan \alpha_1) + G_2(y_2 + \delta x \tan \alpha_2).$$

By subtraction we have

$$\delta M = G_2 \delta x \tan \alpha_2 - G_1 \delta x \tan \alpha_1 ,$$

and hence

$$\frac{\delta M}{\delta x} = G_2 \tan \alpha_2 - G_1 \tan \alpha_1 = C'D\left(\frac{G_2}{C'B'} - \frac{G_1}{A'C'}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a maximum value of M, $\frac{\delta M}{\delta x}$ or $\frac{G_2}{C'B'} - \frac{G_1}{A'C'}$ must change from positive to negative, that is, must pass through zero, as the loads are moved to the left. The values of G_2 and G_1 can change only when a load passes A, C, or B. A load passing A decreases G_1 and a load passing B increases G_2, but a load passing C increases G_1 and decreases G_2; therefore it is only by this last method that $\frac{G_2}{C'B'} - \frac{G_1}{A'C'}$ can be changed from positive to negative and the moment made a maximum. For a maximum moment, then, a load should be at C such that when considered as part of G_2 the expression $\frac{G_2}{C'B'} - \frac{G_1}{A'C'}$ is positive, and when considered as part of G_1 this expression is negative. Or, stated in another way, when $\frac{G_1}{A'C'}$ can be made equal to $\frac{G_2}{C'B'}$ by considering a part of the load as located on one side of C and the remaining part as located on the other side. If a distributed load is passing C this condition can be definitely satisfied.

From the equality $\frac{G_2}{C'B'} = \frac{G_1}{A'C'}$ we have, by composition, $\frac{G_2 + G_1}{A'B'} = \frac{G_1}{A'C'}$, or, if $G = G_1 + G_2$, we have

$$\frac{G_1}{A'C'} = \frac{G}{A'B'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This is the most convenient form of the criterion for maximum moment. Expressed in words it is that *the average unit load on the left of the point must be equal to the average unit load on the whole span.* The unit of length may be taken as a foot or a panel length. The latter is the more convenient for trusses of equal panels.

For a given set of wheel-loads there are usually two or more positions which will satisfy this criterion. The moment for each position must be computed and the greatest value taken.

91. **The Point of Maximum Moment in a Beam or Plate Girder.**—Under any *given position* of the loads the point of maximum moment is the point of zero shear, as proved in Mechanics,* but as the loads are shifted, this point moves and the moment at the point

* See also Chap. VIII.

varies. The problem is to find that point in the beam at which this moment is the greatest that can occur in the beam, and the corresponding position of the loads.

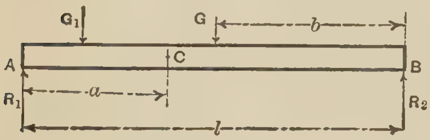

FIG. 124

Let G, Fig. 124, represent the sum of the loads on the beam AB; b, the distance from B to their centre of gravity; C, the required point of maximum moment; and G_1, the sum of the loads to the left of C. Then the condition of zero shear requires that

$$R_1, = G\frac{b}{l}, = G_1, \quad \text{or} \quad \frac{G}{l} = \frac{G_1}{b}.$$

If the moment at C is the greatest which can occur in the beam, then it must be the greatest which can occur at the point C. But for a maximum moment at any point we have, by eq. (4),

$$\frac{G_1}{a} = \frac{G}{l}.$$

Hence for the point C

$$\frac{G}{l} = \frac{G_1}{b} = \frac{G_1}{a}, \quad \text{whence } a = b. \quad \dots\dots\dots \quad (5)$$

The point C will lie near the centre of the beam and under some wheel when that wheel and the centre of gravity of the loads are equidistant from A and B. This condition, together with eq. (4), will serve to determine this point of maximum moment.*

92. Influence Line and Position of Loads for Maximum Floor-beam Concentration.—Let AB and BC, Fig. 125, be two consecutive panels of any lengths d_1 and d_2. A load of unity moving from C to B produces a load at B proportional to the distance of the load from C. The line $C'D$, with ordinate DB' equal to unity, is then the influence line for floor-beam load at B when the loads are between C and B. Likewise DA' is the influence line for loads on the portion BA. The load at B, produced by any load P, is equal to Py'; and the load produced by a uniform load p per unit length extending from A to C is equal to

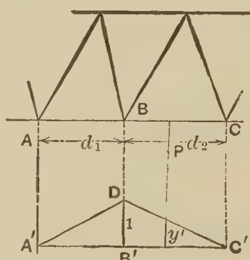

FIG. 125

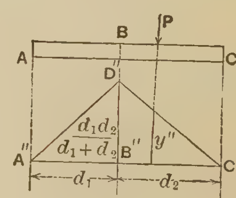

FIG. 126

$p \times$ area $A'DC' = p \times \frac{d_1 + d_2}{2}$, a result evident by inspection.

The influence line in Fig. 125 is of the same *form* as that in Fig. 123, and as the criterion for maximum moment in no way involves the length of the ordinate $B'D$, the same criterion must hold good for maximum panel reaction. That is, *the average unit loads on the two panels must be equal to each other and to the average unit load on both.* If the panels are equal, then the total loads in the two panels must be equal.

If $A''D''C''$ (Fig. 126) be the influence line for moment at B in a beam AC, of length $l, = d_1 + d_2$, the ordinate $D''B''$ will be equal to $\frac{d_1 d_2}{d_1 + d_2}$, and the ratio of any two corresponding ordinates y' and y'' in Figs. 125 and 126 is equal to the ratio of DB' to $D''B'' = 1 \div \frac{d_1 d_2}{d_1 + d_2} = \frac{d_1 + d_2}{d_1 d_2}$. Whence the floor-beam reaction at B in Fig. 125 due to any load P is equal to the moment at B, Fig. 126, due to the same load, multiplied by $\frac{d_1 + d_2}{d_1 d_2}$. Moreover, since the maximum floor-beam concentration occurs for the same position of the loads as does the maximum moment, we have only to find the maximum moment in a beam of length equal to $d_1 + d_2$ at a distance d_1 from one end and multiply this moment by $\frac{d_1 + d_2}{d_1 d_2}$ and we shall

* See p. 83 for a numerical example.

have the maximum floor-beam concentration for panel lengths of d_1 and d_2. If $d_1 = d_2 = d$, the factor is $\frac{2}{d}$.

Shear.

93. Influence Line and Position of Loads for Maximum Shear at any Point in a Beam.—The shear at C in the beam AB, Fig. 127, due to a load unity moving from B towards A increases from zero for the load at B to $+\frac{l-a}{l}$ for the load just to the right of C. As the load passes C the shear becomes $-\frac{a}{l}$ and increases to zero at A. Any movement of loads to the left, therefore, increases the positive shear at C until some load P passes C, when the shear is suddenly decreased by an amount equal to $P\left(\frac{l-a}{l}+\frac{a}{l}\right) = P$. For concentrated loads, therefore, the shear reaches a maximum each time a load reaches C. The greatest of these maximum values evidently occurs when one of the wheels near the head of the train is just to the right of C.

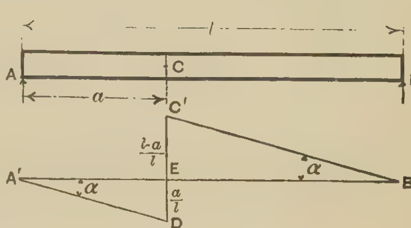

FIG. 127.

To determine which of two consecutive wheels, P_1 and P_2, causes the greater shear when placed just to the right of C, let b be the distance between these wheels, and G' the total load on the beam when P_1 is at C. Let the loads advance a distance b from this position, thus bringing P_2 at C. The effect upon the shear is first to decrease it suddenly by an amount P_1, and then to increase it gradually by an amount equal to $G'b$ tan α, $= \frac{G'b}{l}$. The total increase is therefore $\frac{G'b}{l} - P_1$. According as this expression is negative or positive will wheels P_1 or P_2 give the greater shear. For equal shear,

$$\frac{G'}{l} = \frac{P_1}{b}. \qquad (6)$$

The slight increase in shear due to additional loads that may come upon the beam from the right has been neglected. If G'' be the total load on the beam when P_2 is at C, then the increase in shear will be somewhere between $\frac{G'b}{l} - P_1$ and $\frac{G''b}{l} - P_1$. When the former expression is negative and the latter positive, then both positions should be tried. This will occur only for a short distance, to the left of which both are positive and to the right both are negative.

94. Influence Line and Position of Loads for Maximum Shear in any Panel of a Truss.—Let AB, Fig. 128, be any truss and FC any panel. Let n = total number of panels, m = number on the left of C, and m' the number on the right. Then $m' = n - m$. The shear in FC for a load unity moving from B to C will be equal to the left abutment reaction and will increase from zero at B to $\frac{m'}{n}$ at C. For the same load moving from F to A the shear is negative and equal to the right abutment reaction, thus increasing from a value $-\frac{m-1}{n}$ at F to zero at A. Between C and F the load is carried to these two points by the floor-stringer, the amount transferred to each being proportional to the distance of the load

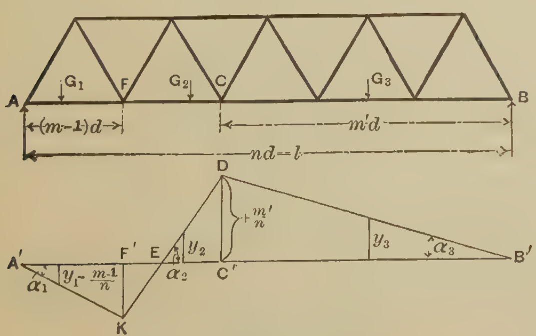

FIG. 128.

from the other. The shear thus varies uniformly and the influence line for this portion is the straight line DK, giving the complete influence line $A'KDB'$.

For a maximum positive shear due to a single load the load should evidently be placed at C. A uniform load will give the maximum positive shear when extending from B' to E, and the shear will then be equal to $p \times$ area EDB'. Now $EC' : DC' :: F'C' : DC' + F'K$, whence $EC' = \frac{m'}{n} \times d \times \frac{n}{m' + m - 1} = \frac{m'd}{n - 1}$, a result already found in Art. 69. The $shear = p \times \text{area } EDB' = p \times \frac{1}{2}\frac{m'}{n} \times \left(m'd + \frac{m'd}{n-1}\right) = \frac{1}{2} pd\frac{m'^2}{(n-1)}$, the same as eq. 17 ($a$), page 51.

For concentrated loads, let G_1 represent the portion from A to F, G_2 that in the panel FC, and G_3 that on the right of C. We then have the total positive shear

$$S = G_3 y_3 + G_2 y_2 - G_1 y_1.$$

Let the loads move a small distance δx to the left. The shear will then be

$$S + \delta S = G_3(y_3 + \delta x \tan \alpha_3) + G_2(y_2 - \delta x \tan \alpha_2) - G_1(y_1 - \delta x \tan \alpha_1).$$

By subtraction and division we have

$$\frac{\delta S}{\delta x} = G_3 \tan \alpha_3 - G_2 \tan \alpha_2 + G_1 \tan \alpha_1$$

$$= G_3 \frac{1}{nd} - G_2 \frac{n-1}{nd} + G_1 \frac{1}{nd}.$$

For a maximum, $\frac{\delta S}{\delta x} = 0$; whence

$$G_3 - G_2(n - 1) + G_1 = 0,$$

or

$$G_2 = \frac{G_1 + G_2 + G_3}{n} = \frac{G}{n}. \qquad (7)$$

That is, *for a maximum shear in any panel the load in the panel must be equal to the load on the truss divided by the number of panels.* The only way $\frac{\delta S}{\delta x}$ can be made zero by passing from positive to negative, and so giving a maximum and not a minimum as the loads are moved to the left, is by a load passing C or A. But for the greatest positive shear there should as a rule be no loads on the portion AF. Hence the maximum shear will occur with some wheel near the head of the train at C, such as will satisfy the above criterion.

Since a wheel passing C causes a large relative increase in G_2, it will be found that the maximum shears in several panels will occur with the same wheel at the panel point to the right. To find for what panels any particular wheel P placed at the panel point to the right will give a maximum shear, let G_2 represent the wheels in the panel other than the wheel P. Then if P at the right-hand panel point gives a maximum, the criterion requires that $G > nG_2$ and $< n(G_2 + P)$. Wheel P at the right-hand panel point will then give a maximum so long as the entire load upon the bridge lies between nG_2 and $n(G_2 + P)$.

COMPUTATION OF STRESSES. PLATE GIRDERS. TRUSSES WITH PARALLEL CHORDS AND VERTICAL WEB BRACING.

95. Wheel-loads; Tabulation of Moments.—A diagram such as is shown in Fig. 129 is of great assistance in finding stresses due to actual wheel-loads.* The diagram in the figure

* See p. 108*b* for a similar moment table for Cooper's Conventional Loads.

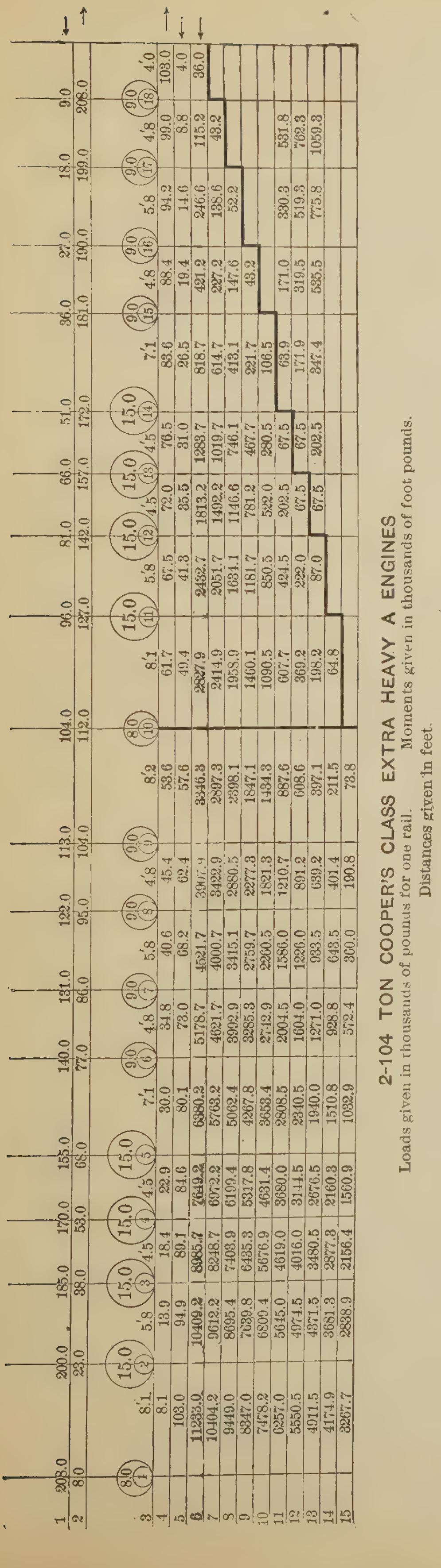

FIG. 129.

is for Cooper's Class "Extra Heavy A." Such a diagram should be made up for each system of loads in use.

All loads are given in thousands of pounds and all moments in thousands of foot-pounds. The loads given are one half the total loads, being those on one rail. All distances are given in feet and tenths, instead of feet and inches as in Cooper's specifications.

The various horizontal columns contain the following data, beginning at the top:

1. Summation of loads from right end.
2. Summation of loads from left end.
3. Distance between wheels.
4. Summation of distances from left end.
5. Summation of distances from right end.

6 to 15. Summation of moments about the wheel vertically over the step in the heavy stepped line.

The weight on each wheel is given in heavy figures on the respective wheels and is in thousands of pounds. The figure in the small circle is the number of the wheel from the left end.

96. Application of the Foregoing Principles to a Pratt Truss.—Let us take a truss of 200 ft. span with eight panels and a height of 32 ft., Fig. 130. Live load only will be considered here.

Chord Stresses.

The maximum moments at b, c, d, and e divided by 32 ft. will give us the maximum chord

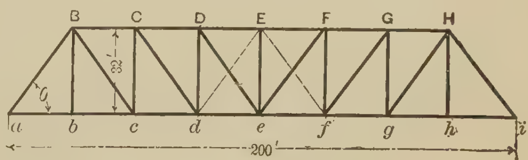

FIG. 130.

stresses. These maximum moments will now be found by the aid of the diagram Fig. 129. The criterion for maximum moment is that the average load per unit length on the left must equal the average load per unit length on the whole bridge. A panel length will be taken as the unit.

1st. *Moment at b.*—It is evident that, in general, the bridge should be loaded with the heaviest possible load, and that heavy concentrations will have the most effect when near the point of moments.

Let us try wheel 2 at b. The bridge will extend 175 ft. to the right of 2, and hence $175 - 94.9, = 80.1$ ft., of uniform load will be on the bridge. Total load then $= G = 208 + 80.1$

$\times$ 1.5 = 328.1, and $\frac{G}{ai} = \frac{328.1}{8} = 41$. Load to the left divided by ab, $= \frac{G_1}{ab} = \frac{G_1}{1}$, = 8, considering wheel 2 as on the right; or = 23, considering wheel 2 as on the left. Since both these values of G_1 are less than $\frac{G}{ai}$, the moment at b will be increased by moving the loads to the left.

Try wheel 3 at b.

$$\frac{G}{ai} = \frac{208 + (175 - 89.1)1.5}{8} = 42.1,$$

$$\frac{G_1}{ab} = \frac{23}{1} \text{ to } \frac{38}{1},$$

and hence the moment will be increased by moving the loads to the left.

Try wheel 4 at b.

$$\frac{G}{ai} = \frac{208 + (175 - 84.6)1.5}{8} = 42.9,$$

$$\frac{G_1}{ab} = \frac{38}{1} \text{ to } \frac{53}{1}.$$

One of these values being less and the other greater than $\frac{G}{ai}$, wheel 4 at b gives a maximum moment.

To find this moment, get first the abutment reaction at a. Line 6 gives the moment of the wheels about the point 19 at the left end of the uniform train load. It is 11233. The right abutment is 175 − 84.6, = 90.4 ft., to the right of 19. Hence the total moment of the load about the right abutment is $11233 + 208 \times 90.4 + \frac{1.5 \times (90.4)^2}{2} = 36168$. Left abutment reaction $= \frac{36168}{8 \times 25}$. Moment of left reaction about $b = \frac{36168}{8 \times 25} \times 25 = \frac{36168}{8} = 4521$. The moment of the wheel-loads to the left of b about b is given in line 12 between wheels 10 and 11, and is to be subtracted from the moment of the abutment reaction. Hence, moment at $b = 4521 - 369 = 4152$, thousand ft.-lbs.

Let us see if there are other maximum values. With wheel 5 at b, $\frac{G}{8} = 43.8$ and $G_1 = 53$ to 68, thus showing that this position does not give a maximum. As the loads are moved farther to the left, G_1 will be made less than $\frac{G}{8}$ by some wheel passing a, but this causes a minimum, not a maximum. As the loads are moved still farther, G_1 will again be made greater than $\frac{G}{8}$ by some heavy wheel 11, 12, or 13 passing b. This will give other maxima, but not so great as that found above, since now we have a uniform train-load on the right, in place of heavy engine-loads. The actual moments for wheels 11, 12 and 13 at b are 3802, 3923, and 3801, respectively. Wheel 4 at b therefore gives the greatest possible moment.

2d. *Moment at c.*—Try wheel 6 at c.

$$\frac{G}{ai} = \frac{208 + (150 - 73)1.5}{8} = 40.4,$$

$$\frac{G_1}{ac} = \frac{68}{2} \text{ to } \frac{77}{2} = 34 \text{ to } 38. \text{ Hence no maximum.}$$

Wheel 7 at c.

$$\frac{G}{ai} = \frac{208 + (150-68.2)1.5}{8} = 41.3,$$

$$\frac{G_1}{ac} = \frac{77}{2} \text{ to } \frac{86}{2} = 38 \text{ to } 43.$$

Hence wheel 7 gives a maximum moment at c. Wheel 8 is found to give no maximum, and hence wheel 7 gives the greatest possible moment.

$$\text{Moment at } c = \frac{11233 + 208 \times 81.8 + \frac{1.5 \times (81.8)^2}{2}}{8} \times 2 - 1460.1 = 6857.4.$$

3d. *Moment at d.*—Wheels 11 and 12 are found to give maximum moments. The corresponding moments are 8530.5 and 8536.0.

4th. *Moment at e.*—Wheels 13 and 14 at e give maxima. The corresponding moments are 9029.0 and 9030.9.

We have, then, by dividing the moments by 32, the following

LIVE LOAD CHORD STRESSES.

Point.	Max. Moment.	Pieces.	Stresses.
b	4152	abc	129.8
c	6857	cd and BC	214.3
d	8536	de and CD	266.8
e	9031	DE	282.2

Web Stresses.

The maximum stresses in all the web members except bB and hH result directly from the maximum shears in the various panels. Those in bB and hH are equal to the maximum panel load, which will be found below. The criterion for maximum shear is that *the total load divided by the total number of panels must equal the load in the panel.*

1st. *Shear in ab.*—For this panel the criterion for shear is the same as for moment at b, hence the same position will give a maximum in both cases. Wheel 4 is at b. Shear = abutment reaction minus panel load at a. This abutment reaction was found to be $\frac{36168}{8 \times 25} = 180.8$. The panel load at a is obtained by finding the moment of the wheels 1, 2, and 3 about b and dividing by the panel length. This moment is given in line 12 between wheels 10 and 11 and is equal to 369.2. Whence shear $= 180.8 - \frac{369.2}{25} = 166.0$.

2d. *Shear in bc.*—Try wheel 3 at c. Total load on the bridge $= G = 208 + (150 - 89.1)1.5 = 299.3$. $\frac{G}{8} = 37.4$. The load in the panel $= G_2 = 23$ or 38. Hence wheel 3 at c gives a maximum. Shear = abutment reaction minus load at b

$$= \frac{11233.0 + 208 \times 60.9 + \frac{1.5 \times (60.9)^2}{2}}{200} - \frac{198.2}{25} = 133.4 - 7.9 = 125.5.$$

Try wheel 4 at *c*.

$$\frac{G}{8} = \frac{208 + (150 - 84.6)\,1.5}{8} = 38.3,$$

$G_2 = 38$ to 53. Hence wheel 4 gives a maximum.

$$\text{Shear} = \frac{11236.2 + 208 \times 65.4 + \dfrac{1.5 \times (65.4)^2}{2}}{200} - \frac{369.2}{25} = 125.4.$$

As the loads are moved still farther to the left, G_2 becomes greater than $\frac{G}{8}$; it is then made less by some wheel passing *b*, thus giving a minimum, and is again made greater by some wheel 11, 12, or 13 passing *c*, thus giving another maximum. Such a maximum is evidently much less than that found above.

3d. *Shear in cd.*—Wheel 3 at *d* gives the only maximum value. Shear = 90.4.

4th. *Shear in de.*—Wheel 3 at *e* gives the only maximum, the value of which is 60.1.

5th. *Shear in ef.*—Wheels 2 and 3 give maxima. With wheel 2 at *f*, the end of the uniform train-load is $94.9 - 75, = 19.9$ ft., to the *right* of the end of the bridge. Wheel 14 is therefore the last wheel to the right that is on the bridge, and is $26.5 - 19.9, = 6.6$ ft., to the left of *i*. Moment about $i =$ moment about wheel $14 + 172 \times 6.6$. Hence

$$\text{shear in } ef = \frac{6257.0 + 172 \times 6.6}{200} - \frac{64.8}{25} = 34.4.$$

With wheel 3 at *f*,

$$\text{shear in } ef = \frac{8347.0 + 190 \times 0.5}{200} - \frac{198.2}{25} = 34.3.$$

6th. *Shear in fg.*—Wheel 2 at *g* gives a maximnm, whose value is 16.27. This positive shear would probably be less than the dead load negative shear and would therefore be needed only to show that fact.

The dead load negative shear to the right of *g* would be much greater than the live load positive shear.

Multiplying the above maximum shears by cosec θ, $= \sqrt{25^2 + 32^2} \div 32, = 1.269$, for the stresses in the diagonals, we have the following

LIVE-LOAD WEB STRESSES.

Panel.	Shear.	Piece.	Stress.	Piece.	Stress.
ab	166.0			*aB*	+210.6
bc	125.5			*Bc*	− 159.3
cd	90.4	*cC*	+ 90.4	*Cd*	− 114.7
de	60.1	*dD*	+ 60.1	*De*	− 76.3
ef	34.4	*eE*	+ 34.4	*Ef*	− 43.7

The proper positions of the loads might have been more quickly found by finding for what panels each of the wheels 2, 3, and 4 would give a maximum shear when placed at the right-hand panel point, as explained in Art. 94. Thus with wheel 3 at this panel point, G_2 varies from 23 to 38 and $8G_2$ lies between 184 and 304, which are then the limiting values of G for wheel 3 to give a maximum. Wheel 3 therefore gives a maximum when the end of the bridge lies between wheel 16 and $(304 - 208) \div 1.5, = 64$ ft., to the right of point 19; that is, when wheel 3 itself is from 74.5 ft. to 153.1 ft. to the left of the right end of the bridge. This includes panel points *f*, *e*, *d*, and *c*, as found above.

The field of wheel 4 is likewise found to lie between 148.6 ft. from the right end, to the left end, thus including points c and b.

The field of wheel 2 is from 14.8 ft. to 80.3 ft. from the right, thus including points h, g, and f.

It is to be noticed that consecutive fields overlap by an amount equal to the distance between wheels.

Floor-beam Reaction; Stresses in Hip Verticals.

These are a maximum when the joint-load is a maximum. By Art. 92 the maximum joint load is equal to the maximum moment at the centre of a beam 50 feet long, multiplied by $\frac{2}{d}$. Wheel 4 at the centre of such a beam is the only wheel giving a maximum moment. The value of this moment $= \frac{1958.9 + 95 \times 2.8}{2} - 369.2 = 743.2$. Hence the maximum floor-beam load $= 743.2 \times \frac{2}{25} = 59.46$. Two such loads applied on the floor-beam at the points of attachment of the stringers will produce the maximum moment and shear in the beam. These are readily computed.

In the diagram, Fig. 129, the floor-beam concentration or stress in hip vertical may be quickly found by placing wheel 13 over the floor-beam and then deducting from the total load on the two adjacent panels the part which is transferred to the two adjacent floor-beams by the stringers. Thus, as wheels 10 to 17 are on the two panels, the total load is 95 and the part transferred to the adjacent beams is, taking moments about wheel 13 and which are given in line 12, $\frac{369.2 + 519.3}{25} = 35.5$. The beam concentration is therefore $95 - 35.5 = 59.5$.

Stringers; Maximum Moment and Shear.

The maximum moment will be near the centre and, by Art. 91, will be under some wheel when that wheel and the centre of gravity of the entire load on the stringer are equally distant from the centre. Length = 25 ft.

It will be simplest to first find what wheels placed at the *centre* will give a maximum moment at that point. These are found by the criterion of eq. 4, p. 75, to be wheels 3 and 4.

With wheel 3 at the centre, wheels 2 to 5 are on the stringer. The distance of the centre of gravity of these four wheels from 5 is found by taking moments about wheel 5 or, as given in line 11 of the diagram, about wheel 14, to be $424.5 \div 60 = 7.1$ ft.; hence wheel 3 should be placed a distance $(9 - 7.1) \div 2 = .95$ ft. to the left of the centre. The moment under wheel 3 is then found to be $233.5, = 60 \times \frac{\overline{11.55}^2}{25} - 87$.

With wheel 4 at the centre, wheels 2 to 6 are on, and the centre of gravity of these five loads is found to be 12.3 ft. to the left of 6, and .7 ft. to the left of 4. With wheel 4 .35 ft. to the right of the centre the moment under 4 is found to be 234.7, the required maximum.

The moments at the *centre* under wheels 3 and 4 are 230.2 and 234.3 respectively, values very nearly as great as the above, thus showing that the moment varies but slightly near the centre of the stringer.

The maximum shear in the stringer occurs at the end, when one of the wheels is about to pass off. Wheel 2 at the left end will evidently give a greater reaction than will wheel 1. With wheel 2 at the left end, wheel 6 will be $(8.1 + 25) - 30.0 = 3.1$ ft. to the left of the right end. Left reaction = shear $= \frac{1090.5 - 8 \times 30 + 69 \times 3.1}{25} = 42.58$. With wheel 3 at the left end the shear is found to be 41.58. The required maximum is therefore 42.58.

97. Plate and Lattice Girders.—The maximum moments and shears are found at several points along the girder, and from these the flange and web stresses are obtained. In addition to these moments the maximum moment possible in the girder may be found as in

the case of the floor-stringer above, by testing a few wheels which give a maximum at the centre.

The other moments are found by the same method as illustrated in the preceding article.

The shear at any point is a maximum with either wheel 1 or 2 just to the right of the point. Wheel 2 at the point will give a maximum whenever $\frac{G'}{l} \gtreqqless \frac{P_1}{b}$, and wheel 1 will give a maximum when $\frac{G''}{l} \lesseqqgtr \frac{P_1}{b}$, where G' = load on the girder with wheel 1 at the point, G'' = load on the girder with wheel 2 at the point, b = distance between wheels 1 and 2, and P_1 = weight of wheel 1. This follows from Art. 93.

Thus for a 60-foot girder, $\frac{P_1}{b} = \frac{8}{8.1} = .99$ and $\frac{P_1}{b}l = 59.4$. Hence when $G' \gtreqqless 59.4$, wheel 2 gives the greater shear; that is, when five or more wheels are on the girder with wheel 1 at the point, which would be for all points 22.9 ft. or more from the right end. When $G'' \lesseqqgtr 59.4$ or when there are four or less loads on the girder with wheel 2 at the point, then wheel 1 gives the greater shear; that is, for all points less than 14.8 feet from the right end. Between 14.8 and 22.9 feet from this end both positions should be tested.

98. Graphical Methods. Load Line and Moment Diagram.—The following graphical methods of computing stresses for actual wheel-loads are mostly due to Dr. Henry T. Eddy. They are given by him in a very elaborate paper discussing the whole subject, in the *Trans. Am. Soc. C. E.*, Vol. XXII, 1890, paper No. 437.*

These methods offer considerable advantage over the analytical methods as regards speed, and if the diagrams are drawn carefully and to a large scale they are sufficiently accurate for computing the stresses in the main members of a truss. For floor-beams and stringers the moment table should be used in getting moments and shears, but the diagram may be used in getting the *position* of the loads.

The diagram, Fig. 131, is constructed as follows: Upon profile paper, or upon specially prepared cross-section paper, is first laid off the wheel-diagram, to a horizontal scale of about 8 ft. to the inch. The wheels are numbered, their distances apart noted, and vertical lines drawn through each. The stepped "load line," 1–2–3–4–, etc., is simply a line whose ordinates, measured from the horizontal reference line 0–0 at the bottom of the sheet, are equal to the summation of the loads to the left. It thus consists, over the wheels, of a series of steps, each step being equal by scale to the load below. Over the uniform load it is a straight line rising at the rate of 1500 lbs. per foot. A convenient scale for loads is about 20,000 lbs. to the inch.

The moment lines, numbered 1, 2, 3, 4, . . . 18, at the right edge, are constructed by beginning at the horizontal reference line at some point near the right end of the sheet and laying off successively on a vertical line the moment of each wheel about that point, beginning with wheel 1. Then the line 1 is drawn from the reference line over wheel 1 to the first point thus found; the line 2 through the intersection of line 1 with the vertical over wheel 2, and the second point so found; then 3 through the intersection of 2 with the vertical over wheel 3, and the third point; etc. A scale of one one-hundredth of the scale for loads will be found convenient. The ordinate at any point, from the reference line to any one of these moment lines, is thus equal to the sum of the moments about the point of the loads up to and including the load corresponding to the moment line taken. Also the ordinate at any point between any two lines, as between 2 and 8, is equal to the sum of the moments of wheels 3 to 8, inclusive, about that point. The broken line AB, formed of segments of moment lines, is evidently an equilibrium polygon for the given loads. The portion BC above B is a parabolic curve and can be easily constructed by computing the moments of the portion of the uniform load between B and points to the right, about those points, and laying off these moments above the line 18.

* The stepped load-line had been used in the office of the Cincinnati Southern Ry. at Cincinnati, since 1881. The moment lines were published by Prof. W. H. Schuerman in 1886. See *Proc. of the Eng. Assoc. of the South*, Vol. IX, p. 35.

99. Application of the Diagram, Fig. 131, to Finding Maximum Moments.—Before using the diagram, the panel points of the truss, or the points along the beam where the

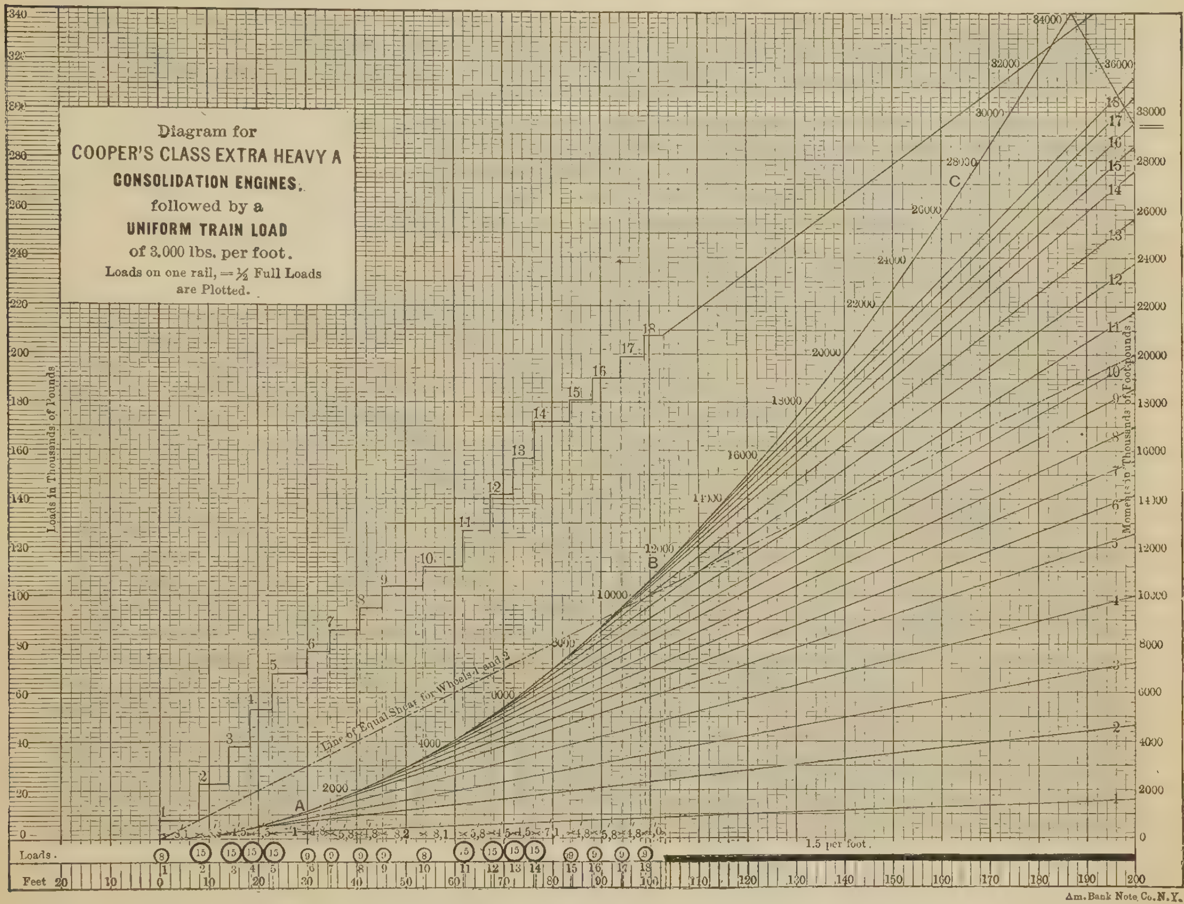

FIG. 131.

moments are desired, are first marked off on a slip of paper to the same scale as the diagram. Suppose 1–2–3–4 . . ., Fig. 132, represent a portion of a load line, and A, C, D, E, B be the panel points of a truss, marked off on a separate slip of paper. Let it be required to find the position of loads for a maximum moment at D. The average load on the left, $= \frac{G_1}{AD}$, must equal the average load on the bridge, $= \frac{G}{AB}$. Try wheel 4 at D by placing D under this wheel. The ordinate $FB = G$, and $\frac{G}{AB}$ is the slope of the line AF. The ordinate ID or $KD = G_1$, and $\frac{G_1}{AD}$ is equal to the slope of AI or AK. The slope of AF is seen to be greater than that of AK and less than that of AI; therefore this position gives a maximum moment at D. Hence, in general, for a maximum moment, if the line whose slope is

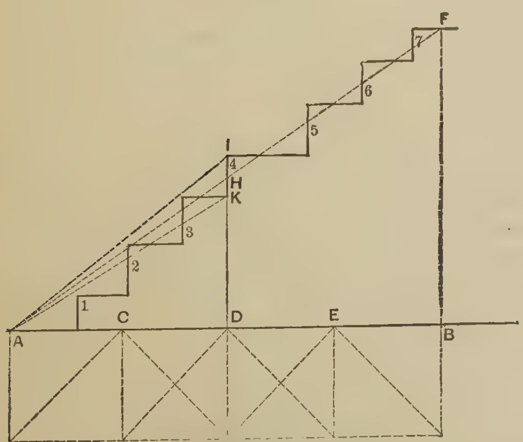

FIG. 132.

equal to $\frac{G}{l}$ cuts the load or step which is above the point, the position is one giving a maximum. This line is conveniently indicated by means of a thread. If there are no loads off the bridge to the left, the line AF starts from the zero line at the end of the bridge at A; but if there are loads to the left, it starts in the load line vertically over the end of the bridge. The right end is the point in the load line vertically over the right end of the bridge. In the above case, if AF should pass above AI, the loads must be moved to the left; and if it should pass below AK, then the loads must be moved to the right.

The moment itself is easily found on the moment diagram by reading off the ordinate at B between the moment lines 0-0 and 7, multiplying this by $\frac{AD}{AB}$, and subtracting the moment of the loads to the left about D, which is given by the ordinate at D between 0-0 and 4. The moment at D is also equal to the ordinate from the closing line to the equilibrium polygon formed by the segments of the moment lines. The extremities of this closing line lie in verticals through the ends of the bridge.

In finding the greatest possible moment in a girder, the centre of gravity of any number of loads is readily found by producing the two segments of the equilibrium polygon including these loads, to their intersection.

100. Application of the Diagram, Fig. 131, to Finding Maximum Shears.—1st. *Shear in Beams or Girders.*—It has been shown in Art. 93 and illustrated in Art. 97 that wheel 1 will cause a maximum shear at all points to the right of the point where $\frac{G''}{l} = \frac{P_1}{b}$, and that wheel 2 will cause a maximum shear to the left of the point where $\frac{G'}{l} = \frac{P_1}{b}$; where G' and G'' are the total loads on the girder with wheels 1 and 2, respectively, at the point; b = distance between wheels 1 and 2; P_1 = weight of wheel 1; and l = length of girder. (Stringers less than about 18 ft. long are an exception, they having a maximum end shear with wheel 5 at the end.)

Let A1–2 . . . 8, Fig. 133, be a load line. Draw the line AT, having a slope of $\frac{P_1}{b}$. Lay off the length of the girder AB on the line 0–0, and at B draw the vertical BT. This vertical ordinate is that load which, divided by l, $= \frac{P_1}{b}$, and therefore is the load G' or G'' above mentioned. In order that this may be the total load on the girder, wheel 5 must be at the right end. Laying off the girder then from B' to the left, the portion $B'C'$ to the right of wheel 2 is the portion of the girder in which the maximum shear is given by wheel 1, and the portion of the girder to the left of C'' has its maximum shears for wheel 2. Between C' and C'' both positions should be tested. The line AT is permanently drawn on the diagram; no other lines need be drawn.

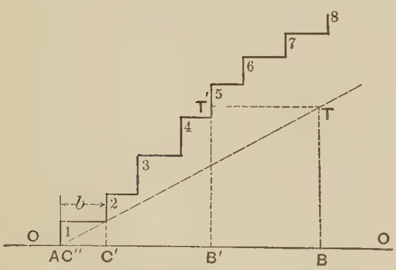

FIG. 133.

2d. *Shear in a Truss.*—The criterion for maximum shear may be stated thus: the total load on the truss divided by its length must equal the load in the panel divided by the panel length. Any wheel P at the panel point to the right will thus give a maximum shear so long as $\frac{G}{l}$ lies between $\frac{G_2}{d}$ and $\frac{G_2 + P}{d}$, where G = total load and G_2 = load in the panel other than P.

Let 1 . . . 15, Fig. 134, be any load line. Mark off on a strip of paper the panel points

of the truss and place it in the position AB, with the first panel point C under wheel 1. Other panel points are D, E, and F. Call the loads P_1, P_2, P_3, etc. Draw the lines AT_1, AT_2, AT_3, etc., having ordinates at C equal to P_1, P_1+P_2, $P_1+P_2+P_3$, etc. These lines will then have the slopes of $\frac{P_1}{d}$, $\frac{P_1+P_2}{d}$, $\frac{P_1+P_2+P_3}{d}$, etc., respectively.

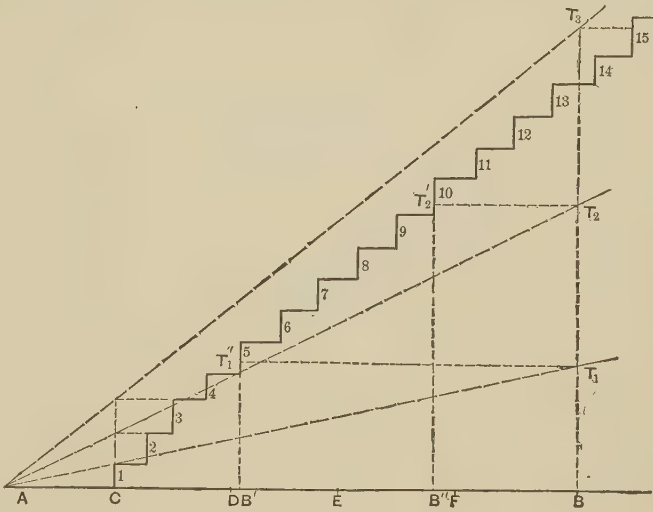

FIG. 134.

Wheel 1 at the right-hand panel point of any panel will give a maximum shear so long as $\frac{G}{l} \leqq \frac{P_1}{d}$. This limiting value of G is the ordinate BT_1 at the right end of the truss. Hence P_1 gives a maximum until P_5 comes on, that is, for the distance $B'C$ at the right end of the truss. Wheel 2 gives a maximum for values of $\frac{G}{l}$ between $\frac{P_1}{d}$ and $\frac{P_1+P_2}{d}$, or for values of G between BT_1 and BT_2. That is, wheel 2 gives a maximum shear in the panel to the left of each joint crossed by it in moving the loads from the position with wheel 5 at B to the position with wheel 10 at B. The strip of paper should be placed with the point B first at B' and then at B'', the point below wheel 2 being marked in each position. The space between these marks is the space in which the shear is dominated by P_2. It is seen to overlap the space $B'C$, dominated by wheel 1, by the distance between wheels, as shown in Art. 96. The ordinate from B to AT_3 being greater than G when P_3 is at C, the last panel point, we may say that P_3 dominates the shear in the truss from a position with wheel 10 at B, to the left end. We thus have, very briefly, the position for maximum shear in all the panels. None of the lines need be actually drawn.

The shears themselves are readily found from the moment lines. The left reaction is equal to the ordinate to the equilibrium polygon at the right abutment, divided by the length of the truss. The panel load to be subtracted is equal to the similar ordinate at the right end of the panel, divided by a panel length.* These ordinates are measured from the line o–o if there are no wheels to the left of the left abutment or the left panel point. If there are, then the ordinates are measured from the moment line corresponding to the wheel nearest the bridge or panel on the left.

EXAMPLE 1. Find the maximum stresses in all the members of the truss of Art. 96.

EXAMPLE 2. Find the maximum moments and shears at points 10 ft. apart in a plate girder 100 ft. long.

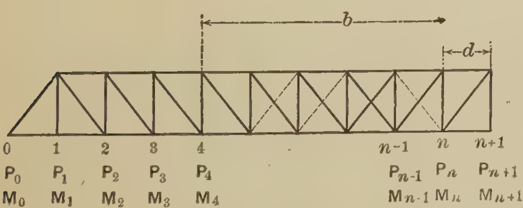

FIG. 135.

101. Computation of Panel Concentrations.—It is frequently required to compute all the panel loads in a truss for some single position of the loading; as, for example, with wheel 3 at the first panel point from the left end. A convenient formula is readily derived as follows:

In Fig. 135, let P_0, P_1, . . . P_n, P_{n+1}, etc., be the panel concentrations; M_0, M_1, . . . M_n, M_{n+1}, etc., the summation of the moments of the loads to the left of each panel point about

* The student must remember that in truss analysis, for both shears and moments, the wheel loads must first be concentrated at the joints before they can come upon the truss, and when the method of sections is employed *joint loads only are to be considered.* The external reactions, however, can be computed from the actual positions of the loads.

that point; b, the distance from panel point n to any other panel point; and d, the panel length. Suppose P_n is required. By equating the moments of the loads about n with the moments of the panel reactions we have

$$\Sigma_0^{n-1} Pb = M_n. \quad \ldots \ldots \ldots \ldots \quad (a)$$

Similarly with centre at $n + 1$,

$$\Sigma_0^{n} P(b + d), = \Sigma_0^{n-1} Pb + d\Sigma_0^{n-1} P + P_n d, = M_{n+1}, \quad \ldots \ldots \quad (b)$$

and with centre at $n - 1$,

$$\Sigma_0^{n-1} P(b - d), = \Sigma_0^{n-1} Pb - d\Sigma_0^{n-1} P, = M_{n-1}. \quad \ldots \ldots \ldots \quad (c)$$

Substituting from (a) and (c) in (b) we readily get

$$P_n = \frac{M_{n+1} - 2M_n + M_{n-1}}{d}. \quad \ldots \ldots \ldots \ldots \quad (8)$$

The values of the M's can be taken from the diagram, or may be computed with the aid of the moment table, Fig. 129. For an example of the application of eq. (8), see Art. 107.

TRUSSES WITH INCLINED WEB MEMBERS AND INCLINED CHORDS.

102. Maximum Moment at Joints in the Unloaded Chord.—Let AB, Fig. 136, be any truss with horizontal or inclined chords, and C any joint of the unloaded chord, not verti-

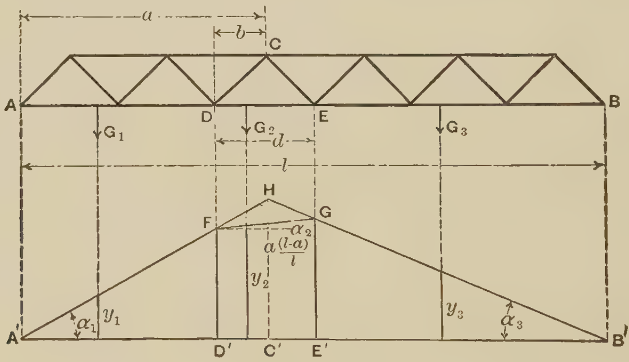

FIG. 136.

cally over a joint of the loaded chord. Let $b =$ the horizontal distance from C to the next loaded chord joint towards A, and $a =$ the horizontal distance from C to A. The other notation is the same as that previously used. The effect upon the moment at C due to a load unity moving from B to E and from D to A is the same as for the moment at a point C in a beam; hence the influence line for that portion is $B'G$ and FA', portions of the influence line AHB', where $C'H = \frac{(l - a)a}{l}$. Between E and D the load is carried by the stringer to these panel points. The influence line for this portion will then be the straight line GF.

To derive the criterion for maximum moment at C, let G_1, G_2, and G_3 represent the portions of the load that lie in the segments AD, DE, and EB, respectively. Let the loads advance a small distance δx to the left. The increase in moment at C will be

$$\delta M = G_3 \delta x \tan \alpha_3 - G_2 \delta x \tan \alpha_2 - G_1 \delta x \tan \alpha_1.$$

For a maximum,

$$\frac{\delta M}{\delta x} = 0 = G_3 \tan \alpha_3 - G_2 \tan \alpha_2 - G_1 \tan \alpha_1. \quad \ldots \ldots \ldots \quad (a)$$

Now we have

$$\tan \alpha_3 = \frac{a}{l}; \quad \tan \alpha_1 = \frac{l-a}{l}; \quad \tan \alpha_2 = \frac{GE' - FD'}{d}.$$

Also

$$GE' = [l - (a - b + d)] \tan \alpha_3 \quad \text{and} \quad FD' = (a - b) \tan \alpha_1.$$

Substituting these values of GE' and FD' in the expression for $\tan \alpha_2$ we have

$$\tan \alpha_2 = \frac{bl - ad}{ld}.$$

Eq. (a) then becomes

$$G_3\frac{a}{l} - G_2\frac{bl - ad}{ld} - G_1\frac{l-a}{l} = 0.$$

If $G = G_1 + G_2 + G_3$, we have, after reduction,

$$\frac{G}{l} - \frac{G_2\frac{b}{d} + G_1}{a} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

or

$$\frac{G}{l} = \frac{G_2\frac{b}{d} + G_1}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For a maximum the left-hand member of eq. (b) must become zero by passing from positive to negative. This can occur only when some wheel passes E or D, as then only can $\dfrac{G_2\frac{b}{d} + G_1}{a}$ be increased. Equation (9) is the criterion required.

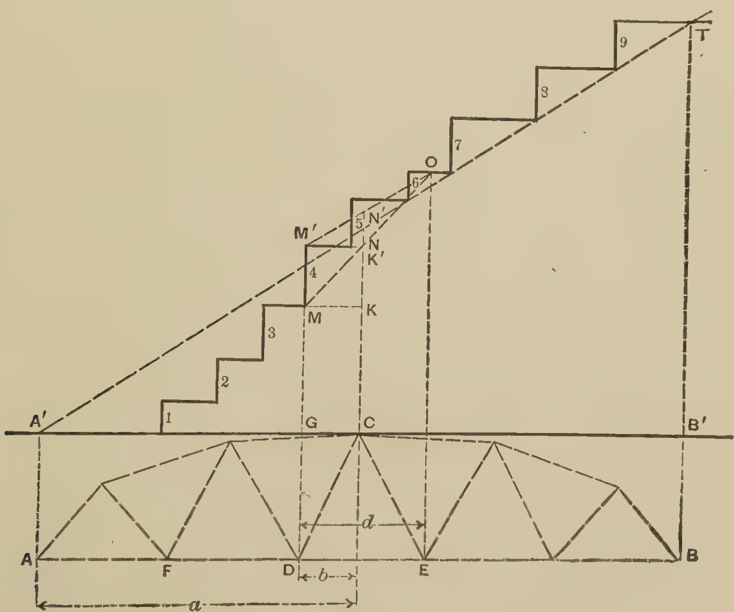

FIG. 137.

Fig. 137 shows a load line and the outline of a truss AB. Try wheel 4 at D for a maximum moment at C. The slope of the line $A'T$ is equal to the left-hand member of eq. (9). G_1

varies between the values GM and GM', or CK and CK', according as wheel 4 is considered in or out of the panel DE. The value of G_2 varies correspondingly between the sum of the loads 6, 5, and 4, and the sum of 6 and 5 only. The corresponding values of $G_2\frac{b}{d}$ are found by drawing MO and $M'O$. They are KN and $K'N'$. The slopes of the lines $A'N$ and $A'N'$, if drawn, would therefore be the two values of the second member of eq. (9). Hence when $A'T$ cuts NN', then this position of the loads gives a maximum moment at C. None of the lines need be drawn, the points N and N' being lightly marked and a thread, $A'T$, stretched from A' to the intersection of the vertical at B with the load line. With a load at E, two points similar to N and N' are obtained by lines drawn from the intersection of the vertical through D with the load line, to the upper and lower limits of the load at E.

The position of the loads having been found, the bending moment at C is most readily found by proportion from the moments at D and E. Thus if M_D and M_E are the bending moments at D and E, respectively, found in the usual way, then the moment M_C, at C, $= M_D + (M_E - M_D)\frac{b}{d}$.

103. Maximum Web Stresses.—Let ED, Fig. 138, be any web number of a truss with inclined chords. If I is the intersection of the chord members EF and CD, then the stress

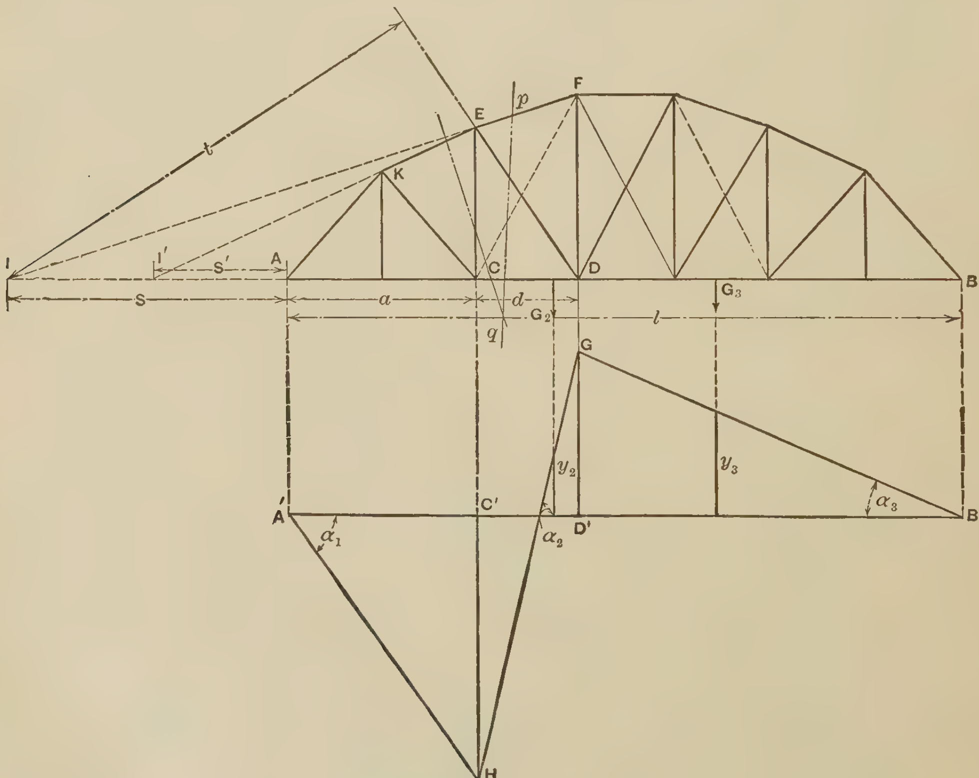

FIG. 138

in ED is equal to the sum of the moments about I of all the external forces acting upon the portion to the left of pq, divided by the lever-arm t. The stress in ED is then a maximum when this moment is a maximum.

The influence line for moment at I is readily drawn, after first finding the values of this moment for a load unity at D and at C. With load unity at D, the moment at $I = \frac{l-a-d}{l}s$,

and with the load at C the moment $= \frac{l-a}{l}s - (a+s)$, a negative quantity. Laying off $D'G$ and $C'H$ equal respectively to these moments, and joining $A'H$, HG, and GB', we have the required influence line for moment at I.

The maximum stress in ED occurs when some of the wheels at the head of the train are in the panel CD, and in exceptional cases only, when some of the loads are to the left of C. In the cases treated we will assume that there are no loads on the portion AC.

Let G_2 represent the total load in the panel CD; G_3, the load in DB; and G, the total load on the bridge. Let the loads advance from any given position, a distance δx towards the left. The moment at I will be increased by an amount

$$\delta M = G_3\delta x \tan\alpha_3 - G_2\delta x \tan\alpha_2;$$

and for a maximum,

$$\frac{\delta M}{\delta x} = 0 = G_3 \tan\alpha_3 - G_2 \tan\alpha_2. \qquad (a)$$

Now

$$\tan\alpha_3 = \frac{D'G}{l-a-d} = \frac{s}{l}, \qquad (b)$$

and

$$\tan\alpha_2 = \frac{D'G - C'H}{d} = \frac{-\frac{ds}{l}+a+s}{d}. \qquad (c)$$

Substituting from (b) and (c) in (a), we have, by putting G for $G_2 + G_3$,

$$\frac{G}{l} - G_2\frac{1+\frac{a}{s}}{d} = 0, \qquad (d)$$

or

$$\frac{G}{l} = G_2\frac{\left(1+\frac{a}{s}\right)}{d}, \qquad (10)$$

as the criterion for maximum moment at I or maximum stress in ED.

This criterion is nearly the same as that for maximum shear in Art. 94; differing only in that here the load in the panel is to be increased by the $\frac{a}{s}$th part of itself before dividing by the panel length. This correction should be added to the ordinate for G_2 when using the diagram, before the thread is stretched.

For the member EC the same panel CD is partially loaded and s is replaced by s', while the other quantities remain the same as for ED.

The actual stress in ED is most readily obtained by first determining the horizontal components in EF and CD, as was done in Chap. IV, Art. 80.

For EC, or for any web member where verticals are not used, it is easier to get the moments, about I, of the abutment reaction at A and the panel load at C, and then divide by the lever-arm of the member. This reaction and panel load are obtained from the summation of moments, or from the diagram, as before.

104. Application of the Foregoing Principles.—The truss of Fig. 109, p. 68, having both inclined web members and inclined chords, will serve to illustrate the principles of the two preceding articles. The stresses in all the members will be found, using the loads of the diagram Fig. 131.

1st. *Upper Chords.*—The truss should first be drawn to a large scale, so that any required lever-arm can be scaled off with sufficient accuracy.

The centres of moments for the upper-chord members are at the lower chord joints. These moments are found as in a beam or a Pratt truss, Art. 96, and hence the computations will not be given here in detail. The table of chord stresses below gives the positions of loads for the greatest maximum moments, the corresponding moments, the lever-arms of the various chord members, and the resulting stresses.

When the piece MN is reached, the centre of moments is at E or F according as EN or

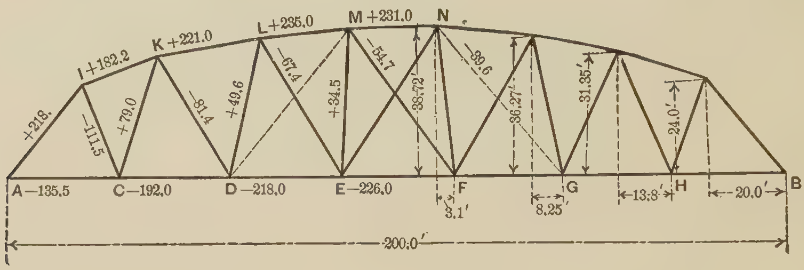

FIG. 139.

MF is in action. Trying the point E, we have as our maximum moment 8950. The corresponding shear in the panel EF is found by subtracting from the reaction at A the loads between A and E and the portion of the loads in EF going to E. It is found to be -12.3; hence EN must be in action and the proper centre of moments for this position of the loads is as we have assumed. The maximum moment at F is 8760, which, being less than 8950, is not considered. Hence the maximum stress in MN is due to a moment at E of 8950.

2d. *Lower Chords.*—The centres of moments for the lower chord members are at the upper chord joints. The positions of the loads giving maximum moments at these joints are found by means of the diagram Fig. 131, as explained in Art. 102.

Moment at I.—This is a maximum for wheel 4 at C. The moment is found by proportion, from the moments at A and C (M_A and M_C). $M_A = 0$. M_C is readily found from the diagram Fig. 131, by reading off the ordinate to the curved moment line at the right end of the truss, dividing by 200, multiplying by 28.57, and subtracting the ordinate to the moment curve at C. Thus,

$$M_C = \frac{35000}{200} \times 28.57 - 360$$

$$= \frac{35000}{7} \times 1 - 360 = 4640.$$

Then $M_I = M_A + (M_C - M_A)\frac{20}{28.6} = 3250$ as given in the table. The lever-arm of $AC = 24$ ft.

Moment at K.—Wheel 9 at D and wheel 4 at C are the two positions giving maximum moments.

With wheel 9 at D

$$M_D = 34500 \times \tfrac{2}{7} - 2400 = 7460,$$

$$M_C = 34500 \times \tfrac{1}{7} - 300 = 4630,$$

and

$$M_K = 4630 + (7460 - 4630)\frac{13.8}{28.6} = 6000.$$

With wheel 4 at C

$$M_D = 35000 \times \tfrac{2}{7} - 2600 = 7400,$$

$$M_C = 35000 \times \tfrac{1}{7} - 360 = 4640,$$

and

$$M_K = 4640 + (7400 - 4640)\frac{13.8}{28.6} = 5970.$$

The maximum moment is therefore 6000, and this divided by the lever-arm 31.35, gives the stress in *CD*, which equals 192 thousand lbs.

Moment at L.—Wheel 8 at *D* and wheel 13 at *E* are the two positions giving maxima. The greatest of the two moments is when wheel 8 is at *D* and is equal to 7900.

Moment at M or N for Stress in EF.—When, for any position of the loads, the moment at *E* is greater than the moment at *F*, the shear in panel *EF* is negative, and hence *EN* is in action; and when the moment at *F* is greater than the moment at *E*, then *MF* is in action. This follows readily from shear = differential of the moment, or from the segment under *EF* of the equilibrium polygon.

Wheels 12 and 13 at *E* are found to give maximum moments at *M*, but in each case the moment at *E* is greater than the moment at *F*, and therefore the centre of moments for *EF* is not at *M*.

Wheels 14, 15, and 16 at *F* are found to give maximum moments at *N*, and moreover with 15 at *F* the moments at *E* and *F* each equal 8760. This is evidently the greatest possible moment at *F* that is not greater than the corresponding moment at *E*, and hence the maximum moment at *N* is also 8760.

The following table gives the chord stresses in thousands of pounds.

LIVE LOAD CHORD STRESSES.

Member.	Centre of Moments.	Position of Loads.	Maximum Moment.	Lever-arm.	Stress.
IK	*C*	4 at *C*	4640	25.5	+ 182
KL	*D*	8 at *D*	7470	33.7	+ 221
LM	*E*	13 at *E*	8950	38.1	+ 235
MN	*E*	13 at *E*	8950	38.7	+ 231
AC	*I*	4 at *C*	3250	24.0	− 135.5
CD	*K*	9 at *D*	6000	31.3	− 192
DE	*L*	8 at *D*	7900	36.3	− 218
EF	*N*	15 at *F*	8760	38.7	− 226

3d. *Web Stresses.*—We first produce the upper chord members to their intersection with the lower chord, and scale off the distances of these intersections from the point *A*. These are the quantities "*s*" of Art. 80. The ratios $\frac{a}{s}$ are given in the third column of the following table of web stresses. They need be computed only to the nearest tenth.

The horizontal component of the maximum stress in *AI* is equal to the maximum stress in $AC = 135.5$. For *IC*, the ratio $\frac{a}{s} = 0$, and the same position of loads gives a maximum stress in this member as in *AC*; that is, wheel 4 is at *C*. The horizontal component of stress in *IC* = hor. comp. *IK* − hor. comp. *AC*. To find these horizontal components we need the moments at *A*, *I*, and *C*. The moments at *A* and *C* are given in the fifth and sixth columns of the table below. The moment at *I*, found by proportion from these two moments, is given in the next column. The hor. comp. in *IK*, found by dividing the moment at *C* by the vertical distance from *C* to *IK*, s given in column 8. The stress in *AC* is given in the next column, and the difference between columns 9 and 8 is the hor. comp. in *IC*. The actual stress is given in the last column.

For the member *CK*, $\frac{a}{s} = 0.6$. In testing for the position of the loads we then multiply

the loads in the panel *CD* by 1.6, and use this product in the place of the loads themselves. (See Art. 80.) Wheels 2 and 3 at *D* are two positions giving maxima. Only the greater one is given in the table. The stress in *CK* is found in the same way as was that in *IC*, that is, by finding the horizontal components of the stresses in *IK* and *CD*. The stresses in *KD*, *LE*, *MF*, and *NG* are found in like manner.

The members *LD* and *ME* being so nearly vertical, it will be more accurate to find the stresses in these pieces by finding their *vertical* components.

The vertical component in $LD =$ shear in $DE -$ vert. comp. KL, $=$ left reaction $-$ load at $D -$ vert. comp. KL. The vertical component in *KL* is readily found from its horizontal component, and the other quantities are easily computed. Likewise the vert. comp. $ME =$ shear in $EF -$ vert. comp. LM.

The *horizontal* components of the stresses in *KL* and *LM* are given in column 8 of the table.

The stresses are given also along the members in Fig. 139.

LIVE LOAD WEB STRESSES.

Member.	Length.	$\frac{a}{s}$	Position of Loads.	Moments at Lower Chord. Left.	Moments at Lower Chord. Right.	Moment at Up Chord.	Hor. Comp. Up. Chord.	Hor. Comp. Lower Ch'd.	Hor. Comp. Web.	Stress in Web.
1	2	3	4	5	6	7	8	9	10	11
AI	31.2		4 at *C*					135.5	135.5	+ 212.
IC	25.5	0	4 at *C*	0	4640	3250	173.1	135.5	37.6	− 111.5
CK	34.3	.6	3 at *D*	3520	6840	5122	131.3	163.1	31.8	+ 79.0
KD	34.8	.3	3 at *D*	3520	6840	5122	197.7	163.1	34.6	− 81.4
LE	41.6	.2	3 at *E*	4846	7069	5487	184.1	151.2	32.9	− 67.4
MF	46.3	0	3 at *F*	4527	5836	4669	150.8	120.7	30.1	− 54.7
NG	50.0	−.2	2 at *G*	2662	3268	2598	92.1	67.1	25.0	− 39.5
									Vt. Comp.	
LD	37.2	.6	2 at *E*	4460			128.9		48.4	+ 49.6
ME	38.8	.3	2 at *F*	4050			105.4		34.3	+ 34.5

105. The Petit Truss.—The maximum stress in the piece *EF*, Fig. 140, is equal to the maximum moment at *D*, divided by its lever-arm. The maximum stresses in the members

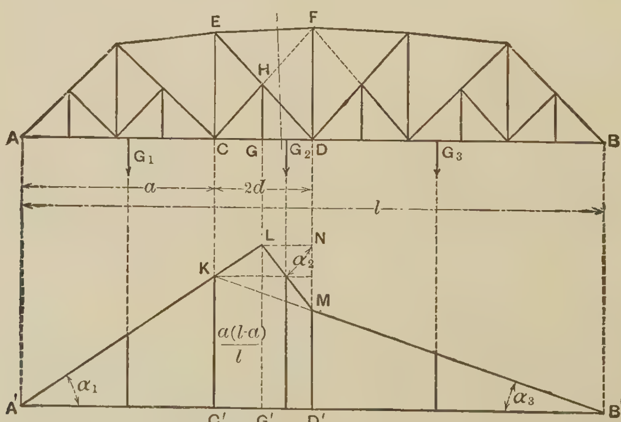

FIG. 140

HG and *HC* are due to the maximum joint load. The piece *HD* when stressed is the only web member in action in the panel *GD*, and hence its maximum stress is determined according to Art. 103. The same applies to the counter, *HF*. There remain then to be treated only the pieces *CD*, *EC*, *EH*, and *CH* when acting with the counter *HF*.

1st. *The piece CD.*—The centre of moments for CD is at E, but the joint G is on the *left* of the section. The influence line for the portions AC and DB will be $A'K$ and MB', where $KC' = \frac{a(l-a)}{l}$, which is the same as the ordinary influence line for moment in a beam; but as the unit load moves from C to G, the moment about E of the forces on the *left* of the section is uniformly increased at the same rate as from A to C. Hence KL will be a prolongation of $A'K$. From G to D the influence line is the straight line LM.

To determine the position for a maximum moment at E: Let G_1, G_2, G_3, and G be respectively the load on AG, GD, DB, and the entire bridge.

A small movement of the loads to the left will increase the moment by an amount equal to

$$\delta M = G_3\delta x \tan \alpha_3 + G_2\delta x \tan \alpha_2 - G_1\delta x \tan \alpha_1 .$$

For a maximum,

$$\frac{\delta M}{\delta x} = 0 = G_3 \tan \alpha_3 + G_2 \tan \alpha_2 - G_1 \tan \alpha_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Now we have

$$\tan \alpha_3 = \frac{a}{l}; \quad \tan \alpha_1 = \frac{l-a}{l};$$

and

$$\tan \alpha_2 = \frac{MN}{d} = \frac{d \tan \alpha_1 + 2d \tan \alpha_3}{d} = \frac{l-a}{l} + \frac{2a}{l} = \frac{l+a}{l}.$$

Substituting these values in (a) and reducing, we have as the condition for a maximum,

$$\frac{G}{l} = \frac{G_1 - G_2}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

To satisfy this criterion a load must be at G.

Eq. (11) is easy of application by means of either the table or diagram.

The moment at E is equal to the reaction at A multiplied by AC, minus the moment of the wheel-loads to the left of C about C, plus the moment of the joint load at G about C. This joint load is, by Art. 91, equal to the bending moment in the centre of a beam of length $2d$, multiplied by $\frac{2}{d}$. Hence the moment of the joint load G about C is equal to twice the bending moment at G if CD were a stringer. In other words, the total moment at E is equal to the bending moment at C in a beam AB, plus twice the bending moment at G in the beam CD.

2d. *The piece EH.*—When the stress in this piece is a maximum, HF is not in action. Now for any position of the loads, the stress in EH would not be altered if HD and ED were to be made separate parallel pieces, as the portion CHD would then form a separate small truss. But this small truss serves merely the office of a floor-stringer in carrying loads to the joints C and D. Hence the stress in EH is the same, and the position of loads for a maximum stress is the same, as for ED were the pieces CH and HG removed. This case reduces then to the case of Art. 103, in which the panel length is CD.

3d. *The piece EC.*—For this piece, as for EH, we may consider CH and HG removed and proceed as in Art. 103.

4th. *The piece CH when acting as a tie.*—Here the member HD is not in action. The same process applies to CH as applied to EH. That is, consider EH and HG removed, and find the maximum stress in CF. This will be the maximum in CH, but for the maximum in HF the panel to be considered is GD.

DOUBLE SYSTEMS.

106. The influence line for either chord or web stress in a truss with a double system of bracing is made up of straight lines of many different inclinations. Since the criteria for maximum values contain as many terms as there are different inclinations in the influence lines, in this case they are very difficult of application.

For example, take the Whipple truss of Fig. 141. The centre of moments for the chord member FH for loads on the full system is at C. For loads at the joints of this system the ordinates for moment at C are ordinates to the influence line $A'IB'$. While for loads on the dotted system, the centre of moments for FH is at D, and the ordinates for moment at D are ordinates to $A'KB'$. Hence the influence line for stress in FH is the broken line $A'abcIKde \ldots B'$. The points a and i are taken half-way between $A'KB'$ and $A'IB'$, assuming the loads at M and N to be equally divided between the two systems.

The influence line for shear in panel CE of the full system is the full broken line of the lower figure, the ordinates to this line being zero for the loads at joints of the dotted system. Loads at M and N are equally divided.

The influence line for shear exhibits in a striking manner the effect of concentrated loads upon the web members when the distance between the loads is an even number of panel lengths. As, for example, two engine concentrations when the panel lengths are twenty-five feet, the length of a locomotive being about fifty feet. Panels of such length are therefore,

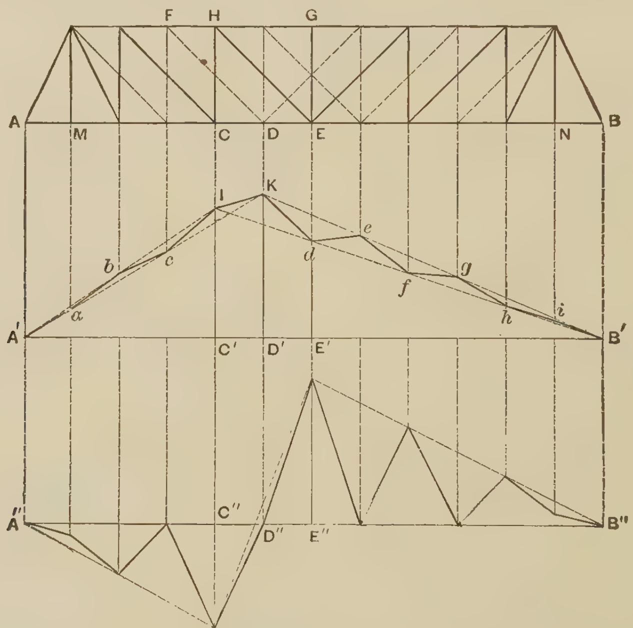

FIG. 141.

by all means, to be avoided in a double system, and a length chosen which will bring the concentrations upon different systems.

The cumulative effect upon chord members is seen to be very small.

In computing stresses in a double-intersection truss, either an equivalent uniform load should be taken, according to the method of Chapter VI, or, as is often done, such a position of the loads should be selected as will give the maximum joint load at the end panel, and this same position retained for all the chord stresses. For web stresses the load should be moved one panel to the right each time, and the portion of the load going to the joint in front neglected. The stresses are then obtained by computing the panel concentrations for these positions by Art. 101 and treating each system separately.

By placing the loads as above indicated only one set of concentrations is required. This method is applicable to either parallel or inclined chords. In the latter the chords are to be considered as straight between successive joints of the system under consideration.

A more accurate method for chord stresses would be to find the *position* of the loading as for moment in a single-intersection truss, then compute the corresponding panel concentrations and find the stress in the member in question, as above explained. This would involve the computation of panel concentrations for each different position, but would probably give stresses very near the exact maximum.

107. Stresses in a Whipple Truss.—Let us take, for example, the truss of Fig. 142. Span = 272 ft.; panel length = d = 17 ft.; height = h = 38 ft. Loading to be as given in the diagram and table. The method of panel concentrations will be used, one position of the loads being taken for all chord stresses.

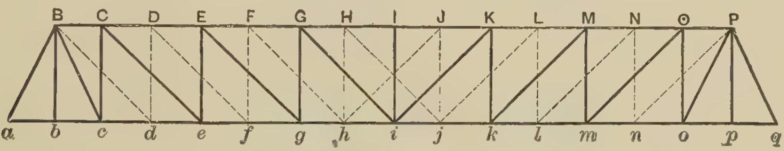

FIG. 142.

1st. *Panel Concentrations.*—Wheel 4 at b is found by trial to give the maximum load at b. By eq. (8), p. 88, any panel concentration, $P_n, = \dfrac{M_{n+1} - 2M_n + M_{n-1}}{d}$, where M_n, M_{n+1}, and M_{n-1} are the summations of moments at the joint in question and the ones to the right and left respectively, and d is the panel length.

With wheel 4 at b we have the following positions:

Wheel.	Joint.	Distance of Wheel to the left.	Wheel.	Joint.	Distance of Wheel to the left.
4	b	0.0	12	e	1.9
7	c	0.6	15	f	2.8
9	d	7.0	end of uniform load	g	0.4

Calling joint a zero, we have the following values of M with wheel 4 at b:

$$M_0 = 0;$$
$$M_1 = 369.2 - 8 \times 18.4 = 222.0;$$
$$M_2 = 1460.1 + 86 \times 0.6 - 8 \times 35.4 = 1228.5;$$
$$M_3 = 2414.9 + 104 \times 7.0 - 8 \times 52.4 = 2723.7;$$
$$M_4 = 4911.5 + 142 \times 1.9 - 8 \times 69.4 = 4626.1;$$
$$M_5 = 7478.2 + 181 \times 2.8 - 8 \times 86.4 = 7293.8;$$
$$M_6 = 11236.2 + 208 \times 0.4 + \frac{(0.4)^2}{2} \times 1.5 - 8 \times 103.4 = 10492.3$$
$$M_7 = 10492.3 + (200 + 0.4 \times 1.5) \times 17 + \frac{(17)^2}{2} \times 1.5 = 14119.3.$$

The values of P, beginning at P_0, are:

$$P_0 = \frac{222.0}{17} = 13.06;$$

$$P_1 = \frac{1228.5 - 2 \times 222.0}{17} = 46.15;$$

$$P_2 = \frac{2723.7 - 2 \times 1228.5 + 222.0}{17} = 28.75;$$

$$P_3 = \frac{4626.1 - 2 \times 2723.7 + 1228.5}{17} = 23.95;$$

$$P_4 = \frac{7293.8 - 2 \times 4626.1 + 2723.7}{17} = 45.02;$$

$$P_5 = \frac{10492.3 - 2 \times 7293.8 + 4626.1}{17} = 31.22;$$

$$P_6 = \frac{14119.3 - 2 \times 10492.3 + 7293.8}{17} = 25.21.$$

Beyond P_6 the concentrations are each equal to a panel length of uniform load $= 17 \times 1.5 = 25.5$.

2d. *Web Stresses.*—The secant of the angle which the members aB and Bc make with the vertical $= \sqrt{17^2 + 38^2} \div 38 = 1.096$; and that of the angle which the remaining diagonals make with the vertical $= \sqrt{34^2 + 38^2} \div 38 = 1.342$. The loads at b and p will be assumed to be equally divided between the two systems.

The stress in aB is found from the shear in ab, with P_1 at b. We have then

$$\text{stress in } aB = 1.096 \times \tfrac{1}{16}[15 \times 46.15 + 14 \times 28.75 + 13 \times 23.95 + 12 \times 45.02 \\ + 11 \times 31.22 + 10 \times 25.21 + (9 + 8 + \ldots + 1) \times 255] = 25.2.7.$$

For Bc, P_1 is placed at c; whence

$$\text{stress in } Bc = 1.096 \times \tfrac{1}{16}[14 \times 46.15 + 12 \times 23.95 + 10 \times 31.22 \\ + (8 + 6 + 4 + 2 + \tfrac{1}{2}) \times 25.5] = 121.2.$$

For cC and Ce, P_1 is at e, and we have

$$\text{stress in } cC = \tfrac{1}{16}[12 \times 46.15 + 10 \times 23.95 + 8 \times 31.22 + (6 + 4 + 2 + \tfrac{1}{2}) \times 25.5] = 85.2;$$
$$\text{stress in } Ce = 1.342 \times 85.2 = 114.3.$$

For Bd, P_1 is at d, and

$$\text{stress in } Bd = 1.342 \times \tfrac{1}{16}[13 \times 46.15 + 11 \times 23.95 + 9 \times 31.22 \\ + (7 + 5 + 3 + \tfrac{1}{2}) \times 25.5] = 129.1.$$

For dD and Df, P_1 is at f; etc.

The maximum stress in $Bb = P_1 = 46.15$.

3d. *Chord Stresses.*—For all chord stresses P_1 is at b.

The stress in ac = hor. comp. $aB = \dfrac{245.1}{1.096} \times \dfrac{17}{38} = 100.1$.

The stress in any other chord member, as DE, is found by obtaining separately the portion due to each system and then adding the two results. For the full system, the centre of moments is at e, and for the dotted system it is at f.

Abutment reaction, full system

$$= \tfrac{1}{16}[\tfrac{1}{2} \times 46.15 \times 15 + 14 \times 28.75 + 12 \times 45.02 + 10 \times 25.21 + (8 + 6 + 4 + 2 + \tfrac{1}{2}) \times 25.5] = 129.0;$$

whence, stress in DE due to loads on the full system

$$= [129.0 \times 68 - (\tfrac{1}{2} \times 46.15 \times 51 + 28.75 \times 34)] \div 38 = 174.2.$$

Abutment reaction, dotted system,

$$= \tfrac{1}{16}[\tfrac{1}{2} \times 46.15 \times 15 + 13 \times 23.95 + 11 \times 31.22 + (9 + 7 + 5 + 3 + \tfrac{1}{2}) \times 25.5] = 101.6,$$

and hence the stress in DE due to loads on the dotted system

$$= [101.6 \times 85 - (\tfrac{1}{2} \times 46.15 \times 68 + 23.95 \times 34)] \div 38 = 164.5.$$

The total stress in DE or fg therefore $= 174.2 + 164.5 = 338.7$.

The stresses in the other chord members are found in a similar manner, using the reactions above found. Near the centre of the truss it may be necessary, in selecting the centre of moments, to find the sign of the shear in one or both of the systems in order to know which diagonal is in action. This is easily done, as the abutment reaction and panel loads are already known.

SKEW-BRIDGES.

108. Fig. 143 illustrates a skew-bridge similar to that of Figs. 119–121, p. 71. As in Art. 86, the loads may be considered as applied along the centre line XY. As an example, let us find the influence line for moment at F. Loads from c to i will evidently have the same effect upon the truss AB as if the bridge were square and equal to AB in length. The influence line from c to i will then be the portion $c''f''i''$, Fig. 144, of the influence line $A''f''B''$ for moment at f in a beam of length AB. Beyond c'' and i'' the line is a straight line with zero ordinates at a' and b'.

From this influence line the criterion for maximum moment may be written out. It is seen, however, that the true influence line differs but slightly from $A''f''B''$, and as regards the *position* of the loads, it may be assumed the same. Hence for a maximum moment at F the average unit load on length AF must be equal to that on length FB.

For moment at the point I the influence line would be straight from i to c, and in that case it would be more exact to make the average load on the length AI equal to that on ib.

The influence line for moment at l, Fig. 143, for finding the stress in CD, is $a'C''D''i''b'$, Fig. 145; and is drawn by drawing the influence line $A''l''B''$ for moment at l in a beam of length AB, and then drawing the straight lines $a'C''$, $C''D''$ and $i''b'$.

For moment at l for stress in AC the influence line is $a'l'''i''b'$. In any case the influence line may be drawn and the exact criterion worked out if desired; or an approximate one found by drawing such a line of the form $A''f''B''$, Fig. 144, as will correspond most closely to the actual influence line. The criterion is then the ordinary one for moments, the segments or lengths to be used in getting average load being in any case the distances corresponding to $A''f'$ and $f'B''$.

The position of loads for maximum shear or web stress may be found in a manner similar to that explained above. The same criterion as for shear in a square truss may generally be employed, using the *truss* length for all panels except the end ones, while for these end panels use the truss length plus $a'A''$, Fig. 144.

It is to be noted that the above approximate methods deal only with the *positions* of the loads and not with actual stresses. It is at most a question of which of two or three different loads shall be at the point, when any one of them would give very close results. In most cases the approximate methods give the correct wheel and hence correct results.

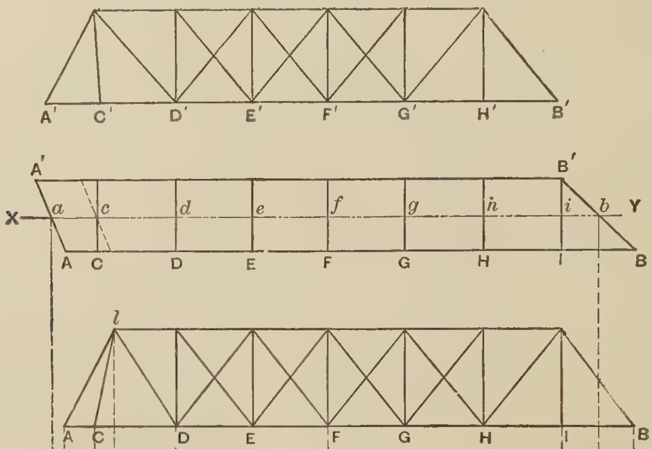

FIG. 143.

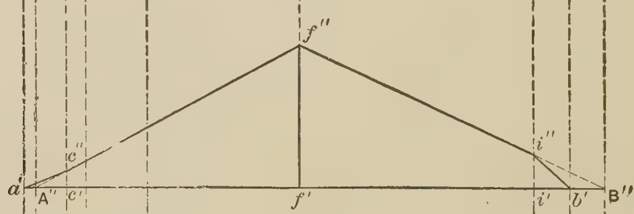

FIG. 144.

FIG. 145.

In getting stresses all loads between c and i are to be treated in the same way as in a square truss. That is, one half acts between C and I of truss AB, and the other half between C' and B' of truss $A'B'$. The loads from a to c and from i to b are best treated by finding the floor-beam reactions at c and i due to these loads, and then transferring one half of each floor-beam load to each truss. The trusses are then treated independently in the ordinary way.

CHAPTER VI.

CONVENTIONAL METHODS OF TREATING TRAIN-LOADS ON TRUSS-BRIDGES OVER 100 FEET IN LENGTH.

109. The Train-load now universally taken in the dimensioning of members in railway bridges in America consists of two of the heaviest engines in use on the line, coupled in a direct position at the head of the heaviest known train-load. The weights of the engines and tenders are assumed to be concentrated at the wheel-bearings, giving definite loads at these points, while the train-load is taken as uniformly distributed. Although engine and train loads have greatly increased in the past twenty years, it is highly probable that the loads now generally assumed will never be materially too small for the actual traffic, except in special cases. These assumed loads are now equivalent to about 4000 lbs. per foot under the engines and their tenders, or over a distance of some 103 feet, followed by a train-load of 3000 lbs. per foot. The only exact solution for the maximum stresses produced by such a load moving across the span has already been given in Chapter V. If each span had to be computed but once, there would be no valid objection to the use of this exact method, which has been almost exclusively employed in America since about the year 1880. It has come to be a common practice, however, to call for bids (either by the span or by the pound) under general specifications, requiring accurate "stress sheets," showing maximum loads and sizes of members, to be submitted with each bid. If on the average there are eight bidders, then the computations have been made independently eight times, and on the average each bidder computes eight spans for each one he succeeds in building. When the laborious method of wheel concentrations is employed the cost of this high-priced labor becomes a considerable portion of the price named for building the bridge. In the end the purchaser pays for the computations eight times over. At present (1892) there is a general desire on the part of the bridge companies to agree upon some conventional method of treating the train-load which will lead to easy and short computations without introducing material errors in the results. The consulting engineers who act for the railway companies are generally inclined to adhere to the more exact methods. From a set of observations taken on bridge members under rapidly moving train loads, given in *Engineering News*, May 9, 1895, it appears that the actual stresses, even on the hip-verticals, are not appreciably greater than those computed from the same loads treated statically, and hence a much greater weight may be given to the computed stresses than has heretofore been done, and there is therefore the greater reason to adhere to the more rigid methods of analysis. It would probably be admitted by all parties that results differing by not more than two or three per cent are practically identical in bridge computations, especially if those differences are both plus and minus and do not appreciably change the total weight. It remains to show that convenient methods of solution can be found which satisfy this requirement for truss-bridges longer than 100 feet.

Five conventional methods of treating the train-load have been employed.

First. The use of two concentrated excess loads, placed fifty feet apart, which may occupy any position in the uniform train load, and which may be conceived as rolling across the span on top of the train load. They would be near the head of the train for shears and one of them at the joint in question for moments.*

* These excess loads to be placed so as to give maximum moments at the several joints, the same as described below for one concentrated load.

Second. The use of one such concentrated excess load in place of two, it always being at the joint in question, while the train load covers the whole span for maximum moments, and reaches to a particular point in the panel in question for maximum shears.

Third. The use of a Uniformly Distributed Excess over about one hundred feet at the head of the train.

Fourth. The use of what is called an Equivalent Uniform Load.

Fifth. The use of a given Uniform Load, all of which over a given space in the middle of the train is to be concentrated on two or four axles.

The first of these methods is apparently a near approximation to the actual loads, and is used by Prof. DuBois in his *Framed Structures.* It is a fair substitute for the wheel concentrations on double-intersection trusses, but has never come into general use, the objection being either that it did not sufficiently shorten the work to warrant its use in place of the more exact method, or that no rational method has been proposed for finding the proper values of the excess loads to use.

The second method was devised by Mr. Geo. H. Pegram, M. Am. Soc. C. E., and published by him in 1886.* It has been introduced into some standard specifications because of its great simplicity and close agreement with the method by wheel loads. A modification of it is given below in detail.

The third approximate method has been used by Mr. C. L. Strobel, M. Am. Soc. C.E., for spans exceeding two hundred feet,† but has been abandoned by him since train loads per foot have so nearly equalled the average load over the engine portion.

The fourth method, of an equivalent uniform load, has been used more or less for many years and seems now likely to come into more general use and to be incorporated into some standard specifications.

The fifth has been used on the Norfolk and Western Railroad since 1889, and is described in *Trans. Am. Soc. Civ. Engrs.*, Vol. XXVI, p. 149.

Since the method of computing stresses from the actual wheel-concentrations will always be considered standard, any conventional method must use an "equivalent" load of some sort, and before it will be accepted by engineers it must be shown to give practically equal stresses for all main truss members for all lengths of span and panel.

110. An Equivalent Load composed of a Uniform Train Load and one Moving Concentrated Load.

PROBLEM.—*Given any system of engine wheel loads followed by a uniform train load (or both preceded and followed by such load), to find what other system of uniform train and single moving excess loads will give practically equivalent stresses in all members of trusses over* 100 *feet long.*

Take the moving concentrated load as approximately equal (in round numbers) to the total excess of the weight of the engines and tenders over that of the train for the same length of wheel base. Thus if

E = total weight of one engine and tender,
b = total length of one engine and tender,
p' = loading per foot of actual train,
Q = single moving concentrated load,
p = equivalent uniform train load per foot to accompany the assumed concentrated load Q,

we have, for two engines coupled, $Q = 2E - 2bp' = 2(E - bp')$. . . . . (1)

Take for the actual Q the nearest even multiple of ten thousand pounds, so that Q would be, for Cooper's Class Extra Heavy A, two engines, 100,000 lbs. A considerable variation can be made in the value of Q without materially affecting the results.

* See Trans. Am. Soc. C. E., Vol. XV, p. 474; also Vol. XXI, p. 575.

† Id., Vol. XXI, p. 594.

Having chosen a value for Q, compute a corresponding value for p, the conventional train loading per foot to be used with Q, by finding from the actual system of wheel loads the moment at the quarter-point of the length of span to be computed (see Art. 96), or of a length of span nearly equal to the one in question. Let $l =$ length of such a span, and $M_{1/4} =$ the maximum bending moment at a point just one fourth the length from the end, regardless of whether this falls at a joint or not. This moment is to be found from a diagram such as that given on p. 85, or in any other way, from the actual wheel loads. Then equate this moment with the moment at this point for the "$p + Q$" loading, and obtain

$$M_{1/4} = \tfrac{3}{32}pl^2 + \tfrac{3}{16}Ql; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

whence

$$p = \frac{32}{3}\frac{M_{1/4}}{l^2} - \frac{2Q}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where l may be taken in round numbers approximately equal to the length of span in question, as the nearest multiple of ten feet. Then use this "$p + Q$" loading as follows:

For Maximum Moments the p loading must extend over the entire span, and the Q load must be at the centre of moments for the member in question, that is, it must be at some particular joint. If Q is placed at the mth joint from the left, counting the end support zero, we would then have for the moment at this joint from live load, for a truss of n equal panels, each d feet in length,

$$M_m = \frac{d(pl + 2Q)}{2n}(n - m)m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The fractional part of this expression is a constant for any given truss, and hence these moments can all be taken out and divided by the height of the truss, thus *giving all chord stresses in a Pratt truss with one setting of the slide-rule.*

For Maximum Shears place Q at the joint on the side of the panel in question from which the load is approaching, and let the uniform load extend into this panel until the maximum shear is produced. This point is such that a load placed at this point gives equal reactions at the next forward joint and at the forward end support. The portion of this panel covered with the uniform load is then $\left(\frac{n - m}{n - 1}\right)d$,* and the maximum shear in this panel is, from live load,

$$S = \frac{pd}{2}\cdot\frac{(n - m)^2}{(n - 1)} + \left(\frac{n - m}{n}\right)Q. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This equation is also very rapidly evaluated by the slide-rule, each term requiring but one setting for the whole span.

Floor System and Hip-vertical.

To find the maximum moment and shear in stringers and floor-beams and the stress in the hip-vertical, which needs to be done but once for any given span, the actual wheel loads may be used, or Q may be taken the same as above, and another p found for a length of span about equal to the panel length. In this case p will be found to be negative, since the load Q is alone more than the maximum load on the stringer in the case of the wheel loads. It would probably be preferable to use the engine diagram for these members.

NOTE.—The authors have tried various expedients for obtaining *a single set of values* of p and Q to be used for all lengths of span for any given set of wheel and train loads, and have made as many as fifty complete sets of computations of stresses in all the members of spans of various lengths and for different lengths of panel (which proves to be a very important consideration), comparing the results with those obtained from

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* See Art. 69, Chap. IV.

the wheel loads,—all for Cooper's Class A loading. They do not find that this can be done with satisfactory results. To use the method with results agreeing closely with those from the wheel loading, it seems to be necessary to obtain the "$p + Q$" loading for a length of span approximately equal to that in hand. Mr. Pegram has advocated single values of p and Q for any given set of wheel loads (see *supra*), and Mr. J. C. Bland, M. Am. Soc. C.E., has proposed the following to correspond with Cooper's four standard loadings:*

"Heavy Grade"........................... $p = 4000$; $Q = 40000$.
"Extra Class A" .... $p = 3400$; $Q = 34000$.
"Class A"................................ $p = 2800$; $Q = 28000$.
"Class B"................................ $p = 2500$; $Q = 25000$.

He has made $Q = 10p$ in every case, but when p and Q are found for a particular length of span, this condition, $p = \frac{1}{10}Q$, makes p too large for longer spans, so that the chord stresses become slightly too great.

How Specified.

It is important to make the specification under which bids are received for building a bridge perfectly clear and definite. The engineer who draws these specifications, therefore, should decide on a set of values of p and Q to be used for the particular length of span and for the loading assumed, and state that such and such lengths of span shall be computed for such and such uniformly distributed and rolling concentrated loads, the web members to be computed for the actual maximum shear on the panel by equation (5).

Tabular Form for Computations.

In making the computations for this conventional loading the work may be conveniently arranged as follows:

TOTAL CHORD STRESSES—ONE TRUSS.

Member.	$\frac{d[(w+p)l+2Q]}{4nh}$	$(n-m)m$	Stress.	Remarks.

The quantity in the second column is a constant for all members in one span. The slide-rule is set once for this as a constant multiplier and all the stresses taken out. If the dead load is desired separately, the w is to be omitted from column 2 and given a column to itself, this heading then being $\frac{dwl}{4nh}$.

In computing the web stresses the live and dead load shears are computed separately, and then combined as in the following tabular form:

WEB STRESSES—ONE TRUSS.

Member.	Dead Load Shear.		Live Load Shear.					Total Shear.	Secant.	Stress.	Member.
	$n+1-2m$	$\times \frac{wd}{4} =$ D. L. Shear.	$\frac{(n-m)^2}{n-1}$	$\times \frac{pd}{4}$	$\frac{n-m}{n}$	$\times \frac{Q}{2}$	Sum = L. L. Shear.				
1	2	3	4	5	6	7	8	9	10	11	12

The total shear in the mth panel, for the uniform load reaching into the panel to give maximum shear, and the concentrated load Q at the loaded panel joint, when w = dead load per foot, is

$$\text{Total shear on one truss} = \frac{wd}{4}(n+1-2m) + \frac{pd(n-m)^2}{4(n-1)} + \frac{Q}{2}\left(\frac{n-m}{n}\right). \quad . \quad . \quad (6)$$

* For the wheel diagrams for these loads see p. 79.

These terms are worked out in the above tabular form. The results in columns 5 and 7 are added to give the live load shear in column 8, and then these results are added algebraically to those in column 3 to obtain the total shear as given in column 9. These are at once the stresses in the verticals, and when multiplied by the secant of the angle the stresses in the diagonals are found. These forms are for the Pratt or Warren type.

METHOD OF EQUIVALENT UNIFORM LOADS.

111. Definition, Equations, and Argument.—An "equivalent load" is one which would produce the same maximum stresses in all the members as are caused by the actual wheel-loads. Evidently no single equivalent *uniform* load can be found to do this. It remains to find a uniform load which will give the nearest approximation to this result. The moment diagram or equilibrium polygon for a uniform load on a jointed structure has its vertices lying in a parabola (Art. 50), while for a plate girder it is a parabola. For excessive wheel concentrations near the head of the load, the polygon joining maximum moment ordinates would be below the parabola at points on the two sides of the centre of the span, somewhat as shown in Fig. 146,* the full line being the moment curve for the equivalent uniform load, and the dotted line being that joining maximum moment ordinates. These curves will cross each other at about the quarter point. We might therefore find the equivalent uniform load, so far as the chord stresses are concerned, by finding what uniform load will give the same moment at the quarter point which is produced there, for that length of span, by the actual wheel loads. To do this we must first find what the maximum moment at this quarter point is for any given length of span, for the actual wheel loads assumed, as described in Chap. V. (It is not necessary, for this purpose, that this point should fall at a joint.) Call this moment $M_{\frac{1}{4}}$. It now remains to find the uniform load p per foot which will give this same moment at this point.

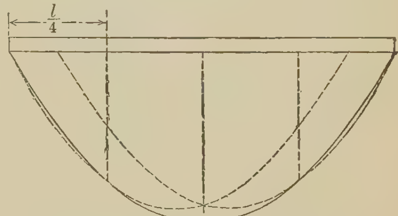

FIG. 146.

For a uniform load p over the entire span of n equal panels, each of length d, the moment at the quarter point is

$$M_{\frac{1}{4}} = \frac{3}{32} pl^2 ; \qquad (7)$$

whence

$$p = \frac{32}{3} \frac{M_{\frac{1}{4}}}{l^2} . \qquad (8)$$

Having found $M_{\frac{1}{4}}$, the moment at the quarter point for the actual wheel loads, eq. (8) gives us at once the equivalent uniform load which will produce the same moment at this point. Having found p, we may write for the

$$\textit{Moment at the } m\text{th} \textit{ joint} = \frac{pd^2}{2}(n - m)m \qquad (9)$$

where the end joint is called zero.

The proportional moment error in this case will be very small. In computing the shear with equivalent uniform live loads, it is customary to treat the load conventionally, *assuming that all the joints on one side of a given panel are fully loaded and all the joints on the other side wholly unloaded.* With this assumption, which cannot possibly be realized, very close agreements can be obtained with the rigid method from actual wheel loads, *when the panel length is not less than one eighth of the span;* when uniform loads are assumed the shears should be always taken out in this way. When there are more than eight panels, and when the engine loading is very much greater than the train loading, the shears are always too small.

* This figure represents the moment diagram, in dotted lines, for two concentrated loads, moving across the span a fixed distance apart.

The equation for maximum shear, due to both dead and live load, for the equivalent uniform load w', found for moments as above, is

$$\textit{Maximum shear in the } m\text{th } \textit{panel} = S_m = \frac{wd}{2}(n-2m+1) + \frac{pd}{2n}(n-m)(n-m+1), \quad (10)$$

where w = dead load per foot,
p = uniform live load per foot,
n = number of panels in span,
m = number of panels in question counting from unloaded end,
d = panel length in feet.

For live load shear only we have

$$S_m' = \frac{pd}{2n}(n-m)(n-m+1). \quad \ldots \ldots \ldots \quad (11)$$

These equations, (9), (10), and (11), are quickly evaluated by the aid of the slide-rule. For one truss take one half the values given by these equations.

112. Application to Systems with Inclined Chords.—*For the Dead Load*, whether this be supposed to be all concentrated on the bottom chord or partly at the unloaded joints, find all the dead load stresses from a single Maxwell diagram as shown in Art. 46, and tabulate these stresses in all the members.

For the Live Load Chord Stresses, cover the entire span with the equivalent uniform live load, and make another diagram for these stresses, and tabulate the stresses found in the chords. Or the better method would be to find these stresses from the dead-load stresses with one setting of the slide-rule.

For the Live Load Web Stresses,* assume a left end reaction of 100,000 lbs., treat the span as a cantilever with the left end free and subjected to this upward reaction, and diagram the stresses in all the members for this one reaction only, and tabulate the stresses in the web members. Now compute the left end reactions for successive positions of the train load as it is backed off from the left towards the right, assuming that the joint at the head of the load may be fully loaded and the next one in advance entirely unloaded. These reactions are found from the formula

$$R_1 = \frac{N(N+1)}{2n} pd, \quad \ldots \ldots \ldots \ldots \quad (12)$$

where N = number of joints loaded,
n = number of panels in the span,
p = equivalent uniform load,
d = panel length.

Having these reactions, and the live load web stresses for a reaction of 100,000 lbs., the actual live load web stresses are found by multiplying those found for the 100,000 lb. reaction by the actual reaction, by means of the slide-rule.

The tabulated form would be as follows:

Chord Members.				Web Members.						
Mem.	Dead Load Stress.	Live Load Stress.	Total Stress.	Mem.	Dead Load Stress.	Stress for $R = 100{,}000$	$N(N+1)$	$\times \frac{pd}{4n} = R_1$	Live Load Stress.	Total Stress.
1	2	3	4	5	6	7	8	9	10	11

Columns 2, 3, 6, and 7 are obtained from the diagrams. Column 10 is found by multiplying together the corresponding results in columns 7 and 9, and dividing by 100,000.

* See also Art. 80.

Columns 9 and 10 are found by the slide-rule—the former from one setting, the latter by separate settings for each result.

Nothing would be gained by obtaining the sum of the dead and live load stresses from one diagram drawn for the sum of the unit loads, since the dead load web stresses must be taken out separately.

By this method all the maximum stresses in the main truss members of a large bridge can be found in a few hours. The results by diagram should always be checked by computing one or two of the simpler cases. Then since the diagram checks itself it may be assumed that if one part is right it is all right.

112*a*. Accuracy of Results by the Conventional Methods.—The following table embodies the results of computations by the two conventional methods here proposed, as compared with the rigid results from the actual wheel loads, all for Cooper's class "Extra Heavy A" loading, as given on p. 79. For engines very much heavier than the train load no satisfactory convention has been found for plate girders, stringers, floor-beams, and hip-verticals. For these the actual wheel loads should be employed.

A careful study of this table, which embodies the best results the authors could obtain for an engine load approximating 4000 lbs. per foot for 100 feet at the head of a train load of 3000 lbs. per foot, will show:

1. That the "$p + Q$" loading gives better results for the web system than the equivalent uniform load. The results on the chord members are identical,* since the "$p + Q$" loading is equal to an equivalent uniform load of $\left(p + \frac{2Q}{l}\right)$ per foot for moments.

2. That the equivalent uniform load gives values very much too small for the web system when the number of panels is greater than eight. If this loading were treated rigidly for shear, the discrepancy would be much greater. This shows that if equivalent loads are used, a different equivalent must be used for shears from that which is used for moments.

3. That the "$p + Q$" loading gives very nearly correct values for both chord and web members, except for the counters where the result is always large by from six per cent in the 300-foot span to 43 per cent in the 150-foot span. If this method should be employed, then the same *computed* fibre stress per square inch could be allowed in the counters as in the mains. When these members are rigidly computed a liberal reduction in the working stress is made for "impact."

4. That since the facilities and aids to wheel-load computation are now so simple and efficient, and since this is acknowledged to most nearly represent the actual conditions of service, such loads should continue to be used in the actual dimensioning of both truss and plate girder simple span bridges when they are prescribed in the specifications.

5. That for estimating cross-sections and weights of bridges for the purpose of bidding on work, the "$p + Q$" loading might well be employed. The total weight found from its use would probably not differ by more than one or two tenths of one per cent from those found from the rigid method. The final sections, however, should be fixed by the wheel-load method of computation.

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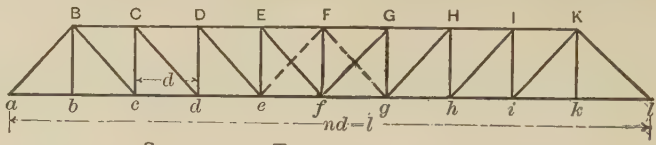
* The discrepancies in the table are all in the last significant figure, due to neglecting the remaining figures.

NOTE.—A Supplemental Note, added to this Chapter in the Third Edition of this Work, will be found on p. 142.

COMPARATIVE RESULTS OF TRUSS COMPUTATIONS BY RIGID AND CONVENTIONAL METHODS.

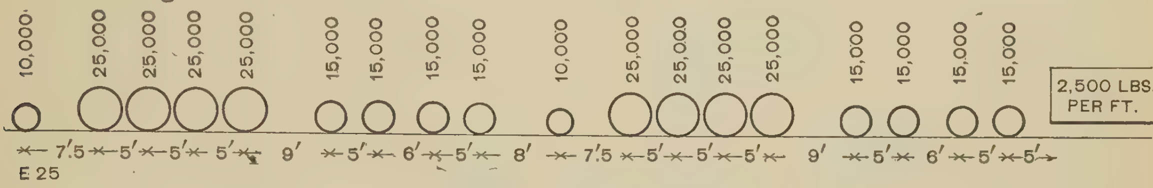


STRESSES IN THOUSANDS OF POUNDS.

Span, Panel Lengths, and Loads.	Members.	(w) Dead-load Stresses.	Wheel Loads. Live Load Stresses.	Wheel Loads. Total Stresses.	($p+Q$) Load. Eqs. (4) and (5). Live Load Stresses.	Total Stresses.	Ratio to Wh. Lds.	(w′) Equivalent Uniform Load. Eqs. (9) and (11). Live Load Stresses.	Total Stresses.	Ratio to Wh. Lds.
$l = 100'$	*ab–bc*	20.2	67.5	87.7	66.9	87.1	99.6	66.9	87.1	99.6
$d = 20'$	*BC–cd*	30.2	94.8	127.2	100.3	130.5	101.5	100.3	130.5	101.5
$w = 1.25$	*CD*	30.2	97.0	127.2	100.3	130.5	101.5	100.3	130.5	101.5
$p = 2.18$	*aB*	32.2	117.9	150.1	106.5	138.7	92.4	106.8	139.0	92.6
$Q = 100$	*Bc*	16.1	62.2	78.3	69.7	85.8	109.6	64.1	80.2	102.4
$w' = 4.18$	*Cd**	0.0	29.2	29.2	39.5	39.5	135.2	32.2	32.2	110.3
$l = 150'$	*ab–bc*	42.0	110.0	152.0	108.1	150.1	98.8	108.5	150.5	99.0
$d = 25'$	*BC–cd*	67.2	169.5	236.7	172.9	240.1	101.4	173.6	240.8	101.7
$w = 1.50$	*CD*	75.6	185.9	261.5	194.5	270.1	103.3	195.3	270.9	103.6
$p = 2.54$	*aB*	63.0	166.5	229.5	162.1	225.1	98.1	162.3	225.5	98.2
$Q = 100$	*Bc*	37.8	110.8	148.6	112.6	150.4	101.2	108.2	146.0	98.2
$w' = 3.88$	*Cd*	12.6	62.6	75.2	71.8	84.4	112.2	64.9	77.5	103.0
	*De**	12.6	31.2	18.6	39.3	26.7	143.5	32.5	19.9	107.0
$l = 200'$	*ab–bc*	56.3	122.7	179.0	117.6	173.9	97.2	117.6	173.9	97.2
$d = 20'$	*BC–cd*	100.0	211.5	311.5	209.1	309.1	99.2	209.1	309.1	99.2
	CD–de	131.3	272.3	403.6	274.5	405.8	100.5	274.5	405.8	100.5
	DE–ef	150.0	312.7	462.7	313.7	463.7	100.2	313.7	463.7	100.2
$w = 1.75$	*EF*	156.3	322.6	478.9	326.8	483.1	100.9	326.8	483.1	100.9
$p = 2.66$										
$Q = 100$	*aB*	96.8	209.9	306.7	202.4	299.2	97.6	202.5	299.3	97.6
$w' = 3.66$	*Bc*	75.3	170.5	245.8	165.4	240.7	97.9	162.0	237.3	96.7
	Cd	53.8	133.8	187.6	132.0	185.8	99.0	126.0	179.8	95.8
	De	32.3	100.8	133.1	102.2	134.5	101.0	94.5	126.8	95.3
	Ef	10.8	72.0	82.8	76.1	86.9	104.9	67.5	78.3	94.6
	*Fg**	10.8	47.2	36.4	53.6	42.8	117.6	45.0	34.2	94.0
$l = 200'$	*ab, bc*	60.2	129.7	189.9	125.1	185.3	97.6	125.3	185.5	97.7
$d = 25'$	*BC, cd*	103.1	214.3	317.4	214.4	317.5	100.0	214.8	317.9	100.2
$w = 1.75$	*CD, de*	128.9	266.7	395.6	268.0	396.9	100.3	268.5	397.4	100.4
$p = 2.66$	*DE*	137.5	282.2	419.7	285.9	423.4	100.9	286.4	423.9	101.0
$Q = 100$	*aB*	97.7	210.7	308.4	203.2	300.9	97.6	203.3	301.0	97.6
$w' = 3.66$	*Bc*	69.8	159.2	229.0	156.1	225.9	98.7	152.5	222.3	96.2
	Cd	41.9	114.7	156.6	115.0	156.9	100.2	108.9	150.8	96.3
	De	14.0	76.3	90.3	79.9	93.9	104.0	72.6	86.6	95.9
	*Ef**	14.0	43.6	29.6	51.2	37.2	125.0	43.6	29.6	100.0
$l = 250'$	*ab, bc*	87.9	161.3	249.2	154.7	242.6	97.4	154.8	242.7	97.4
$d = 25'$	*BC, cd*	156.2	277.4	433.6	275.0	431.2	99.4	275.2	431.4	99.4
$w = 2.00$	*CD, de*	205.1	361.3	566.4	361.0	566.1	100.0	361.2	566.3	100.0
$p = 2.72$	*DE, ef*	234.4	409.2	643.6	412.5	646.9	100.5	412.8	647.2	100.6
$Q = 100$	*EF*	244.1	416.4	660.5	429.7	673.8	102.0	430.0	674.1	102.1
$w' = 3.52$	*aB*	142.8	261.4	404.2	251.3	394.1	97.5	251.1	393.9	97.5
	Bc	111.0	210.9	321.9	204.2	315.2	98.1	200.9	311.9	96.9
	Cd	79.3	165.7	245.0	161.9	241.2	98.5	156.2	235.5	96.1
	De	47.6	125.1	172.7	124.4	172.0	99.6	117.2	164.8	95.4
	Ef	15.9	89.6	105.5	91.6	107.5	101.9	83.7	99.6	94.4
	*Fg**	15.9	59.3	43.4	63.7	47.8	110.1	55.8	39.9	91.9
$l = 300'$	*ab, bc*	120.1	189.2	309.3	183.7	303.8	98.2	183.6	303.7	98.2
$d = 30'$	*BC, cd*	213.5	326.6	540.1	326.5	540.0	100.0	326.4	539.9	99.9
	CD, de	280.5	426.0	706.5	428.6	709.1	100.4	428.4	708.9	100.3
	DE, ef	320.2	476.8	797.0	489.8	810.0	101.6	489.6	809.8	101.5
$w = 2.25$	*EF*	333.6	488.4	822.0	510.2	843.8	102.7	510.0	843.6	102.6
$p = 2.78$										
$Q = 100$	*aB*	193.8	305.2	499.0	296.7	490.5	98.3	295.2	489.0	98.0
$w' = 3.44$	*Bc*	150.7	246.2	396.9	240.1	390.8	98.5	236.2	386.9	97.5
	Cd	107.6	192.9	300.5	189.4	297.0	98.8	183.7	291.3	96.9
	De	64.6	145.9	210.5	144.6	209.2	99.4	137.8	202.4	96.1
	Ef	21.5	105.0	126.5	105.7	127.2	100.5	98.4	119.9	94.8
	*Fg**	21.5	69.7	48.2	72.7	51.2	106.2	65.6	44.1	91.5

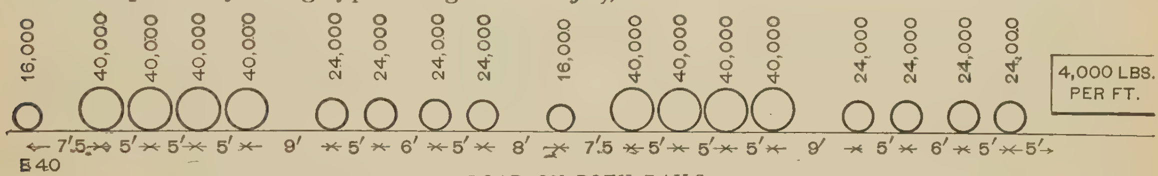
NOTE.—The heights of these spans were as follows: For 100′ span, $h = 25'$; for 150′ and 200′ spans (10 panels), $h = 28'$; for 200′ (8 panels) and 250′ spans, $h = 32'$; for 300′ span, $h = 38'$. The loads given in the first column are for two trusses.

112*b*. Cooper's Conventional Wheel Loads.—In Vol. XXXI of Trans. Am. Soc. C. E., p. 174 (Feb. 1894), Mr. Theodore Cooper, M. Am. Soc. C.E., recommends the following system of engine and train loads for all bridge *computations*, the resulting stresses to be multiplied by a constant factor (by slide-rule) to reduce them to their equivalents for any other system of engine and train loads:



LOAD ON BOTH RAILS.

By using this system of loads as representing the least allowable loads for any railroad bridge, and finding the corresponding live-load stresses, we could then multiply these by factors, as 1.2, 1.4, 1.6, etc., so as to change the stresses to those arising from any correspondingly increased loads. Thus, when the factor 1.6 is used, we shall have 40,000 pounds on the driver axles, in place of 25,000 pounds. Hence we could call this typical engine "E 40" (the one used primarily being typical engine "E 25"), and the loads would become



LOAD ON BOTH RAILS.

This loading corresponds very closely to that of the heaviest Lehigh Valley engines. The live-load stresses for this loading would be obtained by multiplying all the stresses found for the "E 25" loading by multiplying them all by 1.6. This could be done with a single setting of the slide-rule.

This method of treating train-loads has now (1897) come into common use. For the purpose of utilizing this very satisfactory system, the moment table on page 108*b* has been prepared.* It is less elaborate than that on page 79, but it serves the same purpose. It is computed for "Class E 30" loading, this being the mean of the two classes shown above. These moments are given in 1000-lb. units, and for one rail, or for one half the live load. For any other class, as for "Class E 27," take $\frac{27}{30}$ of the stresses found by the use of the table. The particular wheel to put at any joint is found as described in Chapter V. By comparing the table on p. 108*b* with that on page 79 it will be found that the moments given in the former in column *g* correspond to those given in columns 10 and 1 of the latter, reading from the bottom upwards in this case. As these are the only parts of the larger table which are used for finding abutment reactions, and since the panel-load concentrations can be obtained by summing the proper moment-increments in column *f*, it would seem that the smaller table, as here given, were the more convenient. This table also serves to include the moment from uniform loads to a distance of 324 feet from the first wheel of the head engine. The preparation of the table is evident from the headings. The moments given are the moments of the loads on one rail, while the loads given in the diagrams above are the total wheel loads on both rails. Each horizontal line of the table applies to that particular wheel or point in the train loading. The figures set in the upper part of the space are used only in compiling the other results in the table.

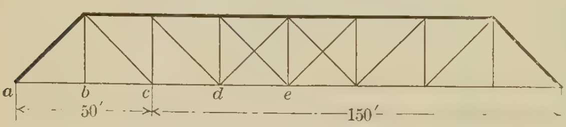
* After one prepared by Mr. C. L. Strobel, and contributed to the *R. R. Gazette* of Dec. 26, 1896, by Mr. T. L. Condron.

COOPER'S E 30-MOMENT TABLE FOR TWO 106½-TON LOCOMOTIVES, FOLLOWED BY 3000 LBS. PER LINEAR FOOT OF TRACK.

(LOADS IN THOUSANDS OF POUNDS. MOMENTS IN THOUSANDS OF FOOT-POUNDS.)

FOR ONE RAIL.

a No. of Wheel or Load.	*b* Distance from 1st Wheel.	*c* Wheel Load.	*d* Sum of Loads.	*e* Dist'ce between Wheels.	*f* Increment of Moment.	*g* Moment.
1	0	7.5	7.5			0
2	8	15	22.5	8	60	60
3	13	15	37.5	5	112.5	172.5
4	18	15	52.5	5	187.5	360
5	23	15	67.5	5	262.5	622.5
6	32	9.75	77.25	9	607.5	1230
7	37	9.75	87	5	386.25	1616.25
8	43	9.75	96.75	6	522	2138.25
9	48	9.75	106.5	5	483.75	2622
10	56	7.5	114	8	852	3474
11	64	15	129	8	912	4386
12	69	15	144	5	645	5031
13	74	15	159	5	720	5751
14	79	15	174	5	795	6546
15	88	9.75	183.75	9	1566	8112
16	93	9.75	193.5	5	918.75	9030.75
17	99	9.75	203.25	6	1161	10191.75
18	104	9.75	213	5	1016.25	11208
19	114	15	228	10	2130	13338
20	124	15	243	10	2280	15618
21	134	15	258	10	2430	18048
22	144	15	273	10	2580	20628
23	154	15	288	10	2730	23358
24	164	15	303	10	2880	26238
25	174	15	318	10	3030	29268
26	184	15	333	10	3180	32448
27	194	15	348	10	3330	35778
28	204	15	363	10	3480	39258
29	214	15	378	10	3630	42888
30	224	15	393	10	3780	46668
31	234	15	408	10	3930	50598
32	244	15	423	10	4080	54678
33	254	15	438	10	4230	59908
34	264	15	453	10	4380	63288
35	274	15	468	10	4530	67818
36	284	15	483	10	4680	72498
37	294	15	498	10	4830	77328
38	304	15	513	10	4980	82308
39	314	15	528	10	5130	87438
40	324	15	543	10	5280	92718



To illustrate how the table is used, the following example is given of a 200-ft. span of eight panels.

With wheel 4 at *c* we have:

Length of bridge to right of *c*	150 ft.
" " train to left of 4 (from table)	18 "
Total length of train on bridge	168 "
Nearest point of train load in table (load 24)	164 "
Distance from load point 24 to end of truss	4 "
Total weight of train to point 24 (from table)	303 lbs. (thousands)
Increment of moment to be added	1,212 ft.-lbs. "
Moment of entire train about point 24 (from table)	26,238 " "
Total moment	27,450 × $\frac{2}{8}$ = 6,862
Negative moment about wheel 4	360
Required moment at *c* (thousands of ft.-lbs.)	6,502

CHAPTER VII.

LATERAL TRUSS SYSTEMS.

113. THE lateral pressure upon a bridge or roof truss arising from wind or the centrifugal force due to loads moving in a curve, are resisted by means of lateral horizontal trusses placed between the chords of the vertical trusses. The chords of the vertical trusses thus form the chords of the lateral trusses. Roof trusses are usually braced laterally in pairs, the end pair taking the greater part of the end pressure, the others being merely stiffened against buckling, according to the judgment of the engineer. The stresses in the end lateral system are computed in the same way as are those in the lateral systems of bridge trusses, which will be discussed in detail.

The lateral systems of a bridge are either of the Howe, Pratt, or Warren type, corresponding usually with the type of the main trusses. Fig. 147 shows the upper and Fig 149 the lower lateral system of a through Pratt truss, Fig. 148. The wind pressure may be from either direction, hence both sets of diagonals are required throughout. The floor-beams

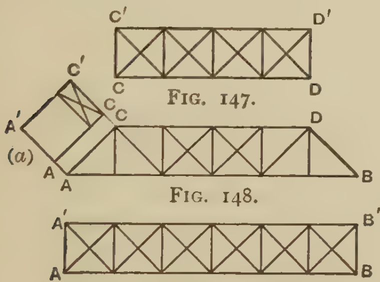


FIG. 147.

FIG. 148.

FIG. 149.

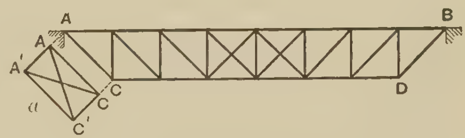


FIG. 150.

usually form the transverse members of the lower lateral system in a through bridge, and of the upper lateral system in a deck bridge. The loads upon the upper laterals of Fig. 148, or upon the lower laterals of a deck bridge supported as in Fig. 150, are carried by them to the points C and D or C' and D', and are then transferred to the abutments at A and B by means of *portal bracing* in the transverse planes of AC and BD. Two forms of such bracing are shown in Fig. 148 (a) and Fig. 150 (a).

STRESSES DUE TO WIND PRESSURE.

114. Upper and Lower Laterals.—The wind pressure upon highway bridges is usually taken at about 30 lbs. per square foot. The exposed area is found by adding the area of the vertical projection of the floor system to twice the vertical projection of one truss, thus assuming no live load upon the bridge at the time of maximum wind pressure. The pressure upon the upper half of the truss is assumed to be taken by the upper laterals, and that upon the lower half by the lower laterals.

Instead of the above loads, specifications sometimes adopt a pressure of from 100 lbs. to 150 lbs. per lineal foot upon the unloaded chord and 200 lbs. to 250 lbs. per foot upon the loaded chord. The wind loads upon the unloaded bridge are usually treated as fixed loads.

For railroad bridges, a pressure of from 30 lbs. to 50 lbs. per square foot is assumed for the unloaded bridge, and about 30 lbs. upon bridge and train, the load due to the pressure upon the train being treated as a moving load. Or, as in Cooper's specifications, a total pressure of 450 lbs. per lineal foot upon the loaded chord, 300 lbs. of which is treated as a moving load; and 150 lbs. per foot upon the unloaded chord.

The adopted unit pressures of 45 lbs. and 30 lbs. of Chap. III agree very well with average practice as regards bridges.

The transference of the pressure upon the train depends upon the disposition of the floor-beams and lateral systems, but usually it may be considered as being all taken up by the lateral system belonging to the loaded chord. This question, together with the effect of the pressure upon the train not being applied in the plane of the floor-beams, will be discussed under the subject of centrifugal force.

In finding stresses, loads on the lateral system of the unloaded chord may be considered as applied equally upon the two sides, windward and leeward; and for the laterals of the loaded chord they may be taken as applied wholly on the windward side. The stresses are then readily found by methods already familiar.

The resulting chord stresses should be combined with those due to dead and live loads if the live load is considered as acting simultaneously with the wind load; or if the live load is considered as not so acting, then they should be added to the dead load stresses. The stress in the windward lower chord is thus often reversed. There is also, as will be seen later, a uniform compressive stress in the windward chord induced by the action of the portal bracing, and this stress is to be combined with the two above. Wherever a reversal of stress occurs, reliance must be placed on the stringers to resist buckling; or where this cannot be done, as in the end panel, the chord must be counterbraced.

Where the lateral rods of the system belonging to the loaded chord are attached to the flanges of the floor-beams, the stresses thus caused in the flanges must be taken into account when they are of the same sign as those caused by the bending moment.

EXAMPLE 1. Find the stresses of the lateral systems of Figs. 148 and 150, where $AB = 120$ ft. and $CC' = 17$ ft. Pratt system of laterals. Pressure on $CD = 100$ lbs. per foot, and upon $AB = 200$ lbs. per foot; all to be taken as a moving load.

EXAMPLE 2. Find the stresses in the lower laterals, Fig. 148, for a pressure of 150 lbs. fixed and 300 lbs. moving load per foot. The diagonals are angle-irons riveted to stringers and floor-beams and are assumed to resist in tension and compression equally.

115. Portal Bracing.*—*Form* 1. A common form of portal bracing is shown in Fig. 151. The distance $AC, = c$, is the length of the end post whether vertical or inclined. With the usual Pratt type of upper laterals, one half the entire load upon the intermediate panels is transferred to the point C, as one abutment of the lateral truss. Call this load R. There is in addition about one-half a full panel load applied at C, and one-half at C'; these are due to the pressure upon the end panel of the upper chord and upon the end posts. Call each $\frac{P}{2}$. These external forces are held in equilibrium by other unknown external forces, in the plane of the portal, applied at A and A'. Let the components of these forces be as represented in the figure.

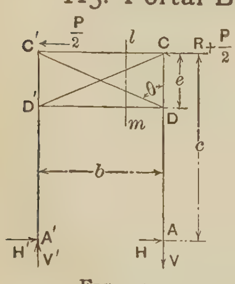


FIG. 151.

From the conditions of equilibrium we have readily,

$$H + H' = R + P, \quad \text{and} \quad V' = -V = (R + P)\frac{c}{b}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Assuming $H = H'$, as is admissible, we have

$$H = H' = \frac{R + P}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* See also pages 159, 288, and 329.

Passing the section lm and taking centre of moments at D, the piece CD' not being in action with wind from right, we have

$$\text{compressive stress in } CC' = \frac{\left(R+\frac{P}{2}\right)e + H(c-e)}{e} = \left(\frac{R+P}{2}\right)\frac{c}{e} + \frac{R}{2}. \quad . \quad . \quad . \quad . \quad (3)$$

If the upper laterals are of the Howe type, transferring their loads to C', the stress in CC' will be less than that above by the amount R.

From Σ vert. comp. $= 0$ we have

$$\text{tensile stress in } C'D = V \sec\theta = (R+P)\frac{c}{b}\sec\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From Σ mom. $= 0$, centre of moments at C', we have

$$\text{compressive stress in } DD' = \frac{Hc}{e} = \frac{R+P}{2}\frac{c}{e}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The force V' produces a uniform compressive stress in the post $A'C'$, and the force V reduces the compressive stress in the portion AD of the post AC. Above D there is no change.

The bending moment at D and $D' = H \times (c-e) = \frac{R+P}{2}(c-e)$, and is the greatest moment in the posts. The maximum compressive fibre stress in the posts occurs on the inner side of the leeward post at D', where the flange stress due to the bending moment adds to the direct stresses due to V' and to live and dead loads. Each post should then be designed to resist this combined stress.

If the posts are inclined, the horizontal components of V and V', ($V\sin\alpha$), (Fig. 152), act directly upon the lower chord as a relief on one side and an additional tension on the other. These stresses are uniform throughout the length of the bridge, and are to be combined with those due to the lower lateral system and to live and dead loads. A reversal of stress often occurs in the end panels of the windward chord, as stated in the previous article.

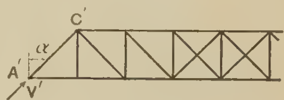


FIG. 152.

A strut, AA', in Fig. 151, is usually inserted to distribute the pressures to the two supports. At the fixed end of the span the stress in this strut may be taken equal to one-half the load brought to A by the lower laterals ($\frac{1}{2}R'$). At the free end it is equal to

$$\tfrac{1}{2}R', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or to

$$(R' + \tfrac{1}{2}P' + H) - \frac{1}{4}\left(\frac{W}{4} - V\cos\alpha\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6')$$

whichever is the greater. In eq. (6'), $\frac{1}{2}P'$ = pressure upon the end panel of the lower laterals applied directly at A; W = total dead load; and $V\cos\alpha$ = upward pull on shoe of windward post. The first parenthesis is thus the total lateral pressure upon A, and the second is the frictional resistance of the post, taking $\frac{1}{4}$ as the coefficient of friction. The difference must be transmitted to A'.

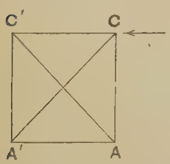


FIG. 153.

When $\frac{W}{4} < V\cos\alpha$, the bridge must be anchored down.

Form 2. The portal of a deck bridge is of the form shown in Fig. 153. The stresses in CC', AC', AA', and the direct stress in $A'C'$ are found as in the previous case. There is no bending moment in any member, nor is there any tension in AC.

Form 3. Fig. 154 shows a common form of portal. The stresses in the main posts are as in Fig. 151, the dangerous section being at D'.

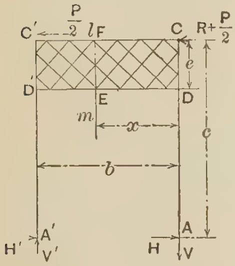


FIG. 154.

Treating the portion $CDD'C'$ as a beam, we may find the stress at any point F of the flange CC', by passing a section FE and then taking moments of the forces on the right, about E. We have then, from eqs. (1) and (2),

$$\text{stress in } CC' = \frac{H(c-e) + \left(R + \frac{P}{2}\right)e - Vx}{e}$$

$$= \frac{R}{2} + \left[\frac{R+P}{2}\frac{c}{e} - (R+P)\frac{c}{b}\frac{x}{e}\right] \quad \ldots \ldots \quad (7)$$

$$= \frac{R}{2} + \frac{V}{e}\left(\frac{b}{2} - x\right). \quad \ldots \ldots \ldots \quad (7')$$

When $x = \frac{b}{2}$, the last term of eq. (7′) is equal to zero; and for $x = 0$ this quantity is equal to $+\frac{V}{e}\frac{b}{2}$, and for $x = b$ it is equal to $-\frac{V}{e}\frac{b}{2}$. The piece CC' may then be considered as having a uniform compressive stress of $\frac{R}{2}$ and an additional stress varying uniformly from zero at the centre to a value of $\frac{V}{e}\frac{b}{2}$ at each end, compressive on the windward side and tensile on the leeward.

Taking moments about F, we find similarly,

$$\text{stress in } DD' = \frac{R+P}{2}\frac{c}{e} - (R+P)\frac{c}{b}\frac{x}{e}, \quad \ldots \ldots \ldots \quad (8)$$

tensile on the windward and compressive on the leeward side. The flange stresses thus differ numerically by $\frac{R}{2}$, as they should, from Σ hor. comp. $= 0$.

The shear on the section $EF = V$, and may be distributed equally over the web members cut.

The stress in the strut AA' is the same as in the first case.

Form 4. In small bridges, for lack of head room, simple knee-braces, DE and $D'E'$ (Fig. 155), are often used.

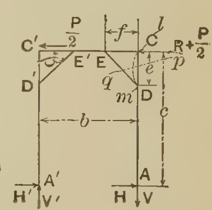


FIG. 155.

With the same notation as before, we have the following stresses:

Passing the section lm through C and D, we have, by taking moments of the forces on the right, about C,

$$\text{hor. comp. tensile stress in } ED = \frac{H \times c}{e} = \frac{(R+P)c}{2e}, \quad \ldots \quad (9)$$

$$= \text{hor. comp. compressive stress in } E'D'.$$

$$\text{Bending moment at } D \text{ and } D' = H \times (c-e) = \frac{R+P}{2}(c-e). \quad \ldots \ldots \quad (10)$$

From Σ vert. comp. $= 0$ on the right of the section pq, we have,

direct compression in CD = vert. comp. $ED - V$,

$$= \frac{R+P}{2}\frac{c}{f} - (R+P)\frac{c}{b} = (R+P)\left(\frac{c}{2f} - \frac{c}{b}\right); \quad \ldots \quad (11)$$

$$\text{The direct compression in } A'D' = V' = (R+P)\frac{c}{b}, \quad \ldots \ldots \ldots \quad (12)$$

a value greater or less than that given by (11) according as f is greater or less than $\frac{b}{4}$. The greater of the two values, (11) or (12), is to be added to the compression due to dead and live loads, and the resulting fibre stress added to that due to the bending moment given in eq. (10). This will give the maximum stress in the end posts.

$$\text{Bending moment at } E = H \times c - V \times f = c(R+P)\left(\frac{1}{2} - \frac{f}{b}\right). \quad \ldots \quad (13)$$

$$\text{Direct compression in } EE' = R + \frac{P}{2} - H = \frac{R}{2}. \quad \ldots \quad (14)$$

$$\text{Direct compression in } EC = R + \frac{P}{2} + \text{hor. comp. } ED - H = \frac{R}{2} + \frac{R+P}{2}\frac{c}{e}. \quad \ldots \quad (15)$$

$$\text{Direct tension in } E'C' = \frac{R+P}{2}\frac{c}{e} - \frac{R}{2}. \quad \ldots \quad (16)$$

The maximum compressive stress in CC' therefore occurs just to the right of E where the stress due to (15) is added to that due to the bending moment of eq. (13). Likewise eqs. (13) and (16) give the maximum tensile stress in CC', which is just to the left of E'.

Form 5. The bending moments on the posts and the stress in the strut CC' may be reduced without reducing head room by the arrangement of Fig. 156. The maximum bending moment in the posts $= H \times (c - e)$ as before.

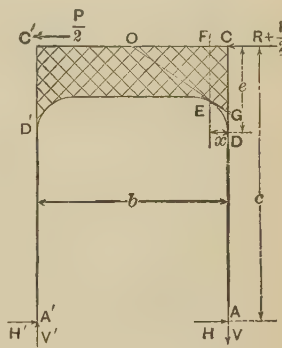


FIG. 156.

The stress at any point F of CC' is given by eq. (7'), in which e is now the variable depth EF of the beam. This stress is a maximum at the section FE where the tangent at E cuts CC' in the centre O; for at that section the variable lever-arm e varies at the same rate as the variable lever-arm $\left(\frac{b}{2} - x\right)$, and hence the rate of change of their ratio is zero.

The horizontal component of the stress in DD' is equal to the stress in CC' at the same section, minus $\frac{R}{2}$, as in eq. (8). This stress is zero at the centre, tensile on the right and compressive on the left. The maximum value of the horizontal component of the stress in DD' is at E, but the maximum value of the stress itself occurs at some point to the right of E. It may be taken as that point near D, to the right of which the flange stress can be assumed as transmitted to the post in a direct line by means of the connecting plate.

The web stresses in the central portion are found by dividing the shear, V — vert. comp. DD', by the number of pieces cut. The two web members meeting at any point of the flange should also be able to take up the difference in the flange stress in the two adjacent panels.

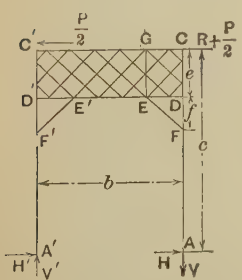


FIG. 157.

Form 6. Fig. 157 shows a modification of Form 5. The maximum moments in the posts occur at F and F'. The maximum stresses in CC' and DD' occur at G and E, and are found as before. The stress in EF may be found by assuming that the stress at D in DD' is equal to that at E, and that the piece EF takes the remaining moment. Thus with centre of moments at C,

$$\text{hor. comp. } EF \times (f + e) = H \times c - \text{stress in } ED \times e.$$

The web members between E and E' take the shear V, and to the right of E they take the shear V, minus the vert comp. of the stress in EF. At E they must take the vert. comp. in EF.

Portals with Fixed End-posts.—In the foregoing discussion the end-posts have been treated

as not capable of resisting bending moment at their bases, A and A' in the figures. Where they are so well anchored to the masonry, however, that the ends are fixed in position, the stresses in the portal and in the posts are very much reduced. With a well-designed portal the upper ends of the posts are also fixed, and we may take the point midway between the lower edge of the portal and the foot of the post as the point of inflexion. Then in Fig. 154, for example, the reactions H and V may be considered as applied at a point half-way between A and D, at which point the moment is zero. The analysis then is the same as already given, the distance c being replaced by $e + \frac{1}{2}(c - e)$. The bending moments on the posts are thus reduced one-half and the other stresses correspondingly.*

The requisite strength of anchorage to give fixed ends is readily found from the bending moments at the feet of the posts. These are the same as at the points D and D'.

The preceding applies also to the sway bracing of elevated structures where the supporting columns are firmly anchored to the foundations.

116. Skew Portals.—Fig. 158 is a plan of the upper lateral system of a skew bridge with vertical end posts, and Fig. 158 (a) is the elevation of the portal upon a parallel plane.

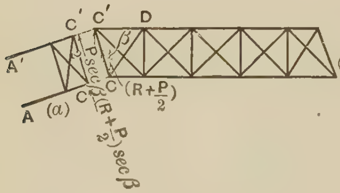


FIG. 158.

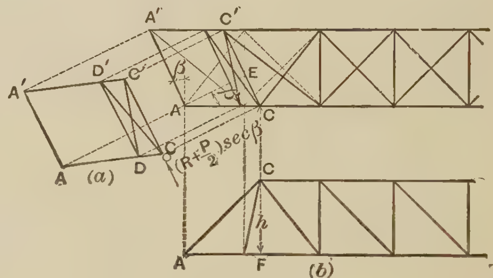


FIG. 159.

The lateral force, $R + \frac{P}{2}$, applied at C (R is the lateral component in CD, and $\frac{P}{2}$ is the wind pressure applied directly to C), is resolved into the components $\left(R + \frac{P}{2}\right)$ sec β and $\left(R + \frac{P}{2}\right)$ tan β, where β = angle of skew. The force $\left(R + \frac{P}{2}\right)$ sec β replaces the force $R + \frac{P}{2}$ in the discussion of the preceding article, and $\left(R + \frac{P}{2}\right)$ tan β, together with a similar force at the right end, act upon the main truss, producing only slight additional stresses. The plan of the upper laterals and inclined portal of a skew bridge are shown in Fig. 159. The lower laterals are represented by dotted lines. Fig. 159 (a) is the projection of the portal on a plane parallel to itself, and Fig. (b) is the elevation of one truss. The lateral force at C in the plane of the portal is, as before, $\left(R + \frac{P}{2}\right)$ sec β. The perpendicular distance from AA' to CC' is equal to $\sqrt{\overline{AE}^2 + h^2}$, where AE is the *horizontal* distance between these two lines, equal to AF cos β, and h is the height of the truss. This distance corresponds to the distance c of the preceding article.

117. Sway Bracing of the same form as portal bracing is usually placed at each panel point of a deck bridge and at each panel point of a through bridge when the height is more than about 25 feet. This bracing is sometimes designed to carry the wind pressure from one chord to the other at each panel point, in which case but one lateral system is needed. The stresses are computed the same as for portal bracing, the external lateral force being equal to the wind load upon one panel. The resulting vertical reactions corresponding

* For the case of end posts fixed at the base with a simple system of diagonal bracing at top, see page 159, Chap. X.

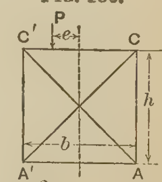
to V and V', Art. 115, act as loads, upward or downward, upon the main trusses. The portal bracing proper is now subjected to the same loads as the intermediate sway bracing.

When both lateral systems are used, the stresses in the sway bracing are still often computed on the same assumptions as the above, although if the two lateral systems have equal lateral deflections these stresses are zero. However, with wind pressure upon the *unloaded* bridge, the lateral system of the chord supporting the floor has, in the case of railway bridges, only about one third of its full load, while the other system is fully loaded. In this case the lateral deflections are not equal; the sway bracing will be distorted, and some stress will be thereby transmitted to the stiffer lateral system. The assumption that one half the wind pressure upon the one system is thus transferred is certainly on the safe side when the portals are properly designed, even with the most rigid form of sway bracing; with a flexible form, as simple knee-braces, very little stress can come upon this bracing from wind pressure. No change need be made in the laterals on account of stresses arising from the sway bracing.

118. Eccentric Loads.—For double-track deck railway bridges and for highway bridges, specifications sometimes require that the sway bracing shall be proportioned to distribute eccentric loads equally upon the two trusses. Such a distribution it is impossible to attain to any extent, without the use of extremely heavy laterals and sway bracing. The stresses in the three systems of trusses—the vertical main trusses, the lateral horizontal trusses, and the sway bracing—due to eccentric loading, depend upon the relative rigidity of the several systems, and will now be discussed.

1st. *The Lateral Horizontal Trusses.*—The sway bracing of a deck railway bridge is shown in Fig. 160. Let P be a panel load upon one track, e the eccentricity, b the width between trusses, and h the height of the trusses. The portions of the load transferred to C' and C are $\frac{1}{2}P + P\frac{e}{b}$ and $\frac{1}{2}P - P\frac{e}{b}$, respectively, there being an excess of $P\frac{2e}{b}$ at C' over that at C. So far as the stresses due to eccentricity are concerned we may then consider a load equal to $P\frac{2e}{b}$ at C' and no load at C.

FIG. 160.



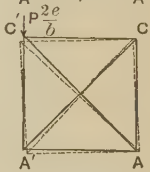


FIG. 161.

If there were no horizontal trusses in the planes AA' and CC' Fig. 161, the effect of the load $P\frac{2e}{b}$ would be to deflect the truss $A'C'$ the full amount, Δ, and to twist the cross-section into the position shown by the dotted lines; for the sway bracing would still preserve the rectangular shape of the cross-section, there being no resistance to the lateral motion of CC' and AA' beyond the slight resistance of the main trusses to becoming warped surfaces. The posts AC and $A'C'$ would thus be inclined as much to the vertical as CC' and AA' are to the horizontal, and if we assume CC' and AA' to have equal lateral movements the movement of each would be $\frac{1}{2}\Delta\frac{h}{b}$. If, however, there were horizontal trusses in the planes AA' and CC', this lateral movement would be somewhat resisted, and with extreme conditions of perfectly rigid sway bracing and very flexible horizontal bracing the lateral deflection of each horizontal truss will be *less* than $\frac{1}{2}\Delta\frac{h}{b}$. Hence the greatest stresses that can occur in the members of the lateral horizontal trusses are less than those due to a deflection of $\frac{1}{2}\Delta\frac{h}{b}$. The stress *per square inch* due to this deflection is independent of the size of the members, but with heavy bracing the deflection, and therefore the unit stress, will be reduced. It will be shown, however, that the stresses due to this maximum deflection are very small.

If Δ_v = deflection of the vertical truss under full live load P,

Δ_h = deflection of lateral truss,

$= \frac{1}{2}\Delta\frac{h}{b}$,

S_v = average unit stress in vertical truss for deflection Δ_v

= unit stress due to full live load,

and

S_h = average unit stress in lateral horizontal truss for deflection Δ_h, we have

$$\frac{\Delta}{\Delta_v} = \frac{P\frac{2e}{b}}{P}, \quad \text{or} \quad \Delta = \frac{2e}{b}\Delta_v.$$

$$\Delta_h = \frac{1}{2}\Delta\frac{h}{b} = \frac{eh}{b^2}\Delta_v.$$

Now for any given case Δ_v can be computed by eq. (6), Chap. XV, using for this purpose a single average value of S_v, or unit stress, to replace p_t and p_c of that equation. Having found Δ_v, the value of Δ_h may be found by means of the above equation. The corresponding value of S_h is then determined by putting $p_t = p_c = S_h$ in the second of eqs. (7), Chap. XV (assuming all the deflection to be due to the web members), and solving for S_h, the deflection being known.

In ordinary cases S_v is about 8000 lbs. per square inch; and the greatest stress on the laterals due to eccentric loading is not usually greater than 2000 lbs. per square inch, an insignificant amount as compared to the great unit stress allowable in these members. In through bridges of average depth the ordinary form of sway bracing is very flexible, and the stress in the laterals is much less than the above.

Take as an example a Pratt truss of 200 ft. span; 12 panels; $h = 33$ ft. 4 in.; $b = 28$ ft.; $e = 6$ ft. The actual computed deflection for full live and dead loads is about 2.5 in. If one half the dead load (double track) be taken at 1300 lbs. per foot, and the live load per track at 3000 lbs. per foot, the deflection due to full live load is equal to

$$\Delta_v = \frac{3000}{4300} \times 2.5 = 1.8 \text{ in.}$$

and

$$\Delta_h = \frac{eh}{b^2}\Delta_v = \frac{6 \times 33\frac{1}{3}}{28 \times 28} \times 1.8 = .46 \text{ in.}$$

Now the actual deflection of the lateral system in terms of the unit stress, computed as in Chap. XV (assuming all the deflection to be due to the *web* members because of the great size of the chords), is equal to

$$\Delta_h = .00025 S_h.$$

Hence

$$S_h = 4000\Delta_h = 1840 \text{ lbs. per square inch.}$$

2d. *The Sway Bracing*—It has been shown above that with no lateral bracing there is no appreciable stress upon the sway bracing; also it is clear that whatever stress is brought upon the sway bracing must be resisted by the laterals at C' and A. But the greatest resistance that is offered by these laterals is that due to a unit stress of possibly 2000 lbs. per square inch, an amount equal to about one-seventh the stress due to wind pressure. Hence the stress in the sway bracing is not more than would be caused by the transferring of one seventh the wind pressure at each panel from one chord to the other, an insignificant amount.

Finally it may be said that, besides transferring perhaps one-half the wind pressure as shown on p. 115, the office performed by the sway bracing is to prevent independent lateral vibration and swaying of the vertical trusses; also to stiffen long posts and to aid in the erection. Its design must be left mainly to the judgment of the engineer.

In deck bridges and deep through bridges the rectangular shape of the cross-section will be almost exactly preserved by the sway bracing, but with shallow sway bracing or with none at all, the floor-beams, when rigidly connected to the posts, are called upon to act as sway bracing, and small bending moments are produced in them with some tension on the connecting rivets. The stresses thus produced are very small, as shown above.

119. **The Centrifugal Force** * F, of a body of weight P, moving in a curve of radius r, is equal to

$$F = \frac{v^2}{gr}P, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where g is the acceleration of gravity $= 32.2$ ft. per second, $= 32.2 \times \frac{(60 \times 60)^2}{5280} = 79{,}100$ miles per hour. Expressed in miles, $r = \frac{5730}{D} \div 5280$, where $5730 =$ radius in feet of a $1°$ curve, and $D =$ degree of curve. Hence

$$F = \frac{v^2D \times 5280}{5730 \times 79100}P = .0000117v^2DP = kP, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where v is in miles per hour and $D =$ degree of curve; P and F are in like units.

Fig. 162 represents a transverse section of a through bridge. The centre of gravity of the panel load is at G, with an eccentricity e. The eccentricity e is the average eccentricity for one panel length and is due partly to the eccentricity of the track and partly to the horizontal displacement of the centre of gravity, caused by the inclination of the track. It will be taken as positive *outwards*. The line DD' is the centre of the floor-beam; AA' and CC' are lateral struts. The stresses caused by the force F and the load P, in the laterals, the main trussses, the stringers and floor-beams, will be discussed in order.

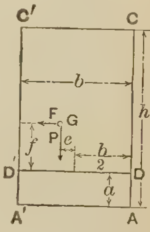


FIG. 162.

The Lateral Trusses.—The loads F and P are given over to the floor-beam and thence to the posts at D and D'. Fig. 163 shows the floor-beam with reactions. The horizontal reactions are each equal to $\frac{F}{2}$, assuming them equal to each other. These horizontal forces are carried by the posts to the laterals, the portion taken by the lower laterals being equal to $F \times \frac{h-a}{h}$, and that taken by the upper being equal to $F \times \frac{a}{h}$, one-half of each being applied at each side. The bending moments at D and $D' = \frac{F}{2}\frac{a(h-a)}{h}$.

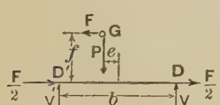


FIG. 163.

If the live load is taken as a uniform load, then P is the same for all panels, and likewise F. But if wheel loads are used, then P and F vary. However, since F is a constant function of P for any one problem, the maximum moments and shears in the lateral systems, due to centrifugal force, will be a constant function of the maximum moments and shears in a vertical truss due to the actual wheel loads. Hence, for the lower laterals, to get moments and shears, multiply those found in a vertical truss for the total wheel loads by $k\frac{h-a}{h}$; and for the upper laterals multiply by $k\frac{a}{h}$. The maximum value of F for one panel is equal to the maximum vertical floor-beam load multiplied by k. This value of F is to be used in getting the moments on the posts.

Usually one of the lateral systems lies practically in the plane of the floor-beams, in which case the whole of F is carried by this one lateral system and the posts receive no bending moment.

* For a fuller discussion of this subject see a paper by Prof. Ward Baldwin, M. Am. Soc. C. E., Trans. Am. Soc. C. E., Vol. XXV, p. 459.

The vertical main trusses supply the reactions V' and V, Fig. 163. Taking moments about D', we have

$$V = \frac{P\left(\frac{b}{2} - e\right) - Ff}{b} = P\left(\frac{1}{2} - \frac{e + kf}{b}\right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and from Σ vert. comp. $= 0$,

$$V' = P\left(\frac{1}{2} + \frac{e + kf}{b}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

From these two equations we see that the inner truss receives its maximum load for a minimum value of k, that is, for a stationary load; and that the outer truss receives its maximum load for a maximum value of k.

Since e varies for the different panels, V and V' are not constant functions of P. However, those portions of V and V' not containing e, i.e., $P\left(\frac{1}{2} - \frac{kf}{b}\right)$ and $P\left(\frac{1}{2} + \frac{kf}{b}\right)$, are constant functions of P, and the stresses in the trusses due to these *portions* of the load may be found similarly to the stresses in the laterals; that is, by finding the moments and shears in a single truss, or in this case the stresses, due to the actual total wheel loads, and then multiplying by the factor $\frac{1}{2}$ for the inner truss ($k = 0$), and by $\left(\frac{1}{2} + \frac{kf}{b}\right)$ for the outer truss.

For the stresses due to the portions $- P\frac{e}{b}$ and $+ P\frac{e}{b}$ it is sufficiently accurate to assume an equivalent uniform load, determine e, and thence $P\frac{e}{b}$ for each panel, and with these panel loads compute the stresses in the ordinary way. A portion of the loads on each truss will be upward.

Stringers.—Each stringer will receive a lateral moment and shear equal to k times the maximum vertical moment and shear due to one-half the actual wheel loads. This moment is taken by the upper flange and adds to its stress, and the shear requires additional rivets to connect the stringer to the floor-beam.

The vertical load upon each stringer is found just as for the trusses, and is given by eqs. (19) and (20) by substituting for b the distance between stringers. If we call this distance b', the load upon the inner stringer is, for $k = 0$,

$$\frac{P}{2}\left(1 - \frac{2e}{b'}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and that upon the outer stringer is

$$\frac{P}{2}\left(1 + 2\frac{e + kf}{b'}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

In this case e may be taken as constant for one panel, using an average value. The maximum moments and shears in the stringers are then found by multiplying those for actual wheel loads symmetrically placed, by the multipliers of $\frac{P}{2}$ in the equations above. The quantity e refers in this case to the axis of the *stringers* and not necessarily to the axis of the bridge; it will thus have different values in different panels, and hence the stresses in the stringers will be different.

Floor-beams.—The *total* maximum floor-beam load is that due to the actual wheel loads moving on a straight track. The portions given over to the two trusses are given by eqs. (19) and (20). With $k = 0$, the value of V is the maximum shear at the inner end; and with k a maximum, the corresponding value of V' is the maximum shear at the outer end of the floor-beam. These shears, multiplied by the distances between the posts and the stringers, give the maximum moments in the floor-beams. The value of e in the above is to be taken as the average value for two panels, referred to the axis of the bridge.

The floor-beams are also subjected to a direct compression from the laterals.

CHAPTER VIII.

FUNDAMENTAL RELATIONS IN THE THEORY OF BEAMS.

120. Historical Sketch.*—For two hundred and fifty years the true theory of the strength of a beam has been a much-mooted question amongst physicists, engineers, and mathematicians.

Galileo was the first of whom we have any record who undertook to discuss the problem. In his famous Dialogues (Leiden, 1638, from which Fig. 164 is taken) he propounds a theory based on an assumed absolute rigidity of the material, and concluded that the fibres of the beam were subjected to a uniform

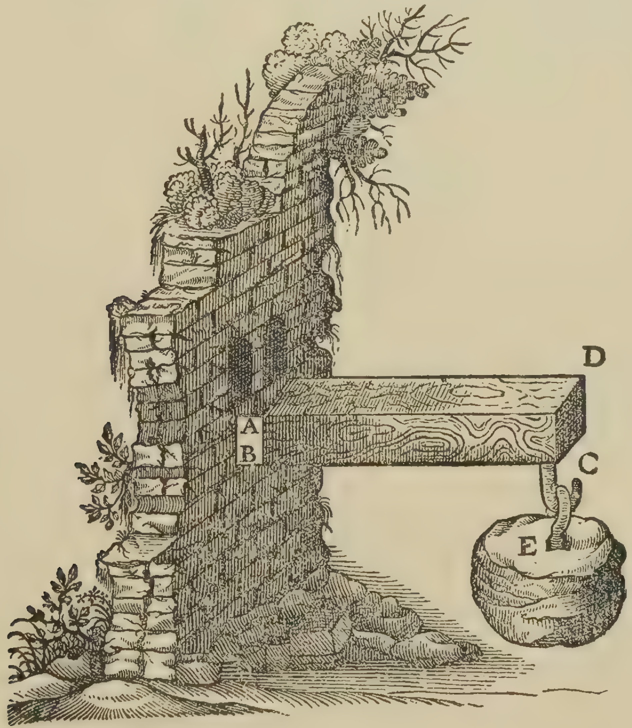


FIG. 164.

tension which acted about the base of the beam as a fulcrum. On this theory the moment of resistance of a solid rectangular beam would be $\frac{fbh^2}{2}$, where f is the ultimate strength of the material in tension.

Robert Hooke first published his famous law of the relation between strain and stress in 1678, discovered by him he says 18 years previously, and kept secret for the purpose of procuring patents on some applications of the principle to springs for watches, clocks, etc. Two years previously he had ventured to publish the law in an anagram at the end of another book, in this form, "*ceiiinosssttuv*," which being interpreted reads, "*Ut tensio sic vis,*" or, "as the extension so is the resistance." Hooke makes this law apply to all "springy" bodies, amongst which he names nearly all ordinary solids. This is still known as *Hooke's Law.*

* This historical review of the development of the true theory of the beam is derived mostly from Saint-Venant's "*Historique Abrégé des Recherches sur la Résistance et sur l'Elasticité des Corps Solides,*" prefixed to his Navier's "Leçons," Third Edition, Paris, 1864, and from Todhunter's "*History of the Theory of Elasticity,*" Cambridge. Eng., 1886.

Mariotte showed by experiment in 1680 that the fibres on one side of the beam were extended and on the other side compressed, and *assumed* that the neutral surface passes through the centre of gravity of the section.

Varignon, in 1702, undertakes to harmonize the theories of Galileo and Mariotte, by admitting the extension of the fibres, but puts the neutral plane at the bottom, as Galileo did, and assumes the tensile stress as uniformly varying from there to the other side. This would make the strength of a solid rectangular beam $\frac{fbh^2}{3}$, which agrees almost exactly with the facts for cast-iron at rupture when f is the tensile strength.

James Bernouilli made an important advance by applying Mariotte's law to obtain deflections of beams (1694 and 1705), and argued that the position of the neutral axis is a matter of indifference, which was a great error. He denied the truth of Hooke's law, which we know is not applicable to all substances, nor to the point of rupture with any substance. He first constructed stress-strain curves, but his work in the field of hydraulics was of even greater importance than in the study of solids.

A. Parent, a French academician, seems to have been the first to perceive (1713) the mechanical necessity of equilibrium between the tensile and compressive stresses, which condition, together with that of a uniform variation of stress, fixes the position of the neutral axis at the centre of gravity of the section. This important discovery seems, however, to have passed unnoticed.

Coulomb reannounced this relation in a memoir to the French Academy in 1773, or sixty years after its first publication by Parent. Saint-Venant credits Coulomb with never having seen Parent's work, as no writer of that century has mentioned it. But even after this second publication of so important a necessary truth, such workers as Girard, Barlow, and Tredgold all misconceived the mathematical necessities in the problem, and resorted to various makeshifts to explain the strength of beams.

Navier finally, in 1824, put the matter on a solid mathematical basis, although he also at first went entirely astray. He stated in his first edition that the moment of resistance varied as the cube of the depth of the beam, and in his second edition this error was corrected, but the moment of the stresses on one side the neutral axis was said to be equal to the moment of the stresses on the other side, about that axis, an equality which does not exist except on symmetrical sections. Navier also fully developed the theory of the deflection of beams as we now use it.

Saint-Venant, a student of Navier's, has finally (1857) in his notes on Navier's *Leçons* given a complete analysis of both the elastic and the ultimate strength of a beam, with suitable equations which will give theoretical results agreeing with the actual tests, when the empirical constants are properly evaluated. This great engineer, physicist, and teacher has done more than any other one to bring theory and practice into harmony and to put both on a thoroughly scientific basis, so far as the strength and elasticity of engineering materials is concerned.*

In spite of these various true expositions of this subject the source of strength in a beam continues still to be very imperfectly understood by many engineers, and even by current writers on applied mechanics, and gross errors in this direction are still common. It is in consideration of this state of the science that the problem is treated so fully here.

121. Elementary Principles of Universal Application.†—Since stress is the internal resistance to distortion produced by the application of external forces to a body, there is always an equilibrium established between the internal stresses and the external forces. When the body is a rigid beam which is acted upon by external forces which produce shear and bending moment at any given section, the equilibrium between the stresses and forces is of exactly the same kind as obtains in the case of a framed structure, as discussed in Chapter II. Hence the three general equations of equilibrium apply to beams the same as they do to trusses. We may therefore pass a section through a beam, replace one portion by the stresses acting at this section, and write the three general equations:

Σ vertical components of stresses $= \Sigma$ vertical components of external forces;
Σ horizontal " " " $= \Sigma$ horizontal " " " "
Σ moments " " $= \Sigma$ moments " " "

* He died January 6, 1886.

† In this chapter few proofs are given for fundamental equations, these being fully developed in modern text-books on Applied Mechanics.

The sum of the vertical components of the stresses is called the shear on the section When the beam is subjected to vertical forces only, there is no resulting horizontal component either of the forces or of the stresses. That is to say, the algebraic sum of the horizontal stresses which make up the resisting moment must be zero, since there is no resultant horizontal force to be resisted. *Therefore the sum of the compressive stresses must always equal the sum of the tensile stresses in simple cross-bending.* This is a mathematical or mechanical necessity, and holds true at rupture the same as at the elastic limit, being independent of the material or of the form of the section. If these two opposing stresses were concentrated at their respective centres of gravity, they would form a couple, the moment of which is the moment of resistance which holds in equilibrium the external forces, and hence it is always equal to the external moment. The horizontal surface where the stresses change from tension to compression is called the neutral surface, or neutral axis, since here there is neither tension nor compression.

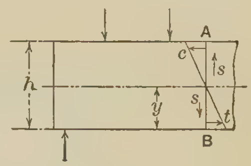


FIG. 165.

If the centre of moments be taken at this surface, the moment of the resisting couple is the arithmetical sum of two moments which are equal to each other under symmetrical conditions. But since these partial moments are added together to make up the total moment of resistance, there is no logical necessity that they should be equal, and when the cross-section is not a symmetrical one they never are equal.

122. Elementary Principles True within the Elastic Limit.—Within the elastic limit the ratio between stress and strain is a constant one, or here Hooke's Law holds true. Beyond this limit the stress does not increase as rapidly as the strain.

Within the elastic limit also a normal section of the beam which is plane before bending is a plane and nearly normal after bending. Since after bending two plane sections which originally were parallel become inclined to each other, it follows that the fibres, or elements, joining these planes have a uniformly varying length across the section even though these planes are no longer normal. If the bending has stopped within the elastic limit, then the stresses are proportional to the strains they are resisting and therefore *for all bending within the elastic limit the stresses are uniformly varying across the section.* This conclusion is a logical or geometrical necessity from the premises which have been experimentally established.

From the two conditions of uniformly varying stress and the absolute equality between the sum of the tensile and the sum of the compressive stresses, it follows (from the laws of mechanics) that *the neutral axis, or neutral plane, traverses the centre of gravity of the section.*

If $I =$ moment of inertia of the cross-section,
$f =$ unit stress on the extreme fibre on either side,
$y_1 =$ distance from neutral axis (c. of gr.) to the same extreme fibre,
$M_0 =$ moment of resistance of beam at any section, = bending moment M, of the external forces on one side of that section about a centre taken at the neutral axis in the plane of the section,

then, so long as f remains inside the elastic limit, we have, for all materials and forms of cross-section, the following exact and universally true relation:

$$M = M_0 = \frac{fI}{y_1}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

which is the general equation of the moment of resistance of a beam, within the elastic limit.

If the section is a symmetrical one, f and y_1 are the same for the extreme fibres on both the tension and the compressive sides. If unsymmetrical, as in Fig. 166, then the extreme fibre on the compressive side, being much farther away from the neutral axis, has a corre-

spondingly larger stress. The unit stresses follow a law of uniform variation across the entire section, but total stress, being the unit stress multiplied by the area over which it acts, might be shown graphically by multiplying the unit stresses by the corresponding widths of the section, which would give a diagram, as shown in Fig. 167, where the areas above and below the neutral axis are equal.

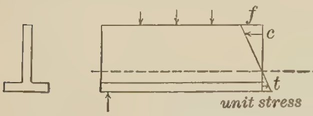


FIG. 166.

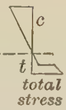
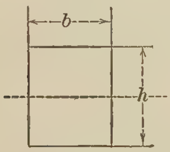
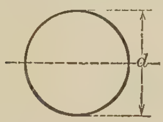
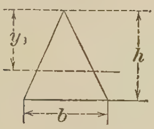
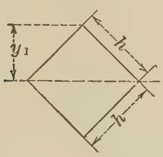
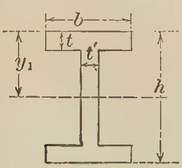
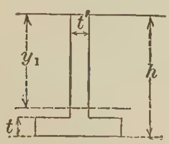
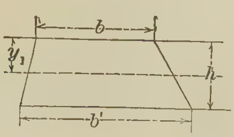


FIG. 167.

The following moments of resistance of solid cross-sections in terms of the unit stress on the most distant fibre, with the corresponding values of I and y_1, are rigidly correct for all bending stresses inside the elastic limit of the material.

Figure.	Distance of Centre of Gravity, or Neutral Axis, from the most distant fibre $= y_1$.	Moment of Inertia about the Centre of Gravity of the Section $= I$.	Moment of Resistance in terms of the Stress in the most distant fibre $= M_0 = \frac{fI}{y_1}$.
168. 	$\frac{h}{2}$	$\frac{bh^3}{12}$	$\frac{1}{6}fbh^2$
169. 	$\frac{d}{2}$	$\frac{\pi d^4}{64}$	$\frac{\pi}{32}fd^3$
170. 	$\frac{2}{3}h$	$\frac{bh^3}{36}$	$\frac{1}{24}fbh^2$
171. 	$\frac{h}{2}\sqrt{2}$	$\frac{h^4}{12}$	$\frac{1}{6\sqrt{2}}fh^3$
172. 	$\frac{h}{2}$	$\frac{bh^3 - (b - t')(h - 2t)^3}{12}$	$\frac{bh^3 - (b - t')(h - 2t)^3}{6h}f$
173. 	$\frac{\frac{1}{2}t'h^2 + t(b - t')(h - \frac{1}{2}t)}{t'h + t(b - t')}$	$\frac{bh^3 - (b - t')(h - t)^3}{3} - Ay_1^2$	$\frac{fI}{y_1}$
174. 	$\frac{b + 2b'}{b + b'} \cdot \frac{h}{3}$	$h^3\left[\frac{3b + b'}{12} - \frac{(b + 2b')^2}{18(b + b')}\right]$	$\frac{fh^2}{6}\left[\frac{3(3b + b')(b + b')}{2(b + 2b')} - (b + 2b')\right]$

123. Elementary Principles true beyond the Elastic Limit and at Rupture.—In Fig. 175 are shown strain diagrams of some of the more common materials used in engineering structures. Their distortions within their respective elastic limits are too small to be shown on this diagram. These limits are therefore at the points where the curves leave the vertical axis. Cast-iron and timber have no definite elastic limits, their diagrams being curved from the start. When these materials are used in beams of simple geometric form, as in solid rect-

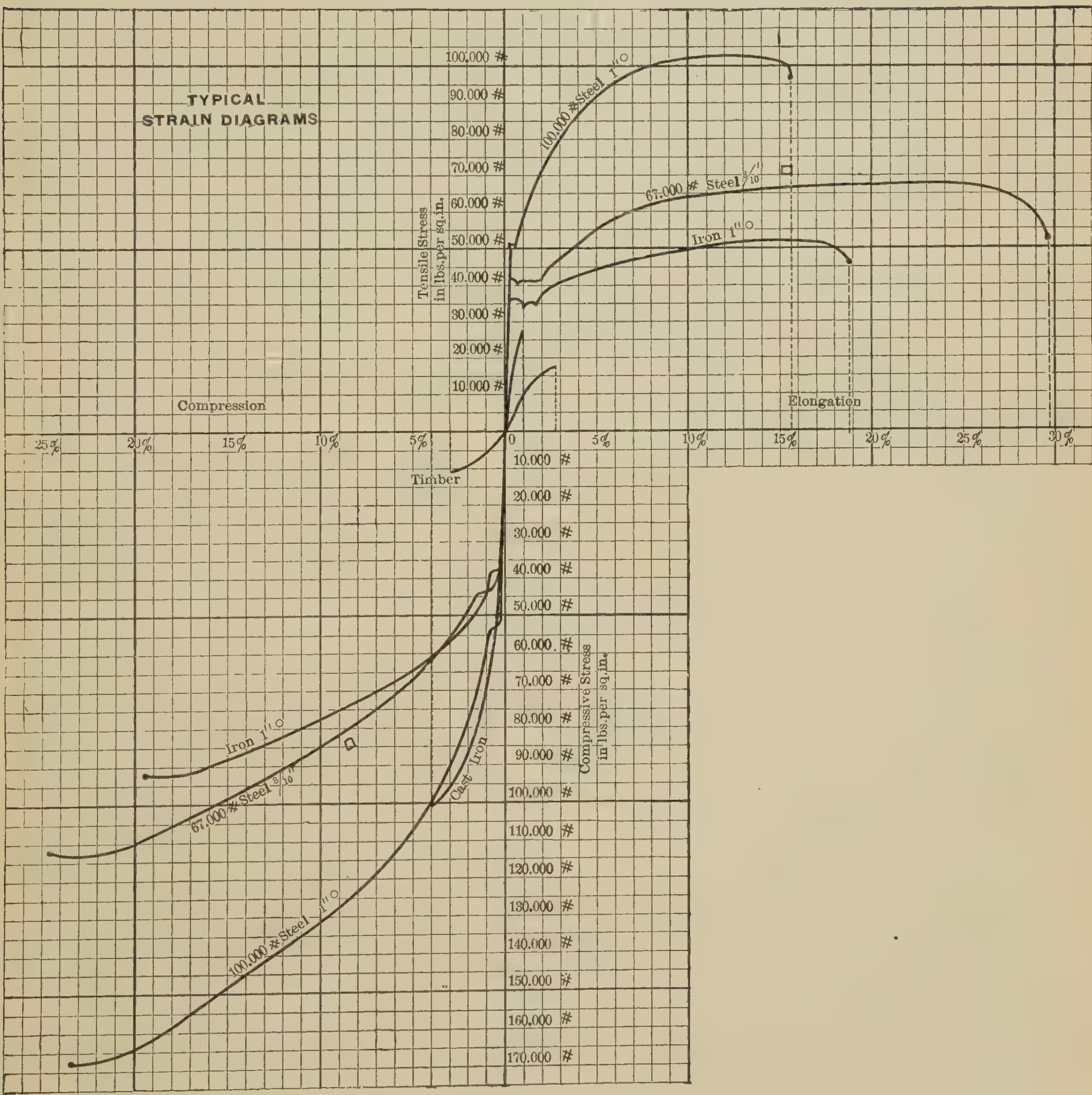


FIG. 175.

angular or circular cross-sections, their breaking strength does not conform to the equations given in the last article. In the case of the ductile metals like wrought-iron and mild steel such a beam will bend cold through an angle of 180° perhaps without rupture, and hence cannot be said to have any assignable "cross-breaking" strength. If such a term is used it should be named "cross-bending" strength, and by this should be understood its strength at its elastic limit. At this point it conforms to the laws of internal distribution of stress as given in the last article. Such materials therefore can scarcely be regarded as having any definite or assignable ultimate cross-breaking strength, or modulus of rupture.

With such materials as timber or cast-iron, however, the case is different. Here the beam does break under definite loads, but these loads are always much greater (perhaps by 100 per cent) than the equations in the previous article would seem to indicate. The fatal mistake writers on mechanics have made has been in ever using such a form of equation as (1) to express the moment of resistance of a beam at rupture. Its field of application lies only within the elastic limit of the material.

124. The Distribution of Longitudinal Stress (Tension and Compression) across the Section of a Beam when loaded beyond its Elastic Limit.—Since a section plane before bending remains sensibly plane after bending, even beyond its elastic limit, it follows that the *strain* or distortion is uniformly varying across the section, under all circumstances. From the neutral axis to the tension side of the beam, therefore, the stress increases just as it does in an ordinary strain diagram, where the increments of strain are plotted to a uniform horizontal scale. If the beam fails in tension, the stress, at rupture, on the extreme fibre on the tension side of the beam is the ultimate tensile strength of the material.

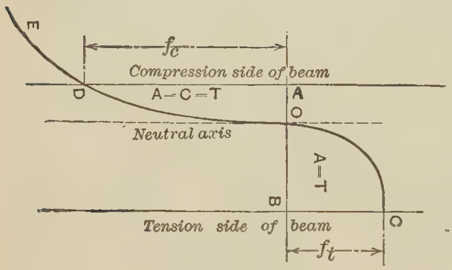


FIG. 176.

Thus let the curves OC and OD in Fig. 176 represent the tension and compression strain diagrams of cast-iron, for instance. The ordinate $CB = f_t$ is the tensile strength of the iron. Since this is the actual stress on the extreme fibre at rupture, we may construct a vertical section, or projection of the beam on this diagram by taking the ordinate BC as the tension side of the beam, the point O marking the neutral axis. Then OB is the distance from the neutral axis to the tension side of the beam to some scale, and for a rectangular cross-section the area OCB would represent the total tensile stress in the beam at that section. But since the total tensile must equal the total compressive stress in simple bending, the area cut off from the indefinite compression strain diagram OE by the line marking the position of the compressed side, AD, must be just equal to the area OCB, or $OAD = OCB$. This condition fixes the position of that side of the beam, AD, to the same scale as obtains for the distance OB, and hence AB represents the total height of the beam, and the scale of the drawing is determined. The point O is then the position of the neutral axis at rupture, and the ordinates to the lines OC and OD from the axis AB represent the tensile and compressive stresses across the section.

The moment of resistance is the moment of the couple formed by these equal and opposite stresses, when concentrated at the centres of gravity of their respective areas, which moment of resistance should equal the bending moment of the external forces at rupture. The fact that this moment, so computed, is not equal to the breaking moment in the case of cast-iron can be attributed only to the existence in cast-iron of high internal stresses, due to the fact that the external surfaces are solidified first. These outer fibres are all in a state of initial compression, which must first be overcome on the tension side of a cast-iron beam before these fibres begin to resist any tensile distortion. In the case of a wooden beam the action is the direct reverse of the above. Timber is much stronger in tension than in compression; hence a wooden beam fails first on the compression side by breaking down the fibres, which results in a continual lowering of the neutral axis, as the load increases, until the beam finally ruptures on the tension side and the failure is complete. Fig. 177 shows a common method of failure of green timber where the tensile stress at rupture AD is about five times the com-

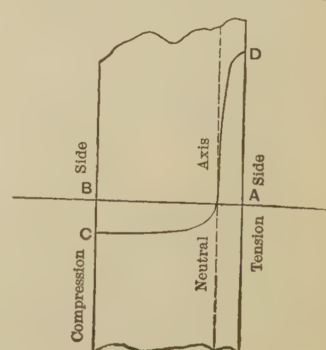


FIG. 177.

pressive stress *CB*. In dry timber the compressive strength is greatly increased and the neutral axis remains nearer the centre of the beam. When the complete tensile and compressive strain diagrams of any material are known the moment of resistance of a beam of that material can be found as here described.

125. Rational Equations for the Moment of Resistance of a Beam at Rupture.— Since a strain diagram is a smooth curve, it would seem possible to obtain some simple equation for such a locus and then proceed to find the enclosed areas and the moment of the couple in terms of the co-ordinates of these curves. M. Saint-Venant, in his notes on Navier's "*La Résistance des Corps Solides*," has offered such equations which prove to be capable of expressing the strength of a beam at rupture very closely. Thus when the strain diagram is fitted to the section of a beam, as shown in Fig. 178,

Let f_t = tensile strength = length of tension ordinate where curve becomes horizontal;

f_c = same in compression;

y_t = distance from neutral axis to tension side, if failure occurs in tension;

y_c = distance from neutral axis to the fibre which would have stress f_c (beyond the limits of the beam if failure occurs in tension);

f = stress in fibre distant y from neutral axis in tension side;

f' = same at distance y' in compressive side.

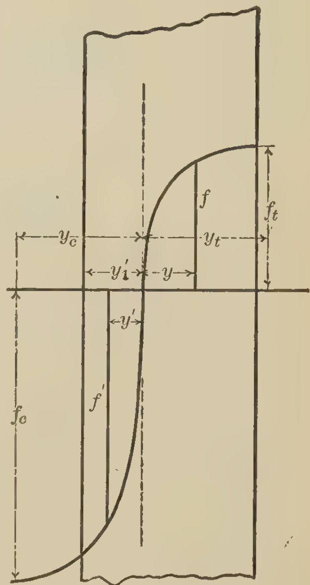


FIG. 178.

Now since nearly all bodies distort or flow somewhat under a constant load at the point of rupture, whether in tension or compression, the strain diagram would be horizontal at this point. Also, as soon as this point is reached, failure is sure to occur under a constant loading. Therefore the strain diagram representing the distribution of the stress on the side which fails may be supposed to come into a horizontal position at the outer fibre on that side. On the other side it comes into a horizontal position at a point beyond the limits of the beam. Thus both of these curves may be fairly represented by parabolas of various degrees, all having the vertices at the point of greatest stress and passing through the origin.

Saint-Venant proposes, therefore, the following equations for these curves:

$$f = f_t\left[1 - \left(1 - \frac{y}{y_t}\right)^m\right] \quad \text{and} \quad f' = f_c\left[1 - \left(1 - \frac{y'}{y_c}\right)^m\right]. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The first of these equations is for the tension and the second for the compression side of the beam. The form of these curves for different values of m is shown in Fig. 179, where they are drawn only for the failing side of the beam. When $m = 1$ we have $f = \frac{f_t}{y_t}y$, which is a linear function, and corresponds to the ordinary equations within the elastic limit. The value of m must be selected so as to make the curve correspond as nearly as possible to the strain diagram of the material, and may be different for the two kinds of stress.

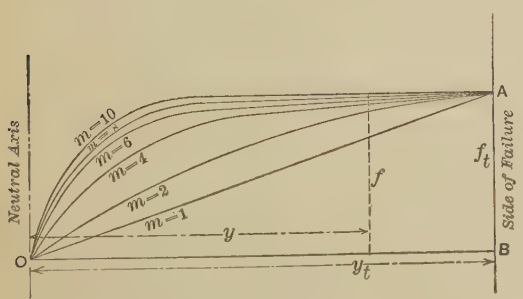


FIG. 179.

Whenever the strength of the material is very much greater in one way than in the other, as in cast-iron, where the compressive strength is some five times what it is in

tension, and in timber, where the reverse holds true, the neutral axis shifts at rupture very far towards the stronger side, as shown in Figs. 177 and 178, and the portion of the strain diagram utilized on this side becomes practically a straight line. In this case the formula expressing moment of resistance is very much simplified, becoming

$$M_0 = \frac{fbh^2}{6}\left(\frac{m\left(\frac{3(m+3)}{m+2} + 4\sqrt{\frac{2}{m+1}}\right)}{m+3+2\sqrt{2(m+1)}}\right)^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here f is the ultimate strength of the material on the side which fails first, as tension in the case of cast-iron and compression in the case of timber. For different qualities of cast-iron, for instance, the tension strain diagrams would point to some one of the various curves shown in Fig. 180, thus indicating what value of m to use in eq. (3). It will be seen that m increases as the material becomes more ductile, and the neutral axis moves farther

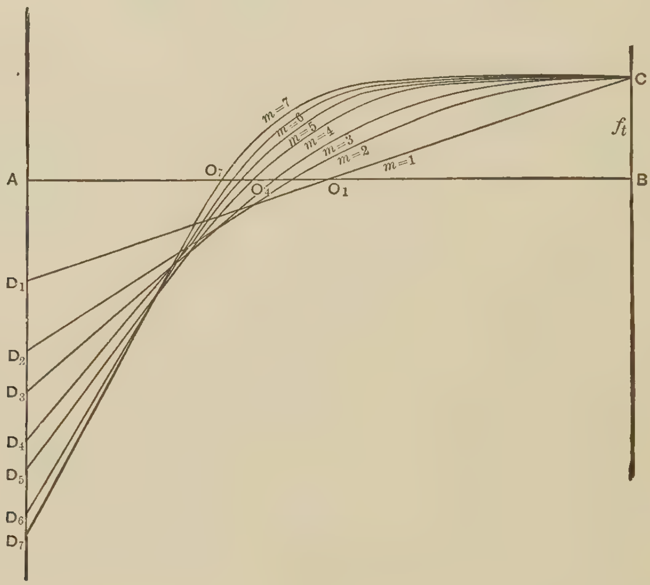


FIG. 180.

and farther towards the compressed side of the beam when the elastic limit in compression is relatively high. When $m = 1$ we have $M_0 = \frac{fbh^2}{6}$, which holds true within the elastic limit for all materials. The following table † gives values of the moment of resistance for various

Value of m.	Dist. of Neutr. Axis from the weaker side $= \frac{y_t}{h}$.	Ratio of Fibre Stress on Stronger Side to that on Weaker Side at Rupture.	Moment of Resistance at Rupture in terms of the weaker strength $= M_0$.
1	0.500	1.000	$1.000 \times \frac{fbh^2}{6}$
2	.550	1.633	1.417 "
3	.586	2.121	1.654 "
4	.613	2.530	1.810 "
5	.634	2.887	1.922 "
6	.655	3.207	2.007 "
7	$\frac{2}{3} = .667$	$\frac{7}{2} = 3.500$	$\frac{56}{27} = 2.074$ "
∞	1.000	∞	3.000 "

* From Saint-Venant's Navier, § 151, p. 182, edition of 1864, Paris. The equation giving moment of resistance for the general case, from which this is derived, is given on p. 179, but is too complicated to be reproduced here.

† Saint-Venant's Navier, Paris, 1864, p. 182.

values of m, and also the position of the neutral axis and the ratio of the fibre stresses on the two sides of the beam at the time of rupture, when the stress varies uniformly from the neutral axis outward on the stronger side, as in Fig. 180.

The value of m for cast-iron varies from 3 to 7, depending on the toughness or ductility of the iron, f being taken as the tensile strength of the metal. For timber f would be taken as the compressive strength, and m taken as 4 or 5.

When the common formula $M_0 = \frac{fI}{y_1}$, or for solid rectangular sections $M_0 = \frac{1}{6}fbh^2$, is used for the ultimate strength of a beam, f becomes the "modulus of rupture in cross-breaking," and its value always lies intermediate between the two moduli in tension and compression. In timber it is nearly an arithmetic mean of these two moduli, while for cast-iron it is from $1\frac{1}{2}$ to 2 times the tension modulus.

For practical purposes it is just as well to use the ordinary equation $M_0 = \frac{fI}{y_1}$ up to rupture, remembering that f in this case becomes the modulus of rupture in cross-breaking, and its value must be determined by cross-breaking tests. The true theory of the ultimate strength of a beam has been given here, in order that the student may not conclude that theory is unable to cope with this problem, as is commonly supposed, and because it is not given in English and American works on applied mechanics.

126. To find the Moment of Inertia of a Section composed of Rectangles.—The moment of resistance of an irregular cross-section in terms of the stress in the extreme fibre can only be found by first finding the centre of gravity of the section which is always traversed by the neutral axis until after the elastic limit is passed, and the moment of inertia of the section about this neutral axis. When the section can be supposed to be made up of a series of rectangles the centre of gravity and moment of inertia can best be found as follows:

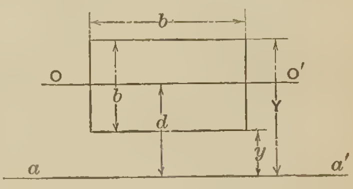


FIG. 181.

In Fig. 181, the moment of inertia of the rectangle about its own centre of gravity axis OO' is $I_0 = \frac{1}{12}bh^3 = \frac{1}{12}b(Y-y)^3$. Its moment of inertia about any other axis, as aa' parallel to OO', is

$$I_a = I_0 + ad^2 = \frac{b}{12}(Y-y)^3 + b(Y-y)\left(\frac{(Y+y)^2}{4}\right) = \frac{b}{3}(Y^3-y^3). \quad . \quad . \quad . \quad . \quad (4)$$

Also the statical moment of the area about the axis aa' is

$$M_a = Ad = b(Y-y)\left(\frac{Y+y}{2}\right) = \frac{b}{2}(Y^2-y^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

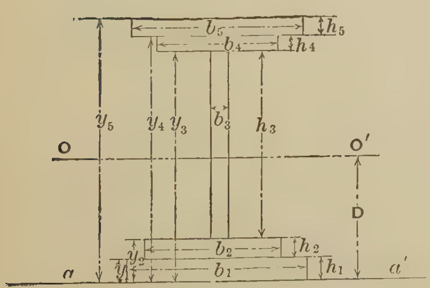


FIG. 182.

In a compound form, made up of rectangles, as in Fig. 182, the total area is the sum of the areas of the several rectangles; the total statical moment about aa' is the sum of the partial moments; and the total moment of inertia about aa' is the sum of the partial moments of inertia about this axis, or we may write

$$A = \Sigma a; \quad M_a = \Sigma M_a; \quad I_a = \Sigma I_a.$$

The most convenient method of computing these values is by arranging the computation in tabular form as follows, where Y represents the larger of two successive values of y.

TABULAR COMPUTATION OF MOMENT OF INERTIA FOR RECTANGULAR FORMS.

b	h	y	A	y^2	$Y^2 - y^2$	$\frac{b}{2}(Y^2 - y^2) = ad$	y^3	$Y^3 - y^3$	$\frac{b}{3}(Y^3 - y^3) = I$
		o		o			o		
b	h_1		b_1h_1		h_1^2	$\frac{b_1}{2}h_1^2$		h_1^3	$\frac{b_1}{3}h_1^3$
		h_1		h_1^2			h_1^3		
b_2	h_2		b_2h_2		$(h_1 + h_2)^2 - h_1^2$	&c.		$(h_1 + h_2)^3 - h_1^3$	&c.
		$h_1 + h_2$		$(h_1 + h_2)^2$			$(h_1 + h_2)^3$		
b_3	h_3		&c.		&c.	&c.		&c.	&c.
		$h_1 + h_2 + h_3$		&c.			&c.		
b_4	h_4		&c.		&c.	&c.		&c.	&c.
		&c.		&c.			&c.		
b_5	h_5		&c.		&c.	&c.		&c.	&c.
		&c.		&c.			&c.		
			$\Sigma = A$			$\Sigma = M_a$			$\Sigma = I_a$.

We now have for the distance to centre of gravity axis, $D = \frac{M_a}{A}$. Also, the moment of inertia about the neutral axis $= I_0 = I_a - AD^2 = I_a - M_aD$, where M_a is the gravity moment of the area about the axis a.

The moment of resistance of the cross-section is

$$M_0 = \frac{fI_a}{y_1},$$

where f is the stress on the extreme fibre which is at a distance y_1 from the neutral axis, either side being taken.

127. To find the Centre of Gravity and the Moment of Inertia of any Irregular Section.

The following graphical method consists essentially in the measurement, in most cases, of a single area easily constructed, and with the aid of a planimeter is very rapid and accurate. The problem will be treated in two parts:

1st. To find the moment of inertia about any given axis.

2d. To find the gravity axis and the moment of inertia about that axis.

1st. Let it be required, for example, to determine the moment of inertia of the rail section, shown in Fig. 183, about any given axis, as AA'. The actual operation would be as follows:

For the upper portion of the figure draw any line OB, perpendicular to AA', and, at some whole number of units k, from AA', draw the parallel BC. Draw also any number of lines through the given area parallel to AA', as lp, kb, jc, etc., spacing them closer where the outline is irregular than where regular, and lay off $B1' = O1$, $B2' = O2$, $B3' = O3$, etc. Then for any point d, draw $d4'$ intersecting BC in m; then Om, thus determining a point d'' on $4d$. In like manner find points e'', f'', etc., corresponding to e, f, etc., and join the new system of points by a smooth curve. The oblique construction lines need not of course be actually drawn, the required intersections only being marked. The new curve for the lower portion is likewise constructed, using a line $B'C'$, at a distance k, below AA' in place of BC.

Then if A'' represent the total area of our new figure above and below, and I_a the required moment of inertia, we shall have

$$I_a = k^2A''.$$

Demonstration.—The general expression for moment of inertia about an axis AA' is

$$I_a = \int b\,dy\,y^2,$$

where b = length of strip parallel to AA', dy its width, and y its distance from the axis. If k is a constant, we may write

$$I_a = k\int\left(b\frac{y}{k}\right)dy\,y. \qquad \text{(1)}$$

$$= k^2\int\left(b\frac{y^2}{k^2}\right)dy. \qquad \text{(2)}$$

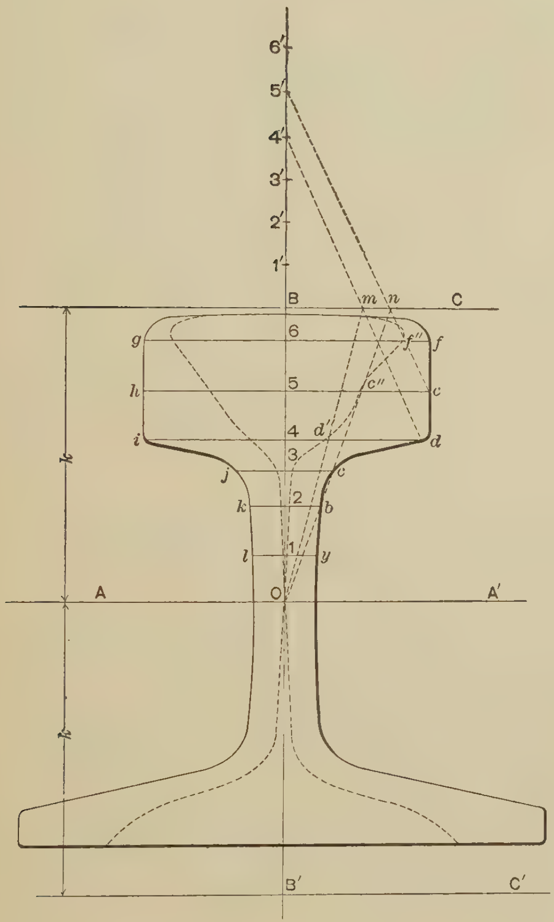


FIG. 183.

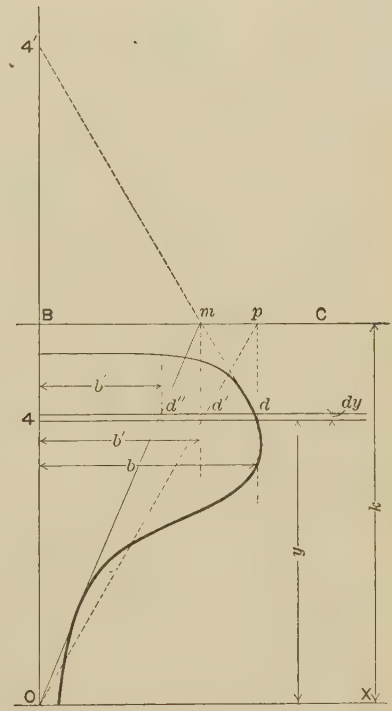


FIG. 184.

Referring now to Fig. 184, let md' be drawn parallel to OB and then Op through d'. Then, since $O4 = B4'$ and $Bm = 4d' = b'$, we have $Bp = 4d = b$. Therefore, by similar triangles,

$$\frac{b}{b'} = \frac{k}{y}, \quad \text{and} \quad \frac{b'}{b''} = \frac{k}{y}. \qquad \text{(3)}$$

Multiplying, we get $b'' = b\frac{y^2}{k^2}$. The above construction being carried out as in Fig. 183 for each horizontal strip, we have, by substituting in (2),

$$I_x = k^2\int b''dy$$

$$= k^2A''. \qquad \text{Q. E. D.}$$

If OB can be made a line of symmetry, it will be necessary to construct but one-half of the new curve; also, k should be made of such a length as to give a fair-sized area to measure and at the same time good intersections.

2d. This problem can usually be reduced to the first by determining the centre of gravity from considerations of symmetry, or by cutting out the section from thick paper and balancing on a knife-edge. Where this cannot readily be done, as in the case of disconnected parts, we may proceed thus:

Assume an axis AA', Fig. 185, parallel to the unknown gravity axis, and construct the area A'' as before; also at the same time project the points m, n, p, etc., upon their corresponding horizontal lines, thus fixing points c', d', e', etc. Join these also by a smooth curve, and let A_1' be that part of the area of this new figure above the axis, and A_2' the lower portion. Only the upper right-hand portion is shown in Fig. 185, but the given area may be of any shape, and the line OB drawn anywhere. As before, we shall have $I_a = k^2 A''$. If now A represent the total original area, we have, by taking moments about AA',

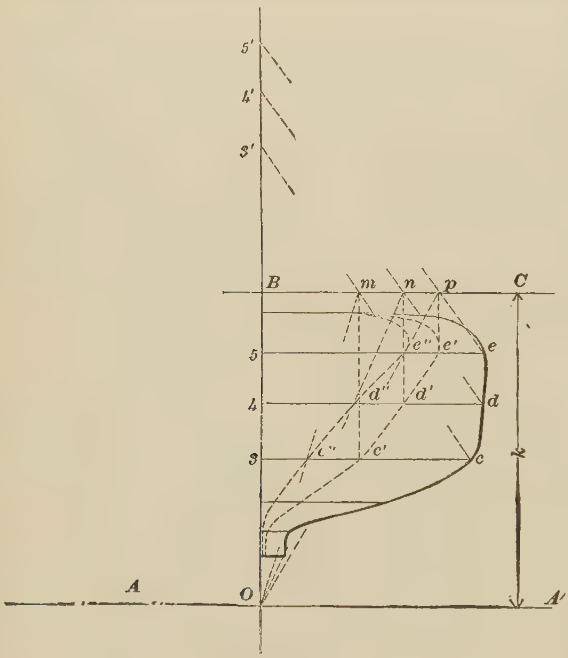


FIG. 185.

Distance of centre of gravity above $AA' = d$,

$$= \int \frac{bdy\,y}{A}$$

$$= \frac{k\int\left(b\frac{y}{k}\right)dy}{A}. \quad . \quad . \quad . \quad (4)$$

From (3), $b' = b\frac{y}{k}$, and considering y negative below the axis, $\int b'\,dy = A_1' - A_2'$. Substituting in (4), we then have

$$d = k\frac{A_1' - A_2'}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

I_0 being the required moment of inertia about a gravity axis, we have

$$I_0 = I_a - d^2 A$$

$$= k^2\left[A'' - \left(\frac{A_1' - A_2'}{A}\right)^2 A\right]$$

$$= k^2\left[A'' - \frac{(A_1' - A_2')^2}{A}\right],$$

in which the areas to be measured are, A, the given area, and A' (A_1' and A_2') and A'', the two "constructed" areas.

If desired, the area A' may be first constructed and the centre of gravity located by eq. (5) then a new axis taken through it and I_0 determined as in the first case. This construction then gives us a method for finding the centre of gravity of a figure without considering the other part of the problem.

Considered as a section through a loaded beam, it is interesting to note that if we make k equal to the distance to one extreme fibre, taking a gravity axis, our area A' will be such

that if a *uniform* stress of the same intensity as that upon this outer fibre were applied, the resulting moment would be the same as that upon the section. This relation is fully discussed and used to some advantage in finding moments of inertia in Sir Benjamin Baker's "Strength of Beams," the above method being in the main an extension and simplification of the one there given.

128. General Relation between Shear and Bending Moment in Beams.—The shear is the summation of all the components of the external forces on one side of the section taken parallel to that section. The bending moment is the sum of the moments of all the external forces on one side of the section about the centre of gravity of the section. There follows at once from these this

PROPOSITION: *The bending moment at any section of a beam or truss is equal to the bending moment at any other section of the beam or truss plus the shear at that section into its arm, plus the products of all the intervening external forces into their respective arms.*

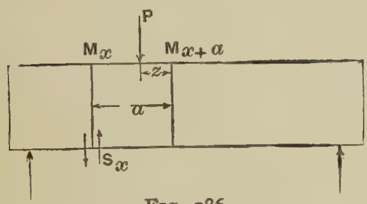


FIG. 186.

Thus in Fig. 186 we have

$$M_{x+a} = M_x + S_x a - Pz. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If these sections be taken very near together, and the intervening load omitted, so that in eq. (6) $a = dx$ and $P = 0$, then M_{x+a} becomes M_{x+dx}, and we have

$$M_{x+dx} = M_x + S_x dx \quad \text{or} \quad M_{x+dx} - M_x = Sdx.$$

But $M_{x+dx} - M_x = dM$; therefore we have

$$dM = Sdx \quad \text{or} \quad \frac{dM}{dx} = S; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

that is to say, the shear at any section is the first differential coefficient of the bending moment at that section. If the bending moment is constant, therefore, the shear must be zero, and, *vice versa*, when the shear is zero the bending moment is constant. But when $\frac{dM}{dx} = 0$ the bending moment is either a maximum or a minimum; therefore when the bending moment passes through a maximum or a minimum the shear is zero, and also when the shear becomes zero the bending moment is either a maximum or a minimum. A knowledge of this relation can often be used with advantage in the analysis of trusses.

129. The Deflection of Beams.—Let Fig. 187 represent a portion of a bent beam one unit in length. The sections which were parallel and normal before bending would now meet if extended. From similar triangles we have

$$\frac{\epsilon}{y_1} = \frac{1}{r}.$$

But $$E = \frac{\text{unit stress}}{\text{unit strain}} = \frac{f}{\frac{\epsilon}{1}} \quad \text{or} \quad \epsilon = \frac{f}{E},$$

also $$M_0 = \frac{fI}{y_1}, \quad \text{or} \quad f = \frac{M_0 y_1}{I}; \quad \text{hence} \quad \epsilon = \frac{M_0 y_1}{EI},$$

and we have

$$\frac{1}{r} = \frac{M_0}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

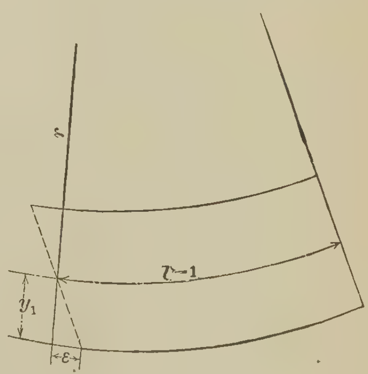
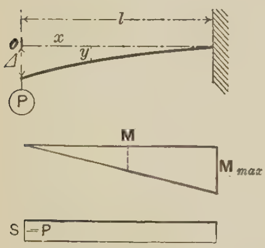
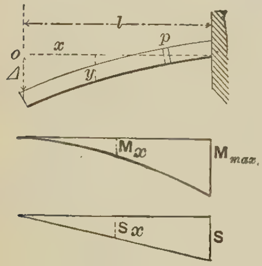
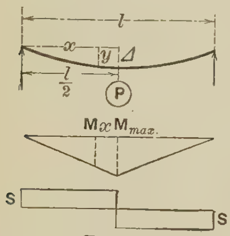
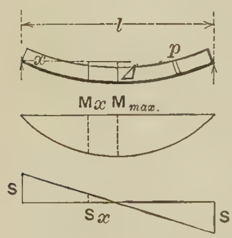
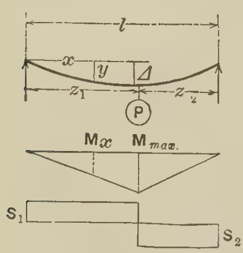


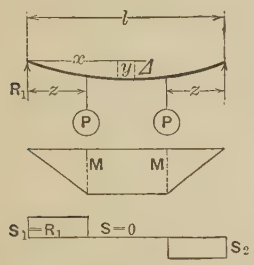
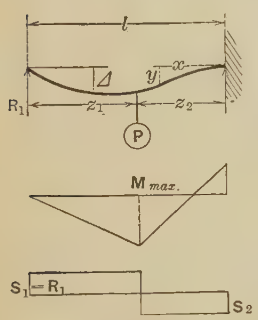
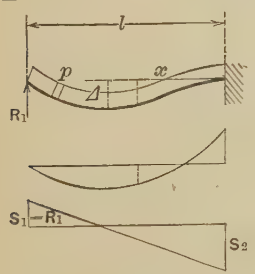
FIG. 187.

But from the calculus we have $\frac{1}{r} = \frac{dxd^2y}{(dl)^3}$. In the case of the deflection of beams the

MOMENTS, STRESSES, AND DEFLECTION OF BEAMS.

The Beam and its Load with the Moment and Shear Diagrams.	Moment Equation and Maximum Moment. M_x and $M_{max.}$	Equation of Elastic Line, and Maximum Deflection in terms of the Loading. y and Δ_w.	Maximum Deflection in terms of Stress on Extreme Fibre of Symmetrical Sections. Δ_f.	Maximum Stress on Extreme Fibre in terms of the Loading. Symmetrical Sections. f.
 FIG. 188.	$M_x = -Px$ $M_{max.} = -Pl$	$y = \frac{P}{6EI}[2l^3 - 3l^2x + x^3]$ $\Delta = \frac{Pl^3}{3EI}$	$\Delta_f = \frac{2fl^2}{3Eh}$	$f = \frac{Plh}{2I}$
 FIG. 189.	$M_x = -\frac{px^2}{2}$ $M_{max.} = -\frac{pl^2}{2}$	$y = \frac{p}{24EI}[x^4 - 4l^3x + 3l^4]$ $\Delta = \frac{pl^4}{8EI}$	$\Delta_f = \frac{fl^2}{2Eh}$	$f = \frac{pl^2h}{4I}$
 FIG. 190.	$M_x = \frac{P}{2}x$ $M_{max.} = \frac{Pl}{4}$	$y = \frac{Px}{48EI}[3l^2 - 4x^2]$ $\Delta = \frac{Pl^3}{48EI}$	$\Delta_f = \frac{fl^2}{6Eh}$	$f = \frac{Plh}{8I}$
 FIG. 191.	$M_x = \frac{px}{2}(l - x)$ $M_{max.} = \frac{pl^2}{8}$	$y = \frac{px}{24EI}[l^3 - 2lx^2 + x^3]$ $\Delta = \frac{5pl^4}{384EI}$	$\Delta_f = \frac{5}{24}\frac{fl^2}{Eh}$	$f = \frac{pl^2h}{16I}$
 FIG. 192.	$x < z_1$ $M_x = \frac{Pz_2x}{l}$ $x > z_1$ $M_x = \frac{Pz_2x}{l} - P(x - z_1)$ $M_{max.} = \frac{Pz_1z_2}{l}$	$x < z_1$ $y = \frac{Pz_2x}{6EIl}[2lz_1 - z_1^2 - x^2]$ $x > z_1$ $y = \frac{Pz_1(l-x)}{6EIl}[2lx - x^2 - z_1^2]$ $\Delta = \frac{Pz_2}{27EIl}\sqrt{3[z_1(2z_2 + z_1)]^3}$ for $x = \frac{1}{3}\sqrt{3[z_1(2z_2 + z_1)]}$	$\Delta_f = \frac{f}{27Eh}\sqrt{3z_1(2z_2 + z_1)^3}$	$f = \frac{Pz_1z_2h}{2lI}$

MOMENTS, STRESSES, AND DEFLECTION OF BEAMS.

The Beam and its Load with the Moment and Shear Diagrams.	Moment Equation and Maximum Moment. *Mx* and *Mmax.*	Equation of Elastic Line, and Maximum Deflection in terms of the Loading. y and Δ_w.	Maximum Deflection in terms of Stress on Extreme Fibre of Symmetrical Sections. Δ_f.	Maximum Stress on Extreme Fibre in terms of the Loading. Symmetrical Sections. f.
 FIG. 193.	$x < z$ $M_x = Px$ $x > z$ $M = Pz = M_{max.}$	$x < z$ $y = \frac{Px}{6EI}[3lz - 3z^2 - x^2]$ $x > z$ $y = \frac{Pz}{6EI}[3lx - 3x^2 - z^2]$ $\Delta = \frac{Pz}{6EI}\left[\frac{3}{4}l^2 - z^2\right]$	$\Delta_f = \frac{f}{3Eh}\left[\frac{3}{4}l^2 - z^2\right]$	$f = \frac{Pzh}{2I}$
 FIG. 194.	$R_1 = \frac{P}{2l^3}[3lz_2^2 - z_2^3]$ $x < z_2$ $M_x = R_1(l-x) - P(z_2 - x)$ $x > z_2$ $M_x = R_1(l - x)$ $M_{max.} = R_1(l - z_2)$ for $x = z_2$	$x < z_2$ $y = \frac{1}{6EI}[R_1x^3 - 3R_1lx_2 + 3Pz_2x^2 - Px^3]$ $x > z_2$ $y = \frac{1}{6EI}[R_1x_1^3 - 3R_1lx^2 + 3Pz_2^2x - Pz_2^3]$ $\Delta = \frac{Pz_2^2}{6EI}(l-z_2)\sqrt{\frac{l - z_2}{3l - z_2}}$ for $x = l\left(1 - \sqrt{\frac{l - z_2}{3l - z_2}}\right)$	$\Delta_f = \frac{2f}{3Eh}\sqrt{(3l - z_2)(l - z_2)}$	$f = \frac{Ph}{4l^3I}(3lz_2^2 - z_2^3)(l - x)$
 FIG. 195.	$R_1 = \frac{3}{8}pl$ $M_x = \frac{p}{4}(4x - l)(l - x)$ $M_{max.} = -\frac{pl^2}{8}$ for $x = 0$	$y = \frac{px^2}{48EI}(l - x)(3l - 2x)$ $\Delta = 0.0054\frac{pl^4}{EI}$ for $x = 0.578l$	$\Delta_f = \frac{0.0864fl^2}{Eh}$	$f = \frac{pl^2h}{16I}$

deviation from a horizontal line is so small that we may call $dl = dx$; hence $\frac{1}{r} = \frac{d^2y}{dx^2}$, and we have, since the bending moment, M, is always equal to the moment of resistance, M_0,

$$\frac{1}{r} = \frac{M}{EI} = \frac{d^2y}{dx^2}, \qquad (9)$$

which are the fundamental equations in the deflection of beams.

From the differential equation $\frac{d^2y}{dx^2} = \frac{M}{EI}$ we can, by giving to M its value in terms of x, and integrating once, obtain $\frac{dy}{dx} = i$, the angle the beam makes with the horizontal. By integrating again we obtain y, the vertical deflection of the beam at any point from its normal

position. The modulus of elasticity E and the moment of inertia I are usually constant. The latter may be made to vary as some function of x, and the integration made with I variable. These problems are worked out in detail in books on applied mechanics, and only a summary will be given here of the results for the more ordinary cases. In solid beams and in plate girders the deflection due to shear is neglected, though it doubtless is something appreciable. These equations do not apply to framed structures, since no account is taken of the deflection due to strains in the web system, which in ordinary bridges is nearly equal to that from the chords. (See Chap. XV.)

130. **The Distribution of Shearing Stress in a Beam.**—It is proved in mechanics that wherever a shearing stress acts along any plane in an elastic solid, there is another shearing stress of the same intensity at that point acting on another plane at right angles to the first. Also, that the effect of these two equal shearing stresses at right angles to each other is to produce two direct stresses, of the same intensity, also at right angles to each other, and at angles of 45° with the former planes, one of these direct stresses being tension and the other compression, as shown in Fig. 196.

The general equation showing the value of the intensity of the shearing stress at any point in the cross-section of a beam of any form is

$$q' = \frac{S}{Ib'}\int_{y'}^{y_1} y\,b\,dy,^{*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where q' = intensity of shearing stress on any plane;
S = total shearing stress on the section;
I = moment of inertia of the section about its neutral axis;
b' = breadth of section where shearing stress $= q'$;
y' = distance of the plane where shearing stress is q', from the neutral axis of the section;
y_1 = distance of extreme fibre on that side of the neutral axis, from that axis;
b = breadth of section at distance y from neutral axis.

Whence it follows that the integral $\int_{y'}^{y_1} y\,b\,dy$ is the statical moment of the area outside the longitudinal plane on which the shearing stress is taken, about the neutral axis of the beam. We may therefore define the intensity of the shearing stress as follows:

The intensity of the shearing stress at any point in a beam of solid section of whatever form is equal to the total shearing force on the entire cross section multiplied by the statical moment of the area of the section outside the longitudinal plane of shear in question about its axis in the neutral plane, divided by the product of the amount of inertia of the entire section into the breadth of the section at that point.

From this there may be deduced the following relations for special cases: For solid rectangular sections the intensity of the shear is zero at the top and bottom sides of the beam, and increases towards the centre as the ordinates to a parabola having its axis coincident with the neutral axis of the beam. Hence the maximum shearing stress is found at the centre of the section, where its value is $\frac{3}{2}$ of the mean intensity, or the shear at the centre of a solid rectangular beam is $\frac{3S}{2A}$, as shown in Fig. 196. If this beam is subjected to a uniform load the total shear is zero at the centre and increases uniformly to the ends. The total shear at any section, as S_1, S_2, S_3, etc., is shown on the lower diagram as the length of the corresponding vertical ordinate. In the upper figure this same total shear appears as the area of the parabola drawn at that section, horizontal ordinates to which represent the shear, at the

* Rankine's Applied Mechanics, § 309, eq. (1).

corresponding point, on a surface one unit long, having the breadth of the beam, this surface being either vertical or horizontal, and normal to the plane of the paper. Since the middle ordinate to a parabola at its vertex is $\frac{3}{2}$ the mean ordinate, it follows that

$$q_0 = \frac{3}{2}\frac{S}{A}. \quad . \quad . \quad (11)$$

This may be derived from equation (10) directly, by taking b constant for a rectangular section, whence we have

$$q' = \frac{6S}{h^2 A}(y_1^2 - y'^2) = \frac{3}{2}\frac{S}{A}, \quad (12)$$

for $y' = 0$, and $y_1 = \frac{h}{2}$.

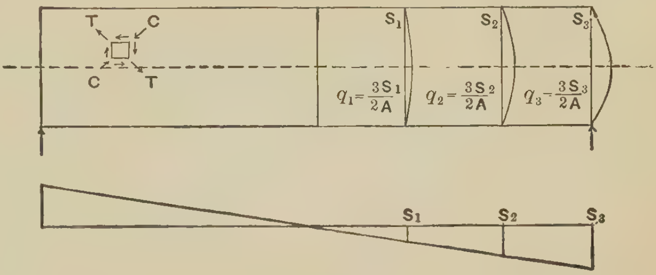


FIG. 196.

For a plate girder, eq. (10) gives directly the shearing stress at any point. In the girder shown in Fig. 197, let the web be $\frac{3}{8}$ in. thick and 48 in. high, while the flanges are $\frac{3}{4}$ in. thick

Uniformly Loaded

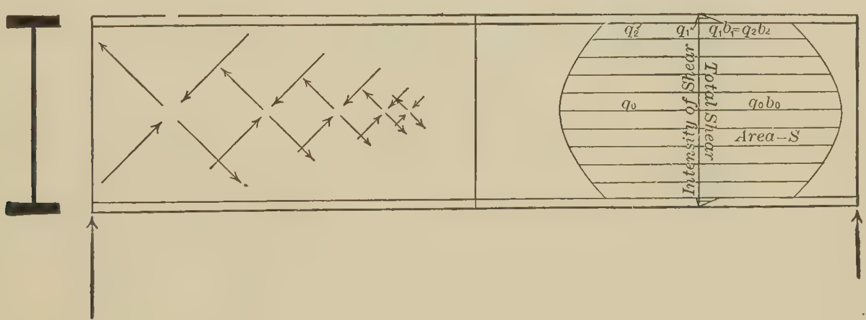


FIG. 197.

and 12 in. wide. Then the neutral axis lies at the centre of the girder, the area of one flange is 9 sq. in., and its statical moment about the neutral axis is 219.4 inch-pounds. The moment of inertia of the entire cross-section is 15,456. Whence we have for the intensity of the shearing stress at the inner planes of the flanges, $q_1 = \frac{S}{Ib_1}\int_{y'}^{y_1} y\,b\,dy = S\left(\frac{219.4}{12 \times 15,456}\right) = 0.00118S$ per sq. in., while in the outer sides of the web we have $q_2 = S\left(\frac{219.4}{\frac{3}{8} \times 15,456}\right) = 0.0378S$ per sq. in. The intensity of stress at the neutral axis is $q_0 = \frac{S}{Ib_0}\int_0^{y_1}(y\,b\,dy) = S\left(\frac{327.4}{\frac{3}{8} \times 15,456}\right) = 0.0565S$ per sq. in.

These three intensities of shearing stress are indicated in the "intensity" shear diagram in Fig. 197, plotted to the left of the plane of shear. The value of the average shearing intensity when the web is assumed to carry it all is $q = \frac{S}{\frac{3}{8} \times 48} = 0.0565S$. This is so near the value of q_0, the true maximum shearing stress, that the ordinary assumptions of web taking all the shear, and thus being uniformly distributed over the web, are seen to be justified.

By taking qb as the total shearing stress on the lamina bdy, we may construct the total shear diagram for this plane, as plotted to the right of the plane of shear in Fig. 197. In this diagram the total shear in a horizontal plane is seen to be the same on the interior sides of the flanges and on the outer edges of the web, and the area of the diagram represents the total shear, S, on the section.

It is important to note that since the shearing stress is nearly constant across the section the resulting direct stresses in the web, which are of equal intensity with the shearing stress, are also nearly uniform in amount across this section, not varying from zero at the top and bottom fibres to $\frac{3}{2}$ the mean at the centre, as in the case with plane rectangular sections.

If the beam be loaded at the centre, or with two concentrated loads, as in the case of a railway bridge floor-beam, then the shear is constant on the outer ends of the beam, and hence over this portion the web is subjected to nearly equal shearing and direct stresses. The compressive stresses, acting at 45° with the vertical, and in a direction downwards towards the end supports, tend to buckle the web plate. Taking a strip aa', Fig. 198, the intensity of the compressive load at the ends of this strip in pounds per square inch, if the dimensions of the cross-section be the same as taken above, would be $0.0378S$, while at the centre it would be $0.0565S$. If there were no tensile stress at right angles to this strip, tending to hold it into its true plane, the strip should be dimensioned in thickness to carry this unit load $0.038S$ as a load on a column of that length, which is $1.4\ h$.

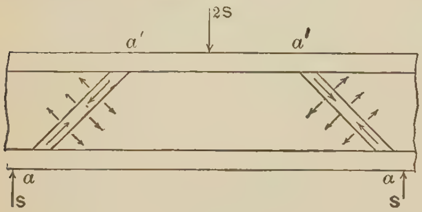


FIG. 198.

This kind of analysis would give an extravagant thickness of metal. Just what the restraining influence of the tensile stress is cannot be determined theoretically, and no adequate experiments have ever been made to show it empirically.

Aside from the strengthening influence of the tensile stress in the web it is common to further stiffen it against buckling by riveting angle-irons on the web in a vertical position, as shown on the left end of the beam in Fig. 199. On the right end these "stiffeners" are

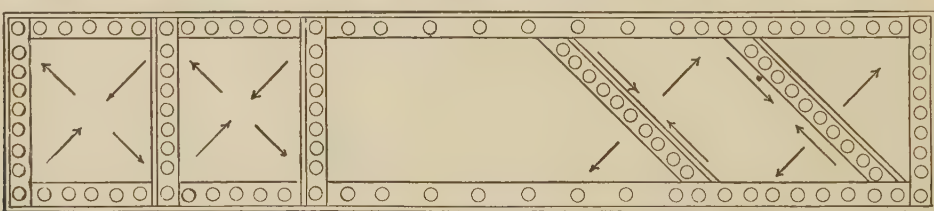


FIG. 199.

placed in an inclined position, directly opposed to the compressive, or buckling, stresses. In this position they are much more efficient in resisting the only strains in the web which are at all likely to cause failure, and in deep girders they might be placed in this position.*

CONTINUOUS GIRDERS.

131. The Continuous Girder is very seldom employed in this country except in swing bridges. The greatest objection to its use is the uncertainty in the stresses resulting either from a settlement in the supports or the impossibility of making these fit exactly to the normal or unstrained profile of the girder. A great deal of literature can be found in standard works on this subject, the original contributions of greatest value being Clapeyron's Theorem of Three Moments (1857), Weyrauch's adaptation to concentrated loads and unequal spans (1873), and Merriman's simplification of the same (1875).

In Fig. 200 (*a*) and (*b*) two consecutive spans are taken from a series of an indefinite number, these two being loaded uniformly in the one case with the unit loads p_{r-1} and p_r, and

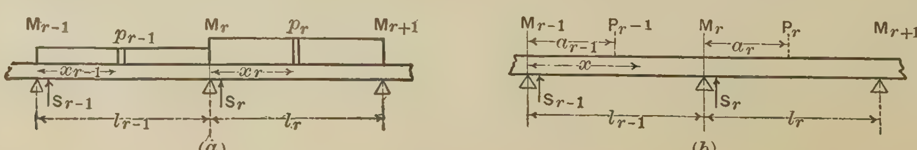


FIG. 200.

* See a full discussion of this question by the author in *Engineering News* of April 25, 1895 (Vol. XXXIII, p. 276).

in the other with the two concentrated loads P_{r-1} and P_r, respectively. Then from the *Proposition* in Art. 128, and eq. (6), we may write at once, for the moment at any section in the $(r-1)$th span, distant x from the left support,

$$M_x = M_{r-1} + S_{r-1}x - \tfrac{1}{2}p_{r-1}x^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

For a concentrated load P_{r-1} distant $a = kl_{r-1}$ from the left support we have

$$M_x = M_{r-1} + S_{r-1}x - P_{r-1}(x - kl_{r-1}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

For more than one load in the span the sign of summation is inserted before the term in P_{r-1}. From these equations, and the general differential equation of the elastic curve, $\frac{d^2y}{dx^2} = \frac{M}{EI}$, we may integrate once and find $\frac{dy}{dx} = \tan i$ at each end of the $(r-1)$th span, by making $x = 0$ and then $x = l_{r-1}$. By changing the subscripts these results would also apply to the rth span, in which case the $\tan_r$ for $x = 0$ would equal the $\tan_{r-1}$ for $x = l_{r-1}$. By equating these two values of $\tan i$ we obtain for uniform loads

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -\tfrac{1}{4}(p_{r-1}l_{r-1}^3 + p_rl_r^3), \quad . \quad . \quad . \quad . \quad (14)$$

while for concentrated loads in these spans we have

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -\Sigma P_{r-1}l_{r-1}^2(k - k^3) - \Sigma P_rl_r^2(2k - 3k^2 + k^3), \quad (14a)$$

no allowance being made for any settlement of supports. (See p. 142 for derivation of eq. 14*a*.)

These are two general forms of Clapeyron's famous *Equation of Three Moments*, first published in "*Comptes Rendus*," Dec. 1857. The forms here given are due first to Weyrauch, Leipzig, 1873, and to Merriman, *Phil. Mag.*, 1875.

When the spans are all equal and load uniform over the whole bridge, the equations become very much simplified, and are as follows:

$$\left.\begin{aligned} &M_1 + 4M_2 + M_3 \\ &= M_2 + 4M_3 + M_4 \\ &\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ &= M_{m-2} + 4M_{m-1} + M_m \end{aligned}\right\} = -\tfrac{1}{2}pl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

When the entire series of equations of three successive moments are written for any series of spans, the number of these is always two less than the number of supports, or than the whole number of M's found in the equations. But the end M's are always zero, which makes the number of unknowns equal to the number of equations, and hence all the intermediate M's can be found. The algebraic reduction is somewhat tedious and will not be given here. After the M's are all obtained, the shears can be found by means of eq. (13), or (13*a*), and hence the supporting forces.

Having found all the external forces acting upon the beam, the moment and shear at any section can be found by the ordinary methods as readily as for a simple beam on two supports. The great value of "the equation of the three moments" consists in its enabling us to find the supporting forces.

*The following formulæ apply only to beams and trusses having a uniform moment of inertia spans all of equal length, and when no settlement occurs at the supports.**

I. Bending Moments at the Supports.

(A) *For Uniform Load over the* r*th Span, Spans all Equal.*—The moment at the Mth support, counting from the left, for any number of spans wholly loaded with the uniform

* This condition is usually stated as "supports on a level." This is very misleading, as the formulæ do apply to supports out of level, provided they are fitted to the profile of the beam or truss in its normal unstrained condition.

load p per foot, the subscripts r applying to all loaded spans, n being the whole number of spans,

$$M_m = -\frac{pl^2}{4(c_{n-1}+4c_n)}\Big[\underbrace{\Sigma(c_r + c_{r+1})c_{n-m+2}}_{\text{(For loaded spans on left.)}} + \underbrace{\Sigma(c_{n-r+2} + c_{n-r+1})c_m}_{\text{(For loaded spans on right.)}}\Big]. \quad . \quad . \quad . \qquad (16)$$

(B) *For Concentrated Loads in the rth Span, Spans all Equal.*—The moment at the mth support, for loads P, at distances $a = kl$ from left of the rth span or spans, when total number of spans $= n$, all equal, is

$$M_m = -\frac{l}{c_{n-1}+4c_n}\Big[\underbrace{\Sigma\{\Sigma P(2k-3k^2+k^3)c_r + \Sigma P(k-k^3)c_{r+1}\}c_{n-m+2}}_{\text{(For loaded spans on left of } m\text{th support.)}}$$

$$+ \underbrace{\Sigma\{\Sigma P(2k-3k^2+k^3)c_{n-r+2} + \Sigma P(k-k^3)c_{n-r+1}\}c_m}_{\text{(For loaded spans on right of } m\text{th support.)}}\Big]. \qquad (17)$$

If both uniform and concentrated loads are found upon the same span, then both formulæ must be used. When more than one span is loaded, the data must be worked out for each, and the sum taken, as indicated by the primary signs of summation. The secondary summation signs for concentrated loads signify the summation for the several joints or concentrated loads.

The following values are to be used for the c coefficients:

$c_1 = 0$	$c_5 = -\ 56$	$c_9 = -\ 10{,}864$
$c_2 = +\ 1$	$c_6 = +\ 209$	$c_{10} = +\ 40{,}545$
$c_3 = -\ 4$	$c_7 = -\ 780$	$c_{11} = -\ 151{,}316$
$c_4 = +\ 15$	$c_8 = +\ 2{,}911$	$c_{12} = +\ 564{,}719$

following the law that $c_m = -4c_{m-1} - c_{m-2}$.

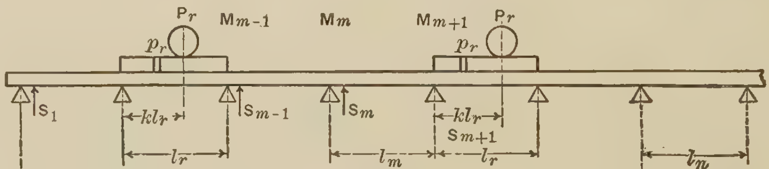


FIG. 201.

Since the concentrated load is usually at a panel point, if there were five panels in the loaded span, k would be $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$, for loads at the first, second, third, and fourth panel points, respectively. The annexed table of the values of the two terms in k, viz., $(k - k^3)$ and $(2k - 3k^2 + k^3)$, has been compiled for a more ready computation of these quantities. They are computed for the aliquot parts of a span-length, or for the joints of a truss of equal panel lengths, for all numbers of panels up to twelve.

When we have but two equal spans, as is commonly assumed to be the case in swing bridges, equations (16) and (17) reduce to the following:

(C) *Two Equal Spans, Uniform Load.*—$M_2 = -\frac{pl^2}{16}$ for full load on either span; or when both spans are fully loaded,

$$M_2 = -\frac{pl^2}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

(D) *Two Equal Spans, Concentrated Loads.*

$$M_2 = -\frac{l}{4}\Big(\underbrace{\Sigma P(k-k^3)}_{\text{(For left span.)}} + \underbrace{\Sigma P(2k-3k^2+k^3)}_{\text{(For right span.)}}\Big). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

VALUES OF $(k - k^3)$ AND OF $(2k - 3k^2 + k^3)$ AT PANEL POINTS.*

Two Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{2}$	.375	$\frac{1}{2}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Three Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{3}$	.296	$\frac{2}{3}$
$\frac{2}{3}$	.370	$\frac{1}{3}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Four Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{4}$	.234	$\frac{3}{4}$
$\frac{2}{4}$	.375	$\frac{2}{4}$
$\frac{3}{4}$	.328	$\frac{1}{4}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Five Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{5}$	.192	$\frac{4}{5}$
$\frac{2}{5}$	.336	$\frac{3}{5}$
$\frac{3}{5}$	.384	$\frac{2}{5}$
$\frac{4}{5}$	.288	$\frac{1}{5}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Six Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{6}$	.162	$\frac{5}{6}$
$\frac{2}{6}$	.296	$\frac{4}{6}$
$\frac{3}{6}$	.375	$\frac{3}{6}$
$\frac{4}{6}$	.370	$\frac{2}{6}$
$\frac{5}{6}$	.255	$\frac{1}{6}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Seven Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{7}$	.140	$\frac{6}{7}$
$\frac{2}{7}$	.263	$\frac{5}{7}$
$\frac{3}{7}$	.350	$\frac{4}{7}$
$\frac{4}{7}$	.851	$\frac{3}{7}$
$\frac{5}{7}$	.349	$\frac{2}{7}$
$\frac{6}{7}$	.228	$\frac{1}{7}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Eight Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{8}$	.123	$\frac{7}{8}$
$\frac{2}{8}$	.234	$\frac{6}{8}$
$\frac{3}{8}$	.322	$\frac{5}{8}$
$\frac{4}{8}$	.375	$\frac{4}{8}$
$\frac{5}{8}$	.381	$\frac{3}{8}$
$\frac{6}{8}$	.328	$\frac{2}{8}$
$\frac{7}{8}$	.205	$\frac{1}{8}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Nine Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{9}$	.110	$\frac{8}{9}$
$\frac{2}{9}$	.211	$\frac{7}{9}$
$\frac{3}{9}$	.296	$\frac{6}{9}$
$\frac{4}{9}$	.356	$\frac{5}{9}$
$\frac{5}{9}$	.384	$\frac{4}{9}$
$\frac{6}{9}$	.370	$\frac{3}{9}$
$\frac{7}{9}$	.307	$\frac{2}{9}$
$\frac{8}{9}$	.186	$\frac{1}{9}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Ten Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{10}$	.099	$\frac{9}{10}$
$\frac{2}{10}$	.192	$\frac{8}{10}$
$\frac{3}{10}$	.273	$\frac{7}{10}$
$\frac{4}{10}$	.336	$\frac{6}{10}$
$\frac{5}{10}$	.375	$\frac{5}{10}$
$\frac{6}{10}$	.384	$\frac{4}{10}$
$\frac{7}{10}$	.357	$\frac{3}{10}$
$\frac{8}{10}$	.288	$\frac{2}{10}$
$\frac{9}{10}$	.171	$\frac{1}{10}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Eleven Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{11}$	.090	$\frac{10}{11}$
$\frac{2}{11}$	.176	$\frac{9}{11}$
$\frac{3}{11}$	.253	$\frac{8}{11}$
$\frac{4}{11}$	.316	$\frac{7}{11}$
$\frac{5}{11}$	.361	$\frac{6}{11}$
$\frac{6}{11}$	.383	$\frac{5}{11}$
$\frac{7}{11}$	.379	$\frac{4}{11}$
$\frac{8}{11}$	.343	$\frac{3}{11}$
$\frac{9}{11}$	.271	$\frac{2}{11}$
$\frac{10}{11}$	.156	$\frac{1}{11}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

Twelve Panels.

$k=\frac{a}{l}$	$k-k^3$	
$\frac{1}{12}$	.082	$\frac{11}{12}$
$\frac{2}{12}$	.162	$\frac{10}{12}$
$\frac{3}{12}$	.234	$\frac{9}{12}$
$\frac{4}{12}$	.296	$\frac{8}{12}$
$\frac{5}{12}$	.344	$\frac{7}{12}$
$\frac{6}{12}$	.375	$\frac{6}{12}$
$\frac{7}{12}$	.385	$\frac{5}{12}$
$\frac{8}{12}$	.370	$\frac{4}{12}$
$\frac{9}{12}$	.328	$\frac{3}{12}$
$\frac{10}{12}$	.255	$\frac{2}{12}$
$\frac{11}{12}$	.146	$\frac{1}{12}$
	$2k-3k^2+k^3$	$k=\frac{a}{l}$

(E) *For a Uniform Load over the Entire Girder, Spans all Equal.*—Equation (16) now reduces to

$$M_m = \left[\frac{c_{n+1}(c_{m+1}-1) - c_m(c_{n+2}-1)}{c_{n+1}}\right]\frac{pl^2}{12}.\dagger \quad \ldots\ldots\ldots \quad (20)$$

This formula is evaluated in the following diagram for all supports for girders of nine spans or less.

* Condensed from Prof. Malverd A. Howe's "Theory of the Continuous Girder," where the student will find the most extended theoretical treatment of the continuous girder in the English language.

† This and the following equation, (21), are taken from Du Bois' "Framed Structures," with some change of form.

MOMENTS AT SUPPORTS; TOTAL UNIFORM LOAD; SPANS ALL EQUAL.

COEFFICIENTS OF $(-pl^2)$.

$$0 \quad 1 \quad \frac{1}{8} \quad 2 \quad 0$$

Λ Λ Λ

$$0 \quad 1 \quad \frac{1}{10} \quad 2 \quad \frac{1}{10} \quad 3 \quad 0$$

Λ Λ Λ Λ

$$0 \quad 1 \quad \frac{3}{28} \quad 2 \quad \frac{2}{28} \quad 3 \quad \frac{3}{28} \quad 4 \quad 0$$

Λ Λ Λ Λ Λ

$$0 \quad 1 \quad \frac{4}{38} \quad 2 \quad \frac{3}{38} \quad 3 \quad \frac{3}{38} \quad 4 \quad \frac{4}{38} \quad 5 \quad 0$$

Λ Λ Λ Λ Λ Λ

$$0 \quad 1 \quad \frac{11}{104} \quad 2 \quad \frac{8}{104} \quad 3 \quad \frac{9}{104} \quad 4 \quad \frac{8}{104} \quad 5 \quad \frac{11}{104} \quad 6 \quad 0$$

Λ Λ Λ Λ Λ Λ Λ

$$0 \quad 1 \quad \frac{15}{142} \quad 2 \quad \frac{11}{142} \quad 3 \quad \frac{12}{142} \quad 4 \quad \frac{12}{142} \quad 5 \quad \frac{11}{142} \quad 6 \quad \frac{15}{142} \quad 7 \quad 0$$

Λ Λ Λ Λ Λ Λ Λ Λ

$$0 \quad 1 \quad \frac{41}{388} \quad 2 \quad \frac{30}{388} \quad 3 \quad \frac{33}{388} \quad 4 \quad \frac{32}{388} \quad 5 \quad \frac{33}{388} \quad 6 \quad \frac{30}{388} \quad 7 \quad \frac{41}{388} \quad 8 \quad 0$$

Λ Λ Λ Λ Λ Λ Λ Λ Λ

$$0 \quad 1 \quad \frac{56}{530} \quad 2 \quad \frac{41}{530} \quad 3 \quad \frac{45}{530} \quad 4 \quad \frac{44}{530} \quad 5 \quad \frac{44}{530} \quad 6 \quad \frac{45}{530} \quad 7 \quad \frac{41}{530} \quad 8 \quad \frac{56}{530} \quad 9 \quad 0$$

Λ Λ Λ Λ Λ Λ Λ Λ Λ Λ

In general it may be said that the moments at the supports next to the ends are always the greatest, and are there about $\frac{1}{10}pl^2$; that they are least at the third supports from the ends, where they are about $\frac{1}{13}pl^2$; and near the centre they are nearly uniform at about $\frac{1}{12}pl^2$.

II. Shears at the Left Ends of the Spans.

Having found the moments at all the supports for the particular loading in question, it remains to find the shears on the left ends of each span, when the stresses in all the members are readily computed. To find these shears, make $x = l_{r-1}$ in eqs. (13) and (13a), when they become

$$M_r = M_{r-1} + S_{r-1}l_{r-1} - \tfrac{1}{2}p_{r-1}l^2_{r-1}$$

for a uniform load over the $(r-1)$th span, and

$$M_r = M_{r-1} + S_{r-1}l_{r-1} - \Sigma P_{r-1}l_{r-1}(1-k)$$

for a series of concentrated loads in the $(r-1)$th span; whence, for the shear at the left end of the rth span, or at the right of the rth support,

$$S_r = \frac{M_{r+1} - M_r}{l_r} + \tfrac{1}{2}pl_r \quad \ldots\ldots\ldots\ldots \quad (21)$$

as the shear at the left end of any span which is uniformly loaded, and

$$S_r = \frac{M_{r+1} - M_r}{l_r} + \Sigma P(1-k) \quad \ldots\ldots \quad (21a)$$

as the shear at the left end of any span carrying concentrated loads.

For unloaded spans the second terms in the right members become zero.

Evidently the shear at the right end of any span is that on the left end minus the intervening load.

The following gives the shears on each side of the supports for the case of uniform load over the entire girder. The supporting force is the sum of the two shears at that support.

SHEARS AT SUPPORTS; TOTAL UNIFORM LOAD; SPANS ALL EQUAL.
COEFFICIENTS OF (wl).

$$\begin{array}{ccc} & 1 & \\ \frac{0\,|\,1}{2} & & \frac{1\,|\,0}{2} \end{array}$$

$$\begin{array}{ccccc} & 1 & & 2 & \\ \frac{0\,|\,3}{8} & & \frac{5\,|\,5}{8} & & \frac{3\,|\,0}{8} \end{array}$$

$$\begin{array}{ccccccc} & 1 & & 2 & & 3 & \\ \frac{0\,|\,4}{10} & & \frac{6\,|\,5}{10} & & \frac{5\,|\,6}{10} & & \frac{4\,|\,0}{10} \end{array}$$

$$\begin{array}{ccccccccc} & 1 & & 2 & & 3 & & 4 & \\ \frac{0\,|\,11}{28} & & \frac{17\,|\,15}{28} & & \frac{13\,|\,13}{28} & & \frac{15\,|\,17}{28} & & \frac{11\,|\,0}{28} \end{array}$$

$$\begin{array}{ccccccccccc} & 1 & & 2 & & 3 & & 4 & & 5 & \\ \frac{0\,|\,15}{38} & & \frac{23\,|\,20}{38} & & \frac{18\,|\,19}{38} & & \frac{19\,|\,18}{38} & & \frac{20\,|\,23}{38} & & \frac{15\,|\,0}{38} \end{array}$$

$$\begin{array}{ccccccccccccc} & 1 & & 2 & & 3 & & 4 & & 5 & & 6 & \\ \frac{0\,|\,41}{104} & & \frac{63\,|\,55}{104} & & \frac{49\,|\,51}{104} & & \frac{53\,|\,53}{104} & & \frac{51\,|\,49}{104} & & \frac{55\,|\,63}{104} & & \frac{41\,|\,0}{104} \end{array}$$

$$\begin{array}{ccccccccccccccc} & 1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 & \\ \frac{0\,|\,56}{142} & & \frac{86\,|\,75}{142} & & \frac{67\,|\,70}{142} & & \frac{72\,|\,71}{142} & & \frac{71\,|\,72}{142} & & \frac{70\,|\,67}{142} & & \frac{75\,|\,86}{142} & & \frac{56\,|\,0}{142} \end{array}$$

$$\begin{array}{ccccccccccccccccc} & 1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 & & 8 & \\ \frac{0\,|\,153}{388} & & \frac{235\,|\,205}{388} & & \frac{183\,|\,191}{388} & & \frac{197\,|\,195}{388} & & \frac{193\,|\,193}{388} & & \frac{195\,|\,197}{388} & & \frac{191\,|\,183}{388} & & \frac{205\,|\,235}{388} & & \frac{153\,|\,0}{388} \end{array}$$

$$\begin{array}{ccccccccccccccccccc} & 1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 & & 8 & & 9 & \\ \frac{0\,|\,209}{530} & & \frac{321\,|\,280}{530} & & \frac{250\,|\,261}{530} & & \frac{269\,|\,266}{530} & & \frac{264\,|\,265}{530} & & \frac{265\,|\,264}{530} & & \frac{266\,|\,269}{530} & & \frac{261\,|\,250}{530} & & \frac{280\,|\,321}{530} & & \frac{209\,|\,0}{530} \end{array}$$

III. Computation of Stresses in the Members.

From the moments at the supports and the shear on the right of each support all the stresses are readily found. Thus for any section distant x from the left end of the mth span, we have, from eqs. (13) and (13a),

$$M_{(mx)} = M_m + S_m x - \tfrac{1}{2} p_m x^2. \quad \ldots \ldots \ldots \quad (22)$$

for a uniform load in this span, and

$$M_{(mx)} = M_m + S_m x - \sum_0^x P_m (x - kl_m). \quad \ldots \ldots \quad (22a)$$

for concentrated loads in the mth panel

If there are no loads in this span, then the last term disappears from each equation.

Also, for shear at any section distant x from the left end of the mth span, we have

$$S_{(mx)} = S_m - p_m x \quad \ldots \ldots \ldots \quad (23)$$

for uniform load, and

$$S_{(mx)} = S_m - \sum_0^x P \quad \ldots \ldots \ldots \quad (23a)$$

for intervening concentrated loads. Or in other words, the shear at any section is equal to the shear at any other section minus the intervening external forces.

Having the moments and shears at any section, the stresses in the members are readily found.

Derivation of Formula (14*a*).

The following derivation of formula (14*a*) is given rather than that of (14), as it is the one commonly used with trussed bridges. The method would be the same for (14).

Using Fig. 200 (*b*) and the notation there given, and calling positive shear upward on the left, and moment producing convexity upward as positive, we may write:

$$a = kl; \quad M_{r+1} = M_r - S_r l_r + P_r(l_r - a); \quad \text{or} \quad S_r = \frac{M_r - M_{r+1}}{l_r} + \frac{P_r(l_r - a)}{l_r}. \qquad (24)$$

Taking the right-hand span, the moment at any section distant x from the left-hand support is

$$M_x = M_r - S_r x + P_r(x - a),$$

whence
$$\frac{d^2y}{dx^2} = \frac{M_x}{EI} = \frac{1}{EI}[M_r - S_r x + P_r(x - a)]. \quad \ldots\ldots\ldots \quad (25)$$

Integrating,
$$\frac{dy}{dx} = \frac{1}{2EI}[2M_r x - S_r x^2 + P_r(x - a)^2] + (C = t_r), \quad \ldots\ldots \quad (26)$$

where $t_r = \tan i$ at the rth support. Integrating again,

$$y = \frac{1}{6EI}[3M_r x^2 - S_r x^3 + P_r(x - a)^3] + t_r x + (C' = h_r), \quad \ldots \quad (27)$$

where $h_r =$ settlement or displacement of the rth support.

If we now make $x = l_r$, y becomes h_{r+1}, or the settlement of the $r + 1$th support. Putting also $a = kl$, and for S_r its value in (24), we have

$$t_r = \frac{h_{r+1} - h_r}{l_r} - \frac{1}{6EI}[2M_r l_r + M_{r+1} l_r - P_r l_r^2(2k - 3k^2 + k^3). \quad \ldots \quad (28)$$

To find t_{r+1}, or tan i, at the $r + 1$th support, evaluate (26) for $x = l_r$, substitute values of $a = kl$ and of S_r from (24), and for t_r its value from (28), and obtain

$$\left(\frac{dy}{dx}\right)_{x=l_r} = t_{r+1} = \frac{h_{r+1} - h_r}{l_r} + \frac{1}{6EI}[M_r l_r + 2M_{r+1} l_r - P_r l_r^2(k - k^3)]. \quad \ldots \quad (29)$$

If we now reduce all these subscripts by unity, this equation will express the tan i at the rth support, or t_r, in terms of M_{r-1}, P_{r-1}, and l_{r-1}, and the movements of the $r - 1$th and the rth supports, this equation then becoming

$$t_r = \frac{h_r - h_{r-1}}{l_{r-1}} + \frac{1}{6EI}[M_{r-1} l_{r-1} + 2M_r l_{r-1} - P_{r-1} l_{r-1}^2(k - k^3)]. \quad \ldots \quad (30)$$

Subtracting (28) from (30), we eliminate t_r and obtain a relation between the moments at three consecutive supports and their intervening loads and spans, which is the Theorem of Three Moments:

$$M_{r-1} l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1} l_r$$
$$= 6EI\left(\frac{h_{r+1} - h_r}{l_r} - \frac{h_r - h_{r-1}}{l_{r-1}}\right) + P_{r-1} l_{r-1}(k - k^3) + P_r l_r^2(2k - 3k^2 + k^3). \quad (31)$$

Since this equation was obtained on the assumption that the bending moment was positive when producing convexity upward, whereas the reverse is assumed in this work, if we will change the signs of one member of this equation it becomes Eq. (14*a*), p. 137.

The first term of the right member of Eq. (31) takes care of all relative settlement of the three supports involved in the equation. This term has been omitted in Eqs. (14) and (14*a*), and the supports assumed to be fixed. This unequal settlement of supports should prevent the use of continuous girders except on absolutely fixed (rock) foundations. There is no necessity, however, for these supports to be "on a level," as is commonly stated in the text-books.

NOTE.—For a complete discussion of the theory of the continuous girder with varying spans and moments of inertia, see "*The Theory of the Continuous Girder*" (120 pp.), by Prof. Malverd A. Howe, Engineering News Publishing Co., 1889. For a complete graphical analysis of the problem, moment of inertia constant, see Prof. Eddy's "*Researches in Graphical Statics*," Van Nostrand, 1878; also the same inserted in Prof. Church's "*Mechanics of Materials*," and in Prof. Burr's "*Bridges*." The continuous girder is now so little employed in America that, in the opinion of the authors, it is no longer necessary to teach the details of this practice in our engineering schools.

CHAPTER IX.

COLUMN FORMULÆ.

132. **Crushing Strength.**—Engineering materials, when tested in compression, divide themselves into two very distinct categories: those which fail absolutely by crushing to pieces along diagonal planes, and those which merely distort, or flow, under the increasing load, and never disintegrate under any pressure, however great. Cast-iron, hard cast-steel, stone, brick, cement, and the like are of the former class, while wrought-iron, all grades of rolled steel, the alloys, and timber, are of the latter. With this class of materials "crushing strength" should be understand to mean the resistance the substance offers to cold flowing, or to permanent distortion under a crushing load. But this point is called the "elastic limit in compression;" therefore *for semi-plastic materials, like the rolled metals, the elastic limit is, or should be regarded as, the ultimate strength.** This has sometimes been called the "crippling strength," but for all practical purposes it should be regarded, in the designing of structures, as the "ultimate strength."

In all grades of rolled iron and steel the elastic limit in compression is practically identical with this limit in tension, and hence *the "elastic limit" as found by a tension test of the materials may be regarded as the "ultimate strength" of that material in compression.* For very short columns of such a material, which are perfectly straight, with exact centering in the testing machine, and with end bearings which resist lateral movement, as square, hinged, or pin ends, it may be possible to place a greater load upon the column, but its length is then permanently shortened, and the chances are greatly in favor of its giving way by lateral deflection for want of perfect fulfilment of one or more of the conditions named. All recorded tests of wrought-iron and steel columns, therefore, which show an "ultimate strength" greater than the elastic limit of the material should be considered as abnormal and misleading, and should be given no weight in any experimental tests of the correctness of any proposed formula, or in the derivation of the constants of a formula to be used for computing the strength of columns.

Unfortunately the elastic limit in column tests has seldom been observed, and notwithstanding the great number of tests made of full-sized members, as well as on laboratory samples, we are still almost entirely devoid of data from which to derive the constants entering into any rational formula for the strength of columns.

THREE METHODS OF COLUMN FAILURE.

133. **I. By Direct Crushing.**—If the column is short, without internal stress, perfectly straight, of uniform size and strength, all its filaments having the same modulus of elasticity and the same elastic limit, the centre of gravity of the imposed load coinciding exactly with the centre of gravity of the cross-section, then all the longitudinal elements of the column will be equally compressed, all will come to their elastic limit at the same time, and all will distort alike. This distortion will continue indefinitely without lateral deflection, the material simply spreading, or flowing, under the imposed load.

This is evidently a purely ideal condition, and can never be realized perfectly even in a

* See paper on *Compressive Strength of Steel and Iron*, by Chas. A. Marshall, M. Am. Soc. C. E., Trans. Am. Soc. C. E., Vol. XVII, p. 53. Consult Plates X and XI for proof of the above statement.

carefully arranged experimental test, to say nothing of the conditions obtaining in actual practice. Here the "ultimate strength" should be regarded as the "elastic limit," notwithstanding greater loads will be resisted after permanent distortion begins. Evidently, so long as the column does not deflect sidewise, its strength per square inch is independent of its length, or of its ratio of length to radius of gyration, and dependent only on the elastic limit of the material. For various lengths, or for increasing values of $\frac{l}{r}$, therefore, failure by crushing only would show a constant unit strength equal to the elastic limit.

134. **II. By Crushing and Bending Combined.**—If any of the above-named conditions are not fulfilled, then the column will bend somewhat under all loads, the bending increasing with the load, and the concave side of the beam at the elastic limit will be subjected in general to compressive stresses from three causes:

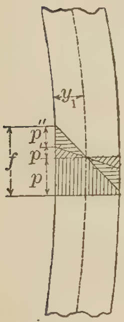


FIG. 202.

First, to a stress $p = \frac{P}{A}$, uniformly distributed over the section.

Second, to a stress $p' = \frac{Pvy_1}{I} = \frac{Pvy_1}{Ar^2} = \frac{pvy_1}{r^2}$, where v is the eccentric displacement of the load, y_1 is the distance of the extreme fibre from the centre of gravity of the cross-section, and r is the radius of gyration of the section. Pv would be the bending moment caused by the eccentric position of the load.

Third, to a stress $p'' = \frac{f-p}{10E}\left(\frac{l}{r}\right)^2 p$ at the elastic limit, due to the bending of the column under the load P, f being the elastic limit stress for that material, and E the modulus of elasticity. This is found as follows: If Δ = lateral deflection under the load P, then the bending moment at the centre is $P(\Delta + v)$. Since the moment at any point is equal to $P(y + v)$, where y is the deflection from a right line at that point, the form of the elastic line of the bent column will be intermediate between a circle, due to the constant moment Pv, and the elastic curve due to the varying moment Py. The latter would be practically that of a beam uniformly loaded, as given on p. 132. Now the centre deflection of a uniformly bent column is, in terms of the stress on the extreme fibre, $\Delta_1 = \frac{p'l^2}{8Ey_1}$, and of a beam uniformly loaded $\Delta_2 = \frac{5p''l^2}{48Ey_1}$ (see p. 132). But since the actual deflection is due to both of these causes, and in practice the former greatly predominates, we may put $\Delta = \frac{(p' + p'')l^2}{8Ey_1}$. But when the total stress on the extreme fibre has reached the elastic limit f, then $p + p' + p'' = f$, or $p' + p'' = f - p$. Hence we may write $\Delta = \frac{(f-p)l^2}{8Ey_1}$. But the bending moment *due to the deflection* is $M = P\Delta$. Hence $P\Delta = \frac{P(f-p)l^2}{8Ey_1}$. Also, in terms of the stress produced on the outer fibres by the deflection eccentricity, $M = \frac{p''I}{y_1}$; therefore we have, since $I = Ar^2$,

$$\frac{p''I}{y_1} = \frac{P(f-p)l^2}{8Ey_1}, \quad \text{or} \quad p'' = \frac{P}{A} \cdot \frac{f-p}{8E} \cdot \left(\frac{l}{r}\right)^2 = \frac{f-p}{8E}\left(\frac{l}{r}\right)^2 p.$$

We may now write, as the total stress on the extreme fibre at the elastic limit,

$$f = p + p' + p'' = p\left[1 + \frac{vy_1}{r^2} + \frac{(f-p)}{8E}\left(\frac{l}{r}\right)^2\right],$$

whence

$$\frac{P}{A} = p = \frac{f}{1 + \frac{vy_1}{r^2} + \frac{f-p}{8E}\left(\frac{l}{r}\right)^2}, \quad \ldots \ldots \ldots \ldots (1)$$

where f = elastic limit of the material in compression;

v = eccentric displacement of the load in inches;

y_1 = distance of outer fibre from centre of gravity of the section in inches;

E = modulus of elasticity;

l = length of column;

r = radius of gyration of the cross-section in inches, $=\sqrt{\frac{I}{A}}$.

This is a nearly rational formula, with no purely empirical constants, for a column free to revolve at the ends, and will give good results.* It must be solved by trial since p is found on both sides of the equation. If $y_1 = \frac{4}{3}r$, which is about the ordinary ratio, and if $f = 34{,}000$ for wrought iron and 42,000 for mild steel, with $E = 27{,}000{,}000$ for iron and 28,500,000 for steel, this formula would become,

For Wrought-iron, $$p = \frac{34{,}000}{1 + \frac{4v}{3r} + \frac{34{,}000 - p}{216{,}000{,}000}\left(\frac{l}{r}\right)^2}, \quad \quad (2)$$

and

For Steel, $$p = \frac{42{,}000}{1 + \frac{4v}{3r} + \frac{42{,}000 - p}{228{,}000{,}000}\left(\frac{l}{r}\right)^2}. \quad \quad (3)$$

These would apply only to columns pivoted on knife-edges. They would not apply to "round-ended" columns, because the point of application of the load shifts as the column bends. They would not apply to a hinged, or pin-connected, column, because the resistance to motion here is very considerable, which greatly increases the strength of the column by preventing lateral deflection.

The second term in the denominator, $\frac{vy_1}{r^2}$, must include all effects of initial bends, or kinks, in the column, and differences in the moduli of elasticity of the elementary forms of which it is composed, as well as eccentric position of load. These are usually unknown functions, and hence this term cannot commonly be evaluated. If this term be omitted,† and an empirical constant coefficient used for $\frac{f-p}{8E}$ in the next term, we have

$$p = \frac{f}{1 + a\left(\frac{l}{r}\right)^2}, \quad \quad (4)$$

which is commonly known as *Gordon's*, or *Rankine's, Formula.*‡

135. III. By Bending alone.—If all the conditions named in I are fulfilled, and we assume the column is loaded somewhat inside its elastic limit, while the length increases, the column will remain in stable equilibrium, and undeflected, until the length reaches a particular point, when it will bend. When the length is less than this critical amount, if the column were bent by a transverse force, it would straighten itself under its load, when this deflecting force is removed. But when the length has reached a certain limit, it will no longer be able to straighten itself under its load, *but will retain any particular deflection which may be given to it.* It is then in unstable equilibrium, and any further increase of load will cause the bending to

* For the rigid derivation of a formula for eccentric loads and pivoted ends by Prof. Marston see Trans. Am. Soc. C. E., Vol. XXXIX, p. 108.

† The authors do not admit the legitimacy of these changes, but they make this supposition here only to show what changes would be necessary to obtain Rankine's formula.

‡ For short columns, with eccentric loads, see *note*, p. 153, and also eq. (11), p. 453.

increase till the elastic limit is reached, when failure is inevitable. For columns pivoted, or free to turn at the ends, this limiting length for a perfectly ideal column is

$$l = \pi\sqrt{\frac{EI}{P}}, \quad \text{or} \quad \frac{l}{r} = \pi\sqrt{\frac{E}{p}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The shortest length of column which could act in this way would be found by making p as large as possible; or when

$$p = f = \text{elastic limit},$$

we have as the minimum length which can fail by bending only

$$\frac{l}{r} = \pi\sqrt{\frac{E}{f}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This lower limiting ratio of l to r for a perfectly ideal column is about 100 for wrought-iron and about 85 for mild steel. A spring-steel column might fail in this way for a ratio of $\frac{l}{r}$ as low as 50.

For the perfectly ideal column, therefore, centrally loaded, failure would occur by methods I and III and never by method II. That is, the strength would be constant and equal to the elastic limit for increasing lengths until the limiting length is reached, when it would fail by bending. The strength of the ideal column, free to turn at the ends, where $\frac{l}{r}$ is greater than this limit, or when failure occurs by bending alone, is

$$\left.\begin{aligned} P &= \frac{\pi EI}{l^2} = \frac{\pi^2 EAr^2}{l^2} = \frac{\pi^2 EA}{\left(\frac{l}{r}\right)^2}, \\ &\text{or} \\ p &= \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This is called *Euler's Formula*,* and is derived as follows:

FIG. 203.

Let OQ, Fig. 203, represent a column, free to turn about its end supports, of such a length that it may be in unstable equilibrium under the load P. That is, under this load the column just begins to deflect, and will under a constant load retain any deflection which may be given to it, within the elastic limit of the material. The bending moment at any section distant x from the origin at O is then

$$M = -Py = EI\frac{d^2y}{dx^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

from the fundamental equation for the deflection of beams. Whence

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}y.$$

Multiplying each side of this equation by dy when x is the independent variable and integrating once, we have

$$\left(\frac{dy}{dx}\right)^2 = -\frac{P}{EI}(y^2 + C).$$

* Contributed to the Berlin Academy by Euler in 1759. For a more rigid derivation of this formula by Henry S. Pritchard, see *Engr. News*, May 6, 1897, and also by Prof. Wm. Cain in Trans. Am. Soc. C. E., Vol. XXXIX, p. 96.

When $\frac{dy}{dx} = 0$, $y = \Delta$, = deflection at the centre; therefore $C = -\Delta^2$, and

$$\left(\frac{dy}{dx}\right)^2 = \frac{P}{EI}(\Delta^2 - y^2), \quad \text{or} \quad dx = \sqrt{\frac{EI}{P}} \cdot \frac{dy}{\sqrt{\Delta^2 - y^2}};$$

whence

$$x = \sqrt{\frac{EI}{P}} \cdot \text{arc sin } \frac{y}{\Delta} + C.$$

When $x = 0$, $y = 0$, $\therefore C = 0$, or

$$x\sqrt{\frac{P}{EI}} = \text{arc sin } \frac{y}{\Delta}. \quad \ldots \ldots \ldots \quad (9)$$

Therefore

$$y = \Delta \sin x\sqrt{\frac{P}{EI}}. \quad \ldots \ldots \ldots \quad (10)$$

This is the equation of the elastic line, the curve being a sinusoid. But for $x = \frac{l}{2}$, $y = \Delta$, and we have from (9)

$$\left.\begin{aligned} &\frac{l}{2}\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \quad \text{or} \quad P = \frac{\pi^2 EI}{l^2}, \\ \text{or}\quad & \\ &p = \frac{P}{A} = \frac{\pi^2 EAr^2}{Al^2}, \quad \text{or} \quad p = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2}, \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (11)$$

which is *Euler's Formula*.

136. The Effect of End Conditions.—When the ends are free to turn, the column bends in single curvature, as in Fig. 204. When both ends are fixed in position it takes the form of a double reversed curve, as in Fig. 205. Here the portion lying between the two points of inflection acts as a whole column on knife-edges, but the length of this portion is only $\frac{l}{2}$. When one end is fixed and the other free to turn, the portion analogous to Fig. 204 is $\frac{2}{3}l$, as shown in Fig. 206. Therefore, when a formula has been derived for the first case it can be used for the other two by putting for l, $\frac{l}{2}$ for fixed ends, and $\frac{2}{3}l$ for one end fixed and the other free to move.

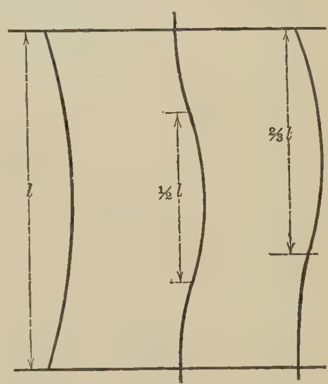


FIG. 204. FIG. 205. FIG. 206

Therefore we may write the following theoretical formulæ for these several conditions:

Name of Formula.	For Pivoted Ends.	For Fixed Ends.	For one end Pivoted and one end Fixed.
Gordon's	$p = \dfrac{f}{1 + a\left(\frac{l}{r}\right)^2}$	$p = \dfrac{f}{1 + \frac{a}{4}\left(\frac{l}{r}\right)^2}$	$p = \dfrac{f}{1 + \frac{4}{9}a\left(\frac{l}{r}\right)^2}$
Euler's	$p = \dfrac{\pi^2 E}{\left(\frac{l}{r}\right)^2}$	$p = \dfrac{4\pi^2 E}{\left(\frac{l}{r}\right)^2}$	$p = \dfrac{\frac{9}{4}\pi^2 E}{\left(\frac{l}{r}\right)^2}$

Unfortunately neither of these end conditions is ever found in practice. The nearest approach to a pivoted end is the ordinary pin connection, but the pin so nearly fills the hole that when the column is loaded the frictional resistance to slipping is a very material source of strength to the column by preventing lateral deflection. The strengthening effect of the pin is greater the larger the ratio of its diameter to the radius of gyration of the column, but even a very small pin well oiled gives a much higher test for long columns than a perfectly frictionless or knife-edge bearing.

The nearest approach to a fixed end commonly found in structures is a squarely abutting end upon a rigid or fixed base. This is equivalent to a fixed end for short lengths, but for long lengths, where the fibres on the convex side come into tension, the square-ended column no longer acts as a fixed end, since the joint cannot usually resist tension.

137. A New Formula.—For theoretically perfect columns and central loading, failure would occur by methods I and III, and along the lines *ABD*, Fig. 207, for pivoted ends, and along *AFH* for fixed ends, if these conditions could be perfectly satisfied.* Any failure in these necessary limitations, either as to the column itself, its loading, or its end bearings, would result in lowering the maximum unit stress, except for very short or for very long lengths. Unless the loading is very eccentric, failure will always occur on very short lengths for $p = f =$ elastic limit of the material. For very long lengths the column fails almost wholly by bending, so that here the only significant condition is the value of the modulus of elasticity, the end conditions, and the ratio $\frac{l}{r}$. But these extremes include all practical lengths of columns, and hence we may say that the actual strength of a given column may be found anywhere within a field of considerable width, depending on a number of indeterminate conditions. Any convenient formula, therefore, having its locus centrally located in this field of experimentally determined results, and satisfying the theoretical requirements for very long and for very short columns, where the unknown functions are relatively unimportant, may be considered as satisfactory. Gordon's formula very fairly satisfies this requirement, being of the form $p = \frac{f}{1 + \frac{1}{a}\left(\frac{l}{r}\right)^2}$. It is not as convenient of application, however, as the one now proposed, which is of the form $p = f - b\left(\frac{l}{r}\right)^2$. If the coefficient b in this formula be evaluated so as to make the locus tangent to that of Euler's curve, and the formula used for all lengths up to this point of tangency, it will give values as near the average of those obtained from actual experiments as possible.

To find the Equation of the Parabola having its Vertex at the Elastic Limit on the Axis of Loads, and Tangent to Euler's Curve.

For hinged and for flat ends an empirical coefficient must be found which will make Euler's curve best fit the observed strength of very long columns. The general form of Euler's formula, for all varieties of end conditions, is

$$\frac{P}{A} = p = \frac{c\pi^2 E}{\left(\frac{l}{r}\right)^2} \qquad (12)$$

For hinged ends we shall use

$$c\pi^2 = 16,$$

and for square or flat ends we will make

$$c\pi^2 = 25.$$

The value of E will be taken as 28,500,000 for steel and 27,000,000 for wrought-iron.

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* See Mr. Marshall's paper referred to in foot-note, p. 143.

We have, therefore, as Euler's formula for Wrought-iron Compression Members,

$$\textit{For Hinged Ends,} \quad p = \frac{432{,}000{,}000}{\left(\dfrac{l}{r}\right)^2}. \quad \ldots \ldots \ldots \quad (13)$$

$$\textit{For Flat Ends,} \quad p = \frac{675{,}000{,}000}{\left(\dfrac{l}{r}\right)^2}. \quad \ldots \ldots \ldots \quad (14)$$

These are the curves BCD and FGH in Fig. 207. The line ABF marks the elastic limit of wrought-iron, and $A'B'F'$ the elastic limit of steel. The tangent curves AC and AG are parabolas, with vertex at A and an axis in AO, drawn tangent to Euler's curve for

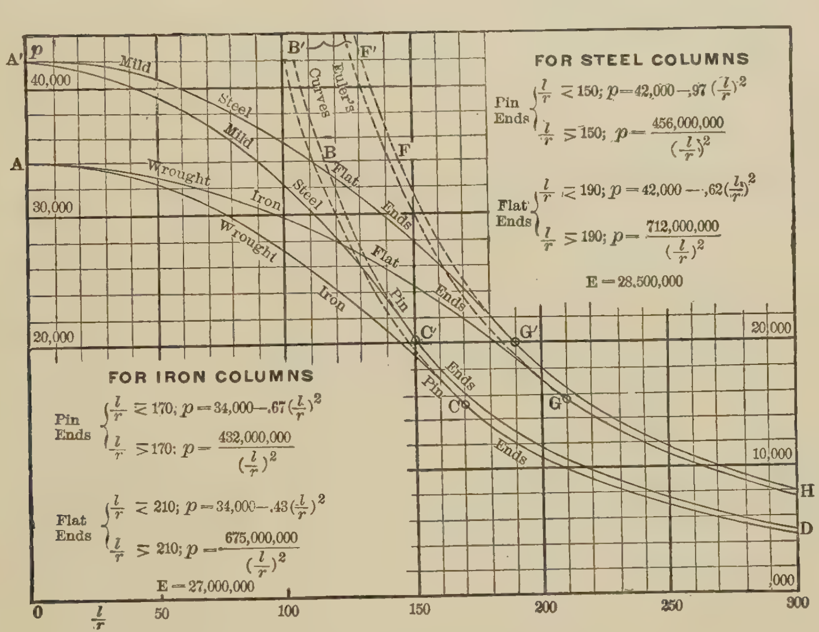


FIG. 207.

hinged and for flat ends at the points C and G, respectively. These are the loci of the wrought-iron formulæ for ordinary lengths, while the analogous curves $A'C'$ and $A'G'$ are the loci of the formulæ for steel columns. The complete loci showing the strength for all lengths are ACD and AGH for wrought-iron, and $A'C'D$ and $A'G'H$ for steel columns. The equation of these tangent parabolas takes this form :

$$\textit{Tangent Parabola,} \quad y = f - bx^2. \quad \ldots \ldots \ldots \quad (15)$$

The equation of Euler's formula takes the form,

$$\textit{Euler's Curve,} \quad y = \frac{k}{x^2}. \quad \ldots \ldots \ldots \ldots \quad (16)$$

We wish now to find the value of $x, \left(= \dfrac{l}{r}\right)$, at the point of tangency, and the value of the coefficient, b, which will pass the parabola through this point. The equations of condition are

found by making $\frac{dy}{dx}$ equal for the two curves, and then equating the two values of y, or making the equations simultaneous for the point of tangency. Thus,

$$\left.\begin{array}{ll} & \frac{dy}{dx} \text{ from eq. (15)} = -2bx \\ \text{and} & \\ & \frac{dy}{dx} \text{ from eq. (16)} = -\frac{2k}{x^3} \end{array}\right\}; \therefore x^4 = \frac{k}{b}. \quad (17)$$

Making the two values of y equal for the point of tangency, we have

$$f - bx^2 = \frac{k}{x^2}, \quad \text{or} \quad b = \frac{f}{x^2} - \frac{k}{x^4}. \quad (18)$$

From (17) and (18), by elimination, we may find

$$x\left(=\frac{l}{r}\right) = \sqrt{\frac{2k}{f}} \quad \text{and} \quad b = \frac{f^2}{4k},$$

which substituted in (15) and (16) we obtain:

$$\textit{Eq. Tang. Parabola,} \quad p = f - \frac{f^2}{4k}\left(\frac{l}{r}\right)^2. \quad (19)$$

$$\textit{Eq. Euler's Curve,} \quad p = \frac{k}{\left(\frac{l}{r}\right)^2}. \quad (20)$$

For wrought-iron, hinged ends,	$k = 16E =$	432,000,000.
For steel, hinged ends,	$k = 16E =$	456,000,000.
For wrought-iron, flat ends,	$k = 25E =$	675,000,000.
For steel, flat ends,	$k = 25E =$	712,000,000.
For wrought-iron, the elastic limit,	$= f =$	34,000.
For mild steel, the elastic limit,	$= f' =$	42,000.

The elastic limit of rolled metals increases with the amount of work put on the bar, or it varies inversely as the thickness of the finished plate and inversely with the temperature when leaving the rolls. Since compression members are made up from sections having thin webs, from $\frac{3}{8}$ to $\frac{3}{4}$ inch thickness, the average elastic limits of wrought-iron and mild steel will be about as here taken.* Hence we may write the following numerical formulæ, remembering that the parabolic law only applies from $\frac{l}{r} = 0$ to $\frac{l}{r} = \sqrt{\frac{2k}{f}}$, as shown above in the value found for x at the point of tangency. With the values of k and x here taken, we have

FORMULÆ FOR THE ULTIMATE STRENGTH OF COLUMNS.

$$\textit{For Wrought-iron Columns, Pin Ends,} \quad \left\{\begin{array}{l} \frac{l}{r} \leqq 170, \quad p = 34{,}000 - .67\left(\frac{l}{r}\right)^2, \\ \frac{l}{r} \geqq 170, \quad p = \frac{432{,}000{,}000}{\left(\frac{l}{r}\right)^2}. \end{array}\right\} \quad (21)$$

* The great change in the elastic limit for different thicknesses of finished sections, and for different conditions of rolling, especially the temperature when leaving the rolls, largely accounts for the extraordinary range of the results of the experimental tests of wrought-iron and steel columns.

For Wrought-iron Columns, Flat Ends,
$$\left\{\begin{array}{l} \frac{l}{r} \overline{\overline{<}} 210, \quad p = 34{,}000 - .43\left(\frac{l}{r}\right)^2 \\ \frac{l}{r} \overline{\overline{>}} 210, \quad p = \frac{675{,}000{,}000}{\left(\frac{l}{r}\right)^2}. \end{array}\right\} \quad (22)$$

For Mild Steel Columns, Pin Ends,
$$\left\{\begin{array}{l} \frac{l}{r} \overline{\overline{<}} 150, \quad p = 42{,}000 - .97\left(\frac{l}{r}\right)^2, \\ \frac{l}{r} \overline{\overline{>}} 150, \quad p = \frac{456{,}000{,}000}{\left(\frac{l}{r}\right)^2}. \end{array}\right\} \quad (23)$$

For Mild Steel Columns, Flat Ends,
$$\left\{\begin{array}{l} \frac{l}{r} \overline{\overline{<}} 190, \quad p = 42{,}000 - .62\left(\frac{l}{r}\right)^2, \\ \frac{l}{r} \overline{\overline{>}} 190, \quad p = \frac{712{,}000{,}000}{\left(\frac{l}{r}\right)^2}. \end{array}\right\} \quad (24)$$

In actual practice $\frac{l}{r}$ is nearly always less than 150, and usually less than 100, so that the formula, $p = f - b\left(\frac{l}{r}\right)^2$ covers the entire range of ordinary practice.

For Cast-iron Columns we have had, until recently, no full-sized tests. The formulæ heretofore used for these columns have been based on crushing tests of small specimens. The tests on full-sized cast-iron columns made at the Watertown Arsenal in 1887 were discredited in the official report of them by the statement that they were of an unusually poor quality of cast-iron. In 1897 another series of tests were made on full-sized cast-iron columns at Phœnixville, Pa., and as these were of the ordinary quality, and their results agreed well with those made in 1887, they have all been accepted now by engineers and architects as indicating the real strength of cast-iron columns.* From these tests the following formula may be used as expressing the average ultimate strength of cast-iron columns with flat ends.

For Cast-iron Columns, Flat Ends $\quad \frac{l}{r} < 120, \quad p = 34{,}000 - 88\frac{l}{r}. \quad (25)$

For White Pine, Flat Ends, $\quad \frac{l}{d} \overline{\overline{<}} 60, \quad p = 2500 - 0.6\left(\frac{l}{d}\right)^2.$†

For Short-leaf Yellow Pine, Flat Ends, $\quad \frac{l}{d} \overline{\overline{<}} 60, \quad p = 3300 - 0.7\left(\frac{l}{d}\right)^2.$

For Long-leaf Yellow Pine, Flat Ends, $\quad \frac{l}{d} \overline{\overline{<}} 60, \quad p = 4000 - 0.8\left(\frac{l}{d}\right)^2.$

For White Oak, Flat Ends, $\quad \frac{l}{a} \overline{\overline{<}} 60, \quad p = 3500 - 0.8\left(\frac{l}{d}\right)^2. \quad (26)$

In Fig. 208 are shown the plotted results of all the most reliable tests of full-sized columns ever made. The loci of the Parabolic Formulæ given above, of the Gorden-Rankine Formulæ, and of Johnson's Straight-line Formulæ are all drawn for both iron and steel.

138. Johnson's Straight-line Formulæ.—In 1886 Mr. Thos. H. Johnson, M. Am. Soc. C.E., presented a paper to the American Society of Civil Engineers in which he showed that a straight-line formula could be made to fit the plotted observations of column tests, as well as any curve, for all the ordinary lengths. He used Euler's curves for the great lengths

* See Johnson's *Materials of Construction*, second and subsequent editions, p. 474.

† See *Watertown Arsenal Experiments*, in Johnson's *Materials of Construction*, pp. 682–4.

to which it is applicable, and made his straight-line loci tangent to these curves. Although these linear equations can be made to fit the observed results for ordinary lengths as well as any other (see these loci in Fig. 208), they give too great values of the ultimate strength for

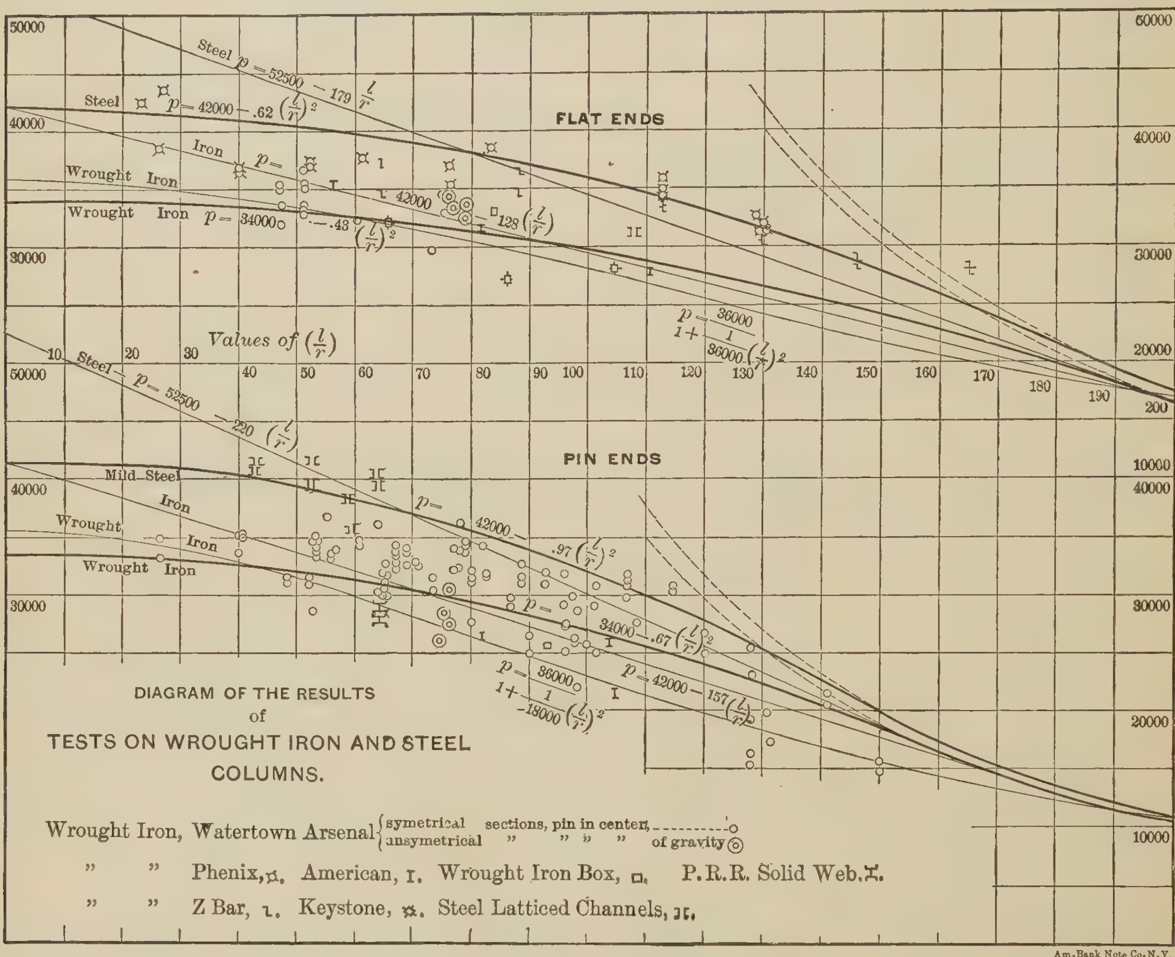


FIG. 208.

the shorter lengths, provided failure be taken at the elastic limit of the material, or where the member takes on an appreciable permanent set. Mr. Johnson's formulæ are, however, the simplest ever yet proposed and have come into very general use. They are as follows:

Wrought-iron—Hinged Ends, $p = 42,000 - 157 \frac{l}{r}$.

" Flat " $p = 42,000 - 128 \frac{l}{r}$.

Mild Steel, Hinged Ends, $p = 52,500 - 220 \frac{l}{r}$.

" " Flat " $p = 52,500 - 179 \frac{l}{r}$.

139. Formulæ to be Used in Dimensioning.*—All working formulæ must include a factor of safety. It has been customary to introduce this as a common factor in the right-hand member of the ultimate strength formula. This is clearly wrong. Referring back to formula (1), p. 144, we see that the second term in the denominator, $\frac{vy_1}{r^2}$, represents not only any known or unknown eccentricity of loading, but also all structural weaknesses, like initial bending, internal stress, unsymmetrical cutting away of parts, etc., and hence some value must be given to this term even when the loads are assumed to be symmetrically placed. The giving to this term an arbitrary value is therefore equivalent to introducing a factor of safety. If, for centrally placed loads, we give this term a value of unity, this is equivalent to making the factor of safety two for short columns, since the third term in this denominator practically vanishes for short lengths. This factor may also be regarded as a factor of safety, for the stress on the most compressed fibre, or as *a factor of safety as to the maximum stress.* If now we arbitrarily introduce a factor of safety of two as common to the entire denominator, we may call this *a factor of safety as to the loads,* which also applies to the stresses as

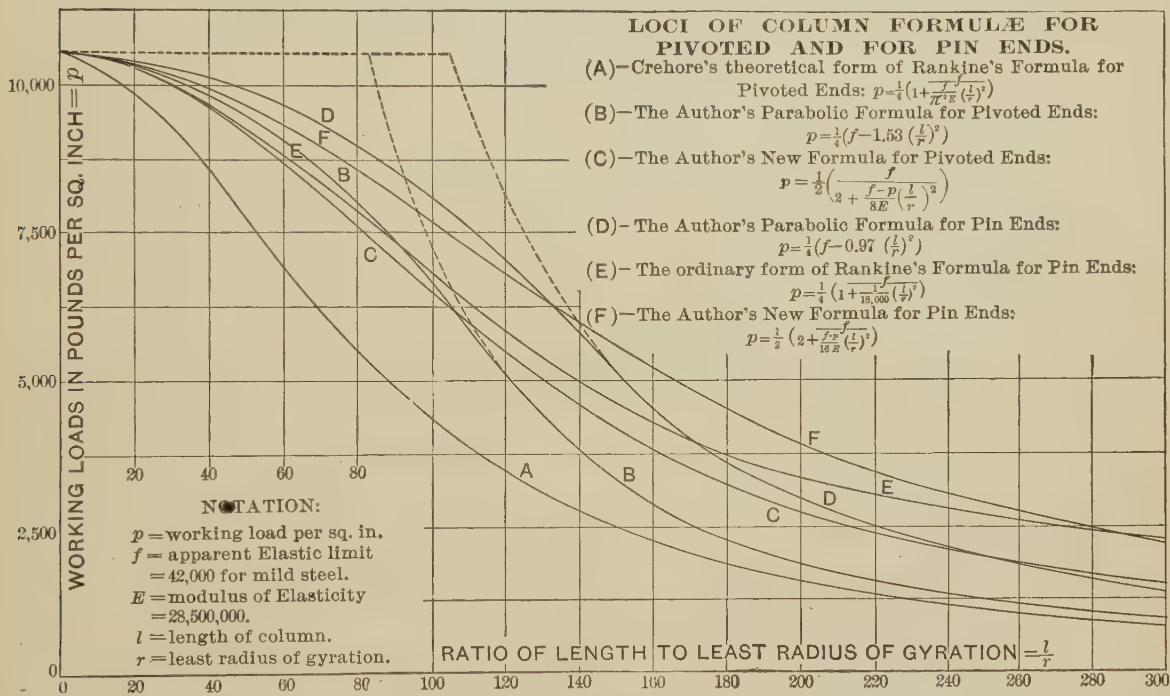


FIG. 208*a*.

a matter of necessity. The result is we now have a factor of safety of two for very long columns (where the terms $1+\frac{vy_1}{r^2}$ are of little relative value), and of four for very short columns (where the last term in $\left(\frac{l}{r}\right)^2$ has little relative value). This is as it should be, since the strength of a very long column is not a matter of stress on the most compressed fibre, but purely a matter of stiffness, or ability to remain straight, or unbent, under its load. Here, therefore, we need only a factor of safety as to the load. By proceeding in this manner we may say:

(1) The column will stand even though, for unknown reasons, the stress on the most compressed fibre is twice as much as it has been computed; and

(2) It will also stand even though the load is twice as great as it has been assumed.

When a factor of safety of four is taken as a common factor for the entire denominator, we are making this wholly a load factor for very long columns, which is unreasonable, as we do not propose to provide for four times as great a *load* as has been assumed. Making these numerical changes, equation (1), p. 144, becomes equation (28) below.

* This article added in the Sixth Edition.

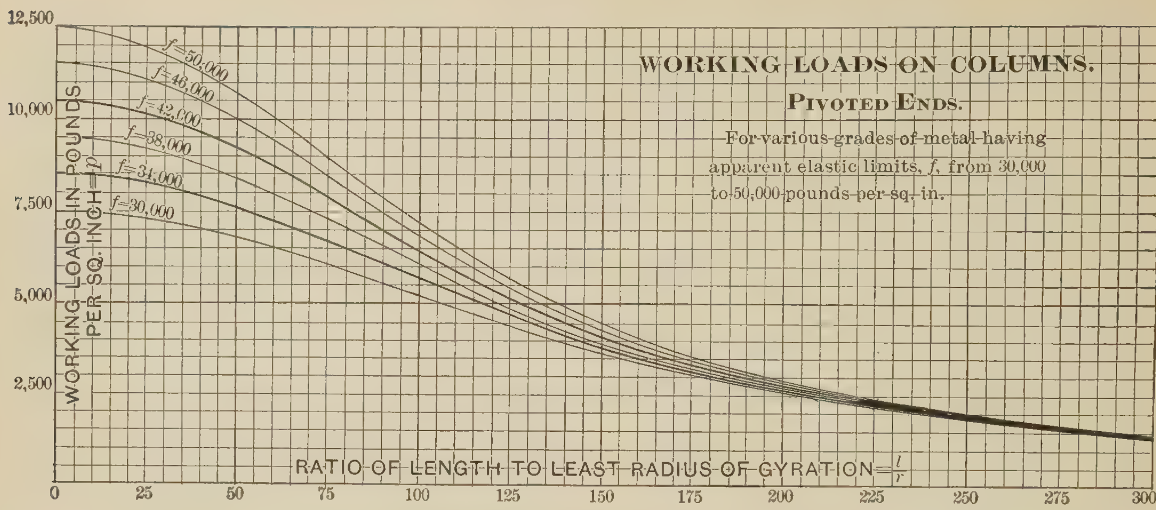


FIG. 208*b*. Equation (28).

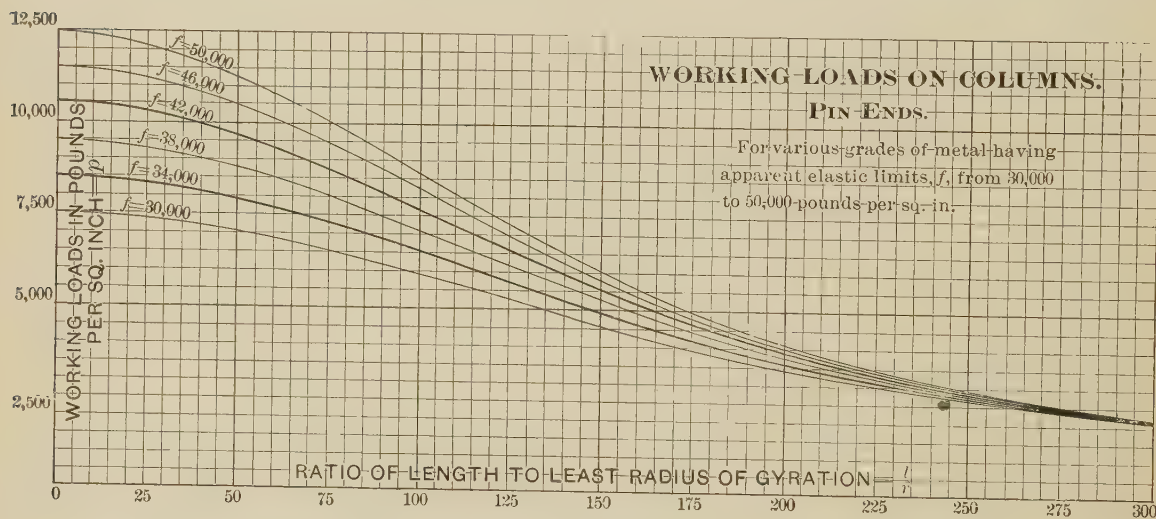


FIG. 208*c*. Equation (29).

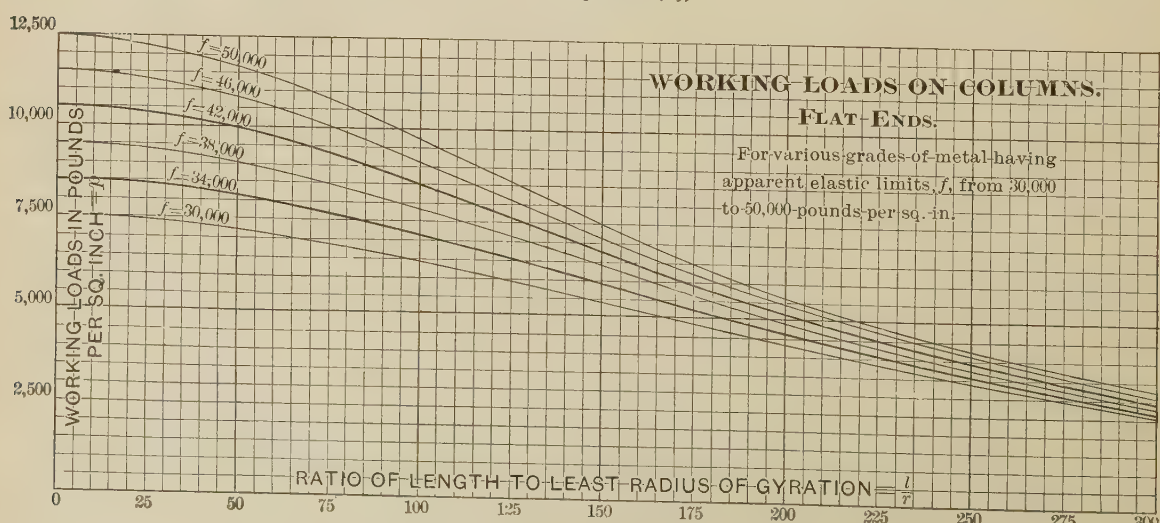


FIG. 208*d*. (Equation 30).

The locus of this equation is curve C in Fig. 208a. It should be compared with the author's parabolic formula as described on the previous pages, here shown as curve B, and also with Crehore's theoretical form of Rankine's formula, here shown in curve A. All these formulæ and loci are for columns with *pivoted ends*.

For pin ends, $8E$ becomes $16E$, by experiment, as described at the bottom of p. 148, and for square ends it becomes $25E$. Making this change for pin ends, we obtain the curve marked F in Fig. 208a, which should be compared with the author's parabolic locus, curve D, and with the ordinary locus of Rankine's equation for pin ends as there given. A study of these curves reveals the following relations:

For Pivoted Ends (comparing with curve C).

(1) The Crehore locus (curve A) is much too low, as shown by the experimental results given in Fig. 208.*

(2) The author's parabolic formula (curve B), having a factor of safety of four throughout, runs too low for long columns, and a little too high for the ordinary lengths. This is also the case to a similar degree with the formulæ for pin ends (curve D), and it is also the case for square ends, not shown in the diagram.

For Pin Ends (comparing with curve F).

(3) The locus of the ordinary Rankine formula lies well below that of the author's new formula F until a value of $\frac{l}{r} = 270$ is reached, where they intersect. This difference is nearly 20 per cent for the ordinary lengths. A similar difference would appear for the formulæ for flat ends if their loci were plotted.

Taking, then, the formulæ C and F, and an analogous one for flat ends, we have the following

WORKING FORMULÆ AND DIAGRAMS.

For Pivoted Ends (Fig. 208b), $$p = \frac{1}{2}\left(\frac{f}{2 + \frac{f-p}{8E}\left(\frac{l}{r}\right)^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (28)$$

For Pin Ends (Fig. 208c), $$p = \frac{1}{2}\left(\frac{f}{2 + \frac{f-p}{16E}\left(\frac{l}{r}\right)^2}\right). \quad . \quad . \quad . \quad . \quad (29)$$

***For Flat Ends* (Fig. 208d),** $$p = \frac{1}{2}\left(\frac{f}{2 + \frac{f-p}{25E}\left(\frac{l}{r}\right)^2}\right). \quad . \quad . \quad . \quad . \quad (30)$$

Giving to f, the "apparent elastic limit," values from 30,000 to 50,000 for different grades of wrought iron and steel ($f = 34{,}000$ for wrought iron and $f = 42000$ for mild steel being safe reasonable values for shape irons in thin sections,† and taking $E = 28{,}500{,}000$ in all cases, there results the various curves in these figures. In finding unit stresses to use in designing, therefore, it is only necessary to know the value of $\frac{l}{r}$ for the given column, and the kind of metal (f) employed. Since r does not change appreciably for different *weights* of section, but only for different *sizes*, when the lateral dimensions of the column are given, the maker's handbooks, or Osborn's *Tables of Radii of Gyration*, will give at once the value of r with sufficient exactness. The working stress (p) can then be taken from the diagrams.

NOTE.—For short heavy columns, such as are used in buildings, the bending of the columns may be neglected and provision may be made for eccentric loads, as indicated on p. 453 in the formula

$$A = \frac{1}{p}(P + 3kP_e), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where A = area of cross-section of columns;
p = maximum crushing stress to be allowed in practice;
P = total load on the column;
P_e = eccentric load;
k = ratio of eccentricity of P_e to the half width of the column in that direction.

* This has been pointed out by the author as being true on both theoretical and experimental grounds. See *Engineering News*, vol. XXXVII (May 20, 1897), p. 311.

† See Johnson's *Materials of Construction*.

CHAPTER X.

COMBINED DIRECT AND BENDING STRESSES. SECONDARY STRESSES.

140. Examples of Combined Stresses.—Whenever a tension or compression member occupies a horizontal position it is evidently subjected to a cross-bending stress from its own weight. If other transverse external forces come upon it, it is still further stressed in this way. In case the direct loading (tension or compression) be not centrally placed over the centre of gravity of the cross-section, or if the member itself be bent from a right line, then there would be a bending moment upon the member equal to the direct loading into its arm, or the deviation of the axial line of the column from the right line joining the centres of gravity of the end sections. In all such cases the member should be dimensioned for both direct and cross-bending stress, and the computation made on the assumption that both kinds of loads act simultaneously.

It is always poor economy to subject a member to bending stress. It must be dimensioned for the stress on the extreme fibres, while the strength of the interior portion of the section remains unused. A framed structure should always be designed so as to obtain only direct stresses in all its members, and then the entire cross-section is available to resist the force coming upon it. It is for this reason that all joints should be designed to bring the centre of gravity lines of all the members to a common point, so as to avoid secondary stresses from bending. The use of knee, portal, and sway bracing also should be avoided whenever possible, as it usually is in buildings, trestles, towers, roofs, and deck bridges.

141. Action of Direct and Bending Stresses.—The effect of any finite cross-bending load upon any member is to produce a corresponding deflection in it. The effect of the direct loading is to increase the bending moment, and therefore the deflection, if acting so as to compress the member, while the reverse is true for an extending force. When the deflection becomes appreciable, the effect of these direct forces upon the cross-bending stresses is too great to be neglected. It is common to assume that these two classes of external forces act independently, and to compute their separate effects, add the resulting stresses together, and to dimension the member accordingly. In the following analysis they are treated as acting simultaneously and the true cross-bending stresses found.

142. Derivation of a General Formula for Combined Stresses.

Let M_1 = bending moment at point of maximum deflection, from cross-bending external forces and from eccentricity of position of longitudinal loading;

v_1 = maximum deflection of member from all causes acting simultaneously;

M_2 = bending moment from the direct loading, P, into its arm, v_1, $= Pv_1$;

P = total direct loading on member, tension or compression;

f_1 = unit stress on extreme fibre from bending alone at section of maximum bending moment, or of maximum deflection, as the case may be, in pounds per square inch;

l = length of member;

y_1 = distance from centre of gravity axis to the extreme fibre under consideration on which the stress from bending is f_1;

E = modulus of elasticity;

I = moment of inertia of the cross-section;
b = breadth of a solid rectangular section;
h = height of section, out to out, in the plane in which bending occurs, $= 2y_1$ for symmetrical sections.
f_2 = unit stress in member from the direct loading, supposed to be uniformly distributed, $= \frac{P}{A}$.
f = total maximum unit stress on extreme fibre, $= f_1 + f_2$.

All dimensions in inches, and forces in pounds.

In all cases of deflection of beams of constant moments of inertia the maximum deflection, *in terms of the stress on the extreme fibre*, is given by the equation

$$v_1 = k\frac{f_1 l^2}{E y_1}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

where k is a numerical factor.

Thus for the extreme cases of a beam supported at the ends and loaded at the centre, and for the same beam loaded uniformly, the value of k is $\frac{4}{48}$ in the former case and $\frac{5}{48}$ in the latter.*

Since almost all cases in practice correspond more closely with the condition of uniform loading, the value of k will be taken as $\frac{5}{50}$ or $\frac{1}{10}$. We then have as the general relation between the deflection of a beam and the stress on its extreme fibre

$$v_1 = \frac{f_1 l^2}{10 E y_1}. \quad \ldots \ldots \ldots \ldots \quad (2)$$

In all cases now under consideration there are the two bending moments acting on the member, M_1 and M_2, which may be of the same or of opposite signs. The moment of resistance, or the moment of the direct stresses, developed at any section must be equal to the algebraic sum of the moments from the external forces, and hence we may write

$$M_0 = \frac{f_1 I}{y_1} = M_1 \pm M_2 = M_1 \pm P v_1, \quad \ldots \ldots \ldots \quad (3)$$

the positive sign to be used for members under compression, and the negative sign for members under tension.

Putting for v_1 its value from (2), we have

$$f_1 = \frac{M_1 y_1 \dagger}{I \mp \frac{P l^2}{10 E}}, \quad \ldots \ldots \ldots \ldots \quad (4)$$

where the negative sign is to be used for compression members, and the positive sign for tension members.

This formula is perfectly general, and applies rigidly to all forms of section and to all forms of loading without material error. The second term in the denominator takes account of the bending moment $M_2 = P v_1$, and can be neglected in all cases where the amount of the bending is known to be inappreciable.

Case I. Tension and Cross-bending.

143. Example 1. *Find the stress in the extreme lower fibres of an eye-bar when used in a horizontal position and subjected to its own weight.*

Here the transverse moment is that due to its own weight. If we take a section at the centre of the

* These factors are given in column four of the table on pages 132 and 133, except that in that table h is used where here y_1 is employed, which is equal to $\frac{h}{2}$; hence the factors here used are one-half of those of the table.

† This is for a member free to turn at the ends. For a member fixed at the ends use $\frac{1}{32}$ or 0.031, and for one end fixed use $\frac{1}{24}$ or 0.043 for k.

bar and equate the moment of resistance of the internal stresses with the algebraic sum of the moments due to the weight of the bar and to the pull upon it into the deflection at the centre, which is the arm of this force, we have

$$M_1 - M_2 = M_0,$$

or

$$\frac{wl^2}{8} - Pv_1 = \frac{f_1bh^2}{6}, \quad \ldots \ldots \quad (5)$$

where w = weight of bar per inch = $0.28bh$.

Putting $v_1 = \frac{f_1l^2}{5Eh}$ from (2), and $P = f_2bh$, and taking $E = 28,000,000$, we have, from (5),

$$f_1 = \frac{4,900,000h}{f_2 + 23,000,000\left(\frac{h}{l}\right)^2} \quad \ldots \ldots \quad (6)$$

as the tensile stress per square inch in the bottom fibres. The total stress in these extreme fibres is therefore

$$f = f_1 + f_2 = \frac{4,900,000h}{f_2 + 23,000,000\left(\frac{h}{l}\right)^2} + \frac{P}{bh}. \quad \ldots \ldots \quad (7)$$

144. *Effect of Height of Bar on Fibre Stress.*—From eq. (7) it is seen that the stress on the extreme fibre from bending of an eye-bar under its own weight is a function of h, f_2, and l. If f_2 and l be considered constant and h allowed to vary, we may find the depth of bar giving maximum fibre stresses by differentiating eq. (6) with reference to f_1 and h, placing the first differential coefficient equal to zero, and solving for h. This gives

$$\frac{df_1}{dh} = 0 = 23,000,000h^2 - f_2l^2,$$

or

$$h = \frac{l}{4800}\sqrt{f_2} \quad \ldots \ldots \quad (8)$$

for height of bar giving maximum fibre stresses from their own weight.

If this value of h be substituted in eq. (6), we have, as the stress on the extreme fibres from bending under their own weight, for the depths giving maximum stresses,

$$f_{1\,(max.)} = \frac{500l}{\sqrt{f_2}}. \quad \ldots \ldots \quad (9)$$

The following table gives the depths of bars having maximum fibre stresses from bending under their own weights, and also the amounts of these stresses, for different working tensile stresses in the bar and for different panel lengths.

TABLE OF DEPTHS AND MAXIMUM FIBRE STRESSES.

Working Tensile Stresses in pounds per square inch.	Length of Eye-bars in Feet.											
	15		20		25		30		35		40	
	Depth.	Fibre Stress.	Depth.	Fibre Stress.	Depth.	Fibre Stress.	Depth.	Fibre Stress.	Depth.	Fibre Stress.	Depth.	Fibre Stress.
	In.	Lbs., Sq. In.	In.	Lbs., Sq. In.	In.	Lbs., Sq. In.	In.	Lbs., Sq. In.	In.	Lbs., Sq. In.	In.	Lbs., Sq. In
8000	3.4	1010	4.5	1340	5.7	1680	6.8	2020	8.0	2350	9.1	2690
10000	3.8	900	5.1	1200	6.3	1500	7.6	1800	8.9	2100	10.1	2400
12000	4.2	820	5.6	1100	6.7	1370	8.4	1640	9.7	1920	11.1	2190
14000	4.5	760	6.0	1020	7.5	1270	9.0	1520	10.5	1780	12.0	2030
16000	4.8	720	6.4	960	8.0	1200	9.6	1440	11.3	1680	12.9	1920

For any given case, where the depth, length, and total pull per square inch are given, use eq. (6) for finding fibre stress from bending.

145. EXAMPLE 2. *Find the stress in the extreme fibres of an eye-bar (or any rectangular tension member) which also carries an external load of w_1 lbs. per linear inch.*

This problem is exactly similar to the former one, except that $(w + w_1)$ is now to be used where w alone was used before. Using this notation, we have, analogous to (5),

$$\frac{(w + w_1)l^2}{8} - Pv_1 = \frac{f_1 bh^2}{6}, \quad \quad (10)$$

from which we have, as before,

$$f_1 = \frac{105{,}000(w + w_1)}{6b\frac{f_2}{1000} + 140{,}000b\left(\frac{h}{l}\right)^2}. \quad \quad (11)$$

Thus if the cross-ties of a bridge should rest directly on the eye-bars, $w_1 l$ would be one-half of a total panel load, and the value of w would be relatively insignificant.

If $w + w_1$ were taken as 900 lbs. per foot, or 75 lbs. per inch, and two eye-bars in each chord with a section of 4 in. × 1 in. be used, and there be a pull of 14,000 lbs. per square inch on these bars, and they be assumed to be 15 ft. long, we have

$$w + w_1 = 75; \quad b = 2; \quad f_2 = 14{,}000; \quad h = 4; \quad \text{and} \quad l = 180,$$

whence $f_1 = 25{,}600$ lbs. per square inch from bending, or $f = f_1 + f_2 = 39{,}600$ *lbs. per square inch total stress* on the extreme lower fibres. This being beyond the elastic limit, the bars would permanently elongate on that side and then be straightened again when the load passed off, and in this way the bars would become "fatigued," and finally would fail.

It may be interesting to note that if the pull in these bars be neglected in the computation of bending moment, and the same transverse load applied, the fibre stress would be about 57,000 *lbs. per square inch from cross-bending alone.* The necessity of taking account of the simultaneous action of the two moments is thus shown.

146. EXAMPLE 3. What is the stress from flexure in a lateral tie-rod 1 in. square, 40 ft. long, hanging freely from end supports, and strained up with an initial pull of 10,000 lbs.?

From eq. (4) we have

$$f_1 = \frac{M_1 y_1}{I + \frac{Pl^2}{10E}} = \frac{8000 \times \frac{1}{2}}{\frac{1}{12} + \frac{10{,}000 \times 480 \times 480}{280{,}000{,}000}} = \frac{4000}{0.08 + 8.23} = 480 \text{ lbs. per square inch.}$$

The deflection is found from the formula

$$v_1 = \frac{5}{24} \cdot \frac{f_1 l^2}{Eh} = 0.82 \text{ in.}$$

This member is so shallow as to act like a wire, the depth giving maximum fibre stress for this span and unit stress being, from eq. (8), equal to 10 in.

CASE II. COMPRESSION AND CROSS-BENDING.

147. EXAMPLE 4. Find the extreme fibre stress in cross-bending arising from its own weight and its compressive load, of a top-chord section, 25 ft. long, composed of two 15-inch channels of 120 lbs. per yard and one 20 in. × $\frac{3}{8}$ in. top plate.

For this case the general equation (4) takes the form

$$f_1 = \frac{M_1 y_1}{I - \frac{Pl^2}{10E}}. \quad \quad (12)$$

The endwise loading is supposed to come upon the section at the centre of gravity axis. The maximum fibre stress will come on the top plate, which is the most compressed side of the section. The value of I for this section is 1155, and of y_1 for the plate side 6.04 in. Taking $E = 28{,}000{,}000$ and $f_2 = 7000$ lbs., P is

$31\frac{1}{2} \times 7000 = 220,000$ lbs. The bending moment at the centre from its own weight is 98,400 in.-lbs. The length is 300 inches. We have, therefore,

$$f_1 = \frac{M_1 y_1}{I - \frac{Pl^2}{10E}} = \frac{98,400 \times 6.04}{1155 - \frac{220,000 \times 90,000}{280,000,000}} = 550 \text{ lbs.}$$

This is the stress on the plate from cross-bending only.

148. EXAMPLE 5. *What is the effect of loading the above column at the centre line of the channel-bars composing its sides?*

This is a common error in construction and deserves careful consideration. The centre of gravity axis of this section is 1.83 in. from the centre line of the channels. If we now omit from consideration the weight of the member, M_1 will be composed wholly of the longitudinal force P_1 into the eccentricity of the loading, which is 1.83 in. Hence $M_1 = 220,000 \times 1.83 = 402,600$ in.-lbs.

In this case the plate side will be convex, and the maximum compressive stress will be found on the latticed side of the member. For this side $y_1 = 9\frac{1}{8}$ in. The other factors are the same as before. Therefore

$$f_1 = \frac{M_1 y_1}{I - \frac{Pl^2}{10E}} = \frac{402,600 \times 9\frac{1}{8}}{1155 - 71} = 3470 \text{ lbs. per square inch.}$$

149. EXAMPLE 6. *What is the combined effect of weight of member and eccentric loading?*

This is a combination of the conditions named in Examples 3 and 4, this combination being a common practice for upper chord members. In this case the algebraic sum of these two effects must be taken. Thus the weight of the member would produce a compression of 550 lbs. per square inch in the upper fibres and a tension of $\frac{9.33}{6.04} \times 550 = 850$ lbs. on the lower fibres. The eccentric loading gave 3470 lbs. compression on these latter, the algebraic sum of the two being 2620 lbs. per square inch compression.

To this must be added the uniformly distributed load $f_2 = \frac{P}{A} = \frac{220,000}{31.5} = 7000$ lbs. per square inch. Making a total of

$$f = f_1 + f_2 = 2620 + 7000 = 9620 \text{ lbs. per square inch.}$$

The total compression in this member is about $37\frac{1}{2}$ per cent greater than the ordinary specification would allow.

150. EXAMPLE 7. *Compute the maximum stress on the extreme fibre of a top-chord section which is used to carry the cross-ties on a railway deck-bridge.*

Let l = 20 ft. = 240 in.;
w = dead load per foot per truss = 500 lbs.;
p = live load per foot per truss = 3400 lbs.;
P = compressive stress in chord = 400,000 lbs.;
M_1 = moment from transverse load = 2,340,000 in.-lbs.

Let the chord be made up of

One top plate,	24 in. × $\frac{7}{8}$ in.,	= 21.0 sq. in.
Two top angles,	4 in. × 4 in.—42 lbs.,	= 8.4 "
Two side plates,	24 in. × $\frac{13}{16}$ in.,	= 39.0 "
Two bottom angles,	6 in. × 4 in.—70 lbs.,	= 14.0 "
	Total area of section	= 82.4 sq. in.

The moment of inertia of this section is 7436, and the neutral axis lies 10.58 in. from the upper side of the chord.

From eq. (4) we have

$$f_1 = \frac{M_1 y_1}{I - \frac{Pl^2}{10E}} = \frac{2,340,000 \times 10.58}{7436 - \frac{400,000 \times 400 \times 144}{280,000,000}} = 3370 \text{ lbs. per square inch.}$$

For this section and compressive load we have

$$f_2 = \frac{P}{A} = \frac{400,000}{82.4} = 4850 \text{ lbs. per square inch.}$$

The total compressive stress on the extreme fibres at top is therefore

$$f = f_1 + f_2 = 3370 + 4850 = 8220 \text{ lbs. per square inch.}$$

The second term in the denominator of formula (4) represents the effect of the direct compressive stress acting with the arm v_1, the deflection of the member. In this case, where the member is very rigid and deflects very little, the effect is very small, this term being but 82, whereas $I = 7436$. The effect of neglecting this term in this case, therefore, would be to give a fibre stress $1\frac{1}{10}$ per cent too small. For more flexible members the effect of neglecting this term is greater.

151. Fixed End Posts with the Upper Ends not Fixed in Direction.—Let Fig. 208*a* represent a portal, or a bent of an elevated railroad or of a steel frame building, having the posts fixed in direction at the ground and having a single system of diagonal bracing as shown. The problem is to find the point of inflection x_0 from the base, and then the values of the reactions H and V upon the columns. These reactions will be the same as in form 1, Art. 115, Chap. VII, when the point of inflection is treated as the base of the column. The problem is simplified by considering a simple beam, as in Fig. 208*b*, fixed at one end and subjected to the forces R and Q, *the determining condition being that the deflections of the beam at D and C shall be equal.* This is the effect of the portal bracing when taken as absolutely rigid as compared with the deflections of the columns.

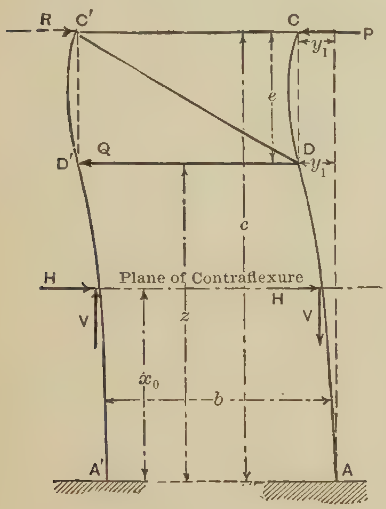


FIG. 208*a*.

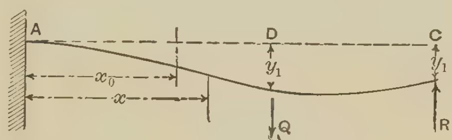


FIG. 208*b*.

Referring now to Fig. 208*b*, we have from the general equation (9), page 133, $M_x = EI\frac{d^2y}{dx^2}$ for any section.

For $x < z$,

$$M_{x<z} = EI\frac{d^2y}{dx^2} = R(c - x) - Q(z - x). \quad \ldots \ldots \ldots \quad (13)$$

For the point of inflection, where $x = x_0$, we have

$$M_{x_0} = 0 = R(c - x_0) - Q(z - x_0). \quad \ldots \ldots \ldots \ldots \quad (14)$$

Integrating eq. (13) between the limits 0 and z, we have

$$EI\frac{dy}{dx} = R\int_0^z (c - x)dx - Q\int_0^z (z - x)dx = R\left(cz - \frac{z^2}{2}\right) - Q\frac{z^2}{2}, \quad \ldots \quad (15)$$

which is EI times the angle of the deflected column at D.

For $x > z$,

$$M_{x>z} = EI\frac{d^2y}{dx^2} = R(c - x),$$

and we have for the portion beyond D,

$$EI\frac{dy}{dx} = \text{angle at } D \text{ from (15)} + \text{angular change beyond } D,$$

$$= R\left(cz - \frac{z^2}{2}\right) - Q\frac{z^2}{2} + R\int_z^x (c - x)dx = R\left(cx - \frac{x^2}{2}\right) - Q\frac{z^2}{2}. \quad \ldots \quad (16)$$

Integrating this again from z to c, we obtain the deflection at C as compared to that at D.

But by the conditions of the problem D and C deflect equally, and hence this last integral is zero, or from (16),

$$EIy = R\int_z^c\left(cx - \frac{x^2}{2}\right)dx - Q\int_z^c\frac{z^2}{2}dx = R\left(\frac{c^3}{3} - \frac{cz^2}{2} + \frac{z^3}{6}\right) + Q\left(\frac{z^3}{2} - \frac{z^2c}{2}\right) = 0; \quad . \quad (17)$$

whence

$$\frac{R}{Q} = \frac{3z^2}{2c^2 + 2cz - z^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

But from (14) we have

$$\frac{R}{Q} = \frac{z - x_0}{c - x_0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

whence

$$x_0 = \frac{z}{2}\left(\frac{z + 2c}{2z + c}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

This gives the point of contraflexure at which H and V (as in Art. 115) are to be applied.

If P is the total applied external force at C, as in Fig. 208*a*, we have

$$H = \frac{P}{2}, \quad \text{and} \quad V = P\frac{(c - x_0)}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$\left.\begin{array}{ll}\text{Maximum bending moment on column (at its base)} & = Hx_0;^* \\ \text{Bending moment at } D & = H(z - x_0);\end{array}\right\} \quad . \quad . \quad . \quad (22)$$

since the horizontal and vertical reactions, H and V, must be supposed to act at the point of inflection.

The remaining analysis is exactly the same as that given in Art. 115, using here $c - x_0$ in place of c.

Thus, with centre of moments at D, we have

$$\text{Stress in } CC' = P + H\left(\frac{z - x_0}{e}\right) = P\left(1 + \frac{z - x_0}{2e}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

With centre of moments at C', we have

$$\text{Stress in } DD' = H\left(\frac{c - x_0}{e}\right) = P\left(\frac{c - x_0}{2e}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

From Σ vertical components $= 0$ we have

$$\text{Stress in } C'D = V\sec\theta = V\frac{\sqrt{b^2 + e^2}}{e}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

If L = compressive stress in the column from the vertical loading, and ϕ = angle column makes with the vertical, then

$$L + V\sec\phi = \text{total direct stress in } A'C',$$

and

$$L - V\sec\phi = \text{total direct stress in } AC.$$

The maximum bending stress in the column by (22) is at the base and is $\frac{P}{2}x_0$. From eq. (12) we then have for the stress in the extreme fibre *from bending*

$$f_1 = \frac{M_1y_1}{I - \frac{Pl^2}{6E}\dagger} = \frac{\frac{P}{2}x_0y_1}{I - \frac{(L + V\sec\phi)l^2}{6E}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The requisite strength of anchorage may be computed from the bending moment at the base of the column (eq. (22)).

This problem is usually solved by assuming the point of inflection $\frac{1}{2}z$ from the base. For

---

* Since x_0 is always greater than $\frac{z}{2}$, it follows that the bending moment at the base of the column is always greater than at D.

† Here the coefficient is $\frac{1}{6}$ instead of $\frac{1}{10}$, as in eq. (12), since the deflection is that of a beam of length $\frac{l}{2}$ fixed at one end and loaded at the other.

the extreme case where $e = z = \frac{c}{2}$, eq. (20) gives $x_0 = \frac{5}{8}z$. When e is less than this the point of inflection approaches the middle point between A and D, so that it may be said this point lies somewhere between $\frac{z}{2}$ and $\frac{5}{8}z$ above the base for all cases of fixed base. But this base is never perfectly fixed in direction, and any flexibility here would lower the point of inflection. Neither is the web bracing above perfectly rigid, and any distortion here would raise the point of inflection, so that these assumptions may be considered as offsetting each other, and the formulæ applied rigidly as above.

152. The Trussed Beam.—A wooden beam is often trussed as shown in Fig. 209. There are usually two tie-rods passing outside the beam and having their nuts bearing upon a cast- or wrought-iron end-plate. At the middle a strut-piece or king-post is inserted, of any desired length. The beam being continuous, the load carried at C will be $\frac{5}{8}$* of the total uniformly distributed load, or $\frac{5}{4}wl$, where $w =$ load per linear inch of beam and $l =$ the half-length AC. The greatest bending moment in the beam occurs at C, and hence here is to be found the greatest fibre stress from bending. For reasonable depths of truss, H, the deflection of C would be inappreciable and may be neglected in computing f_1. The ordinary solution will hold here, therefore, or the fibre stress may be found for the cross-bending alone and added to that from the direct compression.

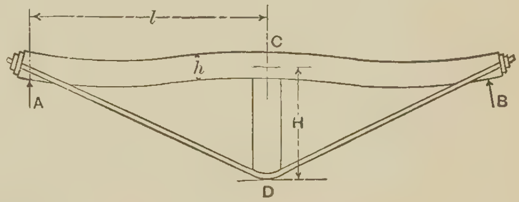


FIG 209.

We have, for the bending moment at C,

$$M_1 = \frac{wl^2}{8}{}^*, \quad \text{or} \quad f_1 = \frac{M_1 y_1}{I} = \frac{3}{4}\frac{wl^2}{bh^2}, \qquad (27)$$

for a beam of solid rectangular section.

The direct compression in the beam is

$$P = \frac{5}{8}\frac{wl^2}{H}, \quad \text{or} \quad f_2 = \frac{P}{bh} = \frac{5}{8}\frac{wl^2}{Hbh}. \qquad (28)$$

Therefore

$$f = f_1 + f_2 = \frac{wl^2}{8bh^2H}(6H + 5h). \qquad (29)$$

The stress in the tie-rod

$$= \frac{5}{8}\frac{wl}{H}\sqrt{l^2 + H^2}. \qquad (30)$$

The Queen-post Truss without Counters.—If the counter-struts shown by dotted lines in Fig. 210 (a) be omitted, as is often done in unscientific construction, especially when a beam is trussed from below by a tie-rod and two posts, any want of symmetry in the loading produces a bending of the loaded beam, as shown in Fig. 210 (b).

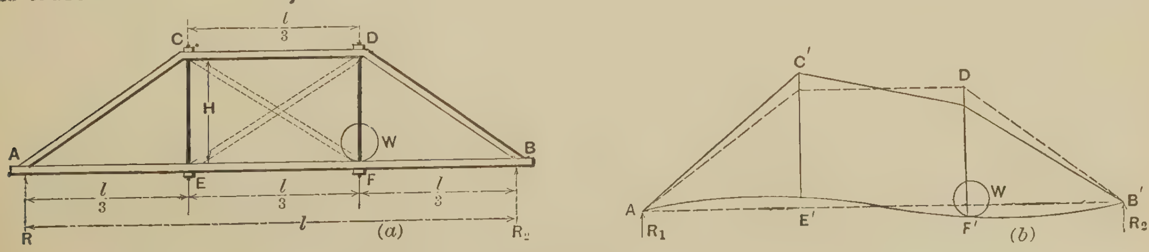


FIG. 210.

Let it be assumed that the beam is supported by the tie-rods at points $\frac{1}{3}l$ from each end. Place the load W at one of these points and find the stresses in all the members.

* See schedules of moments and shears on pages 140 and 141.

Let $H=$ height of truss; $h=$ height of bottom chord; $b=$ breadth of bottom chord; $l=$ length of bottom chord between end joints. Since the combination acts as a whole to carry the load W to the abutments, the same as any beam, the supporting forces are

$$R_1=\frac{W}{3};\quad R_2=\frac{2W}{3}.$$

It is evident at once that the load W divides itself between the truss and the bottom chord acting as a beam, and that the truss will distort as shown in Fig. 210 (b). A part of W therefore, is carried by the tie-rod DF, and the remainder rests on AB at F as on a beam. To find the relative value of these two portions into which W is divided we have—

1. Since the distortion is small and CD remains sensibly horizontal, and the two inclined members AC and DB are sensibly of equal inclination, the horizontal components of these latter are equal, since they are each equal to the stress in CD. But being of equal inclination to the horizon, their vertical components of stress must be equal.

2. But the vertical components in AC and DB are equal to the stresses in the tie-rods CE and DF, respectively, and therefore the stresses in these rods are equal.

3. From the symmetry of the trussing, if the point D drops a certain amount the point C must lift by the same amount, and hence the bottom chord is deflected upwards at E as much as it is downwards at F, and therefore the forces producing these deflections are equal. But the force producing the upward deflection at E is the stress in the tie-rod EC, while the force producing downward deflection at F is the *remaining portion of* W after the part taken by the tie-rod FD is subtracted. That is, the part of W coming directly upon the beam is just equal to the stress in the tie-rod EC, and therefore equal to that in FD.

Therefore the load W divides itself into two equal parts, one half being carried by the truss and the other half by the bottom chord acting as a beam.

The stress in the tie-rods CE and DF is therefore $\frac{W}{2}$, and this is the vertical component in each end-post. Hence

$$\left.\begin{array}{l}\text{Compression in } CD\\ \text{Tension in } AB\end{array}\right\}=\frac{W}{2}\cdot\frac{l}{3H}=\frac{Wl}{6H}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

$$\text{Compression in } AC \text{ and } DB=\frac{W}{2H}\sqrt{\frac{l^2}{9}+H^2}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

Since the central point of the bottom chord is a point of inflection in the bent beam, there is no bending moment at this point, and it may be treated as a free supported end.* The moment at F is then, for a load $\frac{W}{2}$ at F,

$$M_1=\frac{Wl}{18}=\frac{f_1bh^2}{6},\quad\text{or}\quad f_1=\frac{Wl}{3bh^2}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

The direct stress in AB from (31) is

$$f_2=\frac{Wl}{6Hbh},\quad\text{hence}\quad f=f_1+f_2=\frac{Wl}{6bHh^2}(2H+h),\quad . \; . \; . \; . \; . \quad (34)$$

which gives the maximum tensile stress per square inch in the bottom chord.

* Or a vertical section may be passed through this point of inflection and that point taken a centre of moments, whence the stress in $CD=\dfrac{\frac{2}{3}W\frac{l}{2}-W\frac{l}{6}}{H}=\dfrac{Wl}{6H}=$ also tension in AB. Also the shear at the point of inflection in AB $=W-\frac{2}{3}W=\frac{W}{3}$, whence the moment at F and $E=\frac{W}{3}\cdot\frac{l}{6}=\frac{Wl}{18}$. This method is applicable to any truss symmetrical about the panel where the bracing is omitted.

SECONDARY STRESSES.

153. Definition.—Secondary stresses are those bending stresses arising from such causes as the following:

1. The members coming together at a joint do not have their centre-of-gravity lines meeting in a point.

2. Members are not loaded, or attached, symmetrically with reference to the centre-of-gravity lines.

3. Members are not free to rotate at the joints when the live load comes on, and hence they must spring or deflect as the structure deflects.

A few of the more common cases will be investigated.

154. Gravity Lines not Meeting at a Point.—One of the more common instances of this fault can be found in shallow lattice girders, like the New York Elevated Railroads. Here the single intersection Warren girder riveted trusses have joints as shown in Fig. 211. The gravity lines of the web members intersect in C, while the intersections with the upper chord are in A and B. This gives rise to a bending moment at this joint equal to the pull in BD into the arm AC, moments being taken about A. Let the chord be made up of two 3 in. $\times$ 4 in. angles, 25 lbs. per yard, and one plate 12 in. $\times$ $\frac{1}{2}$in. Let the diagonals be composed of two 3 in. $\times$ 4 in. 25 lb. angles each, all riveting coming upon the 4-in. leg. If we assume a unit stress of 7500 lbs. in these web members, or a total stress of 37,500 lbs., and if the inner (4 in.) legs of the angles meet on the web of the chord as shown in the sketch, then the leverage AC would be $5\frac{1}{2}$ in. The bending moment at this joint would then be $5\frac{1}{2} \times 37,500 = 206,000$ inch-pounds.

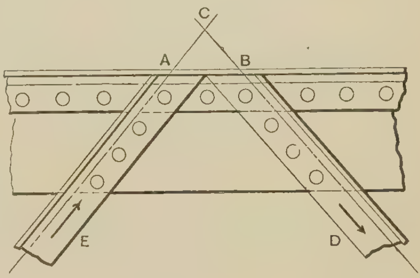


FIG. 211.

If the truss be assumed 5 feet high and the web members at 45°, then these members are 7 feet long and the panel lengths are 5 feet long. Adjacent joints in one chord are subjected to moments of like sign, and all members, both web and chord, are bent in opposite directions at their opposite ends, and by the same amounts approximately. This puts them all in double curvature, and makes a point of contrary flexure occur at their centres. All members meeting at a joint, therefore, resist this bending action of the members from eccentric position, in proportion to their relative rigidities. The angular movement of the joint would be the same for all the members meeting there and would be resisted by all, acting as beams fixed at one end (the joint) and free at the other (the middle section of the member where the moment is zero, it being a point of contrary flexure). The angular change at the joint would then be the deflection of the points of inflection (middle points), divided by the half lengths of the members. Whence if $l =$ this half length, we have for the angle through which the joint has turned, $\alpha = \frac{Pl^3}{3EI} \div l = \frac{Pl^2}{3EI} = \frac{M_1 l}{3EI}$, where M_1 is the moment at the joint carried by one member. Whence we find

$$M_1 = \frac{3EI\alpha}{l}. \qquad (35)$$

Since E and α are the same for all the members, we see that the several members meeting at a rigid joint will share the total bending moment developed at that joint directly as their moments of inertia and inversely as their lengths, or as $\frac{I}{l}$. To divide this moment properly among the members meeting at the joint and rigidly attached, we have only to take out $\frac{I}{l}$ for each of these members and divide the total moment amongst them in proportion to these values.

In the example taken we have for the moment of inertia of the chord 137.6, the length of a chord member being 5 feet. The moment of inertia of one web member (two angles), in the plane of the longer leg, is 8.0 and its length is 7 feet.

We have then

$$\text{For each chord member } \frac{I}{l} = 27.5\,;$$

$$\text{" \quad " \quad web \quad " } \quad \frac{I}{l} = 1.1.$$

There being two members of each kind meeting at this joint, the total sum is 57.2. Therefore each chord section takes $\frac{27.5}{57.2} = 48$ per cent, and each web member takes $\frac{1.1}{57.2} = 2$ per cent of the moment coming at the joint.

This result shows at once that we might have assumed at the start that the moment was wholly resisted by the chord section without appreciable error.

The total moment at the joint has been found to be 206,000 inch-pounds. Forty-eight per cent of this is 98,880 inch-pounds. This is the bending moment resisted by the chord section. The stress on the extreme lower fibre of the web of this member, which lies 8.16 inches from the neutral axis, is

$$f_1 = \frac{M_1 y_1}{I} = \frac{98{,}880 \times 8.16}{137.6} = 5850 \text{ lbs. per square inch}$$

in the extreme bottom fibres of the top chord, being tension on one side of the joint and compression on the other.

Since the chord stress for which this member was designed was probably about 7500 lbs. per square inch, we see that the secondary stress with this apparently favorable arrangement of the web connection gives rise to secondary stresses 78 per cent as large as the primary stresses which were alone taken into account in the computation. In other words, the actual maximum fibre stress is nearly twice as great as that for which the structure was designed. Since it is not uncommon to find riveted structures with joints much more eccentric than the one here computed, the importance of avoiding such eccentric combinations is patent.

In the computation of secondary stresses arising from the non-concurrence of the gravity lines of the assembled members no great error will be made if it be assumed that the heavier and more rigid members take all the moment.

In computing this moment take a centre of moments at the intersection of all the forces but one (if possible), and then the moment developed is the remaining force into its arm. If a centre of moments cannot be chosen so as to leave out but one force, then the products of all the remaining forces into their several arms are found and the algebraic sum taken. This moment is then divided amongst the members in proportion to the quantity $\frac{I}{l}$ for the several members. If some of the members are free to turn at the other ends, then their full lengths are to be taken, while for those members which are rigid at both ends one half their lengths are used for l. If all members are alike in this particular, either all free or all rigid at their opposite ends, then their full lengths can be used, as the reactions will be the same whether the full lengths or the half lengths be taken. The moments of inertia must be computed for the sections at or near the joints.

155. **Members not Loaded Symmetrically with reference to their Centre of Gravity Lines.**—This is a simple case and one not requiring any extended discussion. The bending moment is in all cases equal to the total load (pull or thrust) on the member into its arm, which is the perpendicular distance between the centres of gravity of the applied load and of the cross-section of the member. The bending will be in the plane of this lever arm, and the stress resulting from this eccentric loading is given by the general equation (4), the negative sign in the denominator to be used when the member is in compression and the positive sign when in tension.

156. EXAMPLE 8. *What is the greatest secondary stress in one of the angle-irons shown in Fig.* 211, *due to its being attached by one leg only, when the compressive load upon it is* 7500 *lbs. per square inch or* 18,750 *lbs. on one angle.*

Equation (4) now takes the form

$$f_1 = \frac{M_1 y_1}{I - \frac{Pl^2}{10E}}.$$

Since the load P may be supposed to be concentrated at the centre line of the wide leg, which is riveted to the chord, the eccentricity is therefore for this angle iron but 0.61 inch. The moment is $0.61 \times 18{,}750 = 11{,}440$ inch pounds $= M_1$. The distance from the centre-of-gravity axis of the angle to the most compressed fibre is 0.8 inch, and $I = 1.94$. Hence we have

$$f_1 = \frac{11{,}440 \times 0.8}{1.94 - \frac{18.750 \times 7056}{280{,}000{,}000}} = \frac{9150}{1.94 - .47} = 6220 \text{ lbs. per square inch.}$$

If the two angles forming one member are attached together at intervals throughout their lengths, they would mutually support each other, since they would tend to deflect in opposite directions. This would prevent the development of the secondary stresses here computed. For this reason angle-irons when attached by one leg only should always be placed in pairs, and then attached together throughout their entire lengths.

157. EXAMPLE 9. *Find the shearing stress on each rivet in the attachment shown in Fig.* 212. Let the force P acting in a direction parallel to rivets 2 and 4 be 15,000 lbs. This is a common case in practice where a 1-inch square lateral rod is held with four $\frac{7}{8}$-inch rivets, giving 3750 lbs. to each. Let the eccentricity, a in the figure, be 4 inches. Then the force P may be replaced by the equal force P' acting at the centre of gravity of the rivets, and a moment of $4 \times 15{,}000 = 60{,}000$ inch-pounds. This moment is resisted equally by the four rivets, giving rise to equal shearing stresses in the direction of the several arrows, or circumferentially about the centre of gravity of the rivets. The lever arm of these forces, taking the rivets as placed at the corners of a square 5 inches on a side, would be 3.5 inches. The shearing stress on each rivet, therefore, to resist the moment, would be $\frac{60{,}000}{4 \times 3\frac{1}{2}} = 4300$ lbs. Therefore

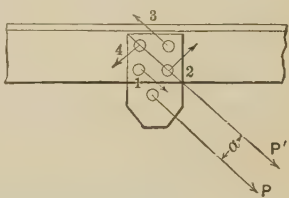


FIG. 212.

The shearing stress on rivet 1	$= 3750 + 4300$	$=$	8050 pounds.
" " " " " 2 and 4	$= \sqrt{(3750)^2 + (4300)^2}$	$=$	5710 "
" " " " " 3	$= 3750 - 4300$	$=$	$-$ 550 "

The minus sign given to the shearing stress in rivet 3 indicates that the shear on this rivet is opposite in direction to that in rivet 1. The shearing stress on rivet 1 is thus shown to be more than twice as much as it was designed to carry, and it is liable to work loose.

158. **Secondary Stresses due to Rigidity of Joints.**—Whenever a framed structure deflects under a load all the angles of intersection at all the joints would be changed by very small amounts if the members were free to turn at the joints. To be perfectly free to turn these members would have to be hung on knife-edges. Pin-connected bridges have generally been supposed free to turn at the joints, but this is not the case. There may be a slight rocking of the member about the pin from the looseness of the pin in the hole, but the maximum play allowed now in good work is so small as to be practically zero. For pin-connected structures, however, the members are comparatively narrow, so that y_1 in formula (4) is small. For the ordinary proportions also of height and panel length to span the angular change is so small that this source of secondary stresses may be neglected. In riveted work, where wide plates are used, they may be much larger. Such stresses need never be computed for pin-connected bridges.

CHAPTER XI.

SUSPENSION-BRIDGES.

159. Introduction.—Suspension-bridges of a crude form have been in use from the earliest times. The first long-span iron bridges were also of this type, the suspension-cables being composed of chains, iron bar links, flat bars with pin connections, and finally of iron and steel wire. Sir Thomas Telford and Sir Samuel Brown greatly developed and improved the designs of suspension-bridges in England from 1814 to 1830, while Mr. J. A. Roebling adapted this style of structure to both highway and railway service in his famous bridge over the Niagara River in 1852–5. Telford's and Brown's bridges were built without the use of stays, as shown in Fig. 213. This is a span of 432 feet, built by Mr. Brown in 1829, the cable

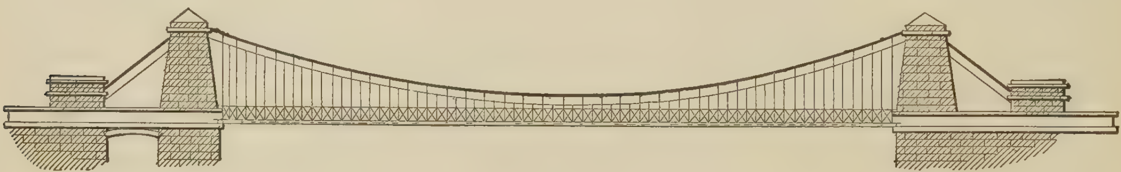


FIG. 213.

being composed of flat wrought-iron bars. Fig. 214 is a view of one half of the Niagara bridge as it was originally constructed, the span being 821 feet. Diagonal stay-cables are introduced,

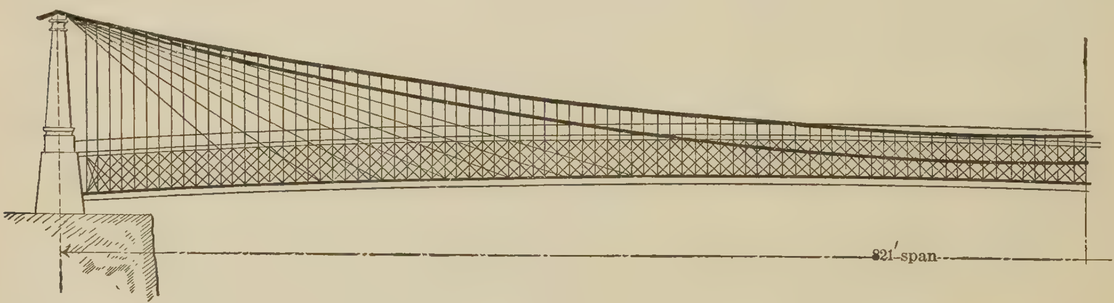


FIG. 214.

and the trusses are exceptionally strong, having a height of 18 feet out to out. The sag of one cable is 54 feet, and of the other 64 feet; each is composed of 3640 No. 9 iron wires.*

When the suspension-cables are made of a high-grade steel wire having an average ultimate strength of some 160,000 lbs. per square inch, and which may be dimensioned for a maximum working load of 40,000 lbs. per square inch, there is great economy in this style of construction. When the material is disposed in small wires, however, presenting a very large ratio of surface to sectional area, it must be protected more perfectly from corrosion than is necessary when the material is disposed in larger members. When so protected, a long span can be built on this principle, for highway traffic, for a much less sum than would be required for any form of truss-bridge to carry the same loads. To stiffen a suspension-bridge sufficiently to carry train loads adds so greatly to the cost as to make it inadvisable to employ

* This bridge has now been reconstructed by Mr. L. L. Buck, M. Am. Soc. C.E., by replacing the stone towers by steel construction, the wooden stiffening truss by one of steel, and omitting the stays altogether.

it for railway purposes except for the very longest spans, as in the case of the proposed bridge over the Hudson River at New York City. Here the span is 2850 feet, and provision is to be made for carrying six tracks, with a possible increase to ten tracks if required. For highway purposes suspension-bridges can be judiciously used for spans exceeding 300 or 400 feet. If the locality is adapted to the use of metallic arches, these could well be employed for spans of from 300 to 500 feet. For all spans of greater length than 500 feet for highway purposes the suspension type of bridge should perhaps always be employed.

THEORY OF SUSPENSION-BRIDGES.

160. Stress in Cable for Uniform Load over the Entire Span.—For this loading the curve of equilibrium is a parabola as shown in Art. 49. It may be proved directly as follows:

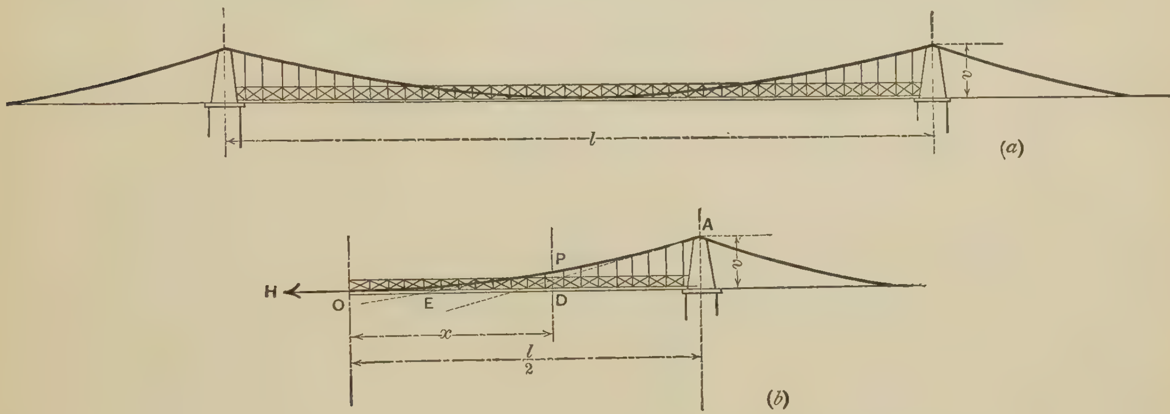


FIG. 215.

Let Fig. 215 (*a*) represent a suspension-bridge uniformly loaded with a dead load w and a live load p per foot. Let $l =$ span, and $v =$ sag, or versed sine of curve. In Fig. (*b*) let the left half of the bridge be removed and replaced by the force H, this being the stress in one cable at bottom where it is horizontal. Let O be taken as the origin of co-ordinates. Take moments about any point in the cable as P, whose ordinates are $PD = y$, and $OD = x$. Then we have, for the external forces acting on the left of the section through P, since the moment here is zero,

$$Hy = \left(\frac{w+p}{2}\right)\frac{x^2}{2}, \quad \text{or} \quad x^2 = \frac{4H}{w+p}y, \quad \ldots \ldots \ldots \quad (1)$$

which is the equation of a parabola referred to its vertex.

To find H take moments about A, the top of the tower, and we obtain

$$Hv = \left(\frac{w+p}{2}\right)\frac{l^2}{8}, \quad \text{or} \quad H = \left(\frac{w+p}{2}\right)\frac{l^2}{8v}, \quad \ldots \ldots \ldots \quad (2)$$

Using this value of H in eq. (1) we have, as the equation of the curve of equilibrium for a load uniformly distributed along the horizontal, referred to its lowest point,

$$x^2 = \frac{l^2}{4v}y, \quad \ldots \ldots \ldots \ldots \ldots \quad (3)$$

In such a curve of equilibrium the stress is simple tension, and *the horizontal component of this stress is constant.* This must be so since the external forces acting on the cable are all vertical, having no horizontal components. Hence

$$\textit{The tension in cable} \qquad = H \sec i = H\frac{EP}{ED} = H\frac{\sqrt{dx^2 + dy^2}}{dx}, \quad . \quad . \quad . \quad (4)$$

$$\textit{The tension in cable at towers} = H \sec i = \left(\frac{w+p}{4}\right) l \sqrt{1 + \left(\frac{l}{4v}\right)^2}, \quad . \quad . \quad . \quad (5)$$

where i = angle of cable with the horizontal.

161. Stiffening Truss for Partial Loads.—When the cable is not loaded uniformly over its entire span it will swing into a new curve of equilibrium for that load, if this action is not resisted in some way. It is resisted by a truss either along the roadway or attached to the cable itself, or both. The object of the truss is only to distribute the load uniformly

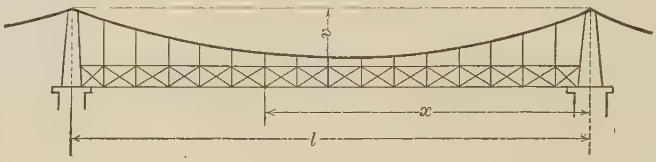


FIG. 216.

over the cable, and not to assist in carrying it to the abutments. The truss should be made to deflect so little that, as compared with the free movements of the cable, it will be relatively rigid; that is to say, its deflection, up or down, at any point under a concentrated load will be very small as compared to the deflection of the cable at that point under this loading, if the truss were not used. Whenever the truss is reasonably efficient in preventing unequal distortions this condition of relative local rigidity holds true, and hence the conclusions based on this assumption are correspondingly correct.

In the following discussion the load is supposed to rest primarily upon the truss, as shown in Fig. 216, and the truss is supposed to distribute this load evenly or uniformly upon the cable. Whatever the contingencies of loading, therefore, *the stress in the hangers is assumed to be constant from end to end of the span.*

A further assumption is that the cable stretches very little for any given unsymmetrical, load, so that when this load is uniformly distributed over it by the truss, its vertical deflection at the centre is insignificant as compared to the vertical deflection of the truss acting alone under this same total load. The former assumption referred to a deflection of the cable at a given point, under a concentrated load there, due to deformation, or change of shape, if acting alone, while this assumption has to do only with the symmetrical deflection of the cable due to its elongation. The deflection of the truss is very small as compared to the former, and very large as compared to the latter.

If, therefore, the cable does not appreciably increase its sag, for the assumed distribution of the concentrated loads, then the cable may be assumed to carry all this live load, while the truss merely serves to distribute it. But the condition of equilibrium of the external forces

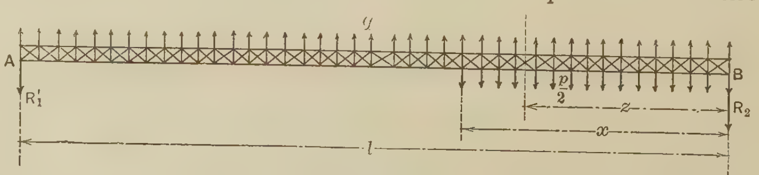


FIG. 217.

acting upon the truss requires that the sum of their vertical components and of their moments shall be zero.

The forces acting on each truss are:

1st. A uniform downward load of $\frac{p}{2}$ lbs. per foot over the distance x from the right support.

2d. A uniform upward pull of q lbs. per foot from each cable, over the entire span.

3d. The two end reactions, R_1 and R_2, for equilibrium. We have assumed that

$$\frac{px}{2} = ql, \quad \text{or} \quad q = \frac{px}{2l}. \qquad (6)$$

This satisfies the condition of equality of vertical components of external forces without reference to the end reactions. But the centre of gravity of the downward forces $\frac{px}{2}$ is $\frac{x}{2}$ from B, Fig. 217, while that of the pull of the hangers is at the centre of the span. These being equal and opposite non-concurrent forces, they form a couple whose arm is $\frac{l-x}{2}$, the moment being $\frac{px}{2}\left(\frac{l-x}{2}\right)$. This couple can only be balanced by another couple composed of the end reactions having the arm l; *hence these reactions are equal and opposite, and equal to*

$$\frac{px}{4l}(l-x),$$

or

$$-R_1 = +R_2 = \frac{px}{4l}(l-x). \qquad (7)$$

We now have all the external forces which act upon the truss and are prepared to find the maximum moments and shears coming upon it, for the uniformly distributed moving load p per running foot. Since the end reactions are both positive and negative for different loadings, the truss must rest upon the piers and be anchored to them sufficiently to resist the maximum negative end shear or reaction. We will first find what position of load gives maximum shears and moments, and then find values for these maxima.

Let the load extend from the right support to a distance x towards the left, as shown in Figs. 216 and 217.

Let z be the distance from the right support to any section, so that z may be greater or less than x, and may vary from 0 to l. The distances x and z are always measured from B.

162. **Discussion for Maximum Shear.**—The general equations for shear (positive shear being that which acts upwards on the left) are, for one truss,

$$\left.\begin{aligned} S_{z<x} &= -R_1 + q(l-z) - \frac{p}{2}(x-z), \\ S_{z>x} &= -R_1 + q(l-z). \end{aligned}\right\} \qquad (8)$$

Substituting $\frac{px}{2l}$ for q and $\frac{px}{2l}\left(\frac{l-x}{2}\right)$ for R_1, we have

$$S_{z<x} = \frac{px}{4l}(x-l-2z) + \frac{pz}{2}; \qquad (9)$$

$$S_{z>x} = \frac{px}{4l}(x+l-2z). \qquad (10)$$

If we take first a given load, $\frac{px}{2}$, and let it remain stationary while our section, z, varies from 0 to l, using eq. (9) from 0 to x, and eq. (10) from x to l, we see by inspection that the shear is a minimum (neg. max.) for $z = 0$ and for $z = l$, and a positive maximum for $z = x$. Thus for $z = 0$ and $z = l$ we have, from eq. (9) and (10) respectively,

$$S_{z=0} = \frac{px}{4l}(x - l) = S_{z=l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

for negative shear,

and for $z = x$ we have
$$S_{z=x} = \frac{px}{4l}(l - x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

for positive shear. From these equations we see that *the shear at the head of the load is always numerically equal to, but of opposite sign from, the shears at the ends of the span*, and that these are the maximum shears on the truss for a continuous load from one support.

If we let the load move across the span, making x vary from 0 to l, we can find from eqs. (11) and (12) the positions of load giving maximum positive and negative shear.

Thus, from either eq. (11) or eq. (12), we have

$$\frac{dS}{dx} = 0 = 2x - l, \quad \text{or} \quad x = \frac{l}{2} \text{ for shear a maximum,}$$

or the greatest shears all occur when the bridge is half loaded.

Substituting this value of x in either of the eqs. (11) or (12), we find *the Maximum Positive and Negative Shears in one truss at both the ends and the centre to be* $\frac{1}{16}pl$; or, in general,

$$\textit{Maximum Shears} = \tfrac{1}{16}pl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

To obtain the maximum positive shear at any point distant z from the right support, we must study the three conditions indicated by equations (9), (10), and (12). Thus for the maximum shear at the head of the load, putting $z = kl$ to adapt our results to all spans, we have

$$S_{z=x} = \frac{pl}{16}(4k - 4k^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

$$= \frac{pl}{16}F \text{ for graphical representation in Fig. } 217a^*.$$

This locus is the parabola ACB, while for a load from the left the shear would be negative and given by the curve $AC'B$.

To find the maximum negative shear for the right half of the span, when the load comes on from the right, we have $z < x$ as given in eq. (9). To find the position of this load which will give a maximum shear of this kind, at any point z under the load, we must solve (9) for S a maximum for x variable, and obtain

$$\frac{dS_{z<x}}{dx} = 0 = 2x - l - 2z, \quad \text{or} \quad x = \frac{l + 2z}{2}.$$

---

* The diagrams in this figure are due to Prof. M. A. Howe.

Substituting this value of x in (9) and putting $z = kl$, we find

$$\text{Max. } S_{z<x} = \frac{pl}{16}(4k - 4k^2 - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

$$= \frac{pl}{16}(F - 1) \text{ for graphical representation.}$$

This is the curve EF' in the figure. Evidently for a load from the left we would have had similarly the positive shear curve DE.

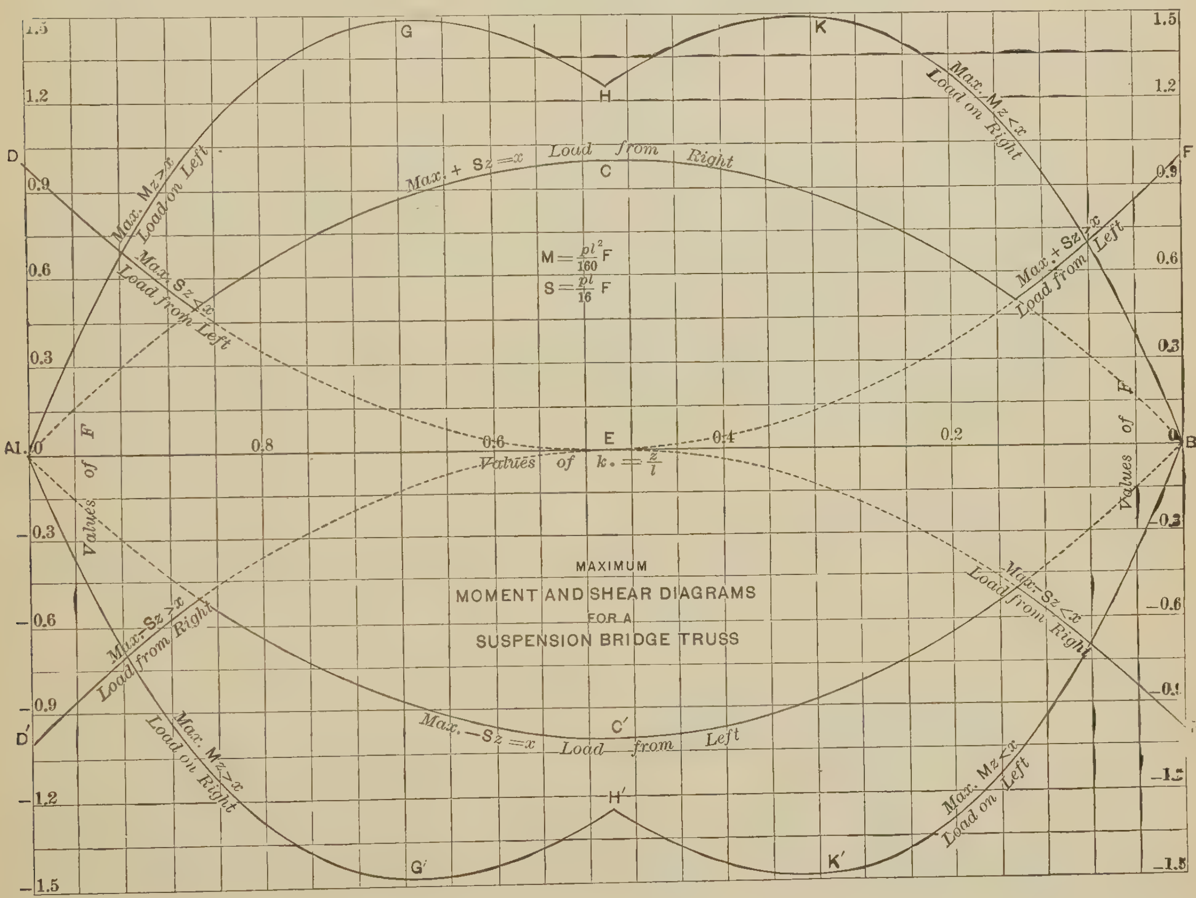


FIG. 217*a*.

To find the maximum negative shear on the left half of the span we would have to take $z > x$, or take our section in front of the load coming on from the right. This is given in equation (10). Treating this as before, we find

$$\frac{dS_{z>x}}{dx} = 0 = 2x + l - 2z, \quad \text{or} \quad x = \frac{2z - l}{2},$$

which substituted in (10), and putting $z = kl$, gives

$$\text{Max. } S_{z>x} = \frac{pl}{16}(4k - 4k^2 - 1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C)$$

which is the same as (B). Therefore this locus $D'E$ is symmetrical with $F'E$, and for a load coming on from the left we have FE symmetrical with DE.

163. Discussion for Maximum Moments.—Referring again to Fig. 217, we may write, for the moments in one truss at any section z,

$$M_{z<x} = -R_1(l-z) + \frac{q(l-z)^2}{2} - \frac{p(x-z)^2}{4}, \quad \ldots \ldots \quad (14)$$

and

$$M_{z>x} = -R_1(l-z) + \frac{q(l-z)^2}{2}. \quad \ldots \ldots \quad (15)$$

Substituting the values of R_1 and q, we obtain

$$M_{z<x} = \frac{pz}{4}\left(1 - \frac{x}{l}\right)(x-z); \quad \ldots \ldots \quad (16)$$

$$M_{z>x} = \frac{px}{4l}(l-z)(x-z). \quad \ldots \ldots \quad (17)$$

By inspection we see that when $z = x$, or when the section is taken at the head of the load, the moment is zero by both equations. Also, that when $z < x$ the moment is positive, and when $z > x$ the moment is negative. Therefore, *the head of the load is always a point of contrary flexure.* And since we have already learned that the shear here is always equal and opposite to that at the two ends, we see that *both the loaded and the unloaded portion of the span can be treated as a simple truss uniformly loaded.* This leads us to conclude that *the maximum moments occur at the middle parts of the loaded and of the unloaded portions*, the former being positive and the latter negative moments. This same conclusion could have been reached by a discussion of eqs. (16) and (17), as was done in the case of shear with eqs. (9) and (10).

These maximum moments are

$$M_{z=\frac{x}{2}} = \frac{px^2}{16}\left(1 - \frac{x}{l}\right); \quad \ldots \ldots \quad (18)$$

$$M_{z=\frac{l+x}{2}} = -\frac{px}{16l}(l-x)^2. \quad \ldots \ldots \quad (19)$$

To find for what position of load the downward bending moment under the load is a maximum, put the first differential coefficient of eq. (18) equal to zero, and find

$$\frac{dM}{dx} = 0 = \frac{px}{8} - \frac{3px^2}{16l}, \quad \text{whence} \quad x = \tfrac{2}{3}l. \quad \ldots \ldots \quad (20)$$

Doing the same for eq. (19) to find position of load for maximum upward bending ahead of the load, we have

$$\frac{dM}{dx} = 0 = l^2 - 4xl + 3x^2, \quad \text{whence} \quad x = \tfrac{1}{3}l. \quad \ldots \ldots \quad (21)$$

Therefore, the maximum downward moment occurs at the middle of the load when it extends over two-thirds of the span, and the maximum upward moment occurs at the middle of the unloaded portion when the load extends over one-third the span.

Inserting these values of x in eqs. (18) and (19), we find

$$\textit{Maximum downward moment in one truss} = \frac{pl^2}{108}; \quad \ldots \ldots \quad (22)$$

$$\text{“} \quad \textit{upward} \quad \text{“} \quad \text{“} \quad \text{“} \quad = \frac{pl^2}{108}. \quad \ldots \ldots \quad (23)$$

For a load p per foot, on two simple trusses of length l, the bending moment at the centre of each would be $\frac{pl^2}{16}$.

In other words, *the maximum upward and downward moments are equal, occur at the one-third and two-thirds points, and are about one-seventh the maximum moment due to the same unit load acting over the same span when unsupported by the cable.*

The maximum shears were found to be $\frac{1}{16}pl$, or just one-fourth what they would be on simple trusses of the same length.

To obtain the maximum positive (downward) moment at any section under the load distant z from the right support, find from eq. (16)

$$\frac{dM_{z<x}}{dx} = 0 = 1 + \frac{z}{l} - \frac{2x}{l}, \quad \text{or} \quad x = \tfrac{1}{2}(l+z).$$

Substituting this in (16), we have

$$\text{Max. } M_{z<x} = \frac{pz}{16l}(l-z)^2 = \frac{pl^2}{16}k(1-k)^2 \quad (\text{where } z = kl)$$

$$= \frac{pl^2}{160}F$$ for graphical representation in Fig. 217*a*. This locus is the curve AGH for a load coming from the left and BKH for a load from the right.

For the maximum negative (upward) bending moment on the unloaded portion we have from eq. (17)

$$\frac{dM_{z>x}}{dx} = 0 = 2lx - 2xz - l^2 + z^2, \quad \text{or} \quad x = \frac{z}{2}.$$

Substituting this in (17), we have

$$\text{Max. } M_{z>x} = -\frac{pz^2}{16l}(l-z) = -\frac{pl^2}{16}k^2(1-k)$$

$$= \frac{pl^2}{160}F$$ for graphical representation in Fig. 217*a*. This locus is the curve $AG'H'$ for a load from the right and $BK'H'$ for a load from the left.

Since all parts of the stiffening truss in a suspension-bridge are subjected to both positive and negative moments and shears, in all parts of its length, the maximum shear occurring at both ends and at the centre, and the maximum moments at the one-third points, it is common to dimension all parts to carry these stresses, thus making uniform sizes throughout the entire truss.

It should also be noted that in this case the shear in the truss from live load is not affected by the amount of the dead load, this latter being suspended wholly from the cable.

If the span is very long, the actual maximum moments and shears may be taken out for all sections by the use of eqs. (11), (12), (18), and (19), and the members proportioned accordingly.

If the stiffening trusses be proportioned to carry safely the moments and shears here found, there will be no necessity for the use of stay-cables. In most cases in practice, however, the trusses are not as strong as here assumed and the use of stay-cables becomes necessary.

164. The Action of Stay-cables.—It is common to use stay-cables reaching from the tops of the towers to the bottom of the stiffening truss out to about one-fourth the span from the towers. At this point the stays become tangent to the main cables at the towers. The stays are superfluous members, and when introduced *the distribution of the load must be deter-*

mined by the principle that it divides itself amongst the systems in direct proportion to their relative rigidity or inversely as their deflections.

We will now find the deflection of any point of attachment of a stay, as B, Fig. 218, resulting from assumed changes in unit stress in the several members of the systems. A load placed at B may pass up the hanger to the cable and thence to the tower, or it may pass

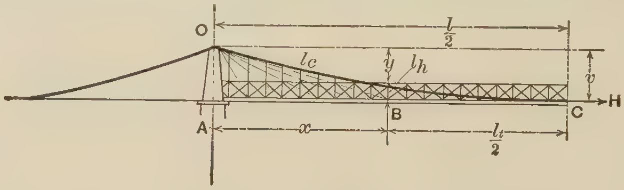


FIG. 218.

up the stay to the tower, causing tensile stress in both the stay and the bottom chord of the truss to the right of B. Since the greatest proportion of the load at B will come upon the stay when the cable deflects the most at this point, which will be when the bridge is fully loaded, we will assume this condition in the following discussion:

The problem is, therefore, to find the deflection of B in terms of the increase in the unit stress in hanger and cable (df_h and df_c) for a full live load over the whole span, and then the deflection of B in terms of the increase in unit stress in stay and bottom chord of truss (df_s and df_t) for the same full load. Then, by placing these equal to each other, since the two systems are rigidly attached, decide how the load at B divides itself between the two systems. It will remain then to compute a table of numerical coefficients for different positions of B on the outer quarters of the span. The following notation will be employed:

Let l = length of span;
v = sag at centre = the vertical projection of the stays;
(xy) = coördinates of the parabolic curve referred to O;
l_c = length of cable between towers;
l_s = length of stay;
l_h = length of hanger;
l_t = length of bottom chord of truss between symmetrical hangers;
dy = deflection of cable at B;
dv = deflection of cable at centre;
dh = stretch of hanger;
dv_s = vertical deflection at B from stretch of stay;
dv_t = vertical deflection at B from stretch of lower chord of truss;
df_c, df_h, df_s, df_t = changes of unit stress in cable, hanger, stay, and truss-chord, respectively, due to live load;
E = the common modulus of elasticity of cable, hanger, and stay;
E_t = modulus of elasticity of truss chord.

165. Deflection of the Cable System for Full Loads.—The length of the cable, in terms of l and v, expressed by a series and the first two terms used, as may be done when v is small as compared to l, is

$$l_c = l + \frac{8v^2}{3l}. \qquad (24)$$

The equation of the curve referred to O, as an origin, is

$$y = \frac{4v}{l^2}(lx - x^2). \qquad (25)$$

To find the relation between small changes of length of cable and of sag at the centre we may use the methods of the calculus. Differentiating eq. (24) for l_c and v variable, we have

$$dl_c = \frac{16}{3} \cdot \frac{v}{l} dv. \qquad (26)$$

Doing the same with (25) for y and v variable, we have

$$dy = \frac{4}{l^2}(lx - x^2)dv, \qquad (27)$$

whence

$$dy = \frac{3}{4lv}(lx - x^2)dl_c. \qquad (28)$$

But dl_c is the change in length of the cable due to a change in unit stress in it of df_c, or

$$dl_c = l_c\frac{df_c}{E} = \left(l + \frac{8}{3}\frac{v^2}{l}\right)\frac{df_c}{E}, \qquad (29)$$

whence

$$dy = \left[\frac{3x}{4v}(l - x) + \frac{2vx}{l}\left(1 - \frac{x}{l}\right)\right]\frac{df_c}{E}. \qquad (30)$$

This is the change of sag, or the deflection, of any point in the cable (xy) due to a change of unit stress in the cable of df_c.

The deflection of B due to the stretch of the hanger is

$$dl_h = l_h\frac{df_h}{E} = (v - y)\frac{df_h}{E} = v\left[1 - \frac{4x}{l}\left(1 - \frac{x}{l}\right)\right]\frac{df_h}{E}. \qquad (31)$$

The total deflection of B due to the cable system is therefore $dy + dl_h$, as given by eqs. (30) and (31).

166. **Deflection of the Stay System for Full Loads.**—The length of a stay in terms of v and x is

$$v^2 = l_s^2 - x^2,$$

whence

$$dv_s = \frac{l_s}{v}dl_s = \frac{l_s^2}{v}\frac{df_s}{E} = \left(\frac{v^2 + x^2}{v}\right)\frac{df_s}{E}. \qquad (32)$$

This is the deflection of B due to a stretch of the stay OB. But the bottom chord also stretches between the corresponding stay attachments at the two ends, which allows of a corresponding diminution in x, for when BC increases, AB decreases by a like amount. If the truss were rigidly fastened, or buttressed at A, we might assume a compression in AB; but although it is anchored down at A, it must be allowed to expand and contract from temperature, and hence the anchor-rods permit of some longitudinal motion here. We are now solving for a full load, and hence the stays will pull symmetrically on the bottom chord. When one end only is loaded, the loaded end of the truss is pinned fast to the pier by the load, while the opposite end is swung free. The stays at the unloaded end also are slack since the truss is here deflected upwards, and hence they cannot counteract the horizontal pull in the stays at the loaded end. In this case the horizontal component of the stress in the stays is taken by the bottom chord in compression from the foot of the stay to the pier at that end, provided it is held to place here. This case will be discussed below.

If we assume, for full loads, that about five-eighths of the length of the bottom chord is in uniform tension, from the stays, of df_t per square inch, the equations will be simplified, and with no

appreciable error. The stretch of bottom chord will be, therefore, $\frac{5}{8}l\frac{df_t}{E_t}$*, and since this will be the measure of the *diminution* of x, we may write

$$-dx = \frac{5}{16}l\frac{df_t}{E_t}.$$

But from the triangle AOB we have $v^2 = l_s^2 - x^2$, whence

$$dv_t = -\frac{x}{v}dx = \frac{5lx}{16v}\cdot\frac{df_t}{E_t}. \quad \ldots \ldots \ldots \quad (33)$$

This is the deflection of B due to the stretch of the lower chord.

The total deflection of B in the stay system is therefore $dv_s + dv_t$, as given in eqs. (32) and (33). We are now prepared to compare the deflections of the two systems.

167. Comparison of Deflections in the Cable and Truss Systems for Full Load.—By giving to x and v values in terms of l in eqs. (30), (31), (32), and (33), we can obtain numerical coefficients for the expressions $l\frac{df_c}{E}$, $l\frac{df_h}{E}$, $l\frac{df_s}{E}$, and $l\frac{df_t}{E_t}$, these being the values of the stretch from full live loads in the cable, the hangers, the stays, and the lower truss chord, respectively. In the following table such coefficients are given for $x = \frac{1}{16}, \frac{1}{8}, \frac{3}{16}$, and $\frac{1}{4}$ of l, and also for $\frac{l}{v} = 12$ and $\frac{l}{v} = 16$, these being about the limiting values of the ratios of length to versed sine for good practice for long spans.

TABLE OF DEFLECTION COEFFICIENTS OF CABLE AND STAY SYSTEMS FOR FULL LOADS.

Items.	Coefficients of $l\frac{df_c}{E}$, $l\frac{df_h}{E}$, $l\frac{df_s}{E}$, and $l\frac{df_t}{E_t}$ for dy, dh, dv_s, and dv_t.							
	$v = \frac{1}{12}l.$				$v = \frac{1}{16}l.$			
	$x = \frac{1}{16}l.$	$x = \frac{1}{8}l.$	$x = \frac{3}{16}l.$	$x = \frac{1}{4}l.$	$x = \frac{1}{16}l.$	$x = \frac{1}{8}l.$	$x = \frac{3}{16}l.$	$x = \frac{1}{4}l.$
Deflection of cable $= dy$	0.54	1.01	1.42	1.72	0.71	1.33	1.89	2.27
Stretch of hanger $= dl_h$	0.06	0.05	0.03	0.02	0.04	0.03	0.02	0.01
Deflection from stay $= dv_s$	0.13	0.27	0.50	0.83	0.12	0.31	0.62	1.06
Deflection from truss chord $= dv_t$	0.24	0.47	0.70	0.92	0.31	0.62	0.94	1.25

Before we can use this table we must decide what the changes in unit stress will be in the several parts. The dead load is supposed to rest wholly on the cable and hangers, the stays and truss being without stress. The dead load may also be taken as equal to one half the live load, or $s = \frac{w}{2}$. Hence the change of stress in the hangers and cable for full live load will be two-thirds of the total stress, or say 24,000 lbs. per square inch. The stays may be stressed to 40,000 lbs. per square inch, all of which is due to live load. The bottom chord of the truss, if of structural steel, may have a tensile stress of, say, 10,000 lbs. per square inch added to it for live load from the stays, since there is then no bending moment in the truss to speak of. If the truss is of timber, then it may have 500 lbs. per square inch added to the total lower chord section, thus making the stretch of this member the same in both cases, since

$$\frac{10{,}000}{28{,}000{,}000} = \frac{500}{1{,}400{,}000} = 0.00036 \doteq \frac{df_t}{E_t}.$$

* The truss may be of timber, and hence it is necessary to use a different modulus of elasticity here.

If we now put

$$\left.\begin{aligned} \frac{df_h}{E} = \frac{df_c}{E} = \frac{24{,}000}{28{,}000{,}000} &= 0.00086, \\ \frac{df_s}{E} = \frac{40{,}000}{28{,}000{,}000} &= 0.00143, \\ \frac{df_t}{E_t} = \frac{10{,}000}{28{,}000{,}000} = \frac{500}{1{,}400{,}000} &= 0.00036, \\ \text{and take } l = 1000 \text{ feet}, & \end{aligned}\right\} \quad \ldots \ldots \quad (34)$$

we obtain the following table of actual deflections:

TABLE OF DEFLECTIONS, IN FEET, OF STAY AND CABLE SYSTEMS.

LENGTH OF SPAN = 1000 FEET.

Items.	Versed sine $= \frac{1}{12}$ span or $v = \frac{l}{12}$.				Versed sine $= \frac{1}{16}$ span or $v = \frac{l}{16}$.			
	$x = \frac{1}{16}l.$	$x = \frac{1}{8}l.$	$x = \frac{3}{16}l.$	$x = \frac{1}{4}l.$	$x = \frac{1}{16}l.$	$x = \frac{1}{8}l.$	$x = \frac{3}{16}l.$	$x = \frac{1}{4}l.$
	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
Deflection of cable $= dy$	0.46	0.86	1.22	1.48	0.61	1.14	1.63	1.95
Stretch of hanger $= dl_h$	0.05	0.04	0.03	0.02	0.04	0.03	0.02	0.01
Deflection of cable system $= dy + dl_h$	0.51	0.90	1.25	1.50	0.65	1.17	1.65	1.96
Deflection from stay $= dv_s$	0.18	0.38	0.72	1.18	0.17	0.44	0.89	1.54
Deflection from truss chord $= dv_t$	0.09	0.17	0.25	0.33	0.11	0.22	0.34	0.45
Deflection of stay system $= dv_s + dv_t$	0.27	0.55	0.97	1.51	0.28	0.66	1.23	1.99
Ratio of cable system to stay system	1.89	1.67	1.29	0.99	2.32	1.77	1.34	0.98

The last line in this table gives the ratios of the deflections of the two systems *for the assumed unit stresses.* But since these deflections must be equal in all cases, we may reason back to the actual stresses and conclude that *the last line in this table shows also the ratio of actual to the assumed stresses in the stays, for the stresses in the cable system remaining constant.* That is, if the change in unit stress in the cable system for live load is to be 24,000 lbs. per square inch, then the stay attached $\frac{1}{16}l$ from the pier will be stressed to 2.0 × 40,000 lbs. per square inch, or 80,000 lbs. per square inch; the one attached $\frac{1}{8}l$ from the pier will be stressed to 1.7 × 40,000 lbs. = 68,000 lbs. per square inch; the one attached $\frac{3}{16}l$ out will be stressed to 1.3 × 40,000 = 52,000 lbs. per square inch; while the one attached $\frac{1}{4}l$ from the pier will be stressed to its assumed amount.

*These results are wholly independent of the absolute or relative sizes of the stays and cable, and of the loads to be carried.** Therefore if these ratios of variation of working stresses can be allowed, the stays may be made of any desired size, independent of all other dimensions. The assumed variation of unit stress in the cable of 24,000 lbs. per square inch due to live-load only is very large. If this be reduced, the stresses in the stays will be reduced accordingly. Since the stays are no part of the main system, it is thought the stresses in those stays next to the piers may very well be about twice the total working stress in the main cable. When the stays are attached farther out from the piers than $\frac{1}{4}$ of the span, they will of necessity have a less unit stress in them than obtains for the cable, whatever their size.

168. Action of the Stays under Partial Load.—If the live load cover only one half the span, for instance, then the truss is supposed to distribute this load evenly upon the cable, and is therefore deflected upward somewhat on the unloaded end. This causes the stays to be slack at that end, and hence they cannot exert the necessary horizontal pull upon the truss to balance that of the stays at the loaded end. In this case the stays act exactly like the sus-

* This is very nearly true for all practicable sizes of stays.

pension members in a cantilever bridge, and the horizontal components of the stresses in them must be resisted by a compressive stress in the bottom chord of the truss back to the pier at the loaded end. To prevent the truss from sliding back over the pier it should have an abutting resistance so arranged as to allow of the necessary expansion but no more. It can then come to a solid bearing and the stays can come into action. Since the cable system does not deflect at any point under a partial load as much as it does under a full load, the stays will be less strained under the load here taken than under the full loads already discussed, and hence this question needs no further consideration.

169. Stresses in Members when Stays are Used.—Since the maximum unit stresses in the stays are independent of their size, so long as they do not carry the whole live and dead load on the parts reached by them, they can be made of any desired size. If they are all of the same size, they carry diminishing loads, as their points of attachment are farther from the piers; or if they are intended to carry equal vertical loads then they must increase in size as the secants of their angles with the vertical.

Let us suppose they are intended to carry one half the live load out to the quarter points. Then if $p =$ total live load per lineal foot of span,

$w =$ " dead " " " " " "
$d =$ spacing between hangers,
$a =$ " " stay attachments,
$P =$ vertical load on one stay,
$j =$ angle stay makes with the vertical,

we have $P = \frac{pa}{4}$ and

$$Stress\ in\ Stay = \frac{pa}{4} \sec j. \quad \quad (35)$$

Since the total load carried by the stays is supposed to be $\frac{p}{2} \cdot \frac{l}{2} = \frac{pl}{4}$, the maximum load on each cable system is $\frac{1}{2}(w + \frac{3}{4}p)l$. Hence the

$$Maximum\ Load\ on\ each\ Hanger = \frac{d}{2}\left(w + \frac{3}{4}p\right). \quad \quad (36)$$

Also, from eq. (5),

$$Maximum\ Stress\ in\ Cable = \frac{l}{4}\left(w + \frac{3}{4}p\right)\sqrt{1 + \left(\frac{l}{4v}\right)^2}. \quad \quad (37)$$

The total tensile stress in the lower chord of the truss for a full load should also be computed. For this loading there is considerable bending moment in the truss, at the centre, when stays are employed. If the stays carry one half the live load on the end portions and reach to the quarter points, there would then be a uniform upward pull on each truss from the cable of $\frac{3}{8}p$ per lineal foot; a uniform downward load on the end sections of $\frac{1}{4}p$ per lineal foot, and a uniform load on the middle portion of $\frac{1}{2}p$ per lineal foot, with end reactions of zero, under our assumptions. These loads would develop a bending moment in the truss at the centre of the span of $\frac{pl^2}{128}$. When the stays are attached to the lower chord the sum of the horizontal components of the stresses in all of them is carried by this chord. The horizontal component of the stress in any one stay which carries a vertical load of $\frac{pa}{4}$ is $\frac{pa}{4} \tan j$. Hence we have

$$Total\ Tension\ in\ Bottom\ Chord = \Sigma\frac{pa}{4} \tan j + \frac{pl^2}{128h}. \quad \quad (38)$$

The total load in each pier is the sum of all the vertical components in the cables and stays leading to it from both sides. If this reaction is vertical, which it always should be, then the horizontal components on the two sides are equal.

If the shore cables and stays are all symmetrical with those on the suspension side (which is likely to be true of the cable but not of the stays), then the vertical components on the two sides are also equal. Now the vertical components on the span side are equal to the load carried; hence we have, for symmetrical arrangement on span and shore sides of pier,

$$\textit{Total Load on Pier} \qquad = 2(p + w)\frac{l}{2} = (p + w)l; \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

and if each pier is composed of two towers, then

$$\textit{Total Load on One Tower} = \tfrac{1}{2}(p + w)l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

170. The Direction and Amount of Pull on the Anchorage.—The horizontal component of the pull on the anchorage is equal to that at the centre of the span, as given in eq. (2). The vertical component is equal to the horizontal component into tan i, where i is the angle the cable makes at the anchorage with the horizontal.

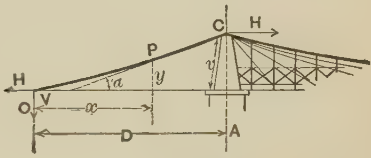


FIG. 219.

Referring to Fig. 219, and taking moments about C, we have

$$Hv - VD - \frac{sD^2}{2} = 0, \quad \text{or} \quad V = H\frac{v}{D} - \frac{sD}{2}, \quad . \quad . \quad . \quad (41)$$

where s is the load per foot horizontal coming upon the cable on the shore side, including its own weight (and which may be simply its own weight); H is the horizontal component of the pull in the cable, which is constant from one anchorage to the other under vertical loads; V is the vertical component of the stress in the cable at the point O, which will be taken as the origin of co-ordinates, at a distance D from the pier, and at a height v below the top of the tower.

Taking moments about any point in the cable, as P, remembering that there is never any moment in the cable itself, we have

$$Hy - Vx - \frac{sx^2}{2} = 0, \quad \text{or} \quad V = H\frac{y}{x} - \frac{sx}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Equating (41) and (42) we find the equation of the curve of the cable on the shore side to be

$$y = \frac{s}{2H} \cdot x^2 - \left(\frac{sD}{2H} - \frac{v}{D}\right)x. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

To find the angle it forms with the horizontal at any point we have

$$\frac{dy}{dx} = \tan i = \frac{s}{H}x - \left(\frac{sD}{2H} - \frac{v}{D}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Hence, for $x = 0$, or at the anchorage, distant D from the pier, we have

$$\tan i_0 = \frac{v}{D} - \frac{sD}{2H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Now $\frac{v}{D}$ is the tangent of the angle a straight line through O and C makes with the horizontal, and when the load on the cable is small on the shore side, or when sD is very small as compared with $2H$, then the cable will deviate very little from a straight line on the shore side of the pier.

The vertical pull on the anchorage is, therefore,

$$H \tan i_0 = \frac{Hv}{D} - \frac{sD}{2}. \quad (46)$$

The angle of the cable on the anchorage side at the tower is found from eq. (44) by making $x = D$, whence

$$tan\ angle\ i\ at\ tower = \frac{sD}{2H} + \frac{v}{D}. \quad (47)$$

This angle should be such as to produce a vertical reaction in the tower. To accomplish this it may be necessary to transfer some horizontal components of stress from stays to cable on the saddle, which may be done through their frictional resistance to sliding without any special means of attachment.

NOTE.—While the above analysis shows that composite structures may be computed and stresses found on any given assumptions, it must be borne in mind that a structure like a wire cable suspension-bridge does not admit of very nice adjustment of initial tension amongst its members, or of very rigid joint connections. Therefore, even though the engineer succeeds in obtaining the proper distribution of loads on the completion of the structure, it is not likely to hold this adjustment any great length of time. Hence it is common practice to dimension the members on the assumption that it is to act as a simple structure, and then the superfluous systems serve as so much additional factor of safety.

CHAPTER XII.

SWING BRIDGES.

171. General Formulæ.—A swing bridge when closed is ordinarily a continuous girder of two or more spans. If the ends of the arms are almost touching their supports, without producing any end reactions from dead load, the span is balanced over the centre, and the continuity of the two arms makes the bridge simply two cantilevers balancing each other. When the live load comes on one arm, that arm is immediately deflected until it finds an end reaction. The unloaded arm rises and still has no end reaction. If the live load covers both arms and is symmetrical about the centre, there may still be no end reactions. The bridge then becomes a tipper, and the condition which then obtains is the same as in a locomotive turn-table. This condition, however, will not be discussed here, as it does not properly come under the head of swing bridges.

In order to simplify the problem, and at the same time make it general in its application, let us assume the bridge closed, with the ends raised by raising their supports; then the reactions become functions of the elasticity. If we assume the supports as unyielding, the distortions of the bridge proper need only be considered. In what follows, the usual assumption of constant moment of inertia is made. This assumption, although not true, is an error on the safe side, and is the only one practicable in computing the stresses.

For a beam continuous over three or more supports, the "Theorem of Three Moments" applies (eqs. 14 and 14*a*, p. 137), and will enable us to find the moments at the supports; from these the reactions, and finally the stresses. Since all loads are given over to the trusses as panel concentrations, the equation (14*a*) for concentrated loads will be used. This equation is

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -\Sigma P_{r-1}l^2_{r-1}(k - k^3) - \Sigma P_r l^2_r(2k - 3k^2 + k^3), \quad (1)$$

in which M_{r-1}, M_r, and M_{r+1} are the moments at the $(r-1)$th, rth, and $(r+1)$th supports, respectively, beginning on the left hand; l_{r-1} and l_r are the lengths of the $(r-1)$th and rth spans; $Pr - 1$ and Pr are any concentrated loads on these spans; and $kl = a$ is the distance from the load to the support on the left.

172. For a Continuous Girder of Two Spans.—Fig. 220, M_1 and M_3 are both zero. Making $r = 2$ in eq. (1), and assuming load P_1 on one arm only, we have

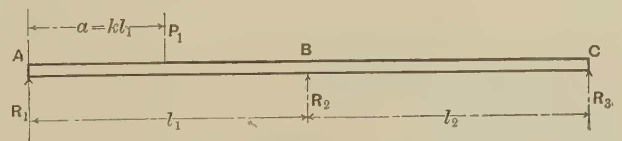


FIG. 220.

$$2M_2(l_1 + l_2) = -P_1l^2_1(k - k^3).$$

$$\therefore M_2 = -\frac{P_1l_1^2}{2(l_1 + l_2)}(k - k^3). \quad \ldots \ldots \ldots \quad (2)$$

Passing a section at B, and equating the moments of the forces to the left with B as a centre, we have also

$$M_2 = R_1 \times l_1 - P_1(l_1 - a); \therefore R_1 = \frac{M_2 + P_1(l_1 - a)}{l_1}.$$

Substituting value of M_2 from eq. (2), we have

$$R_1 = -\frac{P_1 l_1}{2(l_1 + l_2)}(k - k^3) + P_1 - P_1 \frac{a}{l_1}.$$

Let $l_2 = nl_1$; then

$$R_1 = \frac{P_1}{2(1+n)}\{2(1+n) - k(1+2+2n) + k^3\}. \quad \ldots \ldots \quad (3)$$

Similarly, we find

$$R_3 = \frac{P_1}{2(1+n)n}\{-k + k^3\}, \quad \ldots \ldots \quad (4)$$

and from Σ vert. comp. $= 0$ we have

$$R_2 = P_1 - R_1 - R_3;$$

then substituting values for R_1 and R_3, we have

$$R_2 = \frac{P_1}{2n}\{k(1+2n) - k^3\}. \quad \ldots \ldots \quad (5)$$

For loads in span l_2 make $l_1 = nl_2$ and make $a = kl_2 =$ distance from right-hand support. Then R_1 becomes R_3 and R_3 becomes R_1.

The above eqs. (3), (4), and (5) are all that are necessary in computing the reactions for any continuous girder over three supports with unequal spans.

For Equal Spans, $l_1 = l_2 = l$; $\therefore n = 1$.

$$R_1 = \frac{P}{4}\{4 - 5k + k^3\}; \quad \ldots \ldots \quad (6)$$

$$R_2 = \frac{P}{2}\{3k - k^3\}; \quad \ldots \ldots \quad (7)$$

$$R_3 = \frac{P}{4}\{-k + k^3\}. \quad \ldots \ldots \quad (8)$$

The above eqs. (6), (7), and (8) are all that are necessary in computing the reactions for any continuous girder over three supports with equal spans. In any case, it is not necessary to find the pier moments M_2 directly. Having once the reactions, the stresses in the members are easily found by the principles of statics.

For a Continuous Girder over Four Supports (Fig. 221), a condition which frequently obtains in swing bridges, with mid-span l_2 unloaded, we have from eq. (1), making $r = 2$ and making $l_1 = l_3 = l$,

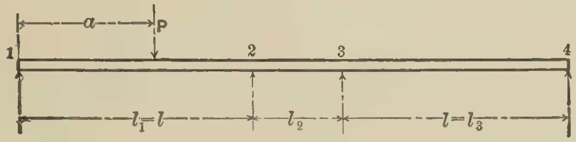


FIG. 221.

$$2M_2(l + l_2) + M_3 l_2 = -Pl^2(k - k^3); \quad \dots\dots\dots \quad (9)$$

and making $r = 3$,

$$M_2 l_2 + 2M_3(l_2 + l) = -P_3 l^2(2k - 3k^2 + k^3). \quad \dots\dots \quad (9a)$$

For convenience make $l_2 = \frac{l}{n}$; $\therefore n = \frac{l}{l_2}$; then $k = \frac{a}{nl_2}$. Make $3 + 8n + 4n^2 = H$; then from eqs. (9), (9a) we can obtain the reactions similarly as for a beam continuous over three supports.

The various steps in deducing the following equations will not be given. We then have

$$R_1 = \frac{P}{H}\{H - (H + 2n + 2n^2)k + (2n + 2n^2)k^3\}; \quad \dots\dots \quad (10)$$

$$R_2 = \frac{P}{H}\{(3 + 10n + 9n^2 + 2n^3)k - (2n + 5n^2 + 2n^3)k^3\}; \quad \dots\dots \quad (11)$$

$$R_3 = \frac{P}{H}\{-(n + 3n^2 + 2n^3)k + (n + 3n^2 + 2n^3)k^3\}; \quad \dots\dots \quad (12)$$

$$R_4 = \frac{P}{H}\{nk - nk^3\}. \quad \dots\dots\dots\dots \quad (13)$$

For loads in span l_3, $a =$ distance from right-hand support. Then R_1 becomes R_4, R_2 becomes R_3, R_3 becomes R_2, and R_4 becomes R_1.

The above equations (10), (11), (12), and (13) are all that are necessary in computing the reactions for any continuous girder over four supports, with equal end-spans, and mid-span unloaded,—conditions usually obtaining in a swing bridge.

When the reactions R_2 or R_3 become minus, eqs. (11) or (12), and the girder is not held down sufficiently by its own weight or otherwise, the condition of the beam is at once changed to that of a beam continuous over three supports. As swing bridges are ordinarily built, it is impracticable to hold down the centre supports. This subject will be taken up again further on in discussing the various forms of swing bridges in common use.

The foregoing equations are all that are needed in computing the stresses for any swing bridge.

CONSTANTS FOR REACTIONS, $P = 1000$ POUNDS, FOR BEAM CONTINUOUS OVER THREE SUPPORTS, WITH TWO EQUAL SPANS.

(For loads on the left span only.)

No. of Equal Panels in each Span.	Values for $k = \frac{a}{l}$.	R_1 +	R_2 +	R_3 −
2	1 ÷ 2	+ 406.25	+ 687.50	− 93.75
3	1 ÷ 3	592.6	481.5	74.1
	2 "	240.7	851.9	92.6
		$\Sigma R_1 = +$ 833.3	$\Sigma R_2 = +$ 1333.4	$\Sigma R_3 = -$ 166.7
4	1 ÷ 4	691.4	367.2	58.6
	2 "	406.3	687.5	93.8
	3 "	168.0	914.0	82.0
		$\Sigma R_1 = +$ 1265.7	$\Sigma R_2 = +$ 1968.7	$\Sigma R_3 = -$ 234.4
5	1 ÷ 5	752.0	296.0	48.0
	2 "	516.0	568.0	84.0
	3 "	304.0	792.0	96.0
	4 "	128.0	944.0	72.0
		$\Sigma R_1 = +$ 1700.0	$\Sigma R_2 = +$ 2600.0	$\Sigma R_3 = -$ 300.0
6	1 ÷ 6	792.8	247.7	40.5
	2 "	592.6	481.5	74.1
	3 "	406.25	687.5	93.75
	4 "	240.75	851.85	92.60
	5 "	103.0	960.7	63.70
		$\Sigma R_1 = +$ 2135.4	$\Sigma R_2 = +$ 3229.25	$\Sigma R_3 = -$ 364.65
7	1 ÷ 7	822.2	212.8	35.0
	2 "	648.7	416.9	65.6
	3 "	484.0	603.5	87.5
	4 "	332.4	763.8	96.2
	5 "	198.2	889.3	87.5
	6 "	86.0	970.9	56.9
		$\Sigma R_1 = +$ 2571.5	$\Sigma R_2 = +$ 3857.2	$R\Sigma_3 = -$ 428.7
8	1 ÷ 8	844.3	186.5	30.5
	2 "	691.4	367.2	58.6
	3 "	544.5	536.1	80.6
	4 "	406.3	687.5	93.8
	5 "	279.8	815.4	95.2
	6 "	168.0	914.0	82.0
	7 "	73.7	977.6	51.3
		$\Sigma R_1 = +$ 3008.0	$\Sigma R_2 = +$ 4484.3	$\Sigma R_3 = -$ 492.3
9	1 ÷ 9	861.5	166.0	27.5
	2 "	725.0	327.8	52.8
	3 "	592.6	481.5	74.1
	4 "	466.4	622.8	89.2
	5 "	348.4	747.6	96.0
	6 "	240.7	851.9	92.6
	7 "	145.4	931.4	76.8
	8 "	64.5	982.2	46.7
		$\Sigma R_1 = +$ 3444.5	$\Sigma R_2 = +$ 5111.2	$\Sigma R_3 = -$ 555.7
10	1 ÷ 10	875.25	149.5	24.75
	2 "	752.0	296.0	48.0
	3 "	631.75	436.50	68.25
	4 "	516.0	568.0	84.0
	5 "	406.25	687.5	93.75
	6 "	304.0	792.0	96.0
	7 "	210.75	878.5	89.25
	8 "	128.0	944.0	72.0
	9 "	57.25	985.5	42.75
		$\Sigma R_1 = +$ 3881.25	$\Sigma R_2 = +$ 5737.5	$\Sigma R_3 = -$ 618.75

Check, for any value of k, $(R_1 + R_2 + R_3) = P = 1000$; also, in any case $\Sigma R_1 + \Sigma R_2 + \Sigma R_3 = \Sigma P = \Sigma 1000$.

173. In Computing the Various Reactions for each truss panel point, the work will be very much simplified by first computing the reactions for an assumed load of, say, 1000 pounds at each point successively. This will give constants for reactions for each panel point, which can then be multiplied by the ratio of the true load to 1000 pounds to obtain the true reaction.

Again, as the panels in each arm are usually of the same length, $k = \frac{a}{l}$ can be expressed by the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.

The table on the opposite page gives the constants for reactions for a beam continuous over three supports of two equal spans, from eqs. (6), (7), and (8) for values of $k \times l$ from 1 to 9 inclusive.

The dead load for railway swing bridges, in general, may be obtained very closely by finding first the weight of a fixed span of the same length, and from this deduct the weight of the turn-table to be used.

For single-track bridges, where the live load is nearly 3000 lbs. per lineal foot.

Total weight of metal $= 5l^2 + 350l$;
" " " turn-table (generally) $= l \times 400$;
" " " metal above turn-table $= 5l^2 - 50l$.

The weight per foot above turn-table may be taken as $w = 5l - 50$, where l here is the total length of the two spans of the swing bridge. To the above weight add 400 lbs. per foot for weight of track.

For double-track bridges add from 70 to 90 per cent to the above weights. The percentage to be added varies indirectly with the length of the span. The live load for highway bridges is the same as given in Chap. IV.

The live load for railway bridges can be assumed as a uniform train load with one or two engine excesses; unless a train of engines is specified, when an equated uniform live load can be used. The former is the more common, and will be used in all the subsequent computations. The excess used in each case is the difference between the maximum panel concentration from the engine and the uniform train load. Where two engines are specified, it is customary to assume the excesses to be equal and placed at the nearest panel points.

174. Centre-bearing Pivot; Three Supports.* Figs. 220 and 222.—In this, the entire weight of the bridge when open is carried by the cross-beam ee' to the pivot P. When closed, the ends of the bridge are raised at a, a', i, and i'. The bridge is thus a continuous girder of two spans for dead load, and for live load so long as the end reactions are positive. The deflection of the ends under dead load is very accurately computed after the bridge is designed, by the method explained in Chap. XV. After raising the ends, wedges are inserted at e and e' and brought to a firm bearing, thus relieving the pivot P from all live load except the panel load at ee'. The ends are not latched down, hence there can be no downward or negative reaction.

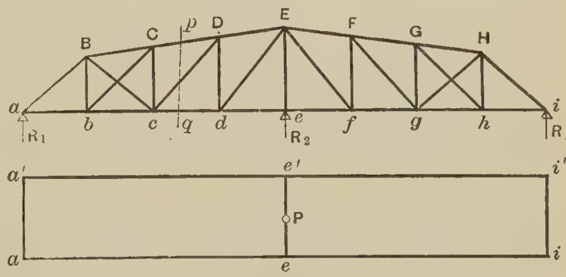


FIG. 222.

To compensate for lack of proper adjustments, and also as unequal chord temperatures affect the deflection at the ends, two assumptions will be made in computing the dead load stresses: 1st, the ends just touching with no positive reactions; and 2d, the end reactions equal to those of a continuous girder over three supports. It is safe to assume that practically

* For this case treated with moment of inertia of the truss variable see Art. 178a, p. 196.

the end reactions will vary between the limits of these assumptions. In fact, it will be shown that in a properly designed bridge the ends should at all times, when the bridge is closed, be raised so that the end reactions will be at least a mean between those existing when there are no positive reactions, and when the end reactions are equal to those of a continuous girder over three supports. The analysis will then consider the following cases:

Case I. Bridge swinging, dead load only acting.

Case II. Bridge closed, ends raised, dead load only acting, continuous over three supports.

Case III. Live load on one arm only, for maximum tension in lower chord and maximum compression in upper chord, also maximum web stresses from end towards centre.

Case IV. Live load on both arms. With a uniform live load on one arm, a train comes on the other arm and advances until the whole bridge is covered, giving maximum tension in the upper chord and maximum compression in the lower chord, also maximum web stresses from the centre to the end.

The stresses to be used will be the largest of each kind obtained by combining Cases III and IV with Cases I or II.

CASE I.—In this case, the two arms are simply cantilevers balancing each other over the centre; and stresses are easily determined by diagram, or otherwise, beginning at the end of the bridge where the only external force is the load at that point.

CASE II.—In this case, the bridge acts as a continuous girder over three supports. As the two arms are equal, the constants for reactions given in the table, Art. 172, can be used. Let W denote the dead load per truss panel $= \frac{w}{2} \times$ panel length; then, as the number of panels $= 4$,

$$R_1 = (1265.7 - 234.4) \times \frac{W}{1000} = \frac{W}{1000} \times 1031.3$$

$$R_2 = (1968.7 + 1968.7) \times \quad " \quad = \quad " \quad \times 3937.4$$

$$R_3 = R_1 \qquad\qquad = \quad " \quad \times 1031.3$$

$$\text{Check: } (R_1 + R_2 + R_3) = \Sigma W = 6W.$$

After finding R_1, the stresses are easily found by diagram or analytically.

CASE III.—In this case, we wish first to find the maximum tension in the lower chord and the maximum compression in the upper chord.

* Before proceeding further it will be necessary to investigate the law of the variation of the moment in a two-span continuous girder due to a load at any point. Let ABC, Fig. 223, represent such a girder. By eq. (6) the reaction at A due to a load P_1 in the first span is

FIG. 223.

$$R_1 = \frac{P_1}{4}\{4 - 5k + k^3\},$$

* The following method of finding the position of live load for maximum chord stresses, and the diagram, Fig. 224, are from a paper by Prof. M. A. Howe on "Maximum Stresses in Draw Bridges," published in the *Journal of the Association of Engineering Societies*, July, 1892.

a positive quantity for all values of k from 0 to 1. The moment at any point N to the left of P_1, at a distance x from A, is equal to $R_1 \times x$, a positive moment. For a point N to the right of P_1 the moment is

$$M = R_1 \times x - P_1 \times (x - kl)$$

$$= P_1 k\left(l - x\frac{5 - k^2}{4}\right). \quad \dots \dots \dots \dots \quad (14)$$

This moment is zero when

$$l = x\frac{5 - k^2}{4};$$

or, if x_0 is that value of x for which the moment is zero, we have

$$\frac{x_0}{l} = \frac{4}{5 - k^2}. \quad \dots \dots \dots \dots \quad (15)$$

To the left of this point of zero moment the moment is positive, and to the right it is negative. For loads in the second span, R_1 is negative; hence the corresponding moment in the first span is at all points negative.

The value of $\frac{x_0}{l}$ in eq. (15) varies from 0.8, for $k = 0$, to 1 for $k = 1$. and hence all loads in the first span cause positive moments at all points to the left of the point where $\frac{x}{l} = 0.8$. The curve of eq. (15) is given in Fig. 224, the scale for values of k and $\frac{x_0}{l}$ being the same as for k and R_1 in the curves for R_1.

The diagram, Fig. 224, contains the plotted curves of eqs. (6) and (8), giving values of R_1 for values of k or k' between 0 and 1, and for a value of P_1 or P_2 equal to unity. The reaction, R_1, due to any panel load, whether from live or dead load, is then found by reading off the ordinate to the proper curve for the value of k or k' corresponding to the panel point in question, and then multiplying by the panel load. The total reaction due to any load is the sum of the partial reactions due to panel loads thus found.

It is seen from the diagram that R_1, for loads on the second span, is always negative, having a maximum negative value of .096 when $k' = .577$. Also, that for loads on the first span R_1 is always positive, varying in value from 1 to 0 as k varies from 0 to 1.

Maximum Positive Moment, or Maximum Compression in the Upper Chord and Maximum Tension in the Lower.—It has just been shown that any load in the second span causes negative moments at all points in the first, and that any load in the first span causes positive moments at all points in the first span to the left of the point where $\frac{x}{l} = 0.8$; hence, for a maximum positive moment at any point to the left of this point, the second span should contain no loads and the first span should be fully loaded.

For a centre of moments to the right of the point where $\frac{x}{l} = 0.8$, those joints in the first span should be loaded for which the point of zero moments is on the right of the centre of moments taken. As many different loadings are required as there are centres of moments in one fifth the span, usually not more than one. The point over the centre support is not used as a centre for positive moments, as the greatest positive moment there is zero.

It will, however, be shown that for all members to the right of the point where $\frac{x}{l} = 0.8$, we can for all practical purposes assume the first span fully loaded; then for a uniform live load one position is all that is necessary to find the maximum compression in the upper chord and maximum tension in the lower.

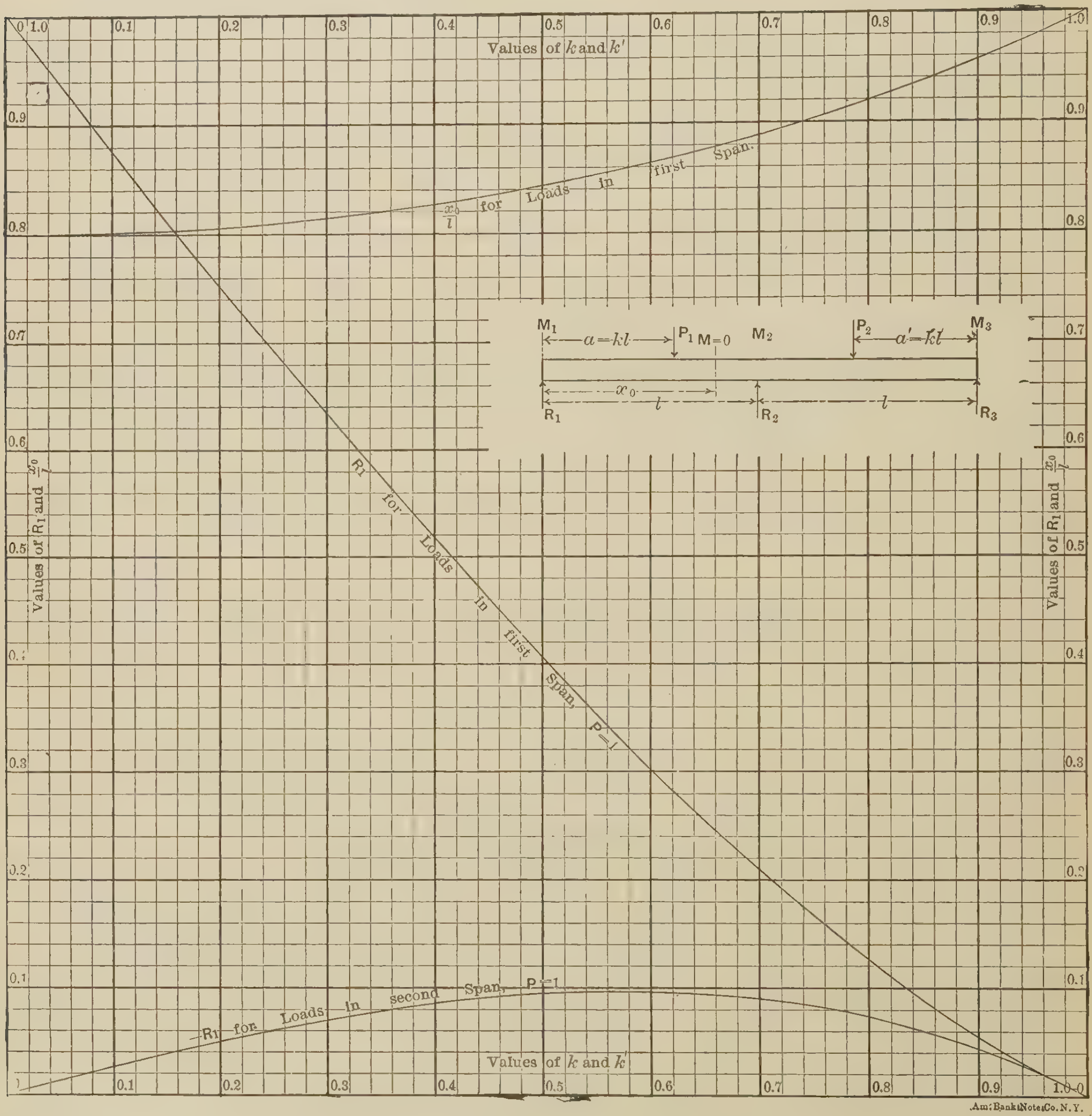


FIG. 224.

The position of the loads having been found in any case, the reaction R_1 is obtained as previously explained, and thence the stresses in the members.

We next want to find the maximum web stresses from the end towards the centre.

Maximum Positive Shear, or Maximum Tension in those Diagonals Inclining Downward toward the Right.—The positive shear in any panel is equal to R_1, minus the loads between the left support and the panel in question. From this it follows, for a maximum positive

shear in any panel, that there should be no loads in the second span, for any load in the second span causes a negative reaction at the left; that all joints in the first span to the right of the panel in question should be loaded, for any load in the first span causes a positive reaction at the left; and that no joints to the left of the panel should be loaded, for any load in the first span causes a less reaction than the load itself.

The proper position of the loads and the corresponding value of R_1 having been found for any member, the stress in the member is perhaps best found by subtracting from or adding to the shear the vertical component of the stress in the upper chord member cut, thus getting the vertical component of the web stress.

The stresses in the verticals corresponding to the stresses in the above system of diagonals should also be found. The loading giving the maximum stress in any diagonal gives also the maximum stress in the vertical meeting the diagonal at the upper chord point.

CASE IV.—In the previous case, the live load was confined to one span. We will now assume two trains on the bridge. One train, which covers only the second span, is a uniform live load. The other train headed by engines is just coming on the first span and advances until the entire bridge is covered.

Maximum Negative Shear, or Maximum Tension in those Diagonals Inclining Downward toward the Left.—The negative shear is equal to the loads to the left of the panel in question, minus the reaction, R_1. For reasons similar to those given in the preceding case, the conditions for a maximum negative shear in any panel are: the second span should be fully loaded, and the first span should be loaded from the left end to the panel in question. The corresponding maximum stresses in the verticals should be found in this case also; and where the same one is treated in this and the preceding case, the greater of the two stresses is to be taken. The actual stress in any web member is found as in the previous case.

Maximum Negative Moment, or Maximum Tension in the Upper Chord and Compression in the Lower Chord.—The second span should be fully loaded, for all centres of moments in the first span. For centres of moments to the left of the point where $\frac{x}{l} = 0.8$, no loads should be in the first span; and for centres of moments to the right of this point, such joints should be loaded for which the point of zero moment is on the left of the centre of moments taken. These positions of loads follow from the reasons given in the previous case.

As previously stated, combine Cases III and IV with Case I or II to obtain the maximum tension or compression in any member. It might here be argued, that when the live load covers the first span only, the second span may rise until the right end is lifted from its support. This happens frequently in swing bridges where the ends are not sufficiently lifted, and the conditions which then obtain are such that the first span is treated as an independent span for live load stresses, and the dead load stresses are the same as for Case I. In no case, however, does this combination give any greater stresses than those obtained by combining Cases III and IV with Case I or II.

NUMERICAL EXAMPLE.*

Let us take a bridge of twelve panels, Fig. 225, with $d = 20$ ft.; $l = 120$ ft.; $bB = 25$ ft., and $gG = 35$ ft.; length of upper chord member $= \sqrt{(20)^2 + (2)^2} = 20.1$ ft. The lengths of the diagonals are as follows:

$$aB = Bc = 32.0 \qquad dE = 36.9$$
$$bC = Cd = 33.6 \qquad eF = 38.6$$
$$cD = 35.2 \qquad fG = 40.3$$

The dead load above turn-table or weight per lineal foot $w = 5l - 50 + 400 = 1550$ lbs. Dead load per truss panel $= \frac{1550}{2} \times 20 = 15{,}500$ lbs. The live load will be taken at 3000 lbs. per lineal foot, headed by two engine excesses of 20,000 lbs. each placed two panels apart.

* For the solution of this case with moment of inertia taken as variable see p. 196.

Live load per truss panel = 30,000 lbs.; live load excess per truss panel = 10,000 lbs.

CASE I. *Bridge swinging, dead load only acting.*—Taking two-thirds of a panel dead load as applied at the lower chord points and one-third at the upper, the joint loads for *b, c, d, e,* and *f* are each equal to 10,300 lbs., and for *B, C, D, E,* and *F* are each equal to 5200 lbs. The load at *a* will be taken at one half of a panel load. (The joint load *a* may be considerably more than this in some cases, owing to the weight of

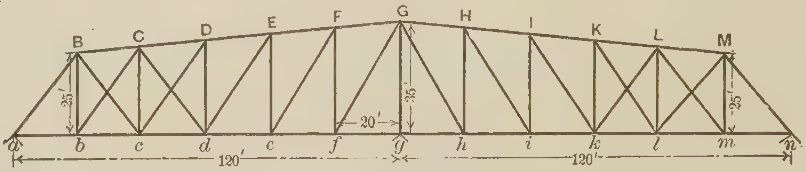


FIG. 225.

locking gear, etc.) The stress diagram, Fig. 226, is then constructed by drawing first the diagram for joint *a*, where the only external force acting is the half panel load, then passing to *B, b, C, c,* etc. The diagonals *Bc* and *Cd* are not in action in this case, and so are omitted. For a check, the stress in *fg* is by moments equal to $[7750 \times 120 + 15{,}500 (1 + 2 + 3 + 4 + 5) \times 20] \div 35 = 160{,}000$ lbs. compression.

The dead load stresses, as scaled from the diagram, Fig. 226, are given in the second column of the table of stresses.

CASE II. *Bridge closed, dead load only acting, continuous over three supports.*—In this case, the loads at *a* and *n* need not be considered, as they are carried directly to the supports. The other joints are loaded as in the previous case. Using the constants for reactions, Art. 173, we have for this span $R_1 = (2135.4 - 364.65) \times 15.5 =$ say $+ 27{,}500$ lbs. With this value of R_1 and with the joint loads distributed as in the previous case, the diagram, Fig. 227, is drawn. For a check, the stress in *fg* may be found by moments. Thus,

$$fg = \{27{,}500 \times 120 - 15{,}500 \times (1 + 2 + 3 + 4 + 5) \times 20\} \div 35 = 38{,}600 \text{ lbs.}$$

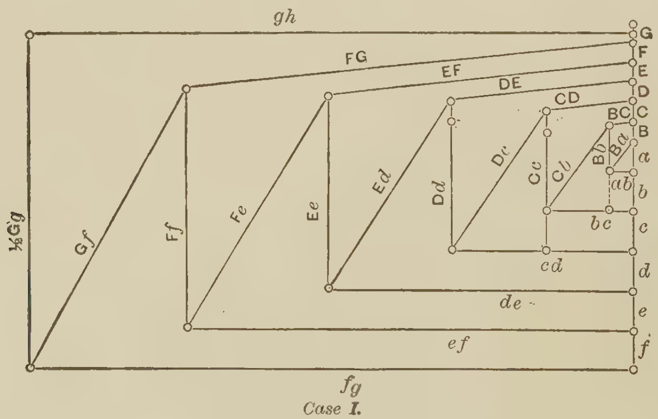


FIG. 226.

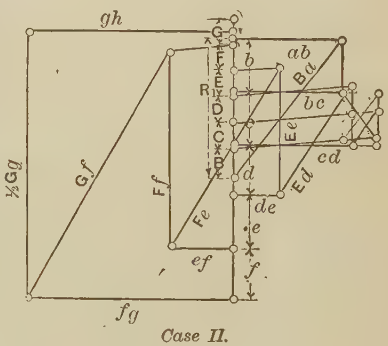


FIG. 227.

The stresses, as scaled from the diagram, Fig. 227, are given in the third column of the table of stresses.

CASE III. *Live load on one arm only, for maximum compression in upper chord and maximum tension in lower chord; also maximum web stresses from end toward centre.*

Maximum Chord Stresses. (Case III.)—First position of live load: engines headed toward left; 30,000 lbs. at *b, c, d, e,* and *f,* with engine excess 10,000 lbs. at *b* and *d*. Using the constants for reactions, Art. 173, we have

$$R_1 = 2135.4 \times 30 + (792.8 + 406.25) \times 10 = \text{say} + 76{,}100 \text{ lbs.}$$

The chord stresses are now readily found by moments; thus,

$$ab = 76{,}100 \times 20 \div 25 = \text{say } 60{,}880 \text{ lbs. tension.}$$

In the panel *bc*, the diagonal *Bc* is in action, since both live and dead load shears in this panel are largely positive. Hence the stress in *bc* is equal to that in $ab = 60{,}880$ lbs. tension.

The centre of moments for BC is at c.

$$BC = \{76{,}100 \times 2 - 40{,}000 \times 1\} \times \frac{20}{27} \times \frac{20.1}{20} = 83{,}500 \text{ lbs.}$$

In the panel cd we will assume cD as acting, then find its stress, and if the result is a tensile stress our assumption is correct; but if not, then Cd must be in action. If cD is in tension, then the chord stress CD must be greater than DE. If CD is less than or equal to DE, then Cd must be in action.

Assuming cD in action, then

$$CD = BC = \{76{,}100 \times 2 - 40{,}000 \times 1\} \times \frac{20}{27} \times \frac{20.1}{20} = 83{,}500;$$

$$DE = \{76{,}100 \times 3 - 40{,}000 \times 2 - 30{,}000 \times 1\} \times \frac{20}{29} \times \frac{20.1}{20} = 82{,}040.$$

As CD is greater than DE, then cD must be acting. Also from Case II we see that cD is in tension for dead load; hence our assumption that cD is in action is correct. Therefore the stress in CD = stress in BC = 83,500 lbs. The centre of moments for DE and cd is at d.

$$cd = \{76{,}100 \times 3 - 40{,}000 \times 2 - 30{,}000 \times 1\} \times \frac{20}{29} = 81{,}630 \text{ lbs.};$$

$$DE = \{\text{"} \times 3 - \text{"} \times 2 - \text{"} \times 1\} \times \text{"} \times \frac{20.1}{20} = 82{,}040 \text{ lbs.}$$

The stresses in de and EF are found in like manner.

For the members ef and FG, the centre of moments is at f. For maximum positive moment at this point, those joints should be loaded for which the value of $\frac{x_0}{l}$ is greater than $\frac{5}{6}$. From the diagram, Fig. 224, we see that $\frac{x_0}{l} > \frac{5}{6}$ for values of k greater than 0.44; hence, for point f, the joints to the right of the point where $k = 0.44$ should be loaded. This includes joints d, e, and f. If the bridge under consideration had only five or less equal panels in each arm, this special position of the live load would not have been necessary; and it will be shown that for all practical purposes this disposition of the live load can be omitted in any case without altering the required sectional area in any member. This is especially true when the live load is headed by heavy engine excesses. However, for this span we will find the stresses for ef and FG when only d, e, and f are loaded, and compare them with the stresses obtained in these members from the second position of the live load. Loading d, e, and f with 30,000 lbs. at d, e, and f and 10,000 lbs. at d and f, we find

$$R_1 = \{(406.25 + 240.75 + 103) \times 30 + (406.25 + 103) \times 10\} = +\ 27{,}590 \text{ lbs.};$$

$$ef = \{27{,}590 \times 5 - (40{,}000 \times 2 + 30{,}000 \times 1)\} \times \frac{20}{33} = -\ 16{,}940 \text{ "}$$

$$FG = \{\text{"} \times 5 - (\text{"} \times 2 \times \text{"} \times 1)\} \times \text{"} \times \frac{20.1}{20} = +\ 17{,}020 \text{ "}$$

Second position of live load; engines headed toward right; 30,000 lbs. at b, c, d, e, and f with engine excess, 10,000 lbs. at d and f.

$$R_1 = 2135.4 \times 30 + (406.25 + 103) \times 10 = +\ 69{,}150 \text{ lbs.};$$

$$ef = \{69{,}150 \times 5 - (30{,}000 \times 10 + 10{,}000 \times 2)\} \times \frac{20}{33} = -\ 15{,}600 \text{ "}$$

$$FG = \{\text{"} \times 5 - (\text{"} \times 10 + 10{,}000 \times 2)\} \times \text{"} \times \frac{20.1}{20} = +\ 15{,}680 \text{ "}$$

Furthermore, these stresses when combined with the dead load stresses, Case II, are again reduced, leaving the combined stresses very small. For instance, ef becomes $-$ 15,600 + 12,700 = $-$ 2900 lbs.; and when joints d, e, and f only are loaded, ef becomes $-$ 16,940 + 12,700 = $-$ 4240 lbs.

Now, as a matter of fact, the compression in ef under Case IV when combined with Case I, really determines the sectional area of the member, and practically it makes no difference whether we take the tension in ef at 2900 lbs. or 4240 lbs.

Maximum Web Stresses. (Case III.)—Beginning at the left hand :—Maximum compression in aB. 30,000 lbs. at b, c, d, e, and f, with 10,000 lbs. excesses at b and d.

$$R_1 = 2135.4 \times 30 + (792.8 + 406.25) \times 10 = \text{say} + 76{,}100 \text{ lbs.};$$

$$aB = 76{,}100 \times \tfrac{32}{25} = + 97{,}400 \text{ lbs.};$$

$$Bb = \text{max. panel concentration} = - 40{,}000 \text{ lbs.}$$

Maximum tension in Bc. 30,000 lbs. at c, d, e, and f, with 10,000 lbs. excesses at c and e.

$$R_1 = \{(2135.4 - 792.8) \times 30 + (592.6 + 240.75) \times 10\} = + 48{,}610 \text{ lbs.}$$

R_1 is the shear in panel bc when b is not loaded. This shear minus the vert. comp. in BC is the vert. comp. in Bc.

$$Bc = \left\{48{,}610 - 48{,}610 \times 2 \times \frac{20.1}{27} \times \frac{2}{20.1}\right\} \times \frac{32}{25} = - 53{,}020 \text{ lbs.}$$

Maximum tension in Cd. For dead load cD has a tension of 9000 lbs., but the end lift may be excessive and so reduce the shear in this panel to zero for dead load, thus giving the maximum tension to Cd, as here found. Then with 30,000 lbs. at d, e, and f, and 10,000 lbs. at d and f,

$$R_1 = \{(406.25 + 240.75 + 103) \times 30 + (406.25 + 103) \times 10\} = + 27{,}590 \text{ lbs.}$$

Any stress in Cd must be accompanied by a corresponding difference between the chord stresses BC and CD, and their difference is a measure of the stress in Cd.

$$\text{Stress in } CD = (27{,}590 \times 3) \times \frac{20.1}{29};$$

$$\text{" \quad " } BC = (\text{ " } \times 2) \times \frac{20.1}{27};$$

$$\text{" \quad " } Cd = (CD - BC) \times \frac{33.6}{20.1}.$$

Therefore, we may write at once :

$$\text{Stress in } Cd = 27{,}590\left(\tfrac{3}{29} - \tfrac{2}{27}\right) \times 33.6 = - 27{,}790 \text{ lbs., and}$$

$$\text{" \quad " } Cc = \text{stress in } Cd \times \frac{29}{33.6}, \text{ or}$$

$$\text{" \quad " } Cc = 27{,}590\left(\tfrac{3}{29} - \tfrac{2}{27}\right) \times 29 = + 23{,}990 \text{ lbs.}$$

CASE IV. *Uniform live load covering the second span.*—A train comes on the first span and advances until the whole bridge is covered.

Maximum Web Stresses. (Case IV.)—Beginning at left hand :—Maximum tension in aB and maximum compression in bB. 40,000 lbs. at a, second span fully loaded.

It is here assumed that the load in the second span lifts the end at a, owing to temperature or lack of adjustment in raising the ends; then any load which comes on at a holds the end down.

$$R_1 \text{ for load in second span} = - 364.65 \times 30 = - 10{,}940 \text{ lbs.}$$

$$\text{Stress in } aB = 10{,}940 \times \tfrac{32}{25} = - 14{,}000 \text{ lbs.}$$

$$\text{" \quad " } bB = \text{ " } \times \tfrac{22}{25} = + 10{,}060 \text{ "}$$

Maximum tension in bC. 40,000 lbs. at b, second span fully loaded.

$$R_1 \text{ for load in second span} = \text{as before} - 10{,}940 \text{ lbs.};$$

$$R_1 \text{ " " at } b = 792.8 \times 40 = +31{,}710 \text{ lbs.};$$

$$\Sigma R_1 = +31{,}710 - 10{,}940 = +20{,}770 \text{ lbs.}$$

Shear in panel $bc = 20{,}770 - 40{,}000 = -19{,}230$ lbs.

This shear plus the vert. comp. in BC, is the vert. comp. in bC.

$$\text{Stress in } bC = \left\{ 19{,}230 + 20{,}770 \times \frac{20.1}{25} \times \frac{2}{20.1} \right\} \times \frac{33.6}{27} = -26{,}000 \text{ lbs.};$$

$$\text{" " } Cc = \text{stress in } bC \times \frac{25}{33.6}; \text{ therefore,}$$

$$\text{" " } Cc = \left\{ 19{,}230 + 20{,}770 \times \frac{20.1}{25} \times \frac{2}{20.1} \right\} \times \frac{25}{27} = +19{,}340 \text{ lbs.}$$

For the maximum tension in cD and maximum compression in Dd. 30,000 lbs. at b and 40,000 at c, second span fully loaded. In this manner we proceed until the centre is reached, the second span remaining fully loaded under all conditions.

Maximum Chord Stresses. (Case IV.)—As before, the second span is fully loaded with uniform live load, and the first span is unloaded for all centres of moments except at f and g. However, if we assume that the load in the second span lifts the end at a, as previously explained, then any load which comes on at a

CHORD STRESSES.

Member.	Dead Load.		Live Load.		Total.	
	Case I.	Case II.	Case III.	Case IV.	+	−
BC	− 6360	+ 29700	+ 83500	− 8800	113200	15160
CD	− 23300	+ 29700	+ 83500	− 16270	113200	39570
DE	− 49300	+ 24400	+ 82040	− 22750	106440	72050
EF	− 82700	+ 9500	+ 61190	− 28370	70690	111070
FG	− 121000	− 12700	+ 15680	− 34860	2980	155860
ab	+ 6360	− 22300	− 60880	+ 8750	15110	83180
bc	+ 23300	− 22300	− 60880	+ 16190	39490	83180
cd	+ 49300	− 24400	− 81630	+ 22640	71940	106030
de	+ 81900	− 9500	− 60890	+ 28230	110130	70390
ef	+ 120800	+ 12700	− 15600	+ 34690	155490	2900
fg	+ 160000	+ 38600	000	+ 80360	240360	000

WEB STRESSES.

Member.	Dead Load.		Live Load.		Total.	
	Case I.	Case II.	Case III.	Case IV.	+	−
aB	− 10000	+ 35500	+ 97400	− 14000	132900	24000
bC	− 28600	000	not max.	− 26600	000	54600
cD	− 46100	− 9000	" "	− 44480	000	90580
dE	− 61000	− 27300	" "	− 70140	000	131140
eF	− 74200	− 43200	" "	− 98020	000	172220
fG	− 86900	− 58300	" "	− 126500	000	213400
Bc	000	− 10700	− 53020	not max.	000	63720
Cd	000	000	− 27790	" "	000	27790
Bb	+ 12700	− 10600	− 40000	+ 10060	22760	50600
Cc	+ 27000	+ 5300	+ 23990	not max.	50990	000
Dd	+ 40500	+ 12700	not max.	+ 34110	74610	000
Ee	+ 53000	+ 25900	" "	+ 55100	108100	000
Ff	+ 65200	+ 40300	" "	+ 78750	143950	000
Gg	+ 180000	+ 108600	" "	+ 210250	390250	000

holds the end down. Then for all centres of moments in the first span, except at f and g, we will assume a load at a.

$$R_1 \text{ for load in second span} = \text{as before} - 10{,}940 \text{ lbs.}$$

The load at a is the max. panel concen. $= 40{,}000$ lbs. Therefore the end tends to lift by the amount of the negative reaction from load in the second span, or $R_1 = -10{,}940$ lbs. This value of R_1 is used for all centres of moments from b to e inclusive.

For centre of moments at f, those joints should be loaded for which the point of zero moment lies to the left of f, i.e., joints b and c. With 30,000 lbs. at b and 40,000 lbs. at c, we have

$$\begin{aligned} R_1 &= \{792.8 \times 30 + 592.6 \times 40\} &&= +47{,}490 \text{ lbs.;} \\ R_1 &\text{ from load in second span} &&= -10{,}940 \text{ “} \\ \Sigma R_1 & &&= +36{,}550 \text{ “} \end{aligned}$$

The stresses in ef and FG are then found with this value of $R_1 = +36{,}550$ lbs.

For centres of moments at g or G the entire bridge must be loaded with the engine excesses at d and f.

$$\begin{aligned} R_1 &\text{ from load in first span} &&= +69{,}150 \text{ lbs.;} \\ R_1 &\text{ “ “ “ second span} &&= -10{,}940 \text{ “} \\ \Sigma R_1 &= 69{,}150 - 10{,}940 &&= +58{,}210 \text{ “} \end{aligned}$$

$$\text{Stress in } fg = \{58{,}210 \times 6 - (30{,}000 \times 15 + 10000 \times 4)\} \times \tfrac{39}{35} = +80{,}220 \text{ lbs.}$$

The maximum stresses for each case are entered in the table of stresses. The total stresses are entered in the last two columns of the table.

In all of the foregoing analysis and computations, we have assumed two conditions which are extreme but not impossible. First, we have assumed two trains on the bridge at the same time—a condition which may or may not occur in the lifetime of a structure; and even then the positions necessary to give maximum stresses, as previously assumed, may not occur. Again, we have assumed the ends insufficiently raised, so as to permit the ends to “hammer,” as it is called. The latter condition obtains frequently in swing bridges where the proper lifting of the ends is neglected. In practice it is customary to assume only one train on the bridge. Then, in Case IV, we assume the train headed by engines coming on one end of the bridge and advancing until the whole bridge is covered. As the train comes on the first span and advances we then obtain a condition for maximum web stresses, or maximum negative shear at the head of the train. When the web members near the centre of the bridge are reached, we must then permit the train to advance until the whole bridge is covered to obtain maximum web stresses. This position of the live load will also give maximum chord stresses near the centre. We see, then, that for all practical purposes it is never necessary to find the distance $\frac{x_0}{l}$, or the point of zero moment for Cases III or IV.

It is also customary to assume that the ends of the bridge are properly raised so as to preclude the possibility of their “hammering,” even under extreme conditions of temperature when the train comes on the bridge. In any event, however, the conditions for dead load under Cases I and II must be assumed.

The cases to be considered should then be stated as follows:

Case I. Bridge swinging, dead load only acting.

Case II. Bridge closed, ends raised, dead load only acting, continuous over three supports.

Case III. Live load on one arm only, for maximum tension in lower chord and maximum compression in upper chord, also maximum web stresses from end towards centre.

Case IV. Live load on both arms, for maximum tension in upper chord and maximum compression in lower chord, also maximum web stresses from the centre towards the end.

Cases III and IV to be combined with Case I or II, as previously explained.

175. Rim-bearing Turn-table—Four Supports.*—Fig. 228.—The most common method of swing-bridge construction provides two supports at the centre, thus dividing the truss into three spans. The bracing of the short central span is usually arranged as in Fig. 228. It is thus capable of resisting shear, and the continuity of the truss is complete.

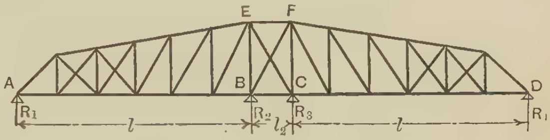


FIG. 228.

With some writers it has been customary to apply to this form of truss the formulæ for a continuous beam of four supports—formulæ based on the assumption of uniform moment of inertia and on a neglect of deflection due to shearing stresses. These formulæ give results closely approximate for beams and long-span trusses, and fairly good results for two-span swing-bridges; but their application to trusses of such short spans as are here considered leads to very erroneous conclusions. With only one span loaded large negative reactions are obtained at B or C, reactions much greater than ever really occur, and which greatly exceed the dead-load positive reactions. To furnish these negative reactions some form of anchorage would have to be provided at B and C, a thing quite impracticable in a swing-bridge.

The true stresses in the diagonals EC and FB, caused by partial loads, and their effect on reactions, may be estimated in the following manner: Assume first that these diagonals are removed and one span, as AB, loaded. The reactions and stresses are now to be computed by treating the bridge as a two-span bridge, or, more accurately, by the formula of Art. 176. Calculate then by the method explained in Chap. XV the deflection of the point *E in the direction CE*. Now if the diagonal CE had been in place its elongation could not have exceeded this movement of E. If the diagonal be small its elongation may be assumed equal to this deflection, and the stress per square inch computed which would result therefrom. If the diagonal be large it would have the effect of reducing somewhat the movement of E, and therefore would have a smaller stress per square inch than a small diagonal. To the total stress as determined above should be added the initial tension due to adjustment. The reaction at C will then be equal to the stress in FC, minus the vertical component of the diagonal stress. Computations for two structures gave in one case a stress of 800 lbs. per sq. in. in the diagonal, and in the other case 1800 lbs. per sq. in., the diagonals in both cases being very small. Increasing the area in the latter case to 12 sq. in., the size actually used, reduced the stress per square inch to 1000 lbs., thus giving a total stress of 12,000 lbs. The stress in FC was 18,000 lbs. and the resulting reaction therefore about 8000 lbs., positive, neglecting initial tension. The formulæ for a four-span continuous girder would give a negative reaction of about 200,000 lbs. and a diagonal stress somewhat greater. The diagonals were apparently designed for this stress.

The above discussion indicates that the effect of bracing in the centre span is small, and in the computations of stresses this bracing can be entirely neglected and the truss treated as as in Art. 176, or approximately as a two-span bridge. Since the purpose of this bracing is merely to stiffen the structure when open, small members should be used. Large ones cause a more unequal distribution of load on the rollers, and should be avoided.

176. Rim-bearing Turn-table; Four Supports; Equal Moments at the Centre Supports.—Fig. 229.—In this the rigid frame $EFBC$ supports the truss by means of the short links EG and FH. The portion BC of the frame is a part of the lower chord of the bridge; in other respects the frame may be considered a part of the pier. There being no diagonals in the panel $EFHG$, there can be no shear transmitted across this panel, and the moments at H and G must be equal. The structure may then be considered a three-span continuous girder, in which the moments at the two centre supports are always equal.

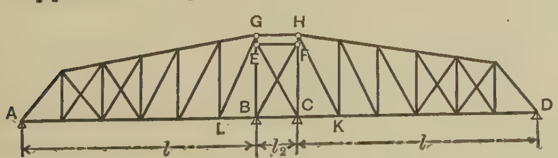


FIG. 229.

* This article changed in the Sixth Edition.

Now if the first and third spans be each loaded with a single load, P, placed symmetrically with reference to the centre, the moments at the second and third supports will be equal to those which would occur in an ordinary continuous girder of three spans, if loaded in the same way, for in the latter case M_2 and M_3 would be equal by symmetry and therefore the shear zero in the second span.

Also these moments M_2 and M_3 are now each produced equally by the two loads P_1 and P_3, hence each may be supposed to be produced wholly by one half the sum of the moment effects of the two loads taken separately. Thus, from eqs. (9) and (9*a*), p. 181, we have

$$M_2 = M_3 = -\frac{l^2}{4l+6l_2}[P_1(K-K^3)+P_3(2K-3K^2+K^3)]. \quad . \quad . \quad . \quad . \quad (17)$$

Now if the load P_3 be removed from the third span we shall have *

$$M_2 = -\frac{Pl^2}{4l_1+6l_2}(k-k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

We also have $M_2 = R_1 \times l - P(l - kl)$, and $M_3 = R_4 \times l = M_2$, whence, by solving for R_1 and R_4 and substituting for M_2 its value in eq. (18), we have

$$R_1 = P\left\{1 - k - \frac{l}{4l+6l_2}(k-k^3)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$R_4 = -P\left\{\frac{l}{4l+6l_2}(k-k^3)\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Using eqs. (19) and (20) in getting the reactions due to the various panel loads, the analysis is otherwise precisely the same as for a two-span bridge, Art. 174.

The two links GE and HF are to be treated as posts of the web system. The maximum stress in GH is equal to the maximum in $LBCK$. The stresses in EB and FC are equal respectively to those in GE and HF. The pieces EF, EC, and BF receive no definite stress; they serve merely to afford stability to the columns EB and FC.

The analysis of the case where light diagonals are inserted in the panel $GHBC$ sufficient only to give stability to the bridge when open is the same as for the preceding case.

177. Rim-bearing Turn-table; Three Supports.—In this the single link GP carries the load at the centre to the rigid frame Pgh. The length of the panel gh is made equal to the width of the truss in order that the weight upon the turn-table may be uniformly distributed. The length of the other panels* in the lower chord is independent of gh. The analysis is precisely the same as for a simple two-span continuous girder.

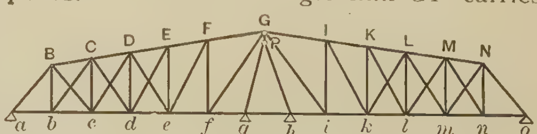


Fig. 230.

Example.—Find the maximum stresses in all the members of the truss in Fig. 230. Length ag = 120 ft.; gh = 18 ft.; ho = 120 ft.; height at B and N = 24 ft., and at G = 36 ft. Total dead load = 1650 lbs. per foot. Take for the live load the equivalent uniform load for a span of 150 ft. (See Chap. VI.)

The span lengths to be considered are 120 + 9 = 129 ft. each.

* This method of arriving at M_2 for a single load is an expedient resorted to here to avoid a recourse to the principle of least work. The problem is indeterminate by the ordinary principles of statics. For an outline of the rigid method see an article by Prof. Merriman in *Engineering News*, Sept. 5, 1895, Vol. XXXIV, p. 150.

The stresses due to dead load when the bridge is open are found by diagram as before.

Joint.	k or k'.	R_1 for $P = 1$.
b	.155	+ .808
c	.310	+ .620
d	.465	+ .444
e	.620	+ .284
f	.775	+ .148
i	.775	− .079
k	.620	− .094
l	.465	− .092
m	.310	− .070
n	.155	− .038

When the bridge is closed and ends raised it is a continuous girder of two spans of 129 ft. each, under dead or live loads. The various values of k or k' for the different joints, together with the corresponding values of R_1 as found from the diagram Fig. 224, are given here. The student is left to complete the analysis.

177*a*. Rim-bearing Turn-table; Four Supports.—*Equal Moments at the Centre Supports; also, Equal Loads on the Turn-table spaced at Equal Distances.*—Fig. 231.—In this the two triangular frames BHC and CID support the truss by means of the short links FH and GI. The portions BC and CD of the frames are a part of the lower chord of the bridge. The point C is common to both frames. The inclinations of the members BH, HC, CI, and

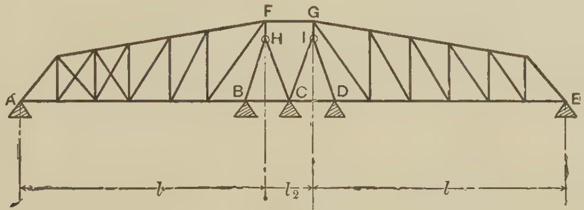


FIG. 231.

ID is such that under maximum loads at H and I the supports B, C, and D are equally loaded. This arrangement brings the weight on the turn-table uniformly distributed over six points. These points can be spaced equal distances apart on the turn-table. Under all conditions, the moments at the two centre supports F and G are always equal. The analysis will then be the same as in Art. 176. However, in all cases of this kind, where the centre span is short as compared with the side spans, it is customary in the computations to assume the centre span left out. The bridge then becomes a swing bridge with two equal spans, as in Art. 174.

LIFT SWING BRIDGES.

Various devices have been suggested whereby the bridge may be made continuous when being opened and be two simple spans when closed. Fig. 231 (*a*) shows a form in which, when the bridge is to be swung, the supports at B and C are lifted far enough to bring the links EF and FG into action and to raise the ends A and D from their supports. All the weight is then at the centre and the bridge is swung on a centre-bearing pivot. When closed, and the supports B and C lowered, the links EF and FG are under no stress. The analysis then consists in finding the dead load stresses

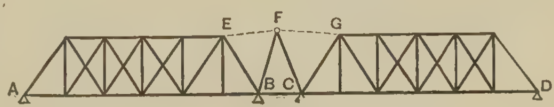


FIG. 231 (*a*).

when open, as in the other forms, and the dead and live load stresses when closed, the bridge then consisting of two simple spans.

178. Wind Stresses.—The analysis for the stresses in the laterals, forming the web systems of the wind trusses, is carried on precisely as for the vertical or main trusses. If the bridge to be considered is a through bridge, the top lateral wind loads must be considered as being transmitted to the supports through bending in the web members. The routes which they select in any case will be such that the internal work done in producing strain will be a minimum. The conditions to be considered are then:

Top Laterals:
Case I. Bridge swinging. Static wind pressure only.
" II. Bridge closed. Ends raised. Static wind pressure only.

Bottom Laterals:
Case I. Bridge swinging. Static wind pressure only.
" II. Bridge closed. Ends raised. Static wind pressure only.
" III. Bridge closed. Live load on one arm only.
" IV. Bridge closed. Live load on both arms.

Combine Cases III and IV with II, or consider I or II alone.

For Cases I and II, when considered alone, it is customary to assume wind pressures of 30 and 50 lbs. per square foot respectively. When the train comes on the bridge, a wind pressure of 50 lbs. would overturn the cars; so that for Cases II, III, and IV, when combined as previously explained, a wind pressure of 30 lbs. per square foot is assumed, which is about the maximum wind pressure that will not overturn a standard box car when fully loaded. It is easy to see that in some bridges Cases I and II, when considered alone, might give a maximum in some of the members.

The chord stresses, resulting from wind loads, should be considered where they increase the chord stresses in the main trusses.

178*a*. Accuracy of the Ordinary Formulæ for Swing-bridges.*—On p. 193 it was noted that there is an error in the ordinary formulæ due to assuming a constant value for the moment of inertia and to the neglect of the shearing distortion, and it was shown that this error is very great in the case there discussed. It is also important to know what this error is in the usual case of the two-span bridge, or the three-span bridge with small diagonals or no diagonals in the centre panel.

After the sections of a bridge have been determined from calculations based on the ordinary formulæ, or on other approximate methods, the reactions can then be very accurately determined by the method of deflections explained below. If too great an error is found, the sections can be corrected accordingly. This would change the reactions slightly, but not enough to affect seriously the stresses. In fact, the reactions as found by the formulæ will usually be accurate enough.

We will consider first a bridge of two equal spans and assume the bridge fully loaded. The reactions are determined as follows: Assume first the bridge loaded with the given load and supported only at the centre, both arms deflecting equally. Determine the deflection of the end point by the formula† $D = \Sigma \frac{Pul}{Ea}$, in which $D =$ deflection of the end point; $P =$ stress in any member due to the assumed load; $u =$ stress in the same member due to a load of one pound applied at the end; $l =$ length of member; $E =$ modulus of elasticity; and $a =$ area of cross-section of the member.

Now find in the same way the upward deflection of the end point due to a load of 1 lb. applied upwards at the end. The stress in each member due to this load, and which corre-

* This article added in Sixth Edition. The student may have to skip the discussion, reading only the conclusions on p. 196*b*, until after Chapter XV has been read.

† See p. 220 for a demonstration of this formula.

sponds to P in the above formula, will be u, the value of which is already known. This upward deflection will then be $d = \Sigma \frac{u^2 l}{Ea}$. The necessary reaction to produce an upward deflection equal to D, or to bring the end back to normal position, will be $R = \frac{D}{d}$.

For a symmetrical truss symmetrically loaded it is necessary to sum the terms $\frac{Pul}{Ea}$ and $\frac{u^2 l}{Ea}$ for one half the truss only. For symmetrical trusses unsymmetrically loaded we can first assume symmetrical loads and determine the corresponding reactions; then from these get the reactions for the unsymmetrical loading by use of the principle that the moment at the centre due to a load on one span is one half the moment due to two such loads symmetrically placed.

Thus let $M =$ centre moment for full load, and $M' =$ centre moment for one span only loaded, $= \frac{M}{2}$. If R and R' are the corresponding reactions, $L =$ span length, and $\Sigma Pa =$ Summation of moments of the loads on one span about the centre, we have

$$M = RL - \Sigma Pa, \quad \text{and} \quad M' = R'L - \Sigma Pa;$$

and since $M' = \frac{M}{2}$, there follows $$R' = \frac{R}{2} + \frac{\Sigma Pa}{2L}.$$

Now, since $\frac{\Sigma Pa}{2L}$ is independent of any errors in reactions, the actual error in R' must be one half that in R. That is, the error in reaction due to the use of the formula is twice as great for bridge fully loaded as for one span only loaded. As these two cases determine the maximum stresses in nearly all the members, it will not usually be necessary to determine true reactions for other partial loads. However, if thought desirable, the reactions can be found for each panel load and the results combined to form influence lines. The additional labor involved is not so great as would seem to be the case, for many of the terms involving P disappear.

For a bridge of unequal spans it is necessary to sum the terms $\Sigma \frac{Pul}{Ea}$ and $\Sigma \frac{u^2 l}{Ea}$ for all members of the bridge. In this case it is convenient to assume the truss supported at the centre and at one end and determine the deflections and reaction at the other.

The foregoing method is applicable not only to a two-span bridge but also to a three-span bridge in which the web members are omitted in the centre span or panel; for, assuming such a truss supported at the centre, or at the centre and at one end, the stresses due to any load are fully determinate.

To get some notion of the accuracy of the ordinary formulas, reactions have been computed for several cases by the above method and also by the use of the formulas; the results are given below. In every case a load of unity per lineal foot extending over the entire bridge has been assumed. The following bridges have been treated:

(A) The Winona Bridge, a description of which will be found in *Engineering News*, Vol. XXVI, 1891, p. 370. This bridge is similar in form to Fig. 229. Each span consists of seven 30-ft. panels; the centre panel is 20 ft. long, centre height 50 ft., and end height 25 ft.

(B) A series of designs with the half-truss containing 6, 5, 4, 3, and 2 panels successively. The general dimensions of these trusses were chosen by taking a corresponding number of panels of the Winona Bridge. Cross-sections of members were properly determined from stresses due to certain assumed dead and live loads. In each of these cases computations were made for trusses with and without a central panel.

(C) The Milwaukee drawbridge of the C., M. & St. P. Ry., which is described in the *Engineering and Building Record*, Vol. XVI, 1887, p. 747. This is of the form shown in Fig. 230. Each span has five 18.5 ft.-panels; the centre panel is 18 feet long, centre height 34 ft., and height at second panel-point from end of 20.4 feet.

The results of these computations are given in the following table:

Truss.	Reactions.				
	True.		By Form.		From Chords.
	(a_1)	(b_1)	(a_2)	(b_2)	
(A)	64.3	65.9	64.3	67.4	52.9
(B) 6 panels	55.7	57.7	53.1	56.3	44.1
(B) 5 "	45.8	47.0	42.0	44.7	34.7
(B) 4 "	35.2	36.1	30.9	33.1	25.3
(B) 3 "	24.3	25.2	20.0	22.1	14.7
(B) 2 "	11.8	11.9	9.4	11.0	8.5
(C)	26.0	....	29.0	....	20.7

Columns headed (a) are the reactions for the several trusses with the central panel omitted, while columns headed (b) are the reactions for the trusses having this panel. The column headed "Reactions from Chords" contains the reactions as determined by deflections, but in the computation of which the chord members are alone considered. These have been figured only for trusses without centre panel, and should therefore be compared with columns (a). *These reactions are the same as would be found by the use of formulas which take strict account of the variation of the moment of inertia of the truss.*

The principal points brought out by this table are:

First, the comparatively close agreement between the true reactions and those found by the ordinary formulæ.

Second, the fact that the formulæ taking account of the central panel give results usually more accurate for these cases than the formulæ which neglect this panel (compare columns (a_2) *and* (b_2) *with* (b_1)).

Third, that the reactions found by treating chords only are in every case much more in error than those found by the ordinary formulæ.

No attempt has been made to determine the law of variation of the error in the ordinary formulæ. Such a generalization would be very difficult to make, and in view of the ease with which the reactions for any particular case can be checked, it would be unprofitable as well.

Temperature Effects.

A very important question, and one of especial significance when discussing the accuracy of working formulæ is that of the effect of a variation of temperature between different members of a swing-bridge. The effect on reactions of any given difference of temperature can very quickly be found from our previous computations.

Let t = change of temperature of any member; c = coefficient of expansion, = .0000065 per 1° F.; l = length of member. Then ctl = total change of length of any member. The deflection of the end of the truss due to the change of length of this member will be $uctl$, where u is the same as in our previous work, and for a change of temperature of any number of members the deflection will be

$$D_t = \Sigma uctl.$$

The reaction necessary to produce a like deflection, or the reaction produced by the assumed temperature changes, will be

$$R_t = \frac{D_t}{d}, \qquad (7)$$

in which d = deflection due to a one-pound load, as before. For example, suppose the lower chord of the Winona Bridge to drop in temperature 1° F. In this case ctl for each member = .00234 and $\Sigma u = 20$, whence

$$D_t = .047, \quad \text{and} \quad R_t = \frac{.047}{.0000334} = 1400 \text{ lbs.},$$

a very large amount for a change of temperature of only 1°. Considering a possible difference of temperature of 30° or even more, the great effect of this element can be appreciated. The effect is so great as to evidently render much refinement in calculation entirely useless, and to indicate that *the ordinary formulæ are accurate enough for all ordinary cases.*

CHAPTER XIII.

CANTILEVER BRIDGES.

179. General Considerations.—The first cantilever bridge built in America was the Kentucky River bridge, by C. Shaler Smith, built in 1876–7. A sketch of this bridge is shown

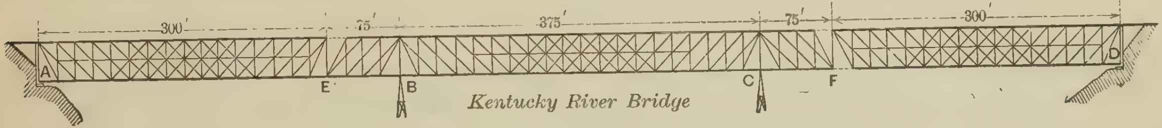


FIG. 232.

in Fig. 232. The bridge is continuous from E to F, and at these points are hung the ends of two simple trusses AE and FD.

Since this bridge was constructed several very large cantilever bridges have been built in America and elsewhere, notably the great Forth bridge, Fig. 233, the Niagara bridge, Fig. 235, the Poughkeepsie, Red Rock, Memphis, and others. In the Niagara bridge the suspended

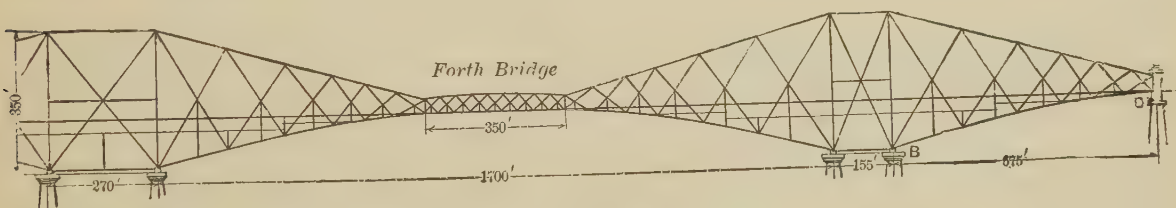


FIG. 233.

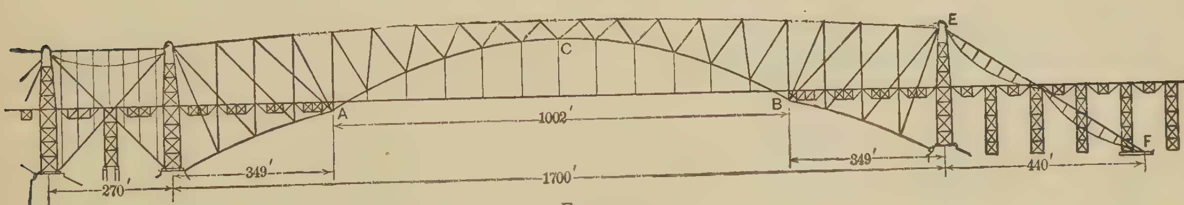


FIG. 234.

span DE is hung from the ends of two double cantilevers, AD and EH, each of which has two points of support, one at the abutment (not shown in the figure) and one at the pier. There being no diagonals in the panels over the piers, the trusses are free to turn at those points as if supported on a single pin. Fig. 234 is a form of bridge proposed by C. B. Bender,* as being far more economical than the Forth bridge as built. The suspended span consists of a three-hinged arch ACB; the arm BD, Fig. 233, is replaced here by the back-stay EF for anchorage, and the roadway is supported on iron trestle-work, the span BD being a land span.

Fig. 236 shows another form, suitable, however, only for short spans.

The chief advantage of the cantilever bridge over a single span bridge is in being able to erect the overhanging arms and the suspended span without the use of false work. In the

* "Principles of Economy in the Design of Metallic Bridges." New York, John Wiley & Sons.

Niagara bridge, for example, the erection is carried on from each pier to the centre, at least three of the four pieces, *DK*, *LM*, *EN*, and *OP*, connecting the suspended span to the cantilever, being provided with adjustable wedges during erection. When connection is made at the centre, these wedges are taken out and the central span swings free on the hangers *DM* and *EO*, except

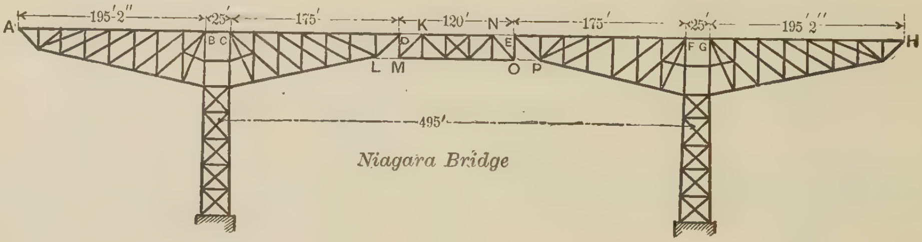


FIG. 235.

as to being prevented from longitudinal vibration by the single remaining piece or a similar device holding it to one of the cantilevers. Where a series of cantilever spans are constructed, each alternate span can thus be erected without falsework. Other than this, the cantilever possesses no advantage over discontinuous spans except perhaps for very long spans.

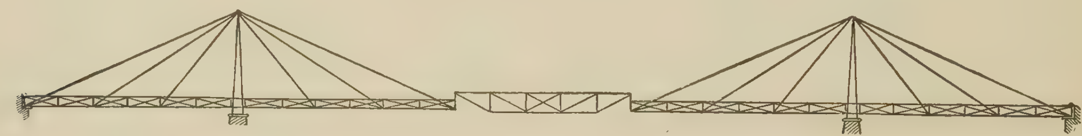


FIG. 236.

For economy the suspended span should be made about four-tenths of the total opening. However, the longer this span, compared to the total span, the less will be the deflection, which is quite an important consideration. Thus the maximum vertical movement of the hinge *E* in the Niagara bridge under the test load was about 9 in., and of the point *E* in the Kentucky River bridge was only 3½ in. The Eighteenth Street viaduct in St. Louis, of the form of Fig. 236, vibrates excessively under the lightest loads.

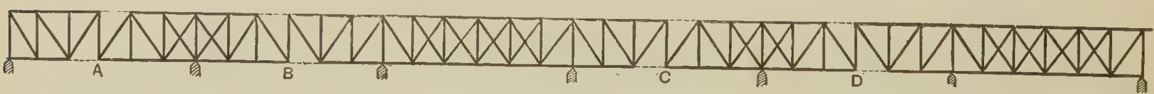


FIG. 237.

Fig. 237 is a proposed form of bridge for a long series of equal spans where erection is difficult. It can be erected entirely without falsework.

Fig. 237 (*a*) shows a bridge proposed for the English Channel, designed by Messrs.

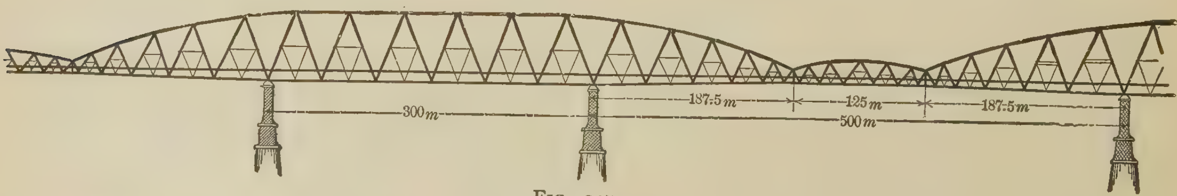


FIG. 237*a*.

Schneider & Co. and H. Hersent, Sir John Fowler and Sir Benjamin Baker consulting engineers.

180. Analysis.—*Dead Load Stresses.*—The stresses in the suspended span are found in the same way as for any single span truss. In both forms of cantilevers, *EF*, Fig. 232, and

EH, Fig. 235, there are but two supports; hence the reactions due to any load are readily found. These being known, the moment and shear at any section can be found, and thence the stresses. Where double systems of web members are used, each load is assumed to be carried by the system to which it belongs. At points of connection between the two systems, the loads applied to the truss may be considered as equally divided between the systems.

Live Load Stresses.—Cantilever bridges being used mainly for long spans, double systems of web members are quite generally adopted for the sake of economy. It has been shown in Chap. V that with double systems in single span bridges it is impracticable to use the exact wheel load method in computing stresses, and that some conventional method, such as an equivalent uniform load, or a uniform load with one or two excesses, should be used instead. It is still more difficult to apply the wheel load method to cantilever bridges with double systems, and hence the following discussion will treat mainly of uniform loads, with or without excess loads.

In finding the proper positions of live load for maximum stresses in the various members, it will be useful to draw the influence lines for moment and shear in a single intersection cantilever of the type shown in Fig. 232. Fig. 238 shows such a truss.

1st. *Moment at G.*—Fig. (*a*).—A load unity at E causes a negative reaction at C equal to

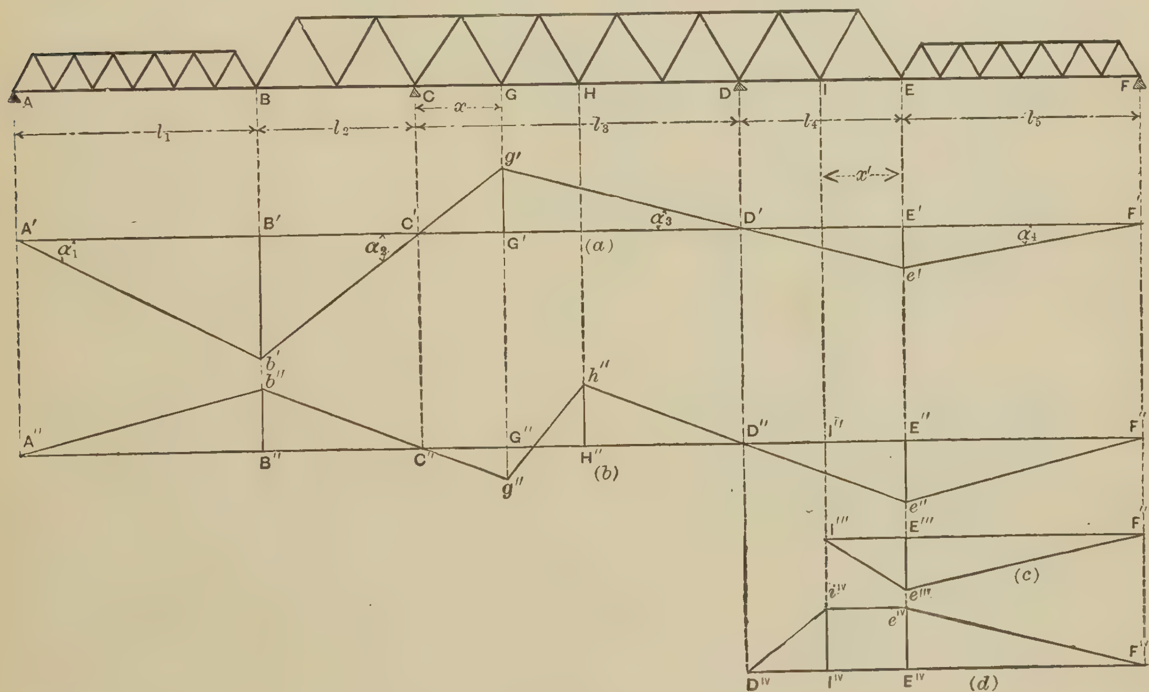


FIG. 238.

$\frac{l_4}{l_3}$, and hence a negative moment at G equal to $\frac{xl_4}{l_3}$, which is laid off as $E'e'$ in Fig. (*a*). As the load moves to D or F, the moment at G decreases uniformly to zero and the influence line for this portion is $F'e'D'$. As the load moves from D to G, the moment increases from zero to a value of $\frac{x(l_3 - x)}{l_3}$ at G; beyond G the moment decreases again, becoming zero when the load is at C, then $\frac{l_2}{l_3}(l_3 - x)$ when at B, and finally zero for load at A. Since the ratio of $e'E'$ to $g'G'$ is equal to $\frac{l_4}{l_3 - x}$, it follows that $g'D'e'$ is a straight line. Similarly, $g'C'b'$ is a straight line.

The influence line shows that for a maximum positive moment at G the span CD should alone be loaded, and that for a maximum negative moment the spans AC and DF should be loaded. A single excess in the two cases should be placed at G, and at E or B according as $E'c'$ or $B'b'$ is the larger. A second excess a fixed distance from the first should be placed in each case on the longer segment of the span from the first excess.

2d. *Shear in the panel GH.*—Fig. (*b*).—The portion $D''h''g''C''$ is the same as for a discontinuous span. Between D and F the shear in GH is equal to the reaction at C caused by the load, and is negative, having a value of $\frac{l_4}{l_3}$ for unit load at E. When the load is between A and C the shear is positive and equal to the negative reaction at D. As before, the lines $b''C''g''$ and $h''D''e''$ are straight lines. The position of loads for a maximum positive or negative shear is evident from the diagram. If full joint loads only are considered, then for maximum positive shear, for example, all joints from H to D in span CD, and span AC, should be fully loaded.

3d. *Moment at I.*—Fig. (*c*).—For loads on EF, the load given over at E, and hence the negative moment at I, varies directly with the distance of the load from F. The moment at I when the load is at I, is zero. Hence the influence line is $I'''e'''F'''$, in which $E'''e''' = 1 \cdot \times x'$ The condition for maximum moment is the same as for moment at E in a discontinuous truss whose span is equal to IF.

4th. *Shear in the panel DI.*—As the load moves from F to E the positive shear increases uniformly from zero to a value equal to unity with unit load at E. The shear then remains constant until the load passes I, then decreases to zero as the load reaches the next panel point to the left (not the abutment, necessarily). The position for a maximum is evident.

The foregoing influence lines show in a general way the positions of loads, whether uniform, or uniform with one or two excesses. If exact methods are desired for a single intersection truss, the conditions for maxima can be readily written out from the influence lines Thus for maximum positive moment at G, if G_1, G_2, G_3, and G_4 are the sums of the loads on AB, BG, GE, and EF respectively, we have at once the condition that

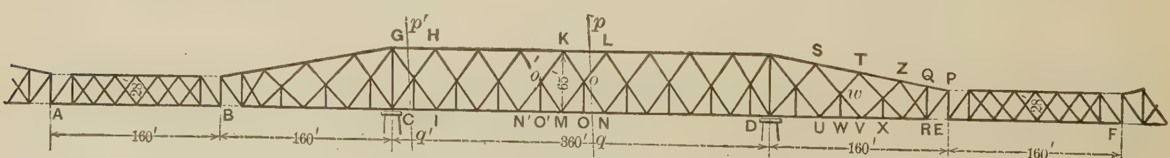
$$G_1 \tan \alpha_1 + G_3 \tan \alpha_3 - G_2 \tan \alpha_2 - G_4 \tan \alpha_4$$

must pass through zero by passing from positive to negative as the loads are moved to the left. The values of the tangents of the angles are readily substituted in any case.

The influence lines for moments and shears in the shore arm GH, Fig. 235, are the same as those for the span CD above considered, except that the portion to the right of D does not exist.

The position of the load having been found in any case, the reactions and stresses are found as for dead load.

181. Example. Indiana and Kentucky Bridge.—Fig. 239.—The span lengths CD and DE are given only approximately.



Indiana and Kentucky Bridge

FIG. 239.

Dead Load Stresses.—The stresses in the span CD due to loads between C and D are found as in Art. 78, Chap. IV.

To find the stresses in CD due to loads on AC and DF, first compute one of the abutment reactions at C or D, and the moment at this abutment. Then the shear just to the right of C, for example, is equal to the reaction at C, minus the loads on BC. This shear is constant from C to D, the loads on CD not being considered, and may be assumed to be equally

divided between the two web members cut by any section. The web stresses due to exterior loads are thus all equal. If AC is symmetrical to DF, then these web stresses are zero. Since the horizontal components of the stresses in KN and LM, for example, are equal and opposite in direction, the stresses in KL and MN are equal and may be found by taking moments about o of the forces between section pq and the section $p'q'$ just to the right of C. The only forces acting on the left of pq, besides the stresses in KL and MN, are the moment and shear at $p'q'$ which are already known. This moment being due to a couple, its value at o is the same as at pq, and adding the moment of the shear about o we have the total moment to be resisted by KL and MN.

The stresses in the arm DE due to loads on that arm, other than the loads at P and R, are found as in Art. 78, Chap. IV; that is, by considering the web systems independent and all the load applied at the main panel points, U, V, etc., the loads at W, X, etc., being considered as carried to the points U, V, etc., by separate small trusses UwV, etc. The stresses in these small trusses are added to the stresses in those main members which coincide with the members of the small trusses.

The stresses in PQ and PR are found from the load at P, which is one-half the weight of the truss EF. The vertical component of the stress in PR together with the load at R may be assumed to be divided equally between the two web members ZR and QR. The stresses in the remaining members are then found by separating the systems and proceeding in the ordinary manner.

Live Load Stresses.—For the maximum positive moments in span CD this span should be fully loaded. If a uniform load is used, the stresses are found in the same way as for dead load. If an excess load is used, the position of such load for maximum stress is to be found. For piece KL, the centre of moments for one web system is at M, and for the other is at N. The excess load should be at that one of these points nearest the centre of the span, the sub-vertical oO not being considered in finding upper chord stresses. If a second excess is employed, it should be on the longer segment of the span from the first.

The position of loads being known, the stress in KL is found by assuming independent systems and neglecting the sub-verticals, or treating them as parts of trussed beams, simply transferring the intermediate panel loads to the main panel points. For the member MN, any single excess should be placed at O, for the tension in MN is due both to the main truss and to the truss MoN. A second excess should be placed on the longer segment of the span from O. The stress in MN due to the main truss is found by considering the systems independent, one-half the load at O going to M and one-half to N. To this stress is added the stress from the truss MoN, with excess at O.

For the maximum negative moments in CD, the spans AC and DF are loaded, with one excess at B or E, and the other (if used) outside or inside these points according as the suspended spans are longer or shorter than the cantilevers. The chord stresses in CD are found as for dead load.

For the maximum tension in Ko and compression in Ko', the span AC should be fully loaded and CD loaded from O to D. One-half the load at O goes to N, and one-half to M and into the other system. Considering all loads to the right of N as applied at the main panel points, the vertical components in Ko and Ko' due to loads on CD are equal to the shear in panel $N'N$ of the system to which these members belong. The stresses in Ko and Ko' due to loads on AC is found as for exterior dead load. Any excess should be placed either at B or N, whichever will give the greater stress as may be found by trial. It will usually be at N, since if at B its effect is divided between Ko and oM. For the piece oN the interior loading should extend from N to D, since a load at O will cause a greater compression in oN due to truss MoN than tension due to the additional one-half panel load at N. The excess if on CD should be at N. For maximum compression in $o'N'$, panel points O' to D inclusive should

be loaded, for the load at O' causes a compression in $o'N'$ as part of the truss $N'o'M$ which is greater than the tension in $o'N'$ due to the additional half load at N'. The excess should be at N. The stresses in oN and $o'N'$ due to loads on AC are the same as that in Ko. Stresses due to negative shear in the span CD are found in a similar way, the span DF and the portion of CD to the left of the members in question being loaded.

For maximum negative moments in the arm DE, the span DF may be considered fully loaded, since the loads to the left of the centre of moments have no effect. Any single excess should be placed at E; and if a second excess is employed, it should be placed to the right or left according as EF or ED is the longer. The stresses are found as for dead load.

The web stresses in DE are also found as for dead load. The excess, if any, should be placed as for positive shear in CD.

182. **Wind Stresses.**—Wind pressure is carried to the abutments by means of horizontal lateral bracing arranged in the same way as the main vertical trusses. The wind stresses are then found in a way precisely similar to that explained in the preceding articles.

CHAPTER XIV.

ARCH BRIDGES.

GENERAL PRINCIPLES.

183. **Arches** of metal may consist of curved beams with solid webs and flanges, or they may be curved trusses with upper and lower chords and web members, either riveted or pin-connected.

With reference to the ordinary modes of support arches may be—

1st. Hinged at the abutments and at the crown.

2d. Hinged at the abutments and continuous throughout.

3d. Fixed rigidly to the abutments and continuous throughout.

In the first case the arch consists of two separate framed structures, and the reactions and stresses can be found by ordinary methods of statics. In the other cases, however, the reactions depend not only upon the loads but also upon the form and material of the arch. In finding these reactions the arch will in all cases be treated as a simple curved beam with a constant or variable moment of inertia as the case may be. (By a curved beam is meant one which has a curved form in its natural or unstrained condition.) All loads will be treated as vertical. The general method of treatment is the same as that employed by Prof. Greene in his "Trusses and Arches," Part III, and by Prof. DuBois in his "Framed Structures."

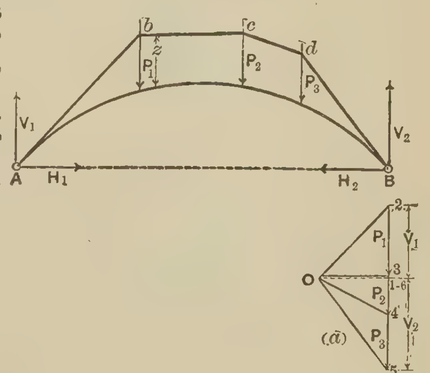


FIG. 240.

184. **Relation between the Equilibrium Polygon and the Stresses at any Section of an Arch.**—Let AB Fig. 240, be an arch of two hinges, with loads P_1, P_2, and P_3. Each abutment reaction will have a horizontal component and a vertical component. From Σ hor. comp. $= 0$ we know that these horizontal components are equal. Suppose that in some way these abutment reactions have been found, the force diagram, 0 1 2 3 4 5 6 0, drawn, and also the corresponding equilibrium polygon $AbcdB$. Notice that here in the case of an arch, the equal and opposite forces H_1 and H_2 are not imaginary, as was the case for the rigid frames treated of in Chapter II.

According to the principles of Chap. II, Art. 28, any segment, bc, of the equilibrium polygon is the line of action of the resultant of all the forces, H_1, V_1, and P_1, to the left (or right), this resultant being given in amount and direction by the ray, 0–3, in the force polygon to which the segment is parallel. The stresses at any section of the arch, therefore, between the loads P_1 and P_2 are the same as would result if H_1, V_1, and P_1 were replaced by a single force 0–3 applied in the line bc. Fig. 241 represents a portion of the arch of Fig. 240, to the left of such a section. The force 0–3, $= R$, applied in the line bc, is in equilibrium with the stresses at the section. These stresses may be considered as consisting of a uniformly distributed direct stress or thrust

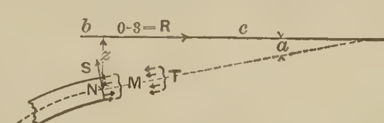


FIG. 241.

T, in the direction of the tangent at N, a shear S, at right angles to T, and a bending moment M. If α is the angle between R and the tangent at N, we have

$$R \cos \alpha = T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$R \sin \alpha = S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also, taking centre of moments at N, the centre of gravity of the cross-section, we have, from Figs. 240 and 241,

$$Hz = M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where H is the pole distance and z is the vertical intercept from the equilibrium polygon to the centre of moments. Thus the thrust, shear, and moment at any section are readily found after the equilibrium polygon is once drawn.

In a solid beam the fibre stresses result directly from the above equations. In a braced arch, however, Fig. 242, the stresses in the members cut by any section are more readily found by the ordinary methods. Thus the stress in AB is equal to the moment of R about C, divided by the lever arm of AB, $= Hz' \div d$. Likewise the stress in $DC = Hz'' \div d$. The component of the stress in AC normal to the arch is equal to the shear, $= R \sin \alpha$. If the

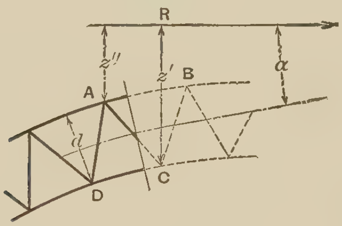


FIG. 242.

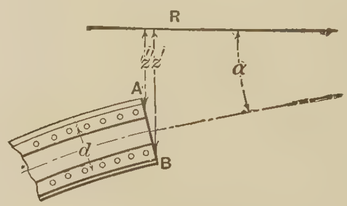


FIG. 243.

chords AB and DC are not parallel, then the stress in AC is found by getting the moment of R about the intersection of AB and DC, and dividing by the lever arm of AC. In a built beam, Fig. 243, in which the direct stress and moment are taken entirely by the flanges and the shear by the web, the stress in flange A is equal to $Hz' \div d$, and in B is equal to $Hz'' \div d$. The shear is as before equal to $R \sin \alpha$.

In Chap. II it has been shown that for a given system of loads an infinite number of equilibrium polygons can be drawn by assuming various amounts and directions for one of the reactions; that is, by assuming various poles. But by Art. 42, Chap. II, three points determine an equilibrium polygon; hence if we have given, besides the loads, three points through which the polygon must pass, or three equations of condition which will determine three points, the equilibrium polygon is at once determined, and the true reactions and stresses may thence be found. The three points required, or the necessary condition equations, for the three kinds of arches will be deduced in what follows. It will be sufficient for our purposes to consider but a single load, that is, our polygon will consist of but two segments.

ARCH OF THREE HINGES.

185. The Equilibrium Polygon.—Fig. 244.—In this case we know that since the arch is free to turn at A, B, and C, the moments at these points must be zero; hence the polygon passes through these points, and to draw it we need only produce BC to the load vertical at k and join Ak.

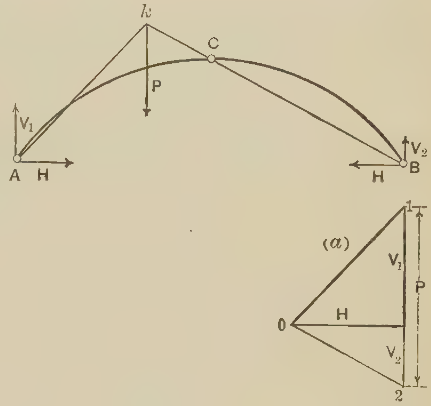


FIG. 244.

186. Position of Loads for a Maximum Stress in any Member.—Let pq, Fig. 245, be any section cutting three members of a braced arch of three hinges, A, B, and C. The centre of moments for DE is G, and a load at k, the intersection of BC and AG, will cause no stress in DE, since the line AG is the line of action of the abutment reaction at A, the only external force on the left of the section. For loads between k and B the reaction line Ak will lie below G, passing through C for loads on CB. The moment of the reaction at A, about G, will then be negative and cause tension in DE. For loads between k and G, inclusive, the reaction line Ak lies above G, and there is then compression in DE. For loads between D and A the only force on the *right* of the section is the reaction at B which acts in the line BC, thus causing compression in DE. Therefore for a maximum tension in DE the load should extend from k to B, and for maximum compression it should extend from A to k. The loads are usually taken as uniformly distributed, and are applied at joints, or at intervals along the upper flange in the case of the flanged beam.

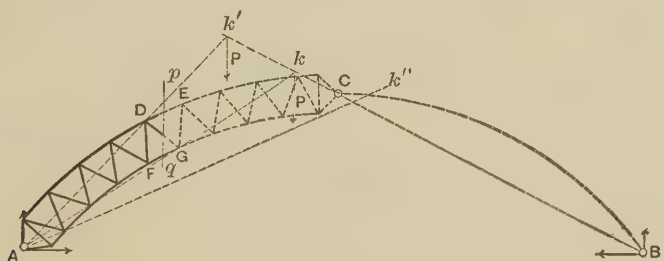


FIG. 245.

The centre of moments for FG is at D, and from considerations similar to the preceding, it is found that the maximum tension and the maximum compression in this member occur when the load extends to the left and right, respectively, of k'.

For the web member DG draw Ak'' parallel to DE and FG if these members are parallel, or to their intersection if not parallel. For all loads between G and B the reaction line at A lies above Ak'', since this reaction line never passes below C. Hence the component of the reaction perpendicular to Ak'' in case of parallel chords, or the moment of the reaction about the intersection of DE, FG, and Ak'' in case of non-parallel chords, produces tension in the member DG. For loads from D to A the right reaction acting in the line BC causes compression in DG. Hence for maximum tension in DG, GB should be loaded, and for maximum compression DA should be loaded. If Ak'' should pass to the left of C, then all loads between its intersection with BC, and B, would cause compression in DG.

In the case of a built beam the centres of moments are taken as in Art. 184, and in finding the loading for maximum shear, the line corresponding to Ak'' is drawn parallel to the flanges at the section considered.

187. Computation of Stresses.—The position of the loading producing the maximum stress in any member having been found according to the previous article, the equilibrium polygon may be constructed for the entire system of loads according to the method of Art. 42, Chap. II, and the stress in the member found as in Art. 184. Or the reactions and stresses may be found analytically. If c is the half-span, Fig. 246, r the rise, and b the distance of any load P from the centre C, measured positively towards the right, we have

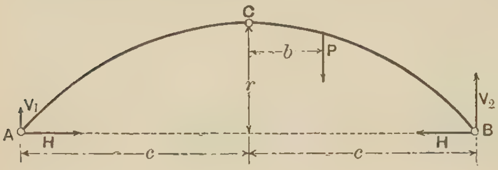


FIG. 246.

$$V_1 = P\frac{c-b}{2c} \quad \text{and} \quad V_2 = P - V_1 = P\frac{c+b}{2c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By taking centre of moments at C and treating the structure AC, we have for loads on CB,

$$Hr = V_1 c,$$

whence

$$H = V_1\frac{c}{r} = P\frac{c-b}{2r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Similarly, for loads on AC we have

$$H = V_2\frac{c}{r} = P\frac{c+b}{2r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

The components of the reactions can thus be computed for each joint load and the results added. The reactions being known, the stresses can be found by the ordinary method of moments.

CURVED BEAMS.

188. Deflection of a Curved Beam.—Before proceeding further it will be necessary to investigate the general case of the deflection of a beam, the beam to be of any shape, and acted upon by forces all lying in the same plane. The following discussion is taken mainly from Prof. Church's "Mechanics of Engineering," pp. 444–9.

Let AB, Fig. 247, be any portion of such a beam in its unstrained form. Suppose now that under the action of certain forces the beam is brought into the position $A'B'$, the change in position being due to any cause whatever. We wish now to find the movement of A and the tangent to the neutral axis at A, with reference to B and the tangent at B, these two points, A and B, being any two points in the beam. This relative motion will be made apparent by making B' coincide with B, and the tangent at B' coincide with the tangent at B. This new position is represented by the dotted outline $A''B$. The absolute movement now shown is the relative movement required. The tangent at A has moved through an angle $\Delta\phi$, making now an angle with the tangent at B of $\theta + \Delta\phi$, θ being the original angle; the point A has also moved in space a distance AA'', the components of which motion, referred to any two rectangular axes with origin at A, will be called Δy and Δx.

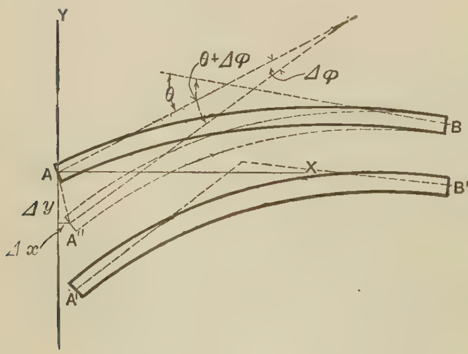


FIG. 247.

(*a*) *Change of Inclination of Tangent*, $=\Delta\phi$.—Let $CEFD$, Fig. 248, be an element of the beam of length ds, whose end faces CE and DF are at right angles to the axis, and whose end tangents make an angle with each other originally equal to $d\theta$. Let $d\phi$ be the change in angle between end faces or end tangents due to bending. The change in length of a fibre at a distance y from the neutral axis will be equal to $yd\phi$, and the corresponding stress per unit area will be equal to $f=E\frac{yd\phi}{ds}$, where E is the modulus of elasticity of the material, and ds is the original length of the fibre. If da is an element of area of the cross-section, the total moment of resistance of the beam is equal to $\int_F^D fday=\int_F^D Ey^2da\frac{d\phi}{ds}$.

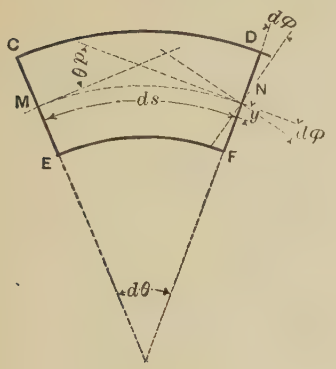


FIG. 248.

But for any particular section, E and $\frac{d\phi}{ds}$ are constant; and if M is the moment of the external forces on one side of the section about N, and I is the moment of inertia of the section, we have

$$M=E\frac{d\phi}{ds}\int_F^D y^2da=EI\frac{d\phi}{ds},$$

from which we have

$$d\phi=\frac{Mds}{EI}, \qquad (6)$$

and in Fig. 247

$$\Delta\phi=\int_A^B d\phi=\int_A^B\frac{Mds}{EI}. \qquad (7)$$

(*b*) *Components of A's motion*, $=\Delta y$ *and* Δx.—Let $AEDCB$, Fig. 249, represent the axis of the unstrained form of the beam, and $A''E''D'CB$ the strained form $A''B$, Fig. 247. Now conceive the beam to pass into its strained form by the successive bend-

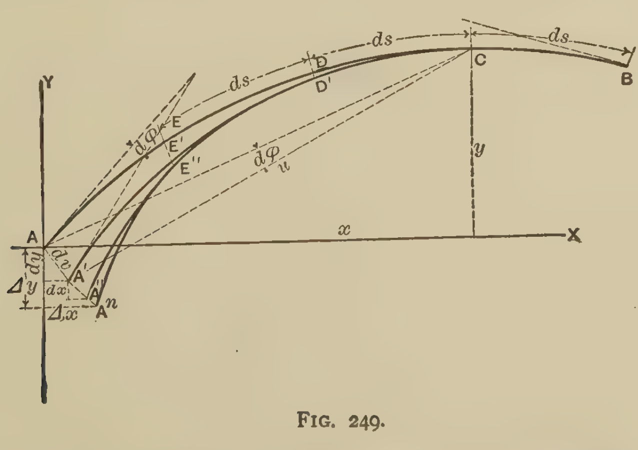


FIG. 249.

ing of each ds in turn. The bending of the element BC through the angle $d\phi$, causes the portion AC to turn through the same angle $d\phi$ about C as a centre, with radius u, the point A moving to A' through a distance dv, having the components dy and dx. Then from

the bending of DC the point A' moves to A'', etc. If x and y are the co-ordinates of any point C, we have, by similar triangles,

$$\frac{dy}{dv} = \frac{x}{u}, \quad \text{and} \quad \frac{dx}{dv} = \frac{y}{u}.$$

Solving for dy and dx and substituting for dv the value $ud\phi$, we have

$$dy = xd\phi \quad \text{and} \quad dx = yd\phi.$$

Substituting the value of $d\phi$ from eq. (6), we have

$$\Delta y = \int dy = \int_A^B xd\phi = \int_A^B \frac{Mxds}{EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$\Delta x = \int dx = \int_A^B yd\phi = \int_A^B \frac{Myds}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

(*c*) *Application of eqs.* (7), (8), *and* (9).—To make clear the application of the above equations, let us take the case of a two-hinged arch ACB, Fig. 250, supporting the loads P_1, P_2, etc. The full line ACB represents the position of the arch when under no load, and the dotted line ACB represents its form when loaded. The tangents to the curve of the arch at B, in the two positions, are BT and BT' respectively. Now the total movement of any point in the arch with reference to B, due to the application of the given loads, may be divided into two parts; one a movement about B as a centre due to the turning of the arch on the hinge B, and the other a movement due to the bending of the arch between the given point and B. These motions occur simultaneously; but if we conceive them to take place successively, the first motion will bring the arch into the position $A'C'B$, the tangent BT moving to BT' through an angle β, and the entire arch turning about B as a centre through the same angle. Now conceive the bending to take place. The arch will then come into the position shown by the dotted line ACB, and the point A' will move to A, making the total movement of A equal to zero. The movement of any point and the tangent at that point with reference to B and BT', due to bending, as for example, the point A' and the tangent at A', is now given by eqs. (7), (8), and (9). Since the point A' comes back to A, its motion due to bending is equivalent to the arc $A'A$, this arc being the path of the first part of its motion. Hence with the axis of X taken parallel to AB, the distance Δx is very small compared to Δy (if Δy were a differential of the first order, Δx would be of the second order and therefore zero compared to Δy); and since Δy is small compared to the dimensions of the arch, we may put Δx equal to zero. Hence from eq. (9) we have for the point A

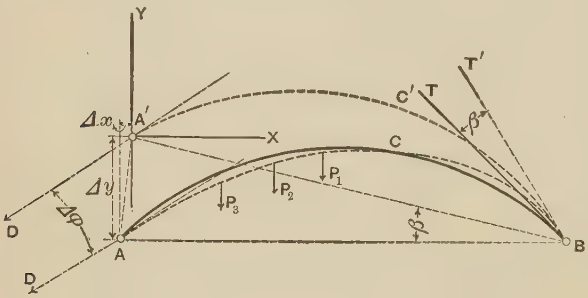


FIG. 250.

$$\Delta x = \int_A^B \frac{Myds}{EI} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This is the only one of the equations (7), (8), and (9) whose value is known beforehand, and hence the only one which is of service in finding reactions.

In the case of an arch with fixed ends, the end tangents are fixed as well as the points A and B; hence, for each end referred to the other, each of the equations (7), (8), and (9) reduce to zero.

PARABOLIC ARCH OF TWO HINGES; VARIABLE MOMENT OF INERTIA.

189. The Equilibrium Polygon.*—In Fig. 251, let r = rise; c = half-span; b = distance of any load from the centre, measured positively towards the right; and x and y = the co-ordinates of any point of the arch referred to A as the origin. Since the moments at A and

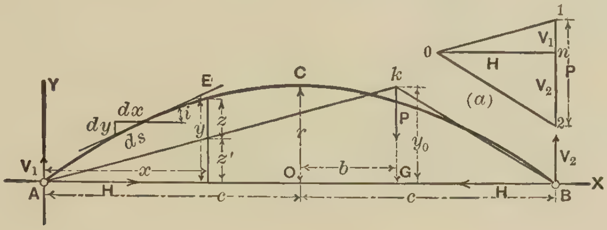


FIG. 251.

B are zero we know that the equilibrium polygon for the load P must pass through these points. We also have the further condition from eq. (10) that $\int_A^B \frac{Myds}{EI} = 0$.

The modulus of elasticity, E, will be taken as constant; and if H is the pole distance and z is the vertical ordinate between the equilibrium polygon and any point E, then $M = Hz$, and hence by substitution

$$\int_A^B \frac{Myds}{EI} = \frac{H}{E}\int_A^B \frac{zyds}{I} = 0, \quad \text{or} \quad \int_A^B \frac{zyds}{I} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If now we make the further assumption that I increases from the crown to the springing line in the same ratio as sec i, where i is the inclination of the arch at any point to the horizontal, we have

$$\frac{I}{\sec i} = \text{a constant} = I_0,$$

where I_0 is the moment of inertia at the crown. This assumption is a reasonable one† and sufficiently exact for practical purposes.

From Fig. 251 we have

$$dx = \frac{ds}{\sec i};$$

hence, by substituting the above value of I in eq. (11), we have

$$\int_A^B \frac{zyds}{I} = \int_A^B \frac{zyds}{I_0 \sec i} = \frac{1}{I_0}\int_A^B zydx = 0;$$

whence

$$\int_A^B zydx = 0, \quad \text{or} \quad \int_0^{2c} zydx = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

From Fig. 251 we have $z = y - z'$; hence eq. (12) becomes

$$\int_0^{2c} y^2dx - \int_0^{2c} z'ydx = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

For a parabola, with origin at A, we have

$$\frac{r - y}{(c - x)^2} = \frac{r}{c^2}, \quad \text{whence} \quad y = \frac{rx}{c^2}(2c - x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

* The following demonstration and that of Art. 194 are from Prof. Greene's "Trusses and Arches," Part III, pp 44, 45, 60-63.

† Since the direct stress in the arch for a full load, and also its cross-section, do increase from crown to springing in accordance with this law.

Substituting this value of y in the first term of eq. (13) and integrating, we have

$$\int_0^{2c} y^2 dx = \frac{r^2}{c^4}\int_0^{2c}(4c^2x^2 - 4cx^3 + x^4)dx$$
$$= \frac{r^2}{c^4}\left(\frac{32}{3}c^5 - 16c^5 + \frac{32}{5}c^5\right) = \frac{16}{15}r^2c. \quad \dots \dots \quad (15)$$

For the second term of eq. (13) we have, from A to G,

$$\frac{z'}{y_0} = \frac{x}{c+b}, \quad \text{whence} \quad z' = \frac{y_0 x}{c+b}. \quad \dots \dots \quad (16$$

Substituting from (16) and (14) in (13) and integrating from 0 to $(c+b)$, we have

$$\int_0^{c+b} z'y dx = \frac{ry_0}{c^2(c+b)}\int_0^{c+b} x^2(2c-x)dx$$
$$= \frac{ry_0}{c^2(c+b)}\left[\frac{2}{3}c(c+b)^3 - \frac{1}{4}(c+b)^4\right]$$
$$= \frac{ry_0}{c^2}\left[\frac{2}{3}c(c+b)^2 - \frac{(c+b)^3}{4}\right]. \quad \dots \dots \quad (17)$$

To integrate the portion on the right of G, we may take our origin at B. The integral will be then the same as eq. (17), but with $(c-b)$ put for $(c+b)$. That is,

$$\int_{c+b}^{2c} z'y dx = \frac{ry_0}{c^2}\left[\frac{2}{3}c(c-b)^2 - \frac{(c-b)^3}{4}\right]. \quad \dots \dots \quad (18)$$

Substituting from (15), (17), and (18), in (13) and reducing, we have

$$\frac{16}{15}r^2c - \frac{ry_0}{6}\left(5c - \frac{b^2}{c}\right) = 0.$$

Solving for y_0 we have

$$y_0 = \frac{32c^2r}{25c^2 - 5b^2}. \quad \dots \dots \quad (19)$$

This equation gives the value of y_0 in terms of known quantities and hence determines the third point in the equilibrium polygon. As the position of the load varies, eq. (19) is the equation of the locus of the point k. Hence if this curve be constructed, the equilibrium polygon for any load is drawn by simply joining the points A and B with the intersection of the load-vertical with this curve.

190. Position of Loads for a Maximum Stress in any Member.—In Fig. 252 the locus of the point k is the curve MN, having an ordinate, y_0, equal to $\frac{32}{25}r$ at the centre, and $\frac{32}{20}r$ at

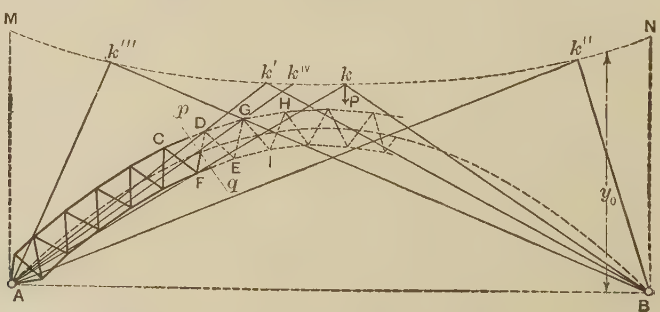


FIG. 252.

the ends of the arch. For the piece CD, with centre of moments at F, a load at k produces no stress, while all loads to the right produce tension and all loads to the left compression. For FE, all loads to the right of k' cause compression and all loads to the left tension. For

the web member DF, draw Ak'' towards the intersection of CD and FE. Then loads between D and k'' cause positive shear on section pq, or compression in DF, while loads to the right of k'' and to the left of F cause tension in DF. For such a piece as EI, with centre of moments at G, loads between k''' and k^{iv} produce positive moment or tension in EI, while loads on the remaining portions produce compression.

The loading for the maximum stress in any member is thus readily found, after having constructed the curve MN. The following table gives enough values of $\frac{y_0}{r}$ to enable this curve to be constructed with sufficient accuracy for this purpose.

TABLE OF VALUES OF $\frac{y_0}{r}$ FOR VARIOUS VALUES OF $\frac{b}{c}$.

$\frac{b}{c}$	0.0	0.2	0.4	0.6	0.8	1.0
$\frac{y_0}{r}$	1.280	1.290	1.322	1.379	1.468	1.600

191. Computation of Stresses.—The stress in each member due to each load may be found graphically by computing y_0 and drawing the equilibrium polygon for each load, the stresses resulting as explained in Art. 184. In this case, however, it will be as easy to determine the stresses analytically.

In Fig. 251 we have, by taking moments about B and then about A,

$$V_1 = P\frac{c-b}{2c} \quad \text{and} \quad V_2 = P\frac{c+b}{2c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Also, by similar triangles in Figs. 251 and 251 (*a*),

$$\frac{H}{V_1} = \frac{c+b}{y_0};$$

whence, from eq. (20),

$$H = V_1\frac{c+b}{y_0} = P\frac{c^2-b^2}{2cy_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

By means of eqs. (19), (20), and (21) the components of the reactions due to each load can be computed and the results added for those loads which act together to cause a maximum stress in any given member. The components of one reaction known, the stress in the member is found by a single equation of moments of the forces upon one side of the section; or if a web member, by a summation of the components of all the forces in a direction normal to the arch at the section considered.

If the arch under consideration is a circular one, the *reactions* may be found with sufficient accuracy in all ordinary cases by using instead, a parabolic arch of the same span, and whose average length of ordinate is the same as that of the given circular arch; that is, one which encloses the same area between the arch and the line joining the springing points. The rise of such an arch is readily found, remembering that the area enclosed by the parabolic arch is equal to $\frac{4}{3}rc$. After obtaining the reactions the stresses should be found for the actual arch.

192. Temperature Stresses.—A rise of temperature of t degrees above the normal, lengthens each element, ds, of the arch, by an amount equal to $teds$, where e is the coefficient

of expansion. The horizontal component of this increment is $teds \cos i$ or $tedx$, and the total change in length of *span* if the arch were free to move would be equal to $\int_0^{2c} tedx = 2cte$. This movement is, however, resisted by horizontal abutment reactions, and stresses are thus developed in the arch.

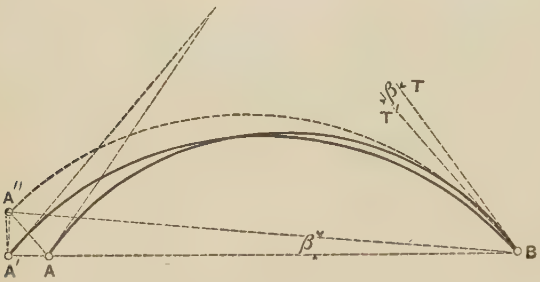


FIG. 253.

Let $A'B$, Fig. 253, be the normal position of the *lengthened* arch, and AB the position when sprung or kept in place by the abutments, $A'A$ being equal to $2cte$. The tangents to the two curves at B are respectively BT' and BT. The movement of A' to A may, as in Art. 188, be conceived as divided into two parts; one a turning about B as a centre through the angle β, the point A' coming to A'', and the other part, that due to bending, the end of the arch moving from A'' to A. As in Art. 188, the horizontal component of $A'A''$ may be put equal to zero, whence the horizontal component of $A''A = A'A = 2cte$, or, from eq. (9),

$$\int_A^B \frac{Myds}{EI}, = \Delta x, = 2cte, \quad \ldots \ldots \ldots \ldots \quad (22)$$

or, according to the assumptions of Art. 189,

$$\frac{1}{EI_0}\int_0^{2c} Mydx = 2cte. \quad \ldots \ldots \ldots \ldots \ldots \quad (23)$$

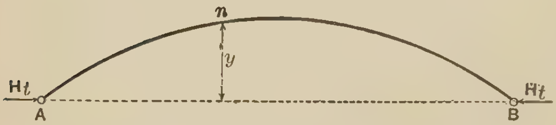


FIG. 254.

Now in Fig. 254 the moment, M, at any point n due to two equal horizontal reactions H_t, is $H_t y$; hence if H_t is the reaction caused by a rise of temperature of t degees, eq. (23) becomes

$$\frac{H_t}{EI_0}\int_0^{2c} y^2dx = 2cte. \quad . \quad . \quad . \quad (24)$$

From eq. (15), p. 210, the value of $\int_0^{2c} y^2dx$ is equal to $\frac{16}{15}r^2c$; hence, substituting in (24) and solving for H_t, we have

$$H_t = \frac{15EI_0te}{8r^2}. \quad \ldots \ldots \ldots \ldots \ldots \quad (25)$$

From this equation the value of H_t can be computed for any change of temperature above or below the temperature for which the arch is designed, and the stresses readily found in all the members by means of a stress diagram. For a *fall* of temperature, t is of course negative and H_t acts outwardly.

193. Stresses due to Change of Length from Thrust.—From the direct thrust due to loads or changes of temperature, each element of the arch is shortened by an amount equal to $\frac{f}{E}ds$, where f = compression per unit area; ds = length of element; and E = modulus of elasticity. This shortening due to compression has precisely the same effect as a fall of temperature. The value of f due to any particular loading or to a change of temperature is not constant along the arch, but it is nearly so and may be so considered for our purposes. Taking for f, then, an average value along the arch for any given loading, the quantity $\frac{f}{E}$ will

replace te of the preceding article. Eq. (25) therefore becomes, if H_s is the horizontal reaction necessary to resist the shortening of the arch,

$$H_s = -\frac{15fI_0}{8r^2}, \qquad (26)$$

the minus sign indicating that H_s acts outwardly, or in the direction of H_t, for a *fall* of temperature.

In getting stresses it will be convenient to assume some value of H_s, as 100,000 lbs. for example, and with this value find the stresses in all the members by means of a stress diagram, the only external forces acting being the two equal horizontal reactions. Then for any particular member, find an average value of f by getting its value at three or four points along the arch, when loaded so as to produce the maximum stress in the member in question; substitute this average value in eq. (26) and determine H_s. Then multiply the stress found from the diagram by this value of H_s and divide by 100,000.

Average values of f due to the maximum rise and fall of temperature may be found once for all, the corresponding values of H_s computed, and these added to the values of H_s as found above.

Stresses due to the shortening of the arch may be prevented to a large extent by constructing it a little longer than the span, and springing it into place. Under certain loads, then, the arch is compressed just enough to make its length normal, and the stresses due to shortening are zero. This initial lengthening of the span by a given amount, Δ, has the same effect as a rise of temperature which lengthens the span by the same amount. Hence, in eqs. (24) and (25), if we substitute Δ for $2cte$, and if H_l is the horizontal thrust due to the initial lengthening, we have

$$H_l = \frac{15EI_0\Delta}{16cr^2}. \qquad (27)$$

This value of H_l is to be added to H_s of eq. (26).

The proper value of Δ to be used in designing the arch may be found by substituting for H_l in eq. (27) an average value of H_s as found from eq. (26), and then solving for Δ.

PARABOLIC ARCH WITH FIXED ENDS; VARIABLE MOMENT OF INERTIA.

194. The Equilibrium Polygon.—Fig. 255 represents such an arch, the ends at A and B being fixed in direction as well as position by the abutments. This being the case, there will

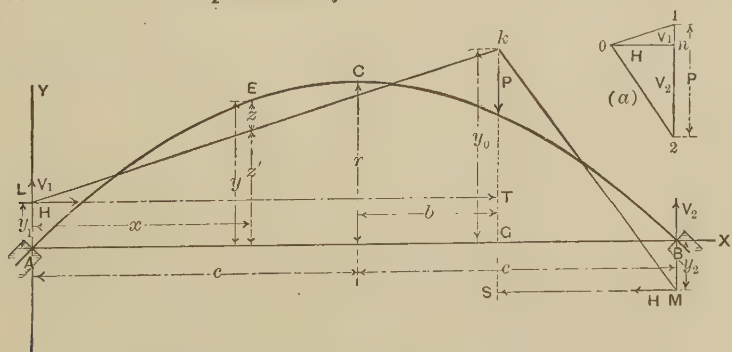


Fig. 255.

in general be some bending moment at A and B, and the equilibrium polygon for a load P will not pass through these two points. Let y_1 be the ordinate from the equilibrium polygon to that abutment farthest from the load P, and y_2 be the ordinate to the other abutment.

Let y_0 be, as before, the ordinate from k, the intersection of the two segments of the polygon, to the line AB. The unknowns are here the three ordinates y_0, y_1, and y_2.

In Art. 188 we have seen that for an arch with no hinges we have the three conditions, from eqs. (7), (8), and (9),

$$\int_A^B \frac{Mds}{EI} = 0, \quad \int_A^B \frac{Mxds}{EI} = 0, \quad \text{and} \quad \int_A^B \frac{Myds}{EI} = 0.$$

Making the same assumptions as in Art. 189 regarding E and I, and putting Hz or $H(y - z')$, for M, the above equations become

$$\int_0^{2c} ydx - \int_0^{2c} z'dx = 0; \quad \ldots \ldots \quad (28)$$

$$\int_0^{2c} xydx - \int_0^{2c} z'xdx = 0; \quad \ldots \ldots \quad (29)$$

$$\int_0^{2c} y^2dx - \int_0^{2c} z'ydx = 0. \quad \ldots \ldots \quad (30)$$

The values of these integrals will now be derived.

Equation (28).—From eq. (14), p. 209, we have $y = \frac{rx}{c^2}(2c - x)$. Substituting this value in the first term of (28) and integrating, we have

$$\int_0^{2c} ydx = \int_0^{2c} \frac{rx}{c^2}(2c - x)\,dx = \frac{4}{3}rc.^* \quad \ldots \ldots \quad (a)$$

The second term of (28) is simply the area of the two trapezoids $ALkG$ and $BMkG$, or

$$\int_0^{2c} z'dx = \frac{y_1 + y_0}{2}(c + b) + \frac{y_2 + y_0}{2}(c - b). \quad \ldots \ldots \quad (b)$$

Subtracting (a) from (b) and reducing, we have from (28)

$$2cy_0 + (c + b)y_1 + (c - b)y_2 - \tfrac{8}{3}rc = 0. \quad \ldots \ldots \quad (31)$$

Equation (29).—Referring to Fig. 255 we see that the first term of (29) is simply the moment of the area between the parabola ACB and the line AB, about the axis AY. This area is equal to $\frac{4}{3}cr$; hence

$$\int_0^{2c} xydx = \tfrac{4}{3}cr \times c = \tfrac{4}{3}c^2r. \quad \ldots \ldots \quad (c)$$

The second term of (29) is likewise the moment of the area $ALkMB$ about AY. This area may be divided into the two rectangles $ALTG$ and $BMSG$, and the two triangles LkT and SMk. Writing out these moments, we have

$$\int_0^{2c} z'xdx = y_1\frac{(c + b)^2}{2} + y_2(c - b)\left[2c - \frac{1}{2}(c - b)\right]$$
$$+ (y_0 - y_1)\frac{c + b}{2} \times \frac{2}{3}(c + b) + (y_0 - y_2)\frac{c - b}{2}\left[2c - \frac{2}{3}(c - b)\right]. \quad . \quad (d)$$

Subtracting (c) from (d), we have after reduction

$$2c(3c + b)y_0 + (c + b)^2y_1 + (c - b)(5c + b)y_2 - 8c^2r = 0. \quad \ldots \quad (32)$$

Equation (30).—From eq. (15), p. 210, we have

$$\int_0^{2c} y^2dx = \tfrac{16}{15}r^2c. \quad \ldots \ldots \quad (e)$$

* This is the well-known area of a parabolic segment and could have been written at once.

For the second term of (30) we have, from A to G,

$$z' = y_1 + \frac{y_0 - y_1}{c + b}x;$$

and if, for the portion from G to B, we take our origin at B, we have for this portion

$$z' = y_2 + \frac{y_0 - y_2}{c - b}x.$$

The value of y is, for both cases, equal to $\frac{rx}{c^2}(2c - x)$ as before. We may then, as in Art. 189, substitute the first value of z' in (30) and integrate from 0 to $(c + b)$, then the second value, and integrate from 0 to $(c - b)$ with origin at B. That is,

$$\int_0^{2c} z'ydx = \int_0^{c+b}\frac{rx}{c^2}(2c - x)\left(y_1 + \frac{y_0 - y_1}{c + b}x\right)dx + \int_0^{c-b}\frac{rx}{c^2}(2c - x)\left(y_2 + \frac{y_0 - y_2}{c - b}x\right)dx. \quad (f)$$

Performing the integrations indicated, collecting terms, combining with (e), and multiplying by $\frac{12c^2}{r}$, we have finally

$$2c(5c^2 - b^2)y_0 + (c + b)^2(3c - b)y_1 + (c - b)^2(3c + b)y_2 - \tfrac{64}{5}rc^3 = 0. \quad . \quad . \quad . \quad (33)$$

We now have three equations, (31), (32), and (33), between three unknown quantities, y_0, y_1, and y_2. To solve these equations, multiply (31) by $(c + b)$ and combine with (32) to eliminate y_1, giving

$$4c^2y_0 + 4c(c - b)y_2 - \frac{8r}{3}(2c^2 - bc) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$

Multiplying (32) by $(3c - b)$ and combining with (33), we have, similarly,

$$4c^2y_0 + 6c(c - b)y_2 - 4r(\tfrac{7}{5}c^2 - bc) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h)$$

Subtracting (g) from (h) and solving for y_2, we have

$$y_2 = \frac{2}{15}\cdot\frac{c - 5b}{c - b}r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Substituting in (g) or (h), we obtain

$$y_0 = \tfrac{8}{5}r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

And finally by substituting in either of the first three equations we have

$$y_1 = \frac{2}{15}\cdot\frac{c + 5b}{c + b}r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Eq. (35) shows that the locus of k is a horizontal straight line at a distance of $\frac{8}{5}r$ above AB. The value of y_1 in eq. (36) varies from $-\infty$ for $b = -c$ to a value of $\frac{6}{15}r$ for $b = +c$. The value of y_2 varies in the opposite way.

195. **Position of Loads for a Maximum Stress in any Member.**—All the reaction lines, Lk, constructed for loads at various points along the arch are tangent to some one curve; like-

wise for the lines kM. In finding position of loads, it will be convenient to construct these curves or envelopes. These curves are hyperbolas, and their equations might be derived; but it will be as convenient to construct them by drawing the reaction lines Lk and Mk for several positions of the load, or for several values of b. The following table gives values of $\frac{y_1}{r}$ and $\frac{y_2}{r}$ for various values of $\frac{b}{c}$, which will enable enough lines to be drawn to locate the curves with sufficient accuracy.

TABLE OF VALUES OF $\frac{y_1}{r}$ AND $\frac{y_2}{r}$ FOR VARIOUS VALUES OF $\frac{b}{c}$.

$\frac{b}{c}$	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	$+0.2$	$+0.4$	$+0.6$	$+0.8$	$+1.0$
$\frac{y_1}{r}$	$-\infty$	-2.0	-0.667	-0.222	± 0.0	$+0.133$	$+0.222$	$+0.286$	$+0.333$	$+0.370$	$+0.400$
$\frac{y_2}{r}$	$+0.40$	$+0.370$	$+0.333$	$+0.286$	$+0.222$	$+0.133$	± 0.0	-0.222	-0.667	-2.00	$-\infty$

In Fig. 256, ac is the locus of k, y_0 being equal to $\frac{6}{5}r$; de is the envelope of the lines Lk and fg is the envelope of the lines kM. If G is the centre of moments for any member DE,

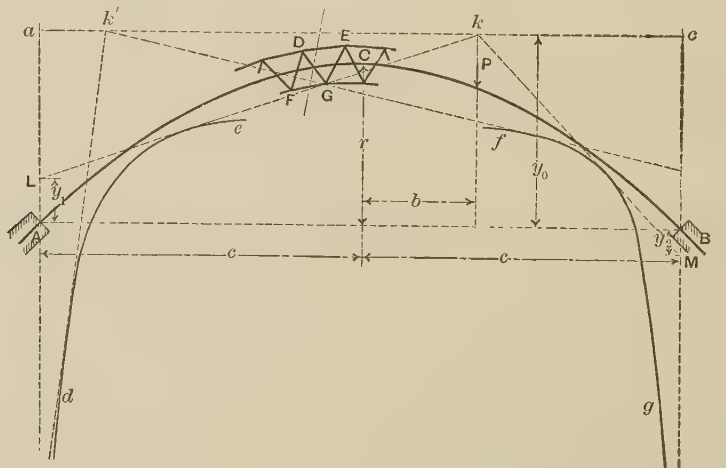


FIG. 256.

the lines Gk and Gk', drawn tangent to de and fg respectively, will determine the position of the loads for a maximum stress of either kind in DE, since loads between k and k' cause positive moment at G, while all other loads cause negative moment. Likewise a line drawn tangent to de and parallel to FG and DE, or towards their intersection, will determine the position of loads for maximum stress of either kind in DG, as in Art. 190.

196. Computation of Stresses.—The stresses can be obtained with sufficient accuracy by graphical methods. A skeleton diagram of the arch should be drawn to a large scale and the equilibrium polygon and force diagram drawn for each joint load, computing y_1 and y_2 by eqs. (36) and (34). The stresses are then found in each member due to each load by the method of Art. 184.

If analytical methods are preferred, the following equations will enable the reactions for any load to be computed:

In Fig. 255 we have, by similar triangles,

$$\frac{V_1}{H} = \frac{y_0 - y_1}{c + b} \quad \text{and} \quad \frac{V_2}{H} = \frac{y_0 - y_2}{c - b} \quad \text{. . . . . (37)}$$

Adding and putting $V_1 + V_2 = P$, we have, after solving for H,

$$H = \frac{P}{\frac{y_0 - y_1}{c + b} + \frac{y_0 - y_2}{c - b}}.$$

Substituting from eqs. (34), (35), and (36) and reducing, we have

$$H = \frac{15}{32}\left[1 - \left(\frac{b}{c}\right)^2\right]^2 \frac{c}{r}.P. \quad \text{. . . . . (38)}$$

By substituting in (37) and reducing, we have

$$V_1 = \frac{1}{4}\left(2 - \frac{b}{c}\right)\left(1 + \frac{b}{c}\right)^2 P \quad \text{. . . . . (39)}$$

and

$$V_2 = \frac{1}{4}\left(2 + \frac{b}{c}\right)\left(1 - \frac{b}{c}\right)^2 P. \quad \text{. . . . . (40)}$$

By means of these equations the components of the reactions due to each load can be computed and the resulting V's added to get the total V due to those loads which act together to cause the maximum stress in any member. The several H's may also be added, and the line of action of the resultant H be found by multiplying each H by the corresponding y_1 or y_2 and dividing the sum of these products by the sum of the H's. The components and line of action of one reaction being known, the stress in the member in question is readily found.

If a circular arch is under consideration, the *reactions* may be found by a similar approximation to that explained in Art. 191.

197. **Temperature Stresses.**—For an arch with fixed ends, the H_t's necessary to resist a lengthening of the span due to a rise of temperature are not applied at A and B, but at some distance y_1 or y_2 above, similarly to the horizontal forces for vertical loads. In this case, for equilibrium, $y_1 = y_2 = h$. The ends of the arch at A and B being fixed in direction, we must have, from eq. (7), when transformed from ds to dx, as in Art. 189, $\int_0^{2c} M dx = 0$, since $M = H_t(y - h)$,

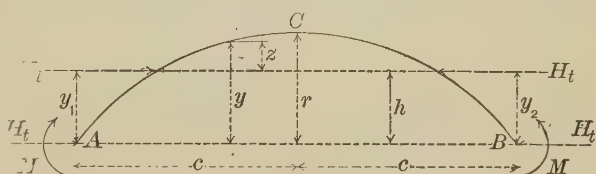


FIG. 257.

$$\int_0^{2c} y dx - \int_0^{2c} h dx = 0. \quad \text{. . . . . (41)}$$

From eq. (a) of Art. 194 we have

$$\int_0^{2c} y dx = \frac{4}{3} rc,$$

and we have also

$$\int_0^{2c} h dx = 2ch;$$

hence substituting in (41) and solving for h, we have

$$h = \tfrac{2}{3} r, \text{ a constant.} \quad \text{. . . . . (42)}$$

Now the lengthening of the span due to a rise of temperature of t degrees above the normal is $2cte$, and we have, as in eq. (23), p. 212,

$$\frac{1}{EI_0}\int_0^{2c} Mydx = 2cte. \quad \ldots \ldots \ldots \quad (43)$$

From Fig. 257 we have, for the moment at any point n,

$$M = H_t z = H_t(y - h).$$

Substituting in (43), we have

$$\frac{H_t}{EI_0}\left[\int_0^{2c} y^2dx - h\int_0^{2c} ydx\right] = 2cte. \quad \ldots \ldots \ldots \quad (44)$$

By eq. (15), p. 210. $\int_0^{2c} y^2dx = \frac{16}{15}r^2c.$

Also we have $\int_0^{2c} ydx = \text{area } ACB = \frac{4}{3}rc.$

Substituting in (44) and solving for H_t, we have

$$H_t = \frac{45EI_0te}{4r^2}. \quad \ldots \ldots \ldots \quad (45)$$

Knowing the amount and line of action of H_t, the resulting stresses may be computed by moments and shears, or in case of a truss the stresses are more readily found by a diagram, the stresses in two or three of the end members being first found by moments.

198. Stresses due to Change of Length from Thrust.—In this case, as in Art. 193, the effect of the shortening of the arch is the same as that caused by a fall of temperature; and, as in that case, we may substitute $\frac{f}{E}$ for te in eq. (45) and we will obtain the outward thrust, H_s, necessary to resist this shortening. Hence

$$H_s = -\frac{45fI_0}{4r^2}. \quad \ldots \ldots \ldots \quad (46)$$

The line of action of this thrust is evidently the same as for H_t, since eq. (42) was derived independently of the cause of the thrust.

The stresses due to change of length are found as in Art. 193.

198*a*. The Full-Spandrel Arch with Two Hinges.—This is coming to be a favorite structure for long spans, especially where stiffness is required, as in railway bridges. The new Niagara arches are of this pattern. The only objection to them has been the great amount of labor involved in their computation. They cannot be solved by the formulæ given in this chapter, since the moment of inertia of the arch increases so enormously from crown to springing, because of the great increase in depth. They can be rigidly solved only by means of the method of least work, which becomes very laborious when applied to so large a structure. The following approximate solution is offered as sufficiently accurate for all practical purposes.

For a load placed at any joint certain reactions are produced, and since these two reactions are in equilibrium with the applied load, the lines of their action lie in the end hinges and intersect upon the vertical line through the loaded point. *The locus of these intersections being found,* all these reaction lines could be drawn, the same as was done in Art. 190.*

In Fig. 257*a* is shown one half of the Niagara railway arch, with the true intersection locus drawn in a dotted line, a', b', ... k', this having been found by the rigid method. If now a parabola be made to fit this true locus as nearly as may be, and its equation found in terms of the constants, span, l, rise of arch, r, and depth of crown, D_c, this may be called the "parabolic intersection locus," and it can be constructed in place, as soon as the general

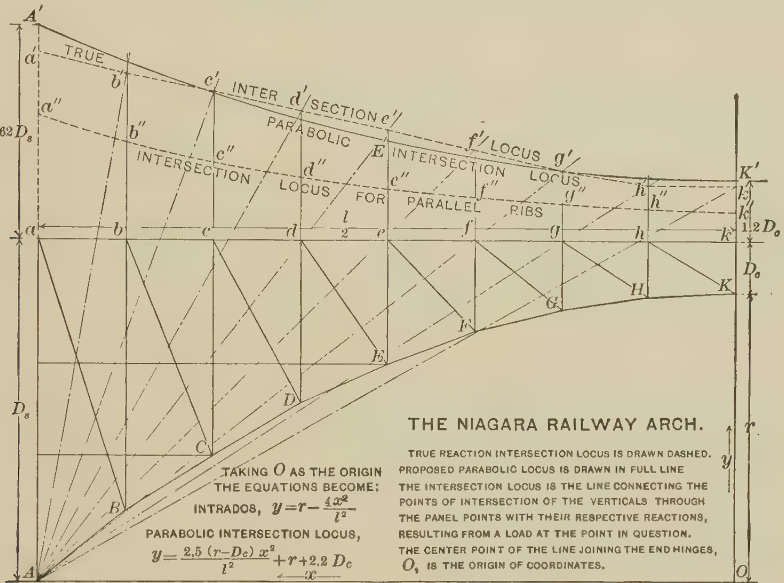


FIG. 257*a*.

dimensions of the arch are known. The reaction lines can then be drawn to the intersections of the verticals with this curve, and the directions, positions, and amounts of these reactions thus at once determined; that is to say, all the peculiar difficulties of the analysis for such a structure would disappear if this intersection locus were known.

In the present example it can be seen by inspection of Fig. 257*a* that the error in the horizontal or vertical components due to an erroneous direction of the reactions, if these were drawn to the parabolic locus instead of to the true one, would be very small. Taking joint E as the worst case, the horizontal component would be in error about 2%, the vertical component about 2½%, and the resultant reaction by less than 2%. The average single error for all the joints would be less than 1%, and these would be compensating, as they are of opposite signs. The resulting effect upon the dimensions of the members, for any combination of loads, would therefore be practically zero. For this one case, therefore, the parabolic locus would have served as well as the true locus, and another curve could be found which would fit the true locus still closer if it were thought necessary.

As to using the equation of this parabolic intersection locus for another bridge, as in a new design, there is this to be said: The intersection locus for a two-hinged arch of constant depth (parallel ribs) is not very different from the parabolic locus here found. The equation of this

* By the rigid method the horizontal components only of these reactions have to be found, since the vertical components are the same as for a beam resting on the same supports.

latter, when referred to an origin at the centre of the line joining the two hinges, or springing points, is (from eq. 19, p. 210)

$$y = \frac{32l^2r}{25l^2 - 20x^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where y = ordinate to locus curve,
l = span of arch,
r = rise of arch,
x = distance out to right or left from the centre.

The curve marked a'', b'', c'', ... k'' is this locus.

The equation of the parabolic locus here used for a full spandrel arch, referred to the same origin, is

$$\textit{Parabolic Locus:} \quad y = \frac{2.5(r - D_c)x^2}{l^2} + r + 2.2D_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

The full-line curve A', E', K' is this locus. It will be seen that if reaction lines were drawn through the springings to the locus, a'', b'', ... k'', these would all have greater horizontal components than those drawn to either of the others. That is to say, the horizontal components of the reactions for an arch of constant depth (but whose moment of inertia increases with the secant of the angle with the horizontal from the crown to the springings, (this condition having been assumed in the derivation of this locus) are all greater than the corresponding horizontal components for a full-spandrel arch; but the difference between actual stresses in a full-spandrel arch and the stresses which would result from using the locus a'', b'', ... k'' would be less than 5% on the average. That is to say, if the reactions were assumed to be the same for a full-spandrel two-hinged arch as they are for such an arch of constant depth or having parallel ribs, the error involved could not exceed 5% on any member.

It is evident, however, that all full-spandrel arches are much more nearly like each other than they are like arches having parallel ribs, and hence it must become evident that in assuming that the Niagara reaction-intersection locus will serve for the solution of all full-spandrel arches, the error involved must be very much less than 5 per cent. Even if the approximate parabolic locus here found be used, the resulting error would certainly never be as much as 2%, and would probably always fall inside of 1%.

When it is considered that other functions of the problem cannot be evaluated nearer than 50%, and that to cover these great uncertainties a factor of safety of 300% or 400% is inserted, is it not very irrational to expend great labor in trying to compute sections for assumed loads to the nearest tenth of 1%?

A somewhat closer approximation to the true locus may be found by using a hyperbola having the equation

$$\textit{Hyperbolic Locus:} \quad y = \sqrt{\frac{8}{3}\left(\frac{x}{l}\right)(Q^2 - K^2) + K^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

where $Q = 1.35r + 1.85D_c$, and $K = r + 2.2D_c$.

This curve coincides at the centre with the parabola here used, but drops a little at the ends.

CHAPTER XV.

DEFLECTION OF FRAMED STRUCTURES AND THE DISTRIBUTION OF LOADS OVER REDUNDANT MEMBERS.

199. **The Deflection of a Framed Structure** for any given loading can be as rigidly computed as the stresses in the members can be found. Although it would seem to be self-evident that the extension or shortening of any main truss member must contribute somewhat to the deflection as a whole, it has long been customary to state that the deflection is almost wholly due to the stresses and strains obtaining in the chords, and but little attention has been given to the web members in this connection. Probably Stoney's two-volume work on "*Theory of Strains in Girders*" (2d ed., London, 1869) has stated this position most emphatically. Indeed he has placed as the frontispiece to the first volume what purports to be a graphical proof of the proposition. He says:

"At first sight it may be thought that the web of the plate girder, or the braced web of the latticed girder, will seriously affect the amount of the deflection curve; but it can be readily shown by carefully constructed diagrams, in which the alterations of length due to the load are drawn to a highly exaggerated scale, that the construction of the web has scarcely any influence on the curvature."

He then constructs a drawing showing this effect on a *very shallow* Warren girder, and seems to prove his proposition. He says of them:

"These diagrams give very interesting results; they show that the curvature of flanged girders is practically independent of change of form in the web and almost entirely due to the shortening of the upper, and the elongation of the lower, flange; and a further inference may be derived from them, viz., that deflection is practically unaffected by the nature of the web, whether it be formed of plates or lattice bars." *

These conclusions prove to be erroneous and have been grossly misleading. In actual structures deflections computed from chord strains only have been found to be about half as great as those actually observed under loads, and the difference has been set down as another illustration of the "universal discrepancy between theory and practice."

In what follows a method will be given for accurately computing the deflection of any point in any framed structure, under any given loading, and it will also be shown that this information may be used to determine stresses in redundant members, otherwise indeterminate.†

200. **Fundamental Propositions.**—Three propositions will first be stated and proved:

PROPOSITION I. *The external work of distortion of a framed structure by a load is equal to the internal work of resistance.*

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* Vol. I., 2d ed., pp. 141, 142.

† The remaining portion of this chapter mostly appeared as a Paper by Prof. Johnson before the Engineers' Club of St. Louis, and was published in the *Jour. Assoc. Eng. Socs.*, for May, 1890.

PROPOSITION II. *The movement of any point in any framed structure subjected to any given load is given by the formula*

$$D = \Sigma \frac{pul}{E},* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $D =$ movement of point under consideration;
$p =$ stress per square inch in any member for any given load;
$l =$ length of any member;
$E =$ modulus of elasticity of any member;
$u =$ factor of reduction;
$\Sigma =$ sign of summation.

That is to say, the quantity $\frac{pul}{E}$ is computed for every member of the truss and the algebraic sum taken as the total movement of the point.

PROPOSITION III. *When there are two or more paths over which a load may travel to reach the support, the load divides itself among the several paths strictly in proportion to the rigidities of the paths.*

The relative rigidities of the paths are indicated by the relative loads required to produce a given deflection, or they are inversely as the deflections produced by a given load which is required to pass wholly over each path in succession. Having established this proposition and computed the rigidities of the paths by Prop. II, we can write enough equations of condition to enable us to solve for any number of redundant members. This proposition applies to all structures, whether framed or composed of masonry, as in the case of a curved masonry dam.

PROOF OF PROPOSITIONS.

Proposition I hardly requires proof. It is simply expressing for the work or energy spent on the structure that which we take for granted as to the force coming upon it; that is to say, action and reaction are equal. The external work of distortion is the product of the load into the deflection of the loaded point divided by two. The internal work of resistance is the sum of the products of the stress produced in each member by its distortion, divided by two.

Thus if W is the external load, acting in any direction,
D the movement of the loaded point *in the direction of the force W*,
P the stress produced in any member,
z the distortion of any member due to the stress P,
then we have

$$\frac{WD}{2} = \Sigma \frac{Pz}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the second member represents the algebraic sum of the quantities $\frac{Pz}{2}$ computed for all the members of the truss for the load W.

From the above it would appear that the total movement D at the *loaded* point is made up of as many parts as there are members in the truss, each one contributing its portion, corresponding to the expression $\frac{Pz}{2}$ for that member. If we represent that portion of D resulting

* Conveniently remembered as the "pull over E" formula.

from the distortion in one member by d, then we may say that the work done at the loaded point corresponding to the work of resistance in any particular member is

$$\frac{Wd}{2} = \frac{Pz}{2}, \quad \text{or } d = \frac{P}{W}z, \qquad (2)$$

In the above discussion we have considered the truss as loaded *only* at the point whose deflection is under consideration, but by so doing we have obtained in (2) a relation between the distortion of any particular member and the linear displacement of our loaded point, which is quite independent of W, since the quantity $\frac{P}{W}$ is a constant ratio for any W whatsoever. In other words, equation (2) is the kinematical relation between the distortion of any member and the movement at the given point produced by that distortion. The ratio $\frac{P}{W}$ may be found by assuming any W at our given point and finding P analytically or graphically for the particular member. Let $\frac{P}{W} = u$, where u may be defined as the stress in the particular member when $W = 1$ lb. Then

$$d = uz.$$

Now suppose the truss loaded in any manner whatsoever and let the resulting distortion of a particular member be z' and its unit stress $= p$. Then $d = uz'$ where u is a known quantity. But we know that $z' = \frac{pl}{E}$, where p has been found analytically or graphically for that particular loading; therefore

$$d = \frac{pul}{E}, \qquad (3)$$

or the movement d at any point due to the distortion $\frac{pl}{E}$ of any one member is equal to that distortion multiplied by a number u, numerically equal to the stress in the member caused by placing 1 lb. at the point in question, *acting in the direction of the movement assumed.*

The student should guard himself here against the too hasty conclusion that the above demonstration applies only to movements of a loaded point. The relation between the movement of *any* point and the deformation of *any* member, from *any* cause, is shown above to be the *same* relation as exists between a force placed at *that* point and the *resulting* stress in *that* member. This latter relation being readily found, it is used for the kinematical relation sought. When the *vertical* movement is desired, the load of 1 lb. at the point acts *vertically.*

The total movement of the point is therefore the sum of all its parts, or

$$D = \Sigma\frac{pul}{E}. \qquad (4)$$

Hence follows Proposition II.

When the vertical deflection is desired of the middle point of a truss, both p and u will always have the same sign for all members when the bridge is fully loaded, and hence their product will always be positive. If any other point be chosen, or if the load be an unsymmetrical one, this product may be negative in a few cases, when the algebraic sum must be taken.

In applying this in practice we always know p, the unit stress in every member for the assumed load on the structure, also its length l, and its modulus of elasticity E.* It remains therefore only to find u for every member. Since this is a pure ratio, equal to the stress in that member for 1 lb. placed at the point, we simply put 1 lb. at the point whose move-

* This modulus of elasticity is now known to be very constant for all grades of wrought iron and steel, and hence the deflection movements can be computed with great accuracy.

ment is desired and find the resulting stress in every member, either algebraically or graphically. This, then, is to be considered as an abstract quantity, being in fact a pure ratio, and the factor of reduction by which the distortion $\frac{pl}{E}$ of each member is reduced to the resulting movement of the point. This point would usually be the end of a cantilevered arm, as in a swing bridge, or the middle point of a truss supported at the ends. It may, however, be any point, and the truss may be of any design, so long as the stresses are all direct tension and compression. The formula can take no account of any bending stresses nor of any lost motion at joints. This formula may be applied to all kinds of trussed forms. The load on the truss may be any assumed load whatsoever for which p, the unit stresses, are computed. But the one-pound load, for finding u for each member, must be put at the point whose movement is desired and must act in the direction of such movement.*

The truth of Proposition III is also nearly self-evident. If the paths be conceived as india-rubber, all taut and ready to act in resisting distortion, then for a given distortion the load will divide itself over the paths in proportion to their several resistances to distortion. But the degree of resistance to a given distortion is a measure of the rigidity of the body. Hence we may say the load divides itself among the paths in proportion to their respective rigidities.

201. Deflection Formula for a Pratt Truss.—For approximate or working values of the deflection of any style of truss we may obtain a formula which is readily evaluated, provided we may assume some average values for intensities of the tensile and compressive stresses. Thus for a Pratt truss, single intersection, of an even number of panels, we may use p_t for the mean tensile stress and p_c for the mean compressive stress, E for the modulus of elasticity, d for the panel length, h for the height of truss, n for the number of panels, and obtain (referring to Fig. 258)

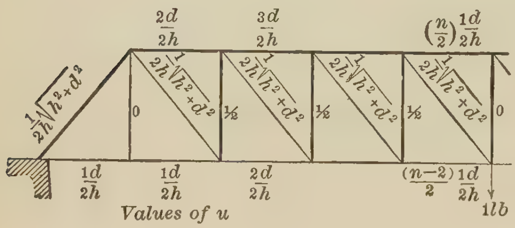

FIG. 258.

$$
\left.
\begin{array}{lll}
\text{For the Upper Chord} & \Sigma\frac{pul}{E} & = \frac{p_c d^2}{8Eh}(n+4)(n-2); \\
\text{For the Lower Chord} & \text{"} & = \frac{p_t d^2}{8Eh}[n(n-2)+8]; \\
\text{For the Web Tension Members} & \text{"} & = \frac{p_t}{2Eh}(n-2)(h^2+d^2); \\
\text{For the Hip Struts} & \text{"} & = \frac{2p_c}{2Eh}(h^2+d^2); \\
\text{For the Verticals} & \text{"} & = \frac{p_c h^2}{2Eh}(n-4).
\end{array}
\right\} \quad . \; . \; . \quad (5)
$$

* This elegant proposition in framed structures was first published in America by Prof. Geo. F. Swain in the *Journal of the Franklin Institute*, April, May, and June 1883. The proposition was first given by Lamé, and afterwards amplified and its application extended by Maxwell and Jenkin, in England, and by Mohr and Winkler, in Germany. In Prof. Swain's treatment the proposition is based on the principle of virtual velocities, and he uses it for finding the stresses in various styles of trusses, particularly arches and continuous girders. The proof given above based on the equality between the external work of deformation and the internal work of resistance is much simpler in its conception and shorter in its method. The proposition may also be proved by the principle of least work. The most frequent use for this proposition probably occurs in connection with the deflection of structures rather than in the computation of stresses.

After the distortions of the several members have been found, the change of position of every joint in the structure may be obtained graphically by means of Williot diagrams, which are themselves an elegant application of the graphical method. See a paper on *The Distortion of a Framed Structure Graphically Treated* by David Molitor in *The Journal of the Association of Engineering Societies*, June 1894. By this method both the horizontal and the vertical displace-

Whence for the whole truss the total deflection of the middle point for a full load is

$$D = \Sigma \frac{pul}{E} = \frac{p_t + p_c}{2Eh}\left[(n+2)\frac{nd^2}{4} + (n-2)h^2\right], \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where p_t = average unit stress of tension members;
p_c = average unit stress of compression members;
E = modulus of elasticity for all members;
h = height of truss in inches;
d = panel length in inches;
n = number of panels in bridge.

It will be noticed that in this case there is nothing to sum but u for each member, as grouped above in eqs. (5), p, l, and E being constant for all the members of a group. Also for such members as give a value of $u = 0$, as for the middle vertical and the end hanger, they are of course omitted, or rather count for nothing in the summation. This means that these two members do not in any way contribute to the deflection of the middle point.

NUMERICAL EXAMPLE.—Take a Pratt truss, 200 feet span, of twelve panels, with a height of 400 inches, or 33 ft. 4 in., the panel length being 200 inches. If the average maximum tensile stress for both dead and live load be taken as 10,000 lbs. per square inch and the average compressive stress as 7000 lbs. per square inch, the total deflection is readily found from eq. (6) to be 2.49 inches.

202. Relative Deflection from Web and Chord Stresses.—By adding the deflection increments due to web members and those due to chord members, we may obtain,

$$\left.\begin{aligned} &\text{For Chord, } \Sigma\frac{pul}{E} = \frac{d^2}{8Eh}\left[(n(n-2)+8)p_t + (n+4)(n-2)p_c\right]; \\ &\text{For Web, } \quad \text{"} \quad = \frac{1}{2Eh}\left[(n-2)(h^2+d^2)p_t + ((n-2)h^2 + 2d^2)p_c\right]. \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

If we should assume that the average stress in the compression members is 0.7 that in the tension members or $p_c = 0.7p_t$, we may write,

$$\left.\begin{aligned} &\text{For Chords, } \Sigma\frac{pul}{E} = \frac{p_t d^2}{8Eh}(1.7n^2 - 0.6n + 2.4); \\ &\text{For Web. } \quad \text{"} \quad = \frac{p_t}{2Eh}\left[(1.7n - 3.4)h^2 + (n - 0.6)d^2\right]. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Whence

$$\frac{\text{Deflection from web}}{\text{Deflection from chords}} = \frac{(6.8n - 13.6)\left(\frac{h}{d}\right)^2 + 4n - 2.4}{1.7n^2 - 0.6n + 2.4} \quad . \quad . \quad . \quad . \quad (9)$$

This ratio increases as $\frac{h}{d}$ increases, and decreases as n, the number of panels, or length of bridge, increases.

For a Pratt truss bridge of 200 feet span, of ten panels, and a height of 30 feet, this fraction becomes $\frac{80}{83}$ or 96.4%.

That is to say, for such a span and for the assumptions made, the deflection from web distortion is about equal to that from chord distortion.

A Whipple truss is simply a pair of Pratt trusses joined into a double intersection system, and hence the deflection of the combination is to be computed by taking one of the systems alone.

---

ments of every joint in the structure are obtained by a single graphical construction. If, however, only the vertical deflection of a single point in a structure is desired, the graphical method offers no advantages over the algebraic method here given.

Thus for such a bridge, 400 feet long, with twenty panels of 20 feet each, the panel length for one system would be 40 feet. Let the height be 60 feet, whence we would have $n = 10$, $d = 40$, $h = 60$.

For this case equation (9) would give exactly the same as before, since n is the same and $\frac{h}{d}$ gives the same ratio.

For the case when $\frac{h}{d}$ is large and n small, as for a bridge, say, of eight panels, of 15 feet each, with a height of 25 feet, we should find

$$\frac{\text{Deflection from web}}{\text{Deflection from chords}} = \frac{142.8}{106.6} = 1.34.$$

That is, for such a case the deflection from the web system is 1.34 times that from the chord system. On the other hand, if the height is about equal to the panel length, and the number of panels is large, as, for instance, $n = 12$ and $h = d$, then we would find that

$$\frac{\text{Deflection from web}}{\text{Deflection from chords}} = \frac{28.4}{60} = 0.47.$$

Or, in this case, the deflection from web would be only about half that from the chords, or one-third the total deflection of the bridge.

In general, it may be said that the deflection of a truss bridge from the web stresses is about equal to that from the chord stresses.* This is directly contrary to the usually received opinions of engineers, who generally assume that the deflection from web stresses is relatively insignificant. It is probable that Mr. Stoney is largely responsible for this generally accredited opinion, as explained above.

203. The Effect on the Deflection of changing the Height of the Truss, everything else remaining the same.—We may differentiate equation (6) for h variable and find

$$\frac{dD}{dh} = \frac{p_c + p_t}{2E}\left[-\frac{(n+2)nd^2}{4h^2} + (n-2)\right],$$

$$\text{"} \quad = \frac{p_c + p_t}{8Eh^2}[(n-2)4h^2 - (n+2)nd^2]. \quad \ldots \ldots \ldots \quad (10)$$

This is the change in the deflection for a change of one inch in the height of the truss.

Putting this quantity equal to zero, and solving for h, we find the height of truss which will give a minimum deflection to be

$$h^2 = \frac{n+2}{n-2} \cdot \frac{nd^2}{4},$$

or

$$h = \frac{d}{2}\sqrt{\frac{n+2}{n-2} \cdot n} \quad \ldots \ldots \ldots \ldots \quad (11)$$

for a minimum deflection.

From eq. (11) we find that for a minimum deflection, or for a maximum stiffness, for given working unit stresses, the height of this stiffest truss has the following values:

TABLE OF HEIGHT OF PRATT TRUSSES OF MAXIMUM STIFFNESS.

Height........	1.73d	1.73d	1.82d	1.94d	2.05d	2.16d	2.27d	2.37d	2.42d
No. of panels..	4	6	8	10	12	14	16	18	20
$\frac{\text{Length}}{\text{Height}}$	2.3	3.4	4.3	5.3	5.9	6.7	7.1	7.7	8.3

* See numerical example in Art. 205.

Or we may say that for maximum stiffness for a given amount of material the height of the truss should vary between $1\frac{3}{4}$ and $2\frac{1}{2}$ times the panel length and from 0.43 to 0.12 of the length of the span, as the number of panels varies from 4 to 20. These heights being very nearly those used in the present practice of bridge designing, there is no new moral to be pointed from this conclusion.

To show that the most economical truss is also the most rigid, let $W =$ actual uniform joint loads, $\Sigma(d) =$ sum of all joint deflections, $P =$ stress in any member due to the loads W, $l =$ length and $A =$ area of cross-section of any member, $p = \frac{P}{A}$, and $E =$ modulus of elasticity. Then the

$$\textit{External Work} = \tfrac{1}{2}W\Sigma(d) = \textit{Internal Work} = \tfrac{1}{2}\Sigma\left(P\frac{Pl}{AE}\right) = \frac{1}{2E}\Sigma(Ppl). \quad . \quad . \quad . \quad (1)$$

If we assume p to be a constant for the entire structure for this full load, then

$$\frac{WE}{p^2}\cdot\Sigma(d) = \Sigma(Al) = \textit{volume of the entire truss}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Evidently when $\Sigma(Al)$ is a minimum $\Sigma(d)$ must also be a minimum. Hence the lightest truss is also the most rigid for constant stress per square inch in the members.

204. Inelastic Deflection.—It must also be understood that neither the general equation, (4), nor the particular one for a Pratt truss, (6), makes any provision for the inelastic deflection due to any slack at the joints from pin-holes being larger than the pin. Since this slack would be only one-half the difference between diameter of hole and that of pin at each end of the member, probably an average value of 0.02 inch would be about right for every member of a well-constructed bridge. Now this affects the web as well as the chord members, and it is equivalent to lengthening every tension member and shortening every compression member, except those in the top chord, by this amount. The top chord would be considered one member from end to end. The effect of such lengthening or shortening of any member on the deflection is given by the same ratio u, so that the deflection caused by this slack on any member is $0.02u$, using the u for that member. In other words, the total inelastic deflection from joints would be

$$\text{Inelastic Deflection} = D' = 0.02\Sigma u, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

remembering to take but one u for the top chord, and that one for the end panel. If the top chord is cut at every panel joint and the sections rest on pins, which is seldom the case, then all the u's must be summed.

For a Pratt truss, of an even number of panels, with top chord provided with riveted cover plates and planed joints, we have

$$\text{Inelastic Deflection} = D' = 0.02\Sigma u = \frac{1}{200h}\left[\left(\frac{n^2}{2} - n + 8\right)d + 2(n-4)h + 2n\sqrt{h^2+d^2}\right]. \quad (13)$$

For the Pratt truss, used in the above example, where $n = 12$, $h = 400$, and $d = 200$, being 200 feet long, we find the inelastic deflection to be equal to 0.494 inch. For the conditions named, therefore, the total deflection of this truss would be $2.49 + 0.49 = 2.98$ inches, or say 3 inches.

This is, of course, the total deflection due to both dead and live load, when the maximum load is on, or it is the amount by which the bridge should be cambered to bring it horizontal under its maximum working load.

205. Numerical Computation of Deflection.—The truss shown in Fig. 259 is one from a highway bridge on Twenty-first Street, St. Louis, erected in 1892. The following is a tabular computation of the deflection of this bridge under its full live load only. The deflection is found for the centre (f), and hence the 1-lb. load is placed at this point for computing

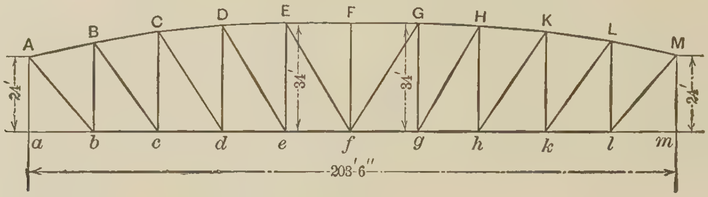

FIG. 259.

u. Two Maxwell diagrams gave both u and the total stress in the members for full load. Since both the loading and the deflection point are symmetrical, we may find the values for one-half only of the truss and then multiply the final result by two for the corresponding members on the other half of the truss.

COMPUTATION OF DEFLECTION AT CENTRE OF TRUSS.

Member.	Length in Inches.	Stress from Live Load.	Area of Section in Square Inches.	Stress per Square Inch. p.	$\frac{pl}{E}$.	u.	Contribution to Truss Deflection $= \frac{pul}{E}$.
							in.
AB	247	+ 169,000	60	+ 2,800	+ .0247	+ 0.38	+ 0.0094
BC	246	+ 274,000	60	+ 4,570	+ .0401	+ .69	+ .0277
CD	245	+ 337,000	77	+ 4,380	+ .0383	+ .96	+ .0368
DE	244	+ 364,000	77	+ 4,730	+ .0381	+ 1.21	+ .0461
EF	244	+ 377,000	77	+ 4,890	+ .0426	+ 1.51	+ .0643
bc	244	− 165,000	31.5	− 5,240	− .0457	− 0.37	+ .0169
cd	244	− 272,000	49	− 5,550	− .0483	− .68	+ .0328
de	244	− 333,000	61.2	− 5,440	− .0474	− .95	+ .0450
ef	244	− 361,000	69.1	− 5,220	− .0455	− 1.20	+ .0546

Half sum = + 0.3336
2

Total Deflection from the distortion of Chords = 0.6672 in.

Member.	Length in Inches.	Stress from Live Load.	Area of Section in Square Inches.	Stress per Square Inch. p.	$\frac{pl}{E}$.	u.	Contribution to Truss Deflection $= \frac{pul}{E}$.
*Ab**	377	− 257,000	49	− 5,240	− .0706	− 0.58	+ .0409
Bc	408	− 228,000	33	− 6,910	− .1007	− .51	+ .0514
Cd	435	− 110,000	24	− 4,580	− .0712	− .49	+ .0349
De	458	− 53,000	15	− 3,530	− .0577	− .47	+ .0271
Ef	476	− 31,000	11.2	− 2,770	− .0471	− .60	+ .0283
Bb	327	+ 148,000	36.5	+ 4,050	+ .0473	+ .44	+ .0208
Cc	360	+ 93,000	24.5	+ 3,800	+ .0489	+ .41	+ .0200
Dd	387	+ 41,000	22.5	+ 1,820	+ .0252	+ .41	+ .0103
Ee	408	+ 7,800	22.5	+ 350	+ .0051	+ .40	+ .0020

+ .2357
2

Total Deflection from distortion of Web = + 0.4714 in.
Total Elastic Deflection of Truss for Live Load = 1.1386 in.
Percentage of Deflection due to Web Members = 41.4

should take account of the deflection which may be attributed to the web system. Otherwise the computed stresses in the chords, for this case, would be about twice their actual amount.

206. Camber of Bridges.—A bridge is cambered partly for appearance and partly that the top chord joints may come to a square bearing when the maximum load is on. It of course adds nothing to the strength of the bridge.

The camber should equal the total deflection, being the elastic deflection due to the maximum dead and live loads and the inelastic deflection due to lost motion at the joints.

The general formulas for these two kinds of deflections are (4) and (12), and for a Pratt truss they are given by equations (6) and (13) respectively.

* The member *Aa* being a heavy tower, it is not included in the table.

A Whipple or any double intersection truss with parallel chords may be considered as two Pratt trusses which must deflect together; and hence equations (6) and (13) may be applied to them, being careful to take n and d for one system only. That is, n would equal one-half the total number of actual panels and d would equal the length of two panels of the double system.

There is no question but that present practice of allowing the upper chord to spread at top where the sections join, relying on its closing up when the maximum load is on, is a poor one. If the cover plates are thin and the number of rivets few, then these joints do actually open and close at top for every passing train. If heavy cover plates on both top and sides and a sufficient number of rivets are used, then as this joint is riveted up, so it will remain. In this case it should be riveted up in a horizontal position, and then cambered up to its final position by bending it as a whole from end to end. This can readily be done if the chord be riveted up before the bridge is swung. It would have to be supported horizontally and all pins inserted except one at bottom. After riveting up the top chord the truss could be jacked up and the last joint closed. The upper chord will then bend bodily from end to end and there will be no movement of the splice. One can see that the upper chord of a 200-foot span will readily bend 3 inches, if this bending is continuous, without serious damage; whereas, if it had all to occur at, say, five or six joints, it would be a source of weakness.

In case the ends of the channels should be cut square so as to come to a full bearing for the maximum load, but riveted up with the camber in, then for all ordinary loads the stress is all thrown on the bottom flanges of the chord channels. It may be asserted that no serious damage would result, but it is at best a very unskilful way of carrying this stress across the spliced section.

207. Errors caused by Neglecting Deflections due to Web Distortions.—In all computations of stresses in metallic structures based on the deflections, or distortions, of the structure from internal stresses, the ordinary formulæ give erroneous results.

Thus in computing the stresses in a continuous girder caused by the settlement of its supports, or the temperature stresses in an arch between fixed abutments, or the stresses in a stiffening truss of a suspension-bridge, the stresses are in all these cases functions of certain assumed or computed distortions. These distortions are always assumed to result in certain bending moments only, and to be wholly provided for by the strains of the chords. No assistance in accommodating this distortion is credited to the web members. By whatever proportion this distortion is absorbed by the web members, the stresses in the chord members are reduced below those now computed for them.

In the case of metallic arches, and stiffening trusses on suspension-bridges, the distortion absorbed by the web is small because the trusses are shallow as compared to their length.

The relative absorption by the web of the horizontal distortion of the St. Louis arches due to a change of temperature is only about one-sixth of the total amount. In the computations it was all assumed to go into the chords, or tubes.

In the case of a continuous girder, however, the depth would be great in proportion to the span, and here the computations of stresses due to a settlement of supports (not supports out of level as usually stated, for if the bridge be built to rest on such supports the formulæ apply)

208. The Determination of Stresses in Redundant Members.*—The above ready method of computing deflections accurately, together with the use of the principle expressed in Proposition III, enables us to find the stresses in redundant members, or in other words to

* For a very complete paper by Prof. Wm. Cain, M. Am. Soc. C. E., on this method of finding stresses in redundant members, based on the principle of least work, see Trans. Am. Soc. C. E., Vol. XXIV., p. 265 (Apr. 1891).

solve any composite system, however many combinations there may be in it. To do this we must compute the deflections of each elementary system for a given load. The reciprocals of those deflections represent the relative rigidities of the different combinations, and since the load is to be divided in proportion to these reciprocals, we thus obtain one less number of equations than we have systems. The other equation results from the sum of the parts equaling the whole. This then gives us as many equations as there are systems, and we can determine what part of the total load passes over each combination, and hence solve for the stresses in such combination. If any one member forms a part of two or more combinations, the total stress in it is the sum of all the stresses caused by the several combinations of which it is a part.*

The application of the formula $\frac{pul}{E}$ and the solution for redundant members will be illustrated by an example.

By putting in the members AG and BG, Fig. 260, the system becomes composite. The

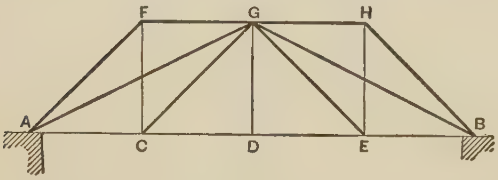

FIG. 260.

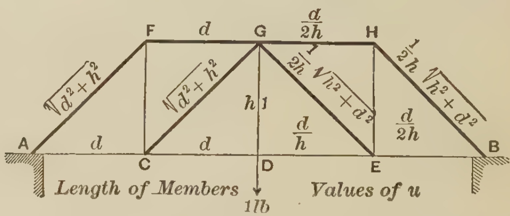

FIG. 261.

first system is shown in Fig. 261, and the second in Fig. 262. The members AB and GD are common to both systems.

The lengths of the members are given on the left half, and the values of u for 1 lb. placed at D are given for all the members on the right half of Figs. 261 and 262.

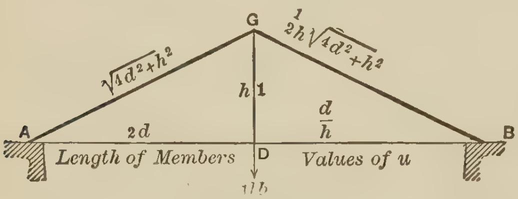

FIG. 262.

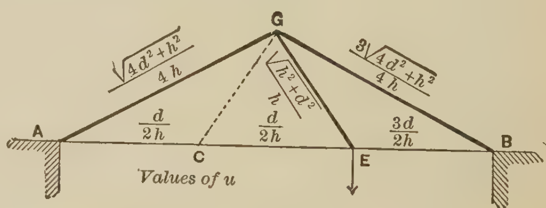

FIG. 262a.

The load at D will divide itself between the systems shown in Figs. 261 and 262; the load at E between the systems 261 and 262a, and similarly with the load at C, provided CG and GE can take both tensile and compressive stress.

By Prop. III the load at D will be divided between the two systems directly as their rigidities, or inversely as their deflections for any given load. But when the joint D is fully loaded we may suppose the whole bridge is fully loaded. In this case all the members would have their working unit stresses which may be supposed to be the same, as p_t for all the tension members and p_c for all compression members in the combination. At least the parts should be proportioned to give nearly these uniform values, and it will be here assumed that they have them.

Since we wish the deflection at the point D, we put the 1-lb. load there for finding u, the resulting stress in each member. These values are given for both systems on the right-hand portion of the figures.

* In the case of initial stress in counters in any panel, the shear in this panel from external loads divides itself between the diagonals by increasing the stress in one and diminishing it in the other. Thus if a_1 and a_2 are the areas of cross-section of the two counters, and S is the shear from external forces, the portion of this shear taken by a_1 is $\frac{a_1}{a_1 + a_2}S$ and the portion taken by a_2 is $\frac{a_2}{a_1 + a_2}S$, being additive in the one case and subtractive in the other, so long as both remain under stress.

The lengths also being there given, and p_t and p_c and E being known, we may write at once the values of $\frac{pul}{E}$ for each member:

Thus for the first truss we have

For lower chord, $\frac{pul}{E} = \frac{3d^2}{h} \cdot \frac{p_t}{E}$.

For upper chord, " $= \frac{d^2}{h} \cdot \frac{p_c}{E}$.

For verticals, " $= 2h\frac{p_t}{E}$.

For inclined struts, " $= 2\left(\frac{d^2 + h^2}{h}\right) \cdot \frac{p_c}{E}$.

Whence the total deflection of the first truss, for a full working load, producing the unit stresses p_t and p_c is

$$\text{Deflection} = \Sigma\frac{pul}{E} = \frac{3d^2 + 2h^2}{Eh}(p_t + p_c). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For the second truss, for a full working load, producing the unit stresses p_t and p_c, we have

For the lower chord, $\frac{pul}{E} = \frac{4d^2}{h} \cdot \frac{p_t}{E}$.

For the vertical, " $= h \cdot \frac{p_t}{E}$.

For inclined struts, " $= \frac{4d^2 + h^2}{h} \cdot \frac{p_c}{E}$.

Whence for the entire truss

$$\text{Deflection} = \Sigma\frac{pul}{E} = \frac{4d^2 + h^2}{Eh}(p_t + p_c). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

These two deflections are equal, as seen in equations (14) and (15), when $3d^2 + 2h^2 = 4d^2 + h^2$, or when $d = h$. In this case the load at D would divide itself equally between the systems.

Next taking the load at E, and the two systems shown in Figs. 261 (load at E instead of at D) and 262a, placing now the 1-lb. load at E in each system, and assuming again that the distribution of this load is desired when the whole bridge is loaded, giving a p_t unit stress in all tension members and a p_c unit stress in all compression members, it can be shown, as before, that

$$\text{Deflection at } E \text{ of first system (Fig. 261)} \quad = \frac{2d^2 + h^2}{Eh}(p_t + p_c). \quad . \quad . \quad . \quad (16)$$

$$\text{Deflection at } E \text{ of second system (Fig. 262}a) = \frac{4d^2 + h^2}{Eh}(p_t + p_c). \quad . \quad . \quad . \quad (17)$$

If $d = h$, these become $\frac{3h}{E}(p_t + p_c)$ and $\frac{5h}{E}(p_t + p_c)$ respectively.

Therefore the load at E divides itself in such a way that $\frac{5}{8}$ of the load at E goes on the first system (Fig. 261) and $\frac{3}{8}$ on the second system (Fig. 262a).

Evidently the same would hold true for the load at C. In other words, for the bridge fully loaded, and for $d = h$, $\frac{1}{2}$ the load at D and $\frac{5}{8}$ of the loads at C and E would be carried by the Pratt truss system, and the remainder by the system $AGBEDC$. It will be noted also that the stress in GC and GE is compression in the former and tension in the latter, and hence the real stress in these members is the algebraic sum of the two.

From the above it is evident that for any given combination of members, and for any given system of loads, it can be determined what portion of the total load goes on each system, and hence what the stresses are in every member.

209. Direct Measurement of Bridge Strains.—To learn the actual strains in members of a bridge truss, or the strains produced by rolling loads at various velocities, some kind of apparatus must be attached directly to the members themselves, and their elongation or compression noted under the various conditions of loading. The apparatus shown in Fig. 263 has been successfully employed in Europe on riveted structures. It has a length of 1 meter between attachments, and registers to 140 lbs. per square inch (0.1 kg. per sq. mm.) when read to one tenth of one division on the scale. Any good instrument-maker could readily construct a similar one to give stress readings by means of a vernier to 100 lbs. per square inch. If the length be taken as five feet, this would require reading this length to the nearest 0.0002 inch. The diameter of the upper disk should be about 10 in. and the length of indicator about 6 in. In working out the relative lengths of the arms take $E = 28,000,000$. The method of attachment is important. It should be fastened by means of pointed steel set-screws. For eye-bars this is an easy matter, but for compression members it is more difficult. The attachment should in all cases be symmetrically made on opposite sides of the member, in the plane of its neutral axis. It should be graduated to read in thousands of pounds per square inch.†

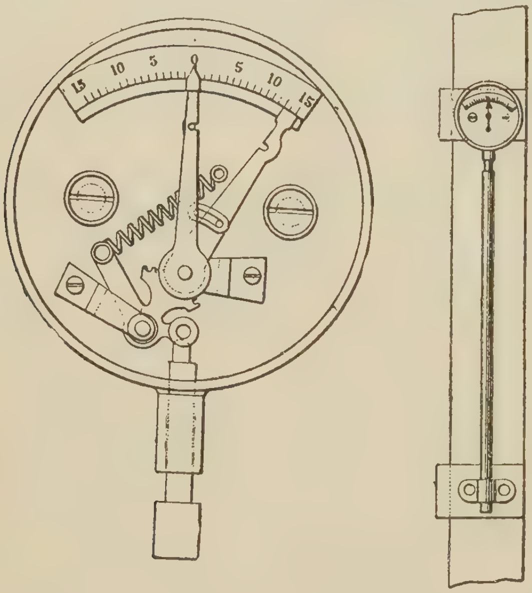

FIG. 263.

209*a*. Vibration of Bridges from Synchronous Impacts.—Every beam or truss carrying a given total load has a definite period of vibration independent of its amplitude. This subject has been investigated theoretically and practically,* and the periodic time of vibration has been found to be

$$t = 0.0093 l^2 \sqrt{\frac{p}{EI}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where t = time of vibration in seconds;
l = length of truss or beam in feet,
p = total load in pounds on one truss or beam, uniformly distributed;
E = modulus of elasticity of the material;
I = moment of inertia of the truss or beam in *foot-units.*

When this periodic time of vibration coincides closely with any synchronous impact, as the stepping of a horse, or the revolution of unbalanced locomotive drivers, or the passing of a low joint on a railway bridge, the amplitude of the vibration rapidly increases until a comparatively small impact may by repetition produce serious deflections with their corresponding stresses in the members. Diagrams showing this action on railroad bridges have been obtained by Prof. S. W. Robinson (Trans. Am. Soc. C. E., Vol. XVI., p. 42), who has also fully discussed the problem.

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* By M. Deslandres, in *Annales des Ponts et Chaussées* for December, 1892. His equation is $t^2 = 6.5 \frac{a^4 \frac{p}{g}}{EI}$, where g is gravity, $a = \frac{l}{2}$, and kilogrammetre units are used. Various diagrams showing the effects of synchronous impacts are given.

† See an excellent paper on this subject, giving results of experiments, in *Engineering News*, May 9, 1895, p. 300. These results show that the actual strains agree very closely with the theoretical statically computed strains, even on the hip-verticals, under a speed of train of 55 miles per hour.

PART II.

STRUCTURAL DESIGNING.

PART II.

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CHAPTER XVI.

STYLES OF STRUCTURES AND DETERMINING CONDITIONS.

210. General Considerations.—The selection of the proper structure to use to fulfil given conditions is a problem which would have to be solved as a special case for any general rules which may be established. The determining factors are so variable that experience in the location and selection of the proper structure is a much safer guide than any rigid formulæ. There are, however, some general principles and approved rules which are worthy of attention and which may be used without any very great error by those who lack the needed experience. The two important problems which confront a constructing engineer at the beginning in the building of a new bridge are, 1st, the best location for the bridge; and 2d, the proper structure to use. The items of first cost and cost of maintenance must be considered together with the probable life of the bridge and its safety in the case of a derailed train. In general, it may be assumed that the item of first cost is the only one which may be varied for any special case as the other items depend upon the details of the construction which would be similar for various locations of the bridge or for different structures. Economy in first cost will then be assumed as the desired result. The first cost of a bridge will vary as the quantities of the material which are necessary in its construction vary. The quantities which may be varied are the masonry in the piers or the *substructure*, and ironwork in the spans or the *superstructure*. The first cost will be a minimum when the combined cost of the substructure and the superstructure is a minimum.

211. The Selection of the most Economical Location for a Bridge.—It will be assumed that the cost of building the road to any of the crossings under consideration is constant, or that the difference in cost on this account may be readily estimated and taken into account in the comparisons. When there are no local conditions limiting the length of span used and if the same style of bridge (i.e., deck or through) may be used, the cheapest bridge is manifestly the shortest. However, if the spans in each of the locations were fixed by local conditions and were different, the shorter bridge requiring the longer spans, it may be possible for the longer bridge to be the cheaper. The amount of iron in a single-track pin-connected span is very closely approximated by the following formula:

$$W = al^2 + kl,$$

where $W =$ total weight of iron in the span, $l =$ length of the span, a is a constant which may be taken as 5, and k another constant which varies with the live load used in proportioning the structure. The term al^2 is usually assumed to represent the weight of the trusses, and kl to represent the weight of the floor system. If this were true, both a and k would vary with the live load used, and no doubt a closer approximation to the weight could be made by varying both a and k, but in an approximate formula little benefit is derived from such refinement. For a live load of 100-ton engines with the usual train load and specifications a may be taken as 5 and k as 350.

The weight per linear foot, w, of a bridge of spans of l length would be, therefore,

$$w = \frac{W}{l} = 5l + 350.$$

If L is the total length of the bridge, the total iron weight would be

$$L(5l + 350).$$

This formula could be used to find the iron required at the various crossings, and therefore the cost of the superstructure. Combining this with the estimated cost of the substructure, the cheapest crossing can be very closely determined.

As an illustration of the application of this formula we will assume alternative single-track crossings, one of which requires five 200-ft. spans, and the other three 300-ft. spans.

For the five 200-ft. spans, ... $w = 1350$, $Lw = 1,350,000$ lbs.
For the three 300-ft. spans, ... $w = 1850$, $Lw = 1,665,000$ lbs.

If the price of iron was five cents per pound, there would be a difference of \$15,750 in favor of the longer bridge in the cost of the iron. If this were more than enough to pay for the two extra piers required the longer bridge would be the cheaper.

212. The Proper Structure to Use at a Given Crossing.

1st. *Bridges of One Span.*—The cost will vary with the kind of bridge used. This question will be considered in detail in a following article on the kinds of bridges, classifying them according to their method of construction into plate girder bridges, riveted truss bridges, and pin-connected truss bridges. Assuming the *kind* of bridge as constant, the cost will depend upon the selection of the cheaper style of bridge, i.e., deck or through. The deck bridge may generally be assumed to be the cheaper style. By referring to Fig. 264, it will be seen that

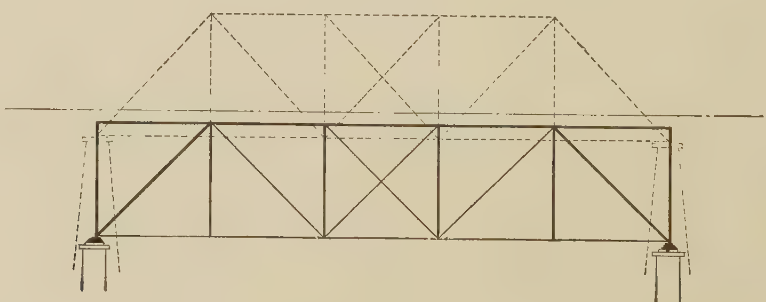


FIG. 264.

there is a saving in the height of the necessary piers about equal to the depth of the truss due to the use of the deck span. There is an increase in the weight of the necessary iron of about ten per cent for the deck span, but this will never offset the saving in the masonry. Fig. 264 represents the case where the abutments are simple piers, the approaches to the span being trestles of wood or iron. Fig. 265 represents the case where the abutments are retaining-walls. There would generally be no increase necessary in the masonry on account of the span being supported on the retaining-walls, so that the choice would be between the deck and through spans, as shown in the figure. There would be very little saving, if any, accomplished by using the deck span except for the shorter spans, say under 180 feet long. For

these shorter spans the trusses of the deck spans may be placed closer together than would be permissible for through bridges on account of the clearance necessary between the trusses of the latter. This would reduce the length of the floor-beams and save some iron.

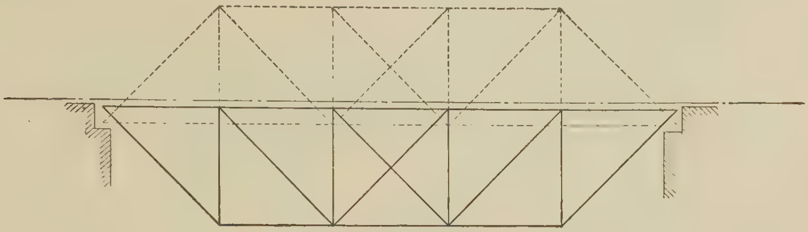


FIG 265.

The iron floor system may be dispensed with in the shorter deck spans by supporting the cross-ties directly on the top chord and a further reduction in the required amount of iron made.

2d. *Bridges which may be of One or More Spans.*—When the span lengths are fixed by local conditions the problem becomes equivalent to that of a series of bridges of one span each, the only open question being the kind of bridge and the style (i.e., deck or through). When the span lengths may be varied *and the style the same,* a very close approximation to the correct length of span to use for economy in total cost can be obtained as follows:

Let A = cost of the two end abutments *in dollars ;*
B = cost of the floor and that part of the iron weight which remains constant *in dollars ;*
C = cost of one pier in dollars assumed as constant;
l = length of the bridge in feet;
x = number of spans ;
p = price of iron per pound *in dollars ;*
y = total cost of bridge *in dollars*;
a = weight per foot of a span b feet in length.

The weight represented by a is that part of the weight per linear foot which varies directly as the length of span (see Art. 211). Then the weight per foot of a bridge with spans of $\frac{l}{x}$ length will be $\frac{a}{b}\frac{l}{x}$, and the total cost of the bridge will be

$$y = A + B + (x - 1)C + l\frac{al}{bx}p.$$

From this we find y to be a minimum when

$$\frac{l}{x} = \sqrt{\frac{b}{ap}C}.$$

For pin-connected truss spans $\frac{b}{a} = \frac{1}{5}$. If iron costs five cents per pound, $\frac{l}{x} = \sqrt{4C}$; and if iron costs four cents per pound, $\frac{l}{x} = \sqrt{5C}$. The economical lengths of spans, in feet, for piers of

various costs are given in the following table, assuming the iron to cost four and five cents per pound respectively.

Cost of One Pier.	Economical Length of Span in Feet.		Cost of One Pier.	Economical Length of Span in Feet.	
	Iron 4 cts. per lb.	Iron 3 cts. per lb.		Iron 4 cts. per lb.	Iron 3 cts. per lb.
$2000	100	115	$10,000	225	258
3000	122	141	12,000	245	283
4000	141	163	14,000	265	305
5000	158	183	15,000	283	316
6000	173	200	18,000	300	346
7000	187	216	20,000		353
8000	200	231	24,000		400

As the length of the bridge is fixed, such a length of span may be readily selected which will make the total cost of the bridge a minimum.

The assumptions made in deriving the formula for the economical length of span are not liable to be in error enough to affect the choice of the proper length of span to use if the total length of the bridge is fixed, as this length can rarely be divided into an even number of spans of the economical length. It is well to err by choosing a *longer* span than the economical one rather than a shorter span.

213. **Kinds of Bridges.**—The metal structures in common use in the railway practice of the United States may be divided into the following kinds according to their method of construction:

* 1st. *Plate Girder Bridges*, used generally up to 75 feet span and often to a little over 100 feet.

2d. *Riveted Truss or Lattice Bridges*, the ordinary lengths of which range between 70 and 110 feet. The extremes of length, however, are not very definitely marked, as some engineers use them as short as 50 feet and others use them for almost any length.

3d. *Pin-connected Truss Bridges*, the lengths of which range from 80 feet to the longest span in use.

A review of the generally accepted merits of each of these kinds of construction will be given for the purpose of aiding the student or engineer in the selection, when occasion demands, of the proper structure. Economy will be the deciding factor in the selection of any particular design, and it is for the engineer to decide what is true economy for his special case. The cost of the maintenance of the floor and the stiffness of the structure against vibration, which is a measure of the probable life of the bridge, together with the degree of safety insured in case of the derailment of a train, are items which affect the true cost in addition to the first cost and should have due consideration. While discussing plate-girder construction some of the floors in general use will be described. A few empirical formulæ for the iron weights of bridges will also be given, from which it is believed that it will be safe to derive the iron weight to be used in the calculation of the stresses, and also which will show *relatively* the amount of iron in the different kinds. These formulæ have been found to give good results for engine loadings of about one hundred tons each with the ordinary train load. They also assume that the structures are carefully designed under some of the approved general specifications now in use, and that the bridge is not askew, that the alignment of track on the bridge is a tangent, and that the design is the most economical in proportions and details.

214. **The Plate Girder Bridge** now generally used for spans of 75 feet or less is deservedly the most popular kind of construction in use. There is a minimum opportunity for error in

* This classification to include rolled I-beams under Plate Girders.

its design and calculation, small chance for defects due to faulty workmanship, and when once put in place requires little attention, except the necessary application of paint to prevent rusting. It is the simplest and cheapest in manufacture for short spans, but for lengths over

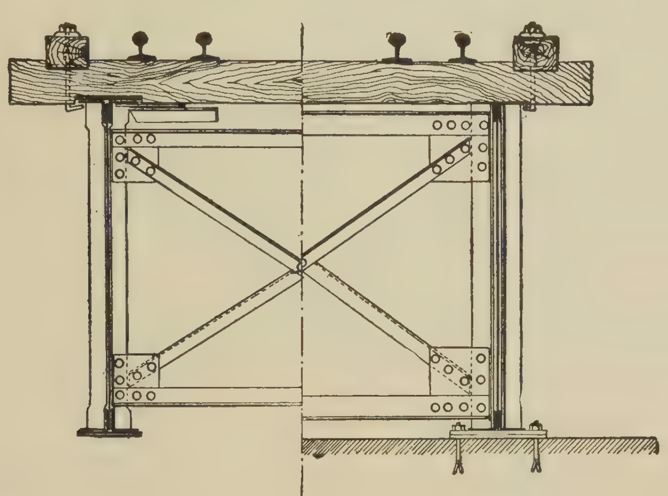


FIG. 266.

60 feet it is the most expensive in first cost of all, due to the fact that it requires more iron in its construction than any other. Its use is gradually extending to longer spans in spite of the cost. For the longer spans over 75 feet the girders become so very heavy that

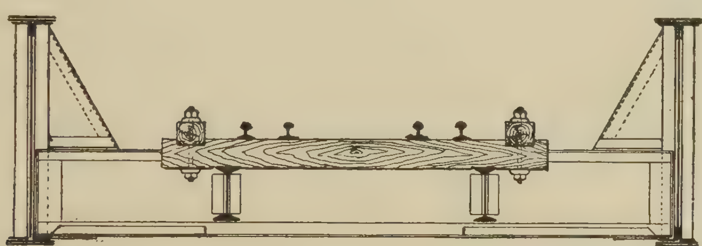


FIG. 267.

transportation becomes difficult. The longer spans require deep girders for economy in iron, and the necessary web plates can only be obtained in short lengths, making frequent web splices necessary, thus adding weight and increasing cost. It is always preferable to have plate

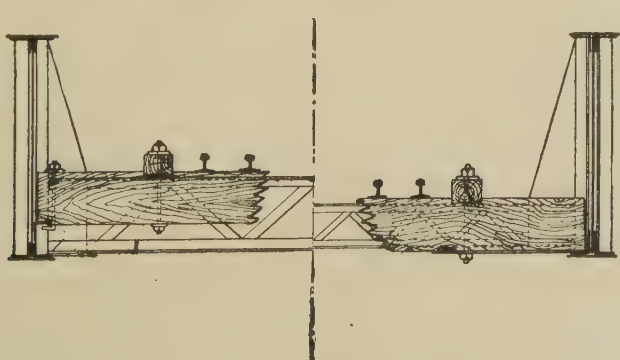


FIG. 268.

girders riveted up completely in the shop by power, leaving the bracing between the girders as the only parts requiring hand riveting after the girders are in place on the piers.

Fig. 266 shows cross-sections at ends and in middle of span for a single-track deck plate girder bridge with the usual floor and mode of attachment. The floor shown is the one most generally used. The detail construction of the ironwork is simple.

Fig. 267 shows a cross-section of the most common style of plate girder through bridge with the same floor and method of attachment as was shown in Fig. 266.

Fig. 268 shows the cross-section of two styles of through plate girder bridge in which the iron floor system shown in Fig. 267 is dispensed with and the track supported on large cross-ties resting on shelf-angles riveted to the webs of the girders or on the bottom flange angles.

Fig. 269 shows longitudinal sectional views of through plate girders with various kinds of the solid iron floor on which a ballasted roadbed is carried over the bridge.

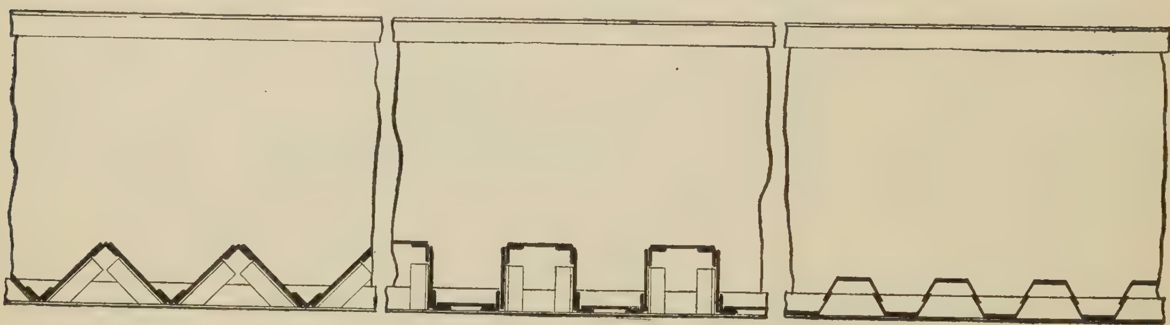


Fig. 269.

The iron weights of the various styles of plate girder bridges* may be approximated by the following formulæ:

Deck Plate Girder Bridge,	$w = 9l + 110$;
Through Plate Girder Bridge,	$w = 8\frac{3}{4}l + 300$ (iron floor system),
" " " "	$w = 9\frac{1}{4}l + 150$ (large ties on shelf or flange angles);
" " " "	$w = 10l + 600$ (solid iron floor);

when w = iron weight per linear foot, l = length over all of girders (i.e., length of necessary bed plates plus distance centre to centre of bed plates).

215. Bridge Floors.—Figs. 266, 267, 268, and 269 show nearly all the floors now much used. Attention will be called, however, to a very popular modification of the floor shown in Fig. 267, which consists of the addition of safety stringers or two additional stringers placed from 2 to 4 feet outside of the main stringers. They are usually made one half the strength of the main stringers, and are used for safety in case of the derailment of the train. Sometimes the main trusses or girders of the bridge are used in place of the safety stringers, as, for example, in Fig. 267 the cross-ties may be extended to rest on shelf-angles riveted to the webs of the girders.

The floor shown in Fig. 268 may be called the standard and is the most generally used of all. The details of the guard-rails and the many various methods of attaching cross-ties and guards to the supporting ironwork will not be considered. The usual attachment is to fasten ties and guards to the stringer by means of a three-quarter-inch hook-bolt passing through every third or fourth cross-tie. The wooden guard-rails are spiked or bolted to every tie. The wooden guard-rail is usually placed with its inside face about 1 foot from inside face of rail or gauge. The inner upper corner is sometimes covered by an angle-iron put on with countersunk-head spikes. Cross-ties are usually spaced with openings of from 4 to 6 inches between them.

* Single track.

The floors shown in Fig. 268 are used when a cheap bridge is wanted and when the distance from the rail to the under side of the bridge is limited. The objection to them is that the cross-tie is under strain from the load and must necessarily be very large and subject to continual inspection, and be promptly removed in case any defect which impairs its strength is noticed. The replacement of the cross-ties is a difficult task, and in order that they may be put in at all the stiffener angles on the inside of the web plate must be so spaced as to allow, room to get them in. The size of cross-tie to use is determined by the load to be carried assuming the heaviest axle load to be supported by three ties, allowing an extreme fibre stress of from 800 to 1000 lbs. per square inch.

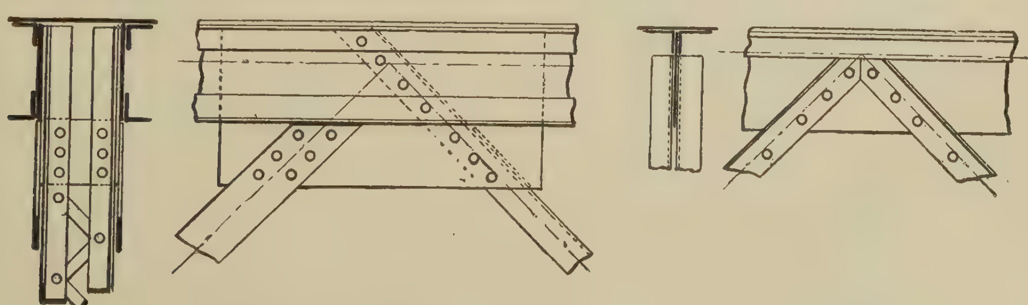
The solid iron floors shown in Fig. 269 are not very generally used as yet, the railroads which do use them much being those in the North and East. They are expensive owing to the amount of iron necessary in their construction, but are undoubtedly the safest, most rigid and permanent of all the various kinds. They can be built when the distance from the rail to the lowest part of the bridge is very small. With this kind of floor the corrugations are filled with concrete, usually bituminous, and the standard ballast on top of this. The cross-tie may be embedded in concrete in the corrugations in case the floor must be very shallow. The iron in corrugated floors alone weighs from thirty to forty pounds per square foot of floor.

216. Riveted Truss or Lattice Bridges are used for the shorter spans on account of their cheapness, the plate girder being the only allowable substitute, while for the longer spans when they sometimes supplant pin-connected trusses it is generally so because of a prevailing belief that they are stiffer structures and safer under a derailed train. There is a range between the limits of about 80 and 100 feet, in which the riveted truss is undoubtedly preferable to the pin-connected, but beyond that it is very doubtful whether it is correct to adopt it in preference to a well-designed pin-connected bridge if the price is in the latter's favor, as it usually will be. Either kind of construction will make satisfactory bridges for the longer spans if well designed.

In the design of riveted trusses the use of multiple systems of web bracing has become generally accepted good practice, and now it is the preferable bracing for all lengths of span. In the multiple intersection trusses the effect of the distortion of the truss under load at the joints of the web and chord members is much reduced by the rigid joints made at the intersections of the web members themselves. For very deep trusses the effect of the distortion is also small, so that single intersection trusses will make good bridges.

The secondary stresses in the riveted truss make the calculation and design of them very much more unsatisfactory than for a pin-connected truss.

The riveted truss may be built with T-chords or box-chords (Fig. 270), the latter being the more expensive, but generally the preferable design.



BOX-CHORDS. FIG. 270. T-CHORDS.

For "pony trusses" or half-through bridges the box-chord will always make the better bridge. The web bracing should always be symmetrical with the plane of the centre of the

truss, and care should be taken to get the neutral axes of all members meeting at a joint to intersect at one point.

The riveted truss requires more metal in its construction than the pin-connected, and its manufacture is nearly as expensive. For long spans the trusses of which cannot be shipped riveted up complete, all connections of web members with the chords must be hand-riveted.

The iron weights of riveted truss bridges* may be approximated by the following formulæ

Deck bridge, cross-ties on top chord.................... $w = 7l + 200$;
Through bridge, iron floor system...................... $w = 7l + 300$;
w = iron weight per linear foot; l = length centre to centre of bearings.

217. **Pin-connected Truss Bridges** are more generally used than the riveted, and owe their popularity to their cheapness, facility of erection, and adaptability to almost all conditions. They are used for all spans over 80 feet, although the general practice now is to limit them to spans over 100 feet. In the early days of bridge building the use of the pin connection was imperative, but of late years the facilities of constructors in general for handling larger pieces and for doing better field work in erecting and riveting, and the capacity of the railroads for transporting heavier and longer pieces than formerly, have made it possible to build better riveted trusses. It is chiefly because of this that the riveted truss is supplanting the pin-connected for the shorter spans where short panels and very light trusses would make the latter expensive. The pin-connected truss can be more satisfactorily designed and proportioned because of its almost absolute freedom from secondary stresses.

The single intersection truss is now the accepted practice in the construction of pin-connected truss spans, the recent use of the inclined top chord correctly and the Petit truss making it possible to build long spans cheaply owing to the lighter and shorter members that the latter types require.

The two general types of truss in common use are the Warren or Triangular truss and the Pratt truss. Another type of truss deserving of mention is the "Pegram truss, designed and patented by Geo. H. Pegram, M.A.S.C.E. It is fully described in Part I of this book.

218. **The Warren or Triangular Truss** usually requires less material in its construction than the Pratt, and for short spans, particularly deck spans, is commonly used. It is also often employed when it is desirable to avoid adjustable members in the truss, because the symmetry of the truss is maintained and a much better looking structure secured than if the Pratt were used with stiffened ties in place of counter ties. An objection often urged against the Warren is that the continual reversal of stress in the web members causes the bearings on the pins to wear or indent the latter. An increase of the bearing surface on the pins would be the remedy for this. A great objection to this truss for through spans is that the floor-beams must either be suspended from the pins or vertical posts introduced at each panel point to rivet the beams to above the pin; the former requirement making the design of an efficient lateral system difficult, and the later adding material with very little compensation. This truss is now rarely used for long spans.

219. **The Pratt Truss**, including in this type all single intersection trusses with vertical intermediate posts, is by far the truss most generally used. It may be termed the standard truss in American practice. In its details it is simpler than any other form, and has stood the test of continual use without any diminution of its popularity. In economy of material and cost it is only second to the Warren for the shorter spans, while for the longer inclined chord and Petit truss spans it requires less material and is less expensive. The recent long spans built in this country are generally Petit trusses.

* Single track.

The iron weight in pin-connected bridges may be approximated by the following formulæ (single track):

Deck span, cross-ties on top chord...... $w = 5l + 250$;
" " iron floor system............ $w = 5.0l + 475$;
Through span, iron floor system........ $w = 5.0l + 350$.

220. Swing Bridges.—For clear openings under 60 feet the plate girder bridge is generally used, the alternative being the riveted truss for these shorter spans. For more than 60 feet clear opening the pin-connected truss is used because of its cheapness, and as it also has necessarily the advantages of stiff top and bottom chords.

221. Cantilever Bridges.—Cantilever bridges are now used only in those cases where it is impossible or impracticable to erect simple spans. They are not economical in material, as they rarely, if ever, require as little iron in their construction as a simple span. Owing to their lack of rigidity, a simple span bridge is preferred if it is practicable to build one at the crossing.

222. The Three-hinged Arch Bridge will very probably be preferred in those cases where the bridge must be erected without any temporary false-work, and where the abutments may easily be made sufficient to take the thrust. A bridge of this kind can readily be designed to erect as a cantilever. The three-hinged arch requires less material in its construction than a simple span.

223. For Long Spans, including those over 600 feet, the cantilever, the arch, the cantilever-arch, and cantilever-suspension are the types from which the engineer has now to select the proper structure. The cantilever-arch and cantilever-suspension bridges require the least quantity of material, and are probably the designs to be depended upon for spans longer than the practical limit set for simple spans.

223*a*. The Essential Elements of Good Bridge Drawing.*—Some of the elements which go into the make-up of good drawings are:

(1) Plain lines not too finely drawn, which will show clearly on a blue print; (2) legible figures which cannot be mistaken, with foot and inch marks clearly shown in their proper places; (3) plain, legible, free-hand, conventional lettering; (4) a simple arrangement of views, dimensions, and notes, which should be strictly adhered to, particularly in the same set of drawings; (5) a careful arrangement of dimension figures, to be shown similarly on all drawings; (6) duplication of dimensions to save patterns and templates and to facilitate manufacture; (7 arrangement of parts, location of rivet and bolt holes, etc., to suit the convenience of shopmen, erectors, painters, and inspectors; (8) a good system of marking the component parts of a structure, for the convenience of shopmen and erectors; (9) a systematic arrangement of bills of material on the drawings; (10) a clear conventional title in the same location on all plans of the same set; (11) a standard size for sheet of drawing paper; (12) proper dates and signatures.

If the foregoing specifications are observed in the making of drawings, others which are essential will naturally follow, resulting in good drawings, and the draughtsman whose rule is to make good drawings will find that each set of plans will show improvements on previous ones.

A draughtsman should be acquainted with the strength of materials, the kinds which are available for his use, and the comparative cost of them. He should be informed on mill, shop, and foundry practice, and the methods employed by masons, carpenters, erectors, and other mechanics who take part in the construction of a bridge. He should be familiar with good specifications for all classes of work which enter into the construction of bridges, and should be thoroughly acquainted with the specifications for the work in hand.

* From a lecture by Onward Bates, M. Am. Soc. C. E.

CHAPTER XVII.

DESIGN OF INDIVIDUAL TRUSS MEMBERS.

224. The Fatigue of Metals.*—Elaborate experiments in Germany and elsewhere have shown that the ultimate strength of metal from a single test is no indication of its ability to resist repeated stresses.

If $f =$ initial unit strength;

$p_1 =$ greatest unit load that may be repeated an unlimited number of times;

$p_2 =$ greatest unit stress that can be reversed an unlimited number of times;

then we may call

f the initial strength;
p_1 the repetition limit;
p_2 the reversal limit.

That is to say, p_1 is the greatest unit stress that can be wholly removed an unlimited number of times, while p_2 is the greatest unit stress which can alternate from tension to compression an unlimited number of times. Both p_1 and p_2 are below the elastic limits for wrought-iron, p_1 being about 26,000 pounds per square inch and p_2 about 16,000 pounds per square inch. The ultimate strength may be put at 50,000 pounds and the elastic limit at 30,000 pounds per square inch.

But in practice the maximum load is seldom wholly removed, and often the reversed stresses are not equal, so that in general we may say that we have a maximum unit stress m and a minimum unit stress n. This minimum stress may or may not be of the same kind (tension or compression) as the maximum stress. For these general cases it is desirable to know what the values of m and n may be for any material.

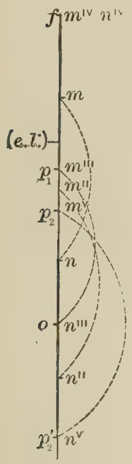


FIG. 271.

In Fig. 271 the relation of these limiting and working stresses is shown graphically. Distance along the line represents stress per square inch, measured either way from o as an origin.

Thus:
$$of = f = \text{initial strength};$$
$$op_1 = p_1 = \text{repetition limit};$$
$$op_2 = p_2 = \text{reversal limit}.$$

The stress of represents the strength of the material from a single test. The point (*e.l.*) is the primary elastic limit, but it plays no important part in this discussion. The stress op_1 may be put wholly on and off an unlimited number of times, and the stress op_2 may be changed to op_2' an unlimited number of times.

If all the stress is not removed each time, but only a part of it, then the maximum stress m may be greater than p_1, so that if a certain portion of om, represented by on, be left on permanently, then m will lie somewhere in the field p_1f; and the greater is the ratio of the fixed to the varying stress, the more nearly will m approach f. Similarly, if only a *part* of the stress is repeated with the opposite sign, then the greater of the two stresses, m_3, will lie somewhere in the field p_1p_2, and the less, n_3, will lie between o and p_2',

* This and the following article are from a paper by Prof. Johnson, read before the Engineers' Club of St. Louis, November 16, 1887. For a much fuller discussion of this question see Chap. XXVII of Prof. Johnson's *Materials of Construction* (John Wiley & Sons, 1897).

and the more nearly n is numerically equal to m, the more nearly will m approach to p_2. Thus we may say that the maximum stress, m, will always be p_1 plus a portion of $p_1 f$ when n is of the same kind of stress, and minus a portion of $p_1 p_2$ when n is of the opposite kind of stress.

It has been found by experiment that the following is approximately true: *The maximum stress is equal to the repetition stress p_1 plus or minus such a part of the adjacent field as the minimum stress is a part of the maximum stress.* Or,

$$m = p_1 + \frac{n}{m}(f - p_1) \text{ for } \textit{repeated stresses}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$m = p_1 - \frac{n}{m}(p_1 - p_2) \text{ for } \textit{reversed stresses}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These are the formulæ to use for determining the breaking stress m when the smaller and fixed stress n is known and when these stresses succeed each other an unlimited number of times.

This is also shown in Fig. 271.

Thus when n lies above o, m is above p_1; when n lies below o, m is below p_1;
" n is at o, m is at p_1; " n is at p_2' $(= -p_2)$, m is at p_2;
" n is at m (static load), m is at f.

Evidently $m - n$ is always the portion of the stress removed each time, corresponding to the movable load on bridges.

For every variation of stress there is a corresponding distortion, and the product of the mean value of the variable stress into the distortion is the work, in foot-pounds, done on the material in distorting it. When the stress is partly or wholly removed the member recovers a corresponding portion of its distortion, and this is work done by the member against the external forces. Now it is this *work* which *wears out* or *fatigues* the material. A given material can recover its length an infinite number of times if the work demanded each time be not too great, and hence it is capable of doing an infinite amount of work if done in sufficiently small amounts. If too much be required at any one time, however, then it wears out or becomes fatigued, and finally breaks down, very much the same as an overworked muscle.

Now the amount of work done at any one time has been shown to be the mean stress into the distortion. In order to keep this maximum single effort a constant, it is evident that, as the mean stress increases, the distortion must diminish. In other words, as the maximum load increases, the *variation in load* $(m - n)$ must decrease. But for m increasing, $m - n$ can decrease only by the more rapid increase in n; therefore, it is only by increasing the static load n that the total load m may be raised above p_1. And since a single effort, equal to f, will rupture the piece, it is evident that as m approaches f the limits between which we can continue to work our specimen indefinitely will become narrower by the approach of n towards m.

Similarly, as the lower limit passes below the zero point and is therefore changed into a stress of the opposite sign, the mean value of the stress diminishes, and hence the distance through which the piece can be worked increases, this maximum range being $2p_2$ when $n = p_2'$ and $m = p_2$.

These "new formulæ" for dimensioning are therefore seen to be very simple in form and rational in conception.

225. **Working Formulæ.**—Formula (1) (named after Prof. Launhardt) may be put in the form

$$m = p_1\left(1 + \frac{f - p_1}{p_1} \frac{\text{min. stress}}{\text{max. stress}}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and formula (2) (named after Prof. Weyrauch) may be put in the form

$$m = p_1\left(1 - \frac{p_1 - p_2}{p_1} \cdot \frac{\text{min. stress}}{\text{max. stress}}\right). \quad \ldots \ldots \quad (4)$$

Experiments made by Wöhler, Spangenberg, Bauschinger, and Baker show that for structural iron and steel p_1 is approximately $\frac{1}{2}f$, and p_2 is approximately $\frac{1}{4}f = \frac{1}{2}p_1$. Therefore we have

For stresses of one kind $$m = p_1\left(1 + \frac{\text{min. stress}}{\text{max. stress}}\right)^*, \quad \ldots \ldots \quad (5)$$

and

For stress of opposite kinds $$m = p_1\left(1 - \frac{1}{2} \cdot \frac{\text{min. stress}}{\text{max. stress}}\right)^*. \quad \ldots \ldots \quad (6)$$

Since p_1 is approximately one half the ultimate strength for wrought-iron and mild steel, and is a common factor in the right-hand members of these equations, the factor of safety may be introduced in p_1, as, for instance, for like stresses, the formula $p = 9000\left(1 + \frac{\text{min.}}{\text{max.}}\right)$ would imply a factor of safety of three for an indefinite number of repetitions of the maximum load in wrought-iron.

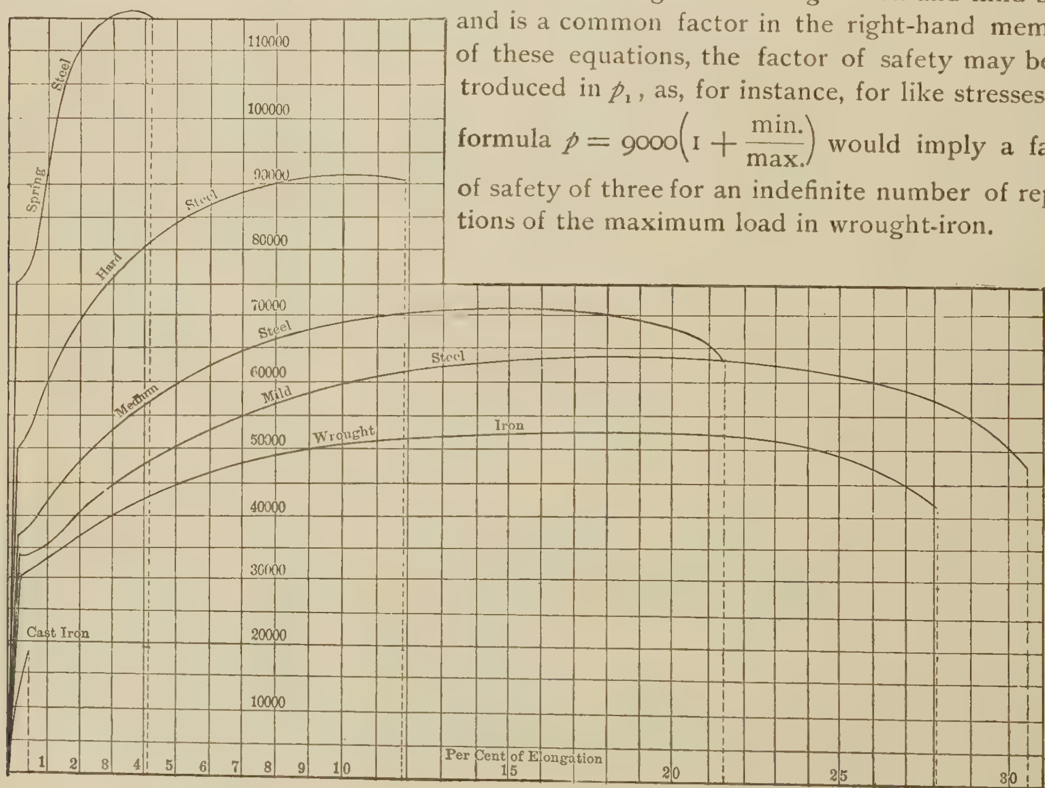


FIG. 272.

226. The Usual Methods of Proportioning Individual Truss Members Subjected to Varying Direct Stresses.—It is only within recent years that the fatigue of the metal due to varying stress has been taken into consideration in proportioning individual truss members in which the stress is always of one kind, either tension or compression. It is not the universal practice at present. If the stress varied in kind, the practice has been to use a lower stress per square inch on such members.

* Prof. Johnson now proposes the formula $m = \dfrac{p_1}{1 - \dfrac{1}{2}\dfrac{\text{min. stress}}{\text{max. stress}}}$ to replace both of these formulæ. See his *Materials of Construction*, p. 545.

The more general practice is to proportion compression members without any reference to the fatigue of the metal, and in many specifications tension members are treated in the same way. The Pennsylvania Railroad Company was probably the first to introduce in its specifications formulæ which were based on the fatigue of the metal from varying stress. For varying stresses of the same kind, all compression or all tension, the company's formula is

$$b = a\left(1 + \frac{\text{minimum stress}}{\text{maximum stress}}\right),$$

when $b =$ allowed stress per square inch in pounds, and a is a constant determined by the quality of the material used and the kind of stress, tension or compression, being 7500 for double rolled iron in tension, 7000 for plates or shapes in tension, and 6500 for plates or shapes in compression. For varying stresses of opposite kinds,

$$b = a\left(1 - \frac{\text{maximum stress of lesser kind}}{2\,.\,\text{maximum stress of greater kind}}\right).$$

For compression b is to be still further reduced by Gordon's formulæ for long struts.

Cooper's specifications allow a unit stress twice as great for dead load as for live load, and thus give practically the same results as those obtained by the Pennsylvania R. R. formulæ. For varying stresses of opposite kinds Cooper specifies that the maximum stress of either kind shall be increased by eight tenths of the maximum stress of the lesser kind and the piece proportioned by the usual formulæ for whichever of these combined stresses would require the larger area of cross-section. Thus, if a piece is subjected to alternating stresses of 100,000 pounds tension and 80,000 pounds compression, it must be proportioned for 164,000 pounds tension or 144,000 pounds compression, whichever requires the larger area of cross-section.

The old method of proportioning a piece subjected to alternating stresses is to increase the maximum stress of either kind by eight tenths of the maximum stress of the other kind, and to proportion the piece by the usual formulæ for whichever of these combined stresses requires the larger area of cross-section. It is evident that numerical formulæ in the form of (5) and (6) are the most rational that can be used.

227. **Tension Members** may be divided into four kinds, according to their method of construction:

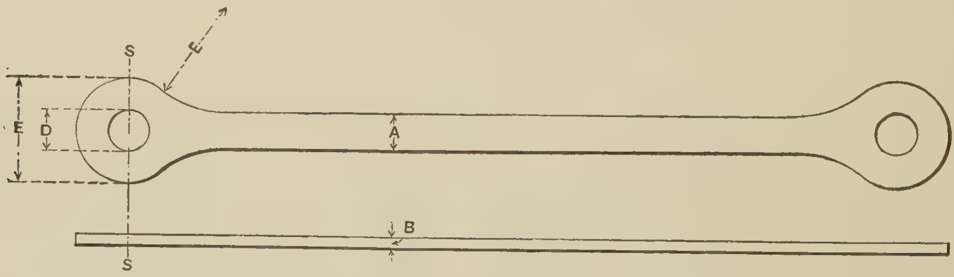
1st. Eyebars.
2d. Square or round rods.
3d. Single shapes.
4th. Compounded sections.

1st. *Eyebars* are used for the main tension members of pin-connected trusses. They are now generally made of low steel of an ultimate strength of from 56,000 to 66,000 pounds per square inch, the methods of manufacture securing a more satisfactory and reliable product from that metal than from iron. Steel eyebars are made by forging or upsetting the eye or head of the bar in a die, and subsequently reheating and annealing the finished bars previous to boring the pin-holes. The quality of the finished bar depends principally on the annealing process. Wrought-iron eyebars are made by piling and welding the head or eye on the body of the bar by various methods. Welding is always an unreliable process, and defects due to imperfect welds and burnt material are difficult to detect.

In the following table are given the standard sizes of steel eyebars (i.e., width of bar and diameter of eye) manufactured by the Edge Moor Bridge Works and which fairly represent the present practice.

It will be noted that there is a specified limiting minimum thickness of bar. This has been found to be advisable on account of the manufacture, and because thin bars will buckle in the head when under strain. This minimum thickness increases with the width of the bar and the diameter of the eye. The thickness of the bar may be made anything greater than this minimum, but a thickness of two inches for bars six inches wide and under is rarely exceeded. As it is desirable to use small pins, and therefore small eyes, on account of the cost of manufacture and the amount of material needed to make the eyes, a thickness of bar will generally be selected that will make these results possible. The thicker the bar the greater will be its leverage and stress, and consequently the greater will be the bending moment on the pin which would result in requiring a larger pin and a larger eye.

EDGE MOOR STANDARD STEEL EYEBARS.



A	B	E	D	C	A	B	E	D	C
Width of Body of Bar.	Minimum Thickness of Bar.	Diameter of Head or Eye	Diameter of Largest Pin hole.	See Note.	Width of Body of Bar.	Minimum Thickness of Bar.	Diameter of Head or Eye.	Diameter of Largest Pin-hole.	See Note.
3	$\frac{3}{4}$	$6\frac{1}{2}$	$2\frac{1}{2}$	33	6	$\frac{7}{8}$	$13\frac{1}{2}$	$5\frac{1}{4}$	37
3	$\frac{3}{4}$	8	4	33	6	$\frac{7}{8}$	$14\frac{1}{2}$	$6\frac{1}{4}$	37
3	$\frac{3}{4}$	9	5	33	6	1	$15\frac{1}{2}$	$7\frac{1}{4}$	37
4	$\frac{3}{4}$	$9\frac{1}{2}$	$4\frac{1}{8}$	33	7	$\frac{15}{16}$	$15\frac{1}{2}$	$5\frac{5}{8}$	40
4	$\frac{3}{4}$	$10\frac{1}{2}$	$5\frac{1}{8}$	33	7	$\frac{15}{16}$	17	$7\frac{1}{8}$	40
4	$\frac{3}{4}$	$11\frac{1}{2}$	$6\frac{1}{8}$	33	8	1	17	$5\frac{3}{4}$	40
5	$\frac{3}{4}$	$11\frac{1}{2}$	$4\frac{5}{8}$	37	8	1	18	$6\frac{3}{4}$	40
5	$\frac{3}{4}$	$12\frac{1}{2}$	$5\frac{5}{8}$	37	8	1	19	8	37
5	1	13	$6\frac{1}{8}$	37	9	$1\frac{1}{8}$	$19\frac{1}{2}$	7	39
5	1	14	$7\frac{1}{8}$	37	9	$1\frac{1}{8}$	$21\frac{1}{2}$	9	39

NOTE.—In column C are given the percentages of excess of area of the eye on line SS over the area of the body of the bar when the largest pin-hole is in the eye.

It is always better to use an eye the diameter of which is about two and one quarter times the width of the bar. In extreme cases the diameter of the eye may be made two and one half times the width of the bar, but it is never desirable to exceed this, as the cost and difficulty of manufacture increase rapidly if larger eyes are used.

The amount of material required to make an eye, or the extra length of bar required beyond the centre of the pin, is approximately $L = \frac{\pi E^2}{4A}$, when L = length of bar required beyond the centre of the pin and E and A as given in the table.

Eyebars are now made as large as 12 × 3 inches with eyes 27 to 30 inches in diameter.

The tests of full-sized eyebars do not, as a rule, give as high results in ultimate strength as the small specimen tests of the material. The small specimen test pieces have usually an area of one half of one square inch. This difference in the results of the tests has caused a great amount of friction between manufacturer and customer. Recent study of the subject shows that a difference in the ultimate strength of

the small test piece and the full size bar is to be expected. Mr. F. H. Lewis, M.A.S.C.E.*, after an exhaustive study of this problem, finds that the losses in ultimate strength of full sized eyebars are due to three distinct causes, viz.:

1. The small specimens are so cut from the original bar as to give results which are in excess of the *average* value of the bar.
2. There is a legitimate loss in ultimate strength due to the annealing of the finished eyebar.
3. The steel is not perfectly homogeneous and the chances of a "soft" or weak place are greater in a large bar than they are in the small test piece.

As the result of his study of this subject, Mr. Lewis recommends 60,000 pounds per square inch as the minimum ultimate strength of the specimen test and 56,000 pounds per square inch as the minimum ultimate strength for full-sized steel eyebars; the maximum ultimate strength to be 70,000 pounds per square inch in both cases.

2d. *Square or Round Rods* are used for all members requiring sleeve-nuts or turn-buckles. They are the most commonly employed for counter-ties, lateral and sway rods, and for the rods of Howe trusses. They are generally made of wrought-iron when adjustable, as the prevailing belief is that steel is weakened very much by the sharp unfilleted corners of the screw-threads. Steel bars with upset screw ends are, however, being made with success, and all rational objection to them is disappearing.

The various ways in which rods are used are shown in the figures on pages 248 and 249. Fig. 273 shows the forged or solid eye, Fig. 274 shows the ordinary loop eye, Fig. 275 shows the rod with two screw ends with either the ordinary nut or the clevis at the ends, and Fig. 276 shows the sleeve-nut or turn-buckle detail which would be used for lengthening or shortening a rod with solid or loop eyes. Tables giving the standard dimensions of these various details are also given.

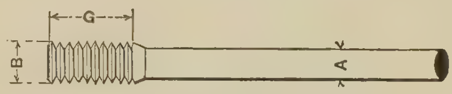
EDGE MOOR STANDARD UPSET SCREW ENDS.

Dimensions in Inches.

SQUARES.

A	B	C	D	G	H
Side of Square.	Diameter of Screw.	Area of Bar.	Area at Root of Thread.	Length of Screw.	Threads per Inch.
¾	1⅛	.56	.694	3¾	7
⅞	1¼	.76	.891	3¾	7
1	1½	1.00	1.295	4	6
1⅛	1⅝	1.27	1.496	4¼	5½
1¼	1⅞	1.56	2.051	4½	5
1⅜	2	1.89	2.302	4½	4½
1½	2¼	2.25	3.023	4¾	4½
1⅝	2⅜	2.64	3.298	5	4
1¾	2½	3.06	3.719	5	4
1⅞	2¾	3.53	4.622	5¼	4
2	2⅞	4.00	4.924	5½	3½
2⅛	3	4.52	5.428	5½	3½
2¼	3¼	5.06	6.510	5¾	3½
2⅜	3½	5.64	7.548	6	3¼
2½	3¾	6.25	8.641	6¼	3
2⅝	4	6.89	9.998	6½	3

ROUNDS.



A	B	C	D	G	H
Diameter of Bar.	Diameter of Screw.	Area of Bar.	Area at Root of Thread.	Length of Screw.	Threads per Inch.
⅝	⅞	.307	.420	3½	9
¾	1	.442	.550	3½	8
⅞	1⅛	.601	.694	3¾	7
1	1¼	.785	.891	3¾	7
1⅛	1⅜	.994	1.057	4	6
1¼	1½	1.227	1.295	4	6
1⅜	1¾	1.484	1.744	4¼	5
1½	1⅞	1.767	2.051	4½	5
1⅝	2	2.073	2.302	4½	4½
1¾	2⅛	2 405	2.651	4¾	4½
1⅞	2¼	2.761	3.023	4¾	4½
2	2⅜	3.141	3.298	5	4
2⅛	2½	3.546	3.719	5	4
2¼	2⅝	3.976	4.159	5¼	4
2⅜	2¾	4.430	4.622	5¼	4
2½	3	4.908	5.428	5½	3½
2¾	3¼	5.939	6.510	5¾	3½
3	3½	7.068	7.548	6	3¼
3¼	3¾	8.295	8.641	6¼	3
3½	4	9.621	9.993	6½	3

NOTE.—Area at root of thread in all cases greater than area of bar.

* See Trans. Am. Soc. C. E., 1892.

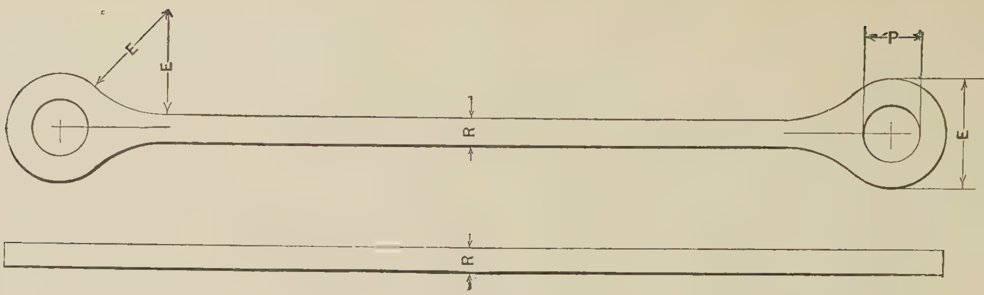


FIG. 273.

Square Rods.			Round Rods.		
Size of Rod.	Diameter of Eye.	Diameter of Largest Pin-hole.	Size of Rod.	Diameter of Largest Eye.	Diameter of Largest Pin hole
$\frac{7}{8}$ sq.	$3\frac{1}{2}$	$2\frac{1}{4}$	$\frac{7}{8}$ diam.	$2\frac{1}{4}$	$1\frac{1}{4}$
1 to $1\frac{3}{16}$	$4\frac{1}{4}$	$2\frac{1}{2}$	1 to $1\frac{1}{8}$	$4\frac{1}{4}$	$2\frac{1}{2}$
$1\frac{1}{4}$ to $1\frac{5}{16}$	$4\frac{1}{2}$	$2\frac{3}{4}$	$1\frac{1}{4}$ to $1\frac{3}{8}$	5	$2\frac{3}{4}$
$1\frac{3}{8}$ to $1\frac{7}{16}$	5	$2\frac{7}{8}$	$1\frac{1}{2}$ to $1\frac{5}{8}$	$5\frac{1}{2}$	3
$1\frac{1}{2}$ to $1\frac{5}{8}$	$5\frac{1}{2}$	$3\frac{1}{8}$	$1\frac{3}{4}$ to $1\frac{7}{8}$	6	$3\frac{1}{4}$
$1\frac{11}{16}$ to $1\frac{15}{16}$	6	$3\frac{1}{4}$	2 to $2\frac{1}{8}$	$6\frac{1}{2}$	$3\frac{1}{2}$
2 to $2\frac{1}{8}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{4}$ to $2\frac{3}{8}$	$7\frac{1}{2}$	4
$2\frac{1}{4}$ to $2\frac{3}{8}$	$7\frac{1}{2}$	4	$2\frac{1}{2}$ to $2\frac{3}{4}$	8	4
$2\frac{1}{2}$ to $2\frac{3}{4}$	8	4			

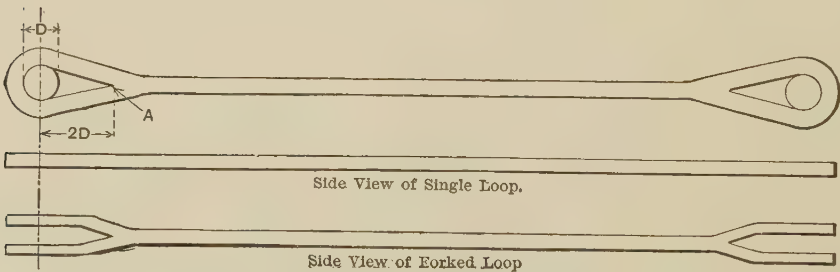
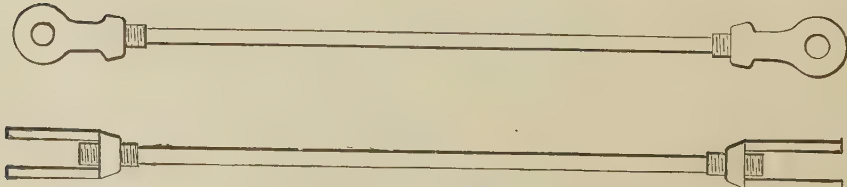


FIG. 274.

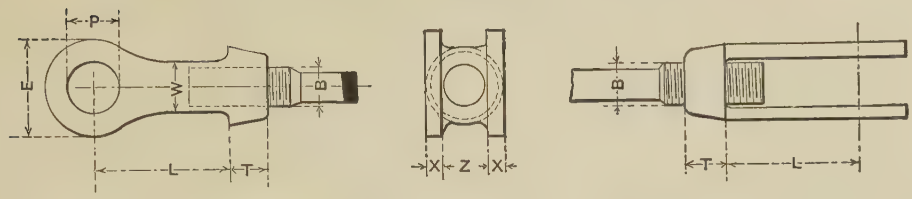
Upset Screw-end Rod with Square or Hexagonal Nuts.



Screw-end Rod with Clevises.

FIG. 275.

EDGE MOOR STANDARD CLEVISES.



B		E	L	P	T	W	X	Z	
Diameter of Screw Ends.		Diameter of Eye.	Length of Fork.	Diameter of Pin.	Length of Thread.	Width of Bar.	Thickness of Bar.	Width in Fork.	Weight.
7/8	1	4	5¼	1⅛ to 2½	1½	2	½	1⅛	5½
1⅛	1¼	4	5¼	1⅜ to 2½	1½	2	½	1⅜	6½
1⅜	1½	4	5¼	1⅝ to 2½	1½	2	½	1⅝	7¼
1⅝	1¾	4	5¼	2	1¾	2	⅝ and ¾	1⅞	9
1⅞	2	4⅞	7	2¼	2	2	¾	2⅛	13¾
2⅛	2¼	5⅞	7	2½	2⅛	2½	¾	2⅜	20¼
2⅜	2½	6⅜	7	2¾	2¼	2½	⅞	2⅝	25¼
2⅝	2¾	6⅞	8¾	3	2¾	3	1	2⅞	

The dimensions marked can be varied. Threads may be right or left. Dimensions in inches; weights in pounds.

EDGE MOOR STANDARD SLEEVE-NUTS.

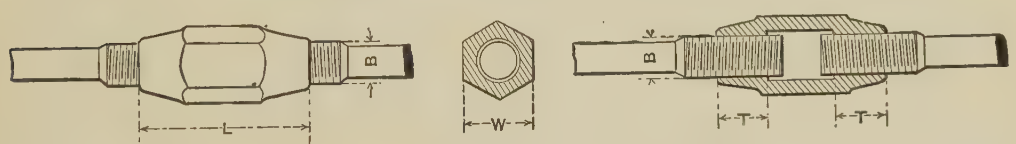


FIG. 276.

B			L	T	W	
Diameter of Screw.			Length of Nut or Screw Ends.	Length of Thread.	Diameter of Hex.	Weight one Nut.
7/8	1	*Ordinary length.*	6	1 7/16	1⅝	1¾
1⅛	1¼		6½	1⅝	2	3
1⅜	1½		7	1⅞	2⅜	4¾
1⅝	1¾		7½	2 1/16	2¾	6¾
1⅞	2		8	2 5/16	3⅛	9¼
2⅛	2¼		8½	2½	3½	12½
2⅜	2½		9	2¾	3⅞	16¾
2⅝	2¾		9½	2 15/16	4¼	21½
2⅞	3		10	3 3/16	4⅝	26½
	3¼		10½	3⅜	5	32
	3½		11	3⅝	5⅜	38¼
	3¾		11½	3 13/16	5¾	45
	4		12	4 1/16	6⅛	53½
1¼		*Extra long.*	12	2⅛	2	
1⅛	1¼		8½	1⅝	2	4
1⅜	1½		9	1⅞	2⅜	6¼
1⅝	1¾		9½	2 1/16	2¾	8¾
1⅞	2		10	2 5/16	3⅛	12¼

These nuts are forged with fibres in direction of stress and are of uniform section. The diameter of hexagonal part of any nut is that of the U. S. standard nut fitting screw of the larger diameter, given in the column B of table. Dimensions in inches; weight in pounds.

For the loop eye, Fig. 274, the diameter of the pin is not limited. The manufacture of the loop makes a weld necessary at the point *A*. As satisfactory welds cannot be generally secured with steel, loop-ended rods are usually made of wrought-iron.

If it were necessary to provide for adjustment in the length of rods with solid or loop eyes, as would be the case for counter-ties, lateral and sway rods, sleeve-nuts or turn-buckles would be used.

It will be noticed that the rods shown in Fig. 275 may be adjusted without using sleeve-nuts.

The tables given are the standards in use by the Edge Moor Bridge Works. Those used by other companies vary somewhat in appearance and in some of the details, but those given fairly represent the important features of all. Some companies use a larger diameter of upset for a given rod than that given in the table. The sleeve-nut or right and left nut shown is often discarded for the open turn-buckle shown in Fig. 277. The advantage of the open turn-buckle is that the ends of the rod are visible, and it may easily be inspected and the positions of the rods noted. An objection to them is that they may be adjusted by running a bar through the link, and are thus liable to be tampered with by incompetent persons. An improvement which seems to be inevitable in the manufacture of the screw ends is the use of smaller threads.

FIG. 277.

3d. *Single Shapes.*—The difficulty met with in the use of single shapes for tension members is to provide proper attachments or connections at the ends. It is assumed that connecting rivets will be so spaced as to develop the required net area of the shape and at the same time produce no eccentricity of stress in the piece if possible. The shapes of iron or steel generally used for these members are plates and angles. For a plate the connection is generally a single lap joint, which, while it produces a small eccentricity of stress, usually developes the strength assumed. For maximum economy of metal, or maximum efficiency for the amount of metal used, the net area must be as great as practicable. The arrangement of rivets shown in Fig. 278 (*a*) secures this result and is preferable to that shown

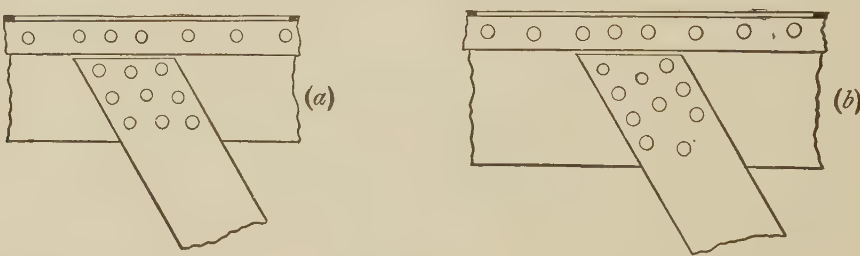


FIG. 278.

in Fig. 278 (*b*), as the rivets are symmetrically placed with respect to the axis of the piece, and the cross-section of the plate is reduced by the cross-section of only one rivet-hole.

For an angle iron, and also for any unsymmetrical shape iron, the great difficulty is to place the rivets so that they will be symmetrical about the neutral axis of the piece. Angle irons should always be attached by rivets through both legs to develop the greatest strength. Cooper specifies that unless this is done only one leg of the angle can be counted as available area. This detail is usually made as shown in Fig. 279.

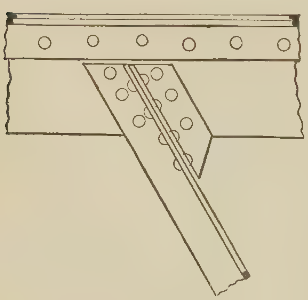


FIG. 279.

Bracing with riveted attachments is much stiffer than rods or bars connecting on pins. The rivets allow no motion which may occur with pin connections; and besides, the elongation of the piece itself is not so much, as it is proportioned for the *net* area, while the stretch must result from the elongation of the *gross* area.

4th. *Compound Sections*, or those composed of two or more simple shapes riveted together, are used for those cases where a stiff tension member is wanted. If the end attachments are riveted, care must be exercised to so arrange the rivets that the stress is not applied eccentrically, and also that the largest net area possible is obtained. If the end attachment is by means of a pin, the ends of the member should be designed as follows. Fig. 280 shows a compound section tension member. There are four conditions which must be fulfilled, viz.:

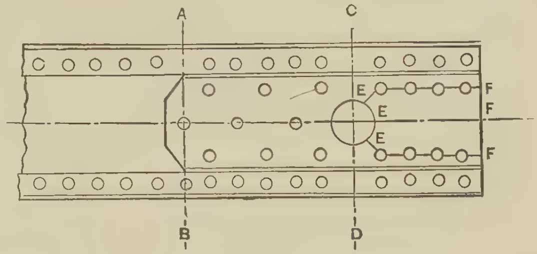


FIG. 280.

1st. The net area of the piece, after deducting the section cut out by the rivet-holes should be the greatest possible. This would be the net section on the line AB in Fig. 280.

2d. The net area on the line CD must exceed the net area of the piece by twenty-five per cent.*

3d. The net area on a line EF between the end of the piece and the edge of the pin-hole must be equal to three fourths of the net area of the piece.

4th. The bearing pressure of the pin must not be excessive.

In the fulfilment of any of the above conditions, care must be taken to so distribute the rivets that the required value of each component piece is developed. Thus, for the net area on line CD the net area of the top angle iron is manifestly the gross area reduced by one rivet-hole, and the value of the angle iron may be determined either by this net area or by the shearing value of the number of rivets *less one* which attach the angle iron to the pin plates between the line CD and the end of the piece. Owing to the fact that steel is not fibrous and is approximately equal in strength in all directions, the better practice is to use steel for the end pin plates in compound tension members. Wrought-iron has a strength perpendicular to its fibre of about two thirds the strength parallel to the fibre.

228. Compression Members.—In Fig. 281 are given some of the more common forms of compression members for framed structures. The factors which usually determine the form of a compression member are cheapness of manufacture and the cost of the component shapes of material, the efficiency of the form as a strut and its suitability as regards dependent details. In Fig. 281 sections A, B, C, D, and E are commonly used for top chords, F, G, H, I, J, K, and L for intermediate posts, and M, N, and O for lateral or sway struts. In all members, whether they are to resist tension or compression, a symmetrical section is to be preferred, as all question of the eccentric application of the stress is done away with. It has become the accepted practice, however, to use the unsymmetrical sections above noted for top chord sections. These sections are stiffer laterally than the symmetrical post sections F, G, H, etc., and it is due to this fact that they are preferred for chords and end posts, which are relied upon to transfer wind stress in bending.

229. The Selection of the Economical Form for a Compression Member.—The shapes of iron commonly used vary in price, so that the designer should always keep well informed as to the market. The engineering journals give very good market reports from which the above information may be obtained. The cost of the manufacture of the finished piece is almost entirely dependent upon the quantity of rivets to be used and the facility with which they can be driven. It will be noted in this connection that E (Fig. 281) requires fourteen lines of rivets; A, C, D, F, and G require eight lines; B, H, I, J, and K require four lines; and that M and N require only two lines. As it is always preferable and cheaper to have the rivets machine-driven, they should be so placed that they may be driven with the

* This is not the universal practice, but is a safe rule. Some engineers make the excess of the area fifty per cent.

usual riveting machine. Four lines of rivets in each of the sections *G* and *I*, or those which attach the lattice bars, are inside of the assembled piece and are seldom in reach of a machine, and are therefore usually hand-driven and expensive.

There are no general rules to guide the designer in the selection of the proper form other

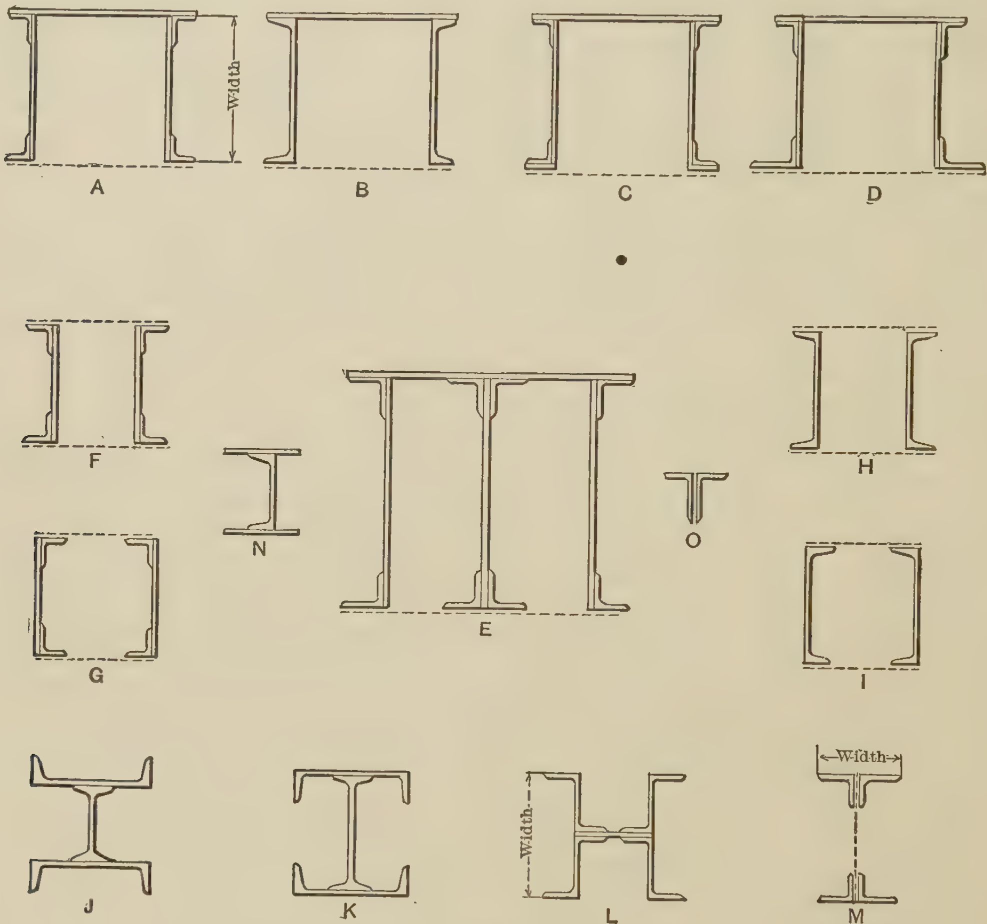


FIG. 281

than those gained by experience. The best guide, when the necessary experience is lacking, is to select that form which will allow the simplest and neatest correct detail for its connections.

230. **The Economical Form of a Compression Member** to select to resist a given stress is dependent on the factors enumerated in the preceding article, particularly on the value of the form in resisting stress. The accepted formulæ for determining the necessary area for a piece of given external dimensions introduce the least radius of gyration as the only variable when the length of the piece and its end connections, whether flat-ended, one flat and one pin end, or two pin ends, are constant. The greater the least radius of gyration the less will be the required area to resist the stress. This requires that such a form be selected as will give the largest least radius of gyration. The various forms shown in Fig. 281 differ in respect to their least radius of gyration even when the width of the section is the same. Thus, it has been found that practically the least radius for *A*, *B*, *C*, *D*, and *E* is four tenths of the width, for *F*, *G*, *H*, and *I* it is three eighths of the width, for *L* it is five sixteenths of the width, and for *M* it is one quarter of the width. The forms *A*, *B*, *C*, *D*, and *E* require the least area and are further economical in material, as they require but one side to be latticed and hence have

less "non-effective" material than most of the others. The form *L* has no latticing, and may often be found to require less material on this account.

231. The Limiting Dimensions of Compression Members.—The natural result of determining the dimensions of a compression member by the usual formulæ would be to select a form giving the largest radius of gyration and which would also be one of large width. If this were carried to extremes, it would result in large members of very thin metal; and there is manifestly a point at which the iron would be too thin to be of any value as a strut. The practice is to limit the thickness of material and the dimensions of the piece, as shown in Fig. 282.

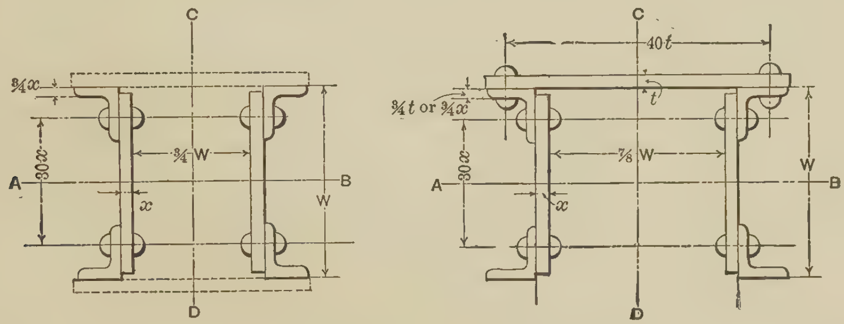


FIG. 282.

The thickness of the top plate should not be less than one fortieth of the distance between the rivets connecting it to the angles; the thickness of the side plates should not be less than one thirtieth of the distance between the rivets connecting it to the angles, and the angles should not be thinner than three quarters of the thickness of the thickest plate riveted to them. The clear width between the side plates should not be less than three fourths the width of the side plate for the post section, and not less than seven eighths of the width for the chord section (Fig. 282), in order that the radius of gyration about the axis *AB* may be equal to or less than that about an axis *CD* perpendicular to *AB*.

The unsupported length of a compression member should not be greater than fifty times the least width of the member or one hundred and fifty times the least radius of gyration. This is necessary in order to secure stiffness, which is as desirable as strength.

It will be noticed that the side plates in Fig. 282 must have a thickness of at least one thirtieth of the distance between the rivets attaching them to the angles, while the top plate is limited to one fortieth of the distance between rivets. This would only be true of pieces which are pin-ended and when the pins are parallel to the axis *AB*. When the piece is square-ended and receives its stress through a butt joint, the thickness of all plates should be at least one thirtieth of the distance between rivets. The top plate of pin-ended members is allowed to be thinner than this, in order to allow more metal to be concentrated in the web plates, which receive their stress directly from the pins, and to secure a section as symmetrical as practicable. It is conceded that the advantages thus obtained are more than the disadvantage due to the use of a thinner top plate.

232. End Details of Compression Members.—In Chapter XXI are given the principles which should govern the designer in proportioning the pin plates and in deciding upon the location and number of rivets to use for a pin-ended piece. The student is referred to that chapter, as the principles can be more readily understood when illustrated by a practical example.

If the member is square-ended no increase of the area is necessary, as all that is required is that the piece be held firmly in position, in order to obtain an even bearing over the entire area of the cross-section.

In all end connections, whether pin or square, arrange the rivets so that their maximum efficiency may be realized and stagger them or space them on zigzag lines, in order to avoid as much as possible the danger of local failure at the ends due to the weakening of the plates by the rivet-holes. In some recent experiments on full-sized compression members, the pieces failed by splitting the plates at the ends where they were weakened by rivet-holes before two thirds of the estimated value of the piece was developed.

Compression members theoretically do not require any metal beyond the centre of the pins at their ends, but it is always better to provide metal enough beyond the pin to resist displacement of the piece by an external force. For the posts of trusses and like members it is customary to run the section of the post at least two inches beyond the pin.

233. **Tie Plates on Compression Members.**—There are many methods of determining the size of tie or batten plates in use, but none give rational results, and the error in all cases is no doubt very much on the side of safety. As to their thickness there is little disagreement, as all concede that they should be able to withstand either compression or tension. In order that they may be able to resist a compression stress the thickness must be at least one fiftieth of the distance between the rivet lines in the segments which they connect, or else the plates may be made of a minimum allowable thickness and stiffened with angle irons. These two cases are illustrated in Figs. 283 and 284. As to the length of the tie-

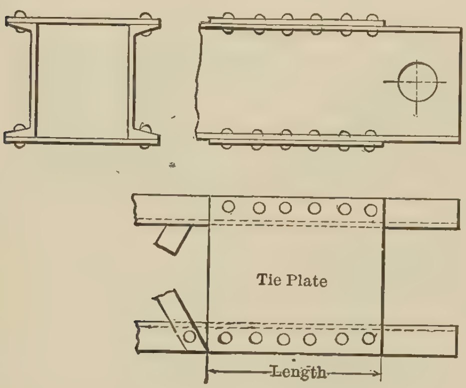


FIG. 283.

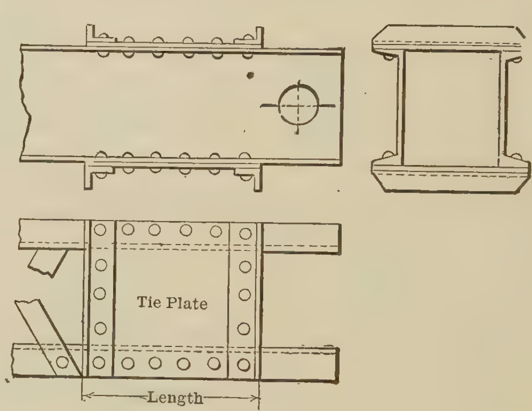


FIG. 284.

plate and the number of rivets necessary in the connection with each segment, there is a great diversity of opinion. Some specifications require that their length shall be one and one half times the depth of the piece, while others require the tie plates to be square, and in either of these cases the number of rivets used would be determined by the number which could be put in using the minimum pitch, usually three inches. Another specification which attempts to be rational requires that the tie plates shall be long enough to take sufficient rivets to transfer one quarter of the total stress on the member from one segment to the other. The duty required of the tie plate is to hold the two segments in line and to take up any bending moment which an eccentric application of the stress or an external force may produce in the segments. In all compression members composed of two channels, either solid rolled channels or compounded of plates and angles, with the pin bearing on the webs of the channels, there is a bending moment produced in each segment due to the fact that the neutral axis of the channel is not coincident with the centre of the web of the channel. The centre of the application of the stress is necessarily at the centre of the web. This bending moment should be resisted by the tie plate, and the rivets connecting the tie plate to the segment should also be able to transfer it from the segment to the tie plate. The amount of this

bending moment can always be accurately determined. Should the member be acted upon by an external force perpendicular to the webs of the channels, and if the tie plate is placed some distance from the end of the member, as is usually the case, a bending moment is produced in the segments, which the tie plate and its connecting rivets should be able to resist. The closer the tie plate is to the end of the member the less the bending moment from the latter cause becomes, hence the desirability of locating the tie plate as near the end of the piece as practicable. If the tie plates were proportioned for the bending moment produced by the eccentric application of the stress at the ends and in addition thereto for any probable external force, such as the pressure of the wind, the result would be more rational and satisfactory than any of the rules now in use. In order to hold the segments in line, all that is necessary is that the tie plates be stiffer against bending than the segments which they connect.

234. The Latticing of Compression Members.—There are no rules other than empirical ones in use by which the size and spacing of lattice bars for compression members are determined. The duty of the latticing is to hold the segments composing the members together so that they may act as one piece, to resist any local tendency to get out of line or to buckle and to transfer any transverse shearing force to the ends of the member. It is very probable that the first of these conditions is fulfilled when the latticing is continuous from end to end and the individual bars have a thickness of at least one fiftieth of their length, so that they may be able to resist compression or tension. The second condition is fulfilled when the distance between the rivets attaching the lattice bars to one segment is less than sixteen times the width of the segment in the plane of the latticing, as experiments on pieces in compression show that a column whose unsupported length is less than sixteen times its least width will fail by crushing instead of buckling. The third condition could easily be provided for if the shearing forces were known. Any probable external forces should be provided for, but in addition to them there are stresses in the bars due to the bending of the strut from the direct stress which can only be estimated. It has been suggested that, as our compression formulæ all assume a certain extreme fibre stress due to the flexure of the strut, from this known extreme fibre stress we find an equivalent uniform load acting in the plane of the latticing which will produce this fibre stress, and from this load find the stress in the lattice bars.

As an illustration of the above, assume the common straight-line formula for struts, $9000 - 40\frac{l}{r}$, where l = length of strut in inches and r = radius of gyration of the strut about an axis perpendicular to the plane of the latticing. The term $40\frac{l}{r}$ is the extreme fibre stress from the bending moment induced in the strut by the direct stress. Then the uniform transverse load, W, which would produce this fibre stress in the strut would be

$$W = \frac{320 \times A}{b},$$

where A = area of the strut and $b \times r$ = distance of the extreme fibre from the neutral axis. If the lattice bars were inclined at an angle of 45° with the axis of the piece, the maximum stress on the lattice bars would be

$$\frac{112A}{b} = \frac{1}{4} \times \frac{320A}{b} \times 1.4.$$

The width of lattice bars with one rivet at each end should be about three times the diameter of the rivet used.

When the lattice bars are long, a saving in material may be made by using small angle irons instead of flat bars, as they are more effective in resisting compression and hence require less area of cross-section.

The common rules for latticing are given in Art. 271.

235. Common Formulæ for Compression Members.—The formulæ in common use for determining the proper area of cross-section for a strut to resist a given compressive stress may be divided into two kinds, viz.: 1st, those that plot in a curve; and 2d, those that plot in a straight line. Within the limits of practical use they will give practically the same results, and as the straight-line formulæ are easier and quicker in application they are generally preferred.

Rankine's Formula, ... $p = \dfrac{8000}{1 + \dfrac{l^2}{ar^2}}$.

$a = 18{,}000$ when the strut has two pin ends;
$a = 24{,}000$ " " " " one pin and one flat end;
$a = 36{,}000$ " " " " two flat ends.

Common Straight-line Formula, $p = 9000 - b\dfrac{l}{r}$.

$b = 40$ when the strut has two pin ends;
$b = 35$ " " " " one pin and one flat end;
$b = 30$ " " " " two flat ends.

The Pennsylvania Lines West of Pittsburgh straight-line formulæ:

Top Chord Sections, $p = 8400 - 84\dfrac{l}{h}$.

Common Post Sections, $p = 8400 - 88\dfrac{l}{h}$.

In all of the foregoing formulæ, p = permissible stress per square inch, l = length of the member in inches, r = least radius of gyration of the member in inches, and h = least width of the member in inches. The formulæ used by the Pennsylvania lines are only to be applied to the chord and post sections given in Fig. 282, the general form of their equation being $p = 8400 - c\dfrac{l}{h}$, when c is dependent upon the *form* of the member. These formulæ also do not take into account the differing end conditions of the member; it being assumed, no doubt, by the engineers of this road that all members of a truss really act as pin-ended members, and that any error in this assumption would be on the safe side.

The constants 8000, 9000, and 8400 in the foregoing formulæ are used to determine the permissible *working* stresses for railway bridges and are for wrought-iron. It is customary to increase p 25 per cent for highway bridges. If the material is steel, p would ordinarily be increased 20 per cent for railway bridges and 50 per cent for highway bridges.

The parabolic formulæ of Prof. Johnson given in (28) p. 153 are the most rational yet advanced, and are to be recommended for use as they are very easy of application and are the nearest approximations to the truth of all the formulæ we have.

CHAPTER XVIII.

DETAILS OF CONSTRUCTION.

236. Riveting.—The diameters of the rivets used in structural iron-work are $\frac{1}{2}''$, $\frac{5}{8}''$, $\frac{3}{4}''$, $\frac{7}{8}''$, and $1''$. Rivets $\frac{3}{4}''$ and $\frac{7}{8}''$ in diameter are most generally used, smaller sizes being used only in unimportant details, or where the clearances in the piece riveted prohibit the use of a larger rivet. Rivets larger than $\frac{7}{8}''$ diameter are only used in pieces where the metal is very thick or where the stresses absolutely require larger rivets. Field rivets, or those to be driven in the structure after it is in place by hand, should never be larger than $\frac{7}{8}''$ diameter, and preferably $\frac{3}{4}''$ diameter, owing to the difficulty of driving tight rivets of the larger size by hand.

Rivets are usually made of the same material as the pieces in which they are driven. If steel rivets are used, they are made of a very soft steel. Field rivets are generally of wrought-iron in all cases, as the difficulty of driving good steel rivets prohibits their use. The range of temperature at which steel can be effectively worked is very small, and as in field riveting considerable time is lost in passing the rivet from the forge to the riveters, a rivet has time to cool to a point below which it is not advisable to do any work upon it such as would be necessary in driving it.

237. Size of Rivet-heads.—The sizes of the heads of rivets vary slightly. The ordinary sizes are shown in Fig. 285.

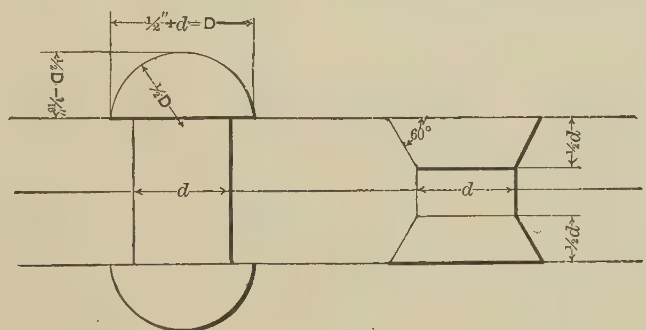


FIG. 285.

The usual weight of a pair of rivet-heads (information usually needed when calculating the finished weight of riveted work) may be obtained from the following table:

Size of Rivet in Inches.	Weight of One Pair of Heads in Pounds.	Length of Rivet required to make one Head.
$\frac{1}{2}$	0.11	$\frac{7}{8}''$
$\frac{5}{8}$	0.22	$1''$
$\frac{3}{4}$	0.27	$1\frac{1}{8}''$
$\frac{7}{8}$	0.44	$1\frac{1}{4}''$
1	0.76	$1\frac{1}{4}''$

The length of the material required to form one head is also given in the foregoing table. Rivets are furnished with one head formed, the other head being made when the rivet is

driven. In ordering rivets first find the grip or distance between the heads of the rivet. The grip of the rivet is the thickness of the plates or parts through which the rivet is to be driven plus $\frac{1}{32}$ of an inch for each joint between the plates to allow for uneven surfaces which prevent closer contact. This grip must be increased in the ratio of the area of the hole to area of the rivet material, the hole usually being $\frac{1}{16}$ of an inch larger in diameter than the rivet. To this add the length required to form one head, and the length of the rivet under the head is obtained. Thus, assuming a $\frac{3}{4}$-inch rivet joining three half-inch plates, the grip would be $3 \times \frac{1}{2} + \frac{1}{16} = 1\frac{9}{16}$. Increasing this in the ratio of 44 to 52 we obtain $1\frac{3}{4}$, nearly. Adding $1\frac{1}{8}$ inches for the head, we get $2\frac{7}{8}$ inches as the length to order. Rivets are ordered in even eighths of an inch.

For a countersunk head add one half the diameter of the rivet for the head.

Besides the full-button head and the countersunk head, in extreme cases rivets with flattened heads may be used. These heads are made by flattening the button heads, and should never be used when the height is less than $\frac{5}{16}$ of an inch.

In calculating clearances it is always better to provide for a head one eighth of an inch higher than the standard head used to allow for discrepancies in the length of material used and in the upsetting of the rivet. This rule should apply to countersunk heads as well as to the other kinds, as the rivet often does not upset sufficiently to bring the head flush with the plates. Chipping countersunk heads in order to make them flush with the plate is to be avoided when possible as it will often loosen the rivets, and is expensive if done well.

Countersunk rivets should never be used in plates of less thickness than one half the diameter of the rivet. Also, as a matter of economy in manufacture, they should not be used in long pieces, as the extra handling of such pieces which would be necessary in order to countersink a few rivets would make them expensive.

238. Determination of the Size of Rivet to Use.—The diameter of the rivet should not be less than the thickness of the thickest plate through which the rivet passes, owing to the difficulty of punching holes the diameters of which are less than the thickness of the plate. While this is not an impossibility, it is expensive owing to the risk of tool breakage. The size of the rivets is often determined by the clearances or space in which the rivet must be driven. Shapes with small flanges will require small rivets, as the practical limits of the distance of the centre of the rivet from the edge of the piece, and of the clearance necessary for the tool in driving the rivet, then determine what is the largest size it is possible to use (see Art. 239). It is customary and cheaper to use one size of rivet throughout one entire piece, as a change of dies is avoided and the punching and riveting done quicker.

When none of the foregoing conditions fix the size of the rivets, it is good practice to use that size of rivet which will amply resist all stresses and be most economical in manufacture. Smaller rivets reduce slightly the cost of punching and driving and weigh less, thus effecting a saving both in the cost of manufacture and of the material.

239. Practical Rules governing the Spacing of Rivets.—The minimum distance from centre to centre of rivets should not be less than three diameters of the rivet. Closer spacing than this is liable to split or otherwise injure the material. The maximum distance from centre to centre of rivets in a compression member, where it is essential that the parts riveted act as one whole, should not be greater than sixteen times the thickness of the thinnest outside plate riveted. In general, a maximum pitch of 6 inches should not be exceeded if it is advisable to have the parts drawn sufficiently close to prevent the entrance of water.

In punching the rivet-holes it has been found that they must be located a certain distance from the edge or end of a piece, in order to avoid all danger of splitting the material. In wrought-iron the danger of splitting is less when the weak section is perpendicular to the fibre of the material than when it is parallel to the fibre, because of the greater strength of the material in the direction of the fibre. Hence a hole may be punched in wrought-iron

closer to the edge than to the end. Some empirical rules for these distances are given in Fig. 286.

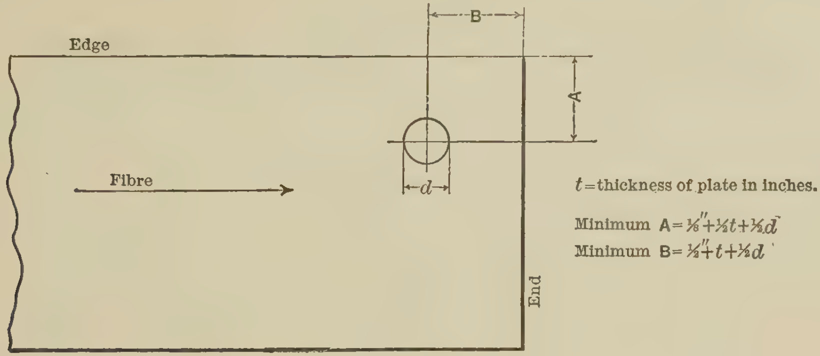


FIG. 286.

It will be noted that the minimum distances *A* and *B* depend upon the thickness of the plate and the diameter of the rivet, but that the distance from the *edge of the hole* to the end or side of the plate depends on the thickness of the plate alone. It is well to exceed these minimum distances whenever practicable, a common rule being to make the end distance twice the diameter of the rivets and the side distance one quarter of an inch less. It must be remembered that the diameter of the hole is always one sixteenth of an inch larger than the nominal diameter of the rivet to allow the latter to be entered while hot. When the rivet is driven it is upset, and then completely fills the hole.

In power-riveting, and also in the best hand-riveting, a certain amount of room is necessary for the tools holding or forming the heads. The ordinary requirements in riveting are shown in Fig. 287.

By referring to Fig. 287, it will be seen that provision must be made for clearing a tool

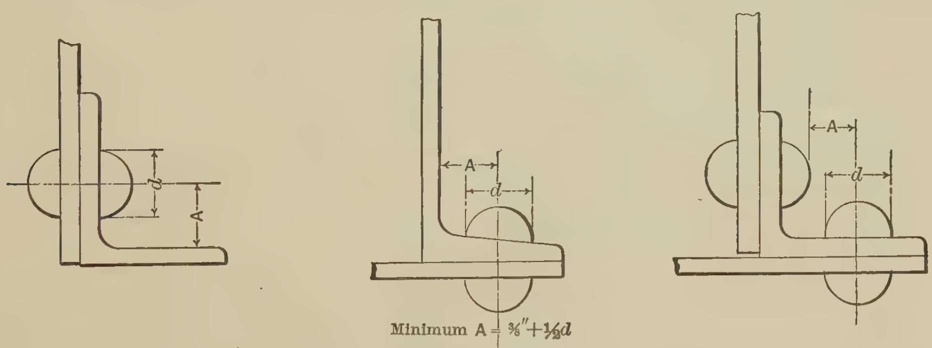


FIG. 287.

whose diameter is three quarters of an inch greater than the diameter of the rivet-head.

240. Length or Grip of Rivets.—It has been found that there is a practical limit to the length or grip a rivet may have, which if exceeded renders it almost impossible to drive tight rivets. Those who have investigated this subject now specify that the maximum grip of a rivet shall never exceed *four times the diameter of the rivet.* This is a very essential factor in good riveting, and should never be overlooked. Examples of bad designing in this respect are very common, particularly so in plate-girder flanges, where a series of thick flange plates are riveted to the flange angles. Often the bearing plates on the ends of a compression member are so thick as to render the rivets which are supposed to attach them firmly to the main section probably useless. There is always a remedy for such cases, and it should be applied.

241. Strength of Rivets.—Rivets may fail by shearing off, by being crushed, or in extreme cases of bad designing by flexure. They may also fail by direct tension, either by the head breaking off or, rarely, by failure of the body of the rivet. The value of a rivet in direct tension is so unreliable that its use to resist such a stress is generally prohibited.

The shearing stress per square inch usually allowed on wrought-iron rivets is three quarters of the allowed tensile stress per square inch on plate or shape iron. The following table gives the shearing value of the common sizes of rivets for stresses of 6000, 7500, and 9000 lbs. per square inch, which are the most common values in use:

TABLE OF SHEARING VALUE OF RIVETS.

Diameter of Rivet in Inches.		Area of Rivet in Square Inches.	Value in Single Shear at		
Fraction.	Decimal.		6000 Lbs. per Square Inch.	7500 Lbs. per Square Inch.	9000 Lbs. per Square Inch.
$\frac{1}{2}$	0.500	0.1963	1178	1472	1766
$\frac{5}{8}$	0.625	0.3068	1841	2301	2761
$\frac{3}{4}$	0.750	0.4418	2651	3313	3975
$\frac{7}{8}$	0.875	0.6013	3608	4510	5412
1	1.000	0.7854	4712	5890	7068

6000 lbs. per square inch is usually allowed on railroad work, and 7500 lbs. per square inch for roadway bridges when the material is wrought-iron; 7500 and 9000 are the usual corresponding values used for steel.

When the rivets are to be driven by hand during erection (i.e., field rivets) it is customary to increase the number of rivets by 25 to 50 per cent to allow for faulty riveting. The diameter of the rivet is taken to be that of the rivet before it is driven. The real diameter of the rivet in place, if it completely fills the hole and if the holes in the pieces match exactly, is one sixteenth of an inch larger than the nominal diameter.

The crushing or bearing value of rivets is usually estimated at so many pounds per square inch on the bearing area of the rivet on the metal through which it passes. This bearing area is assumed to be the diameter of the rivet by the thickness of the piece. It is also customary to take the diameter of the rivet in computing the bearing area as that of the rivet before it is driven.

The bearing or crushing values of rivets for the various thicknesses of plates and the usual allowed stresses of 12,000, 15,000, and 18,000 lbs. per square inch are given in the following tables:

TABLES OF BEARING VALUES OF RIVETS.

For 12,000 lbs. per square inch.

Diameter of Rivet in Inches.	Thickness of Plate in Inches.										
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$
in.											
$\frac{1}{2}$	1500	1875	2250	2625	3000	3375	3750	4125	4500	4875	5250
$\frac{5}{8}$	1875	2340	2810	3280	3750	4220	4690	5150	5625	6100	6560
$\frac{3}{4}$	2250	2810	3375	3940	4500	5060	5625	6190	6750	7310	7875
$\frac{7}{8}$	2625	3280	3940	4590	5250	5900	6560	7220	7875	8530	9190
1	3000	3750	4500	5250	6000	6750	7500	8250	9000	9750	10500

For 15,000 lbs. per square inch.

Diameter of Rivet in Inches.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$
in.											
$\frac{1}{2}$	1875	2340	2810	3280	3750	4220	4690	5160	5620	6090	6560
$\frac{5}{8}$	2340	2920	3510	4100	4690	5270	5860	6450	7020	7610	8200
$\frac{3}{4}$	2810	3510	4220	4920	5625	6320	7030	7740	8420	9130	9840
$\frac{7}{8}$	3280	4100	4920	5740	6560	7370	8200	9020	9840	10600	11490
1	3750	4690	5625	6560	7500	8440	9380	10030	11250	12190	13120

For 18,000 lbs. per square inch.

Diameter of Rivet in Inches.	Thickness of Plates in Inches.										
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$
in.											
$\frac{1}{2}$	2250	2810	3375	3940	4500	5060	5625	6190	6750	7310	7875
$\frac{5}{8}$	2810	3510	4220	4920	5625	6320	7030	7740	8440	9140	9840
$\frac{3}{4}$	3375	4220	5060	5910	6750	7590	8440	9290	10120	10960	11810
$\frac{7}{8}$	3940	4920	5910	6990	7875	8850	9840	10830	11810	12800	13780
1	4500	5625	6750	7875	9000	10120	11250	12375	13500	14525	15750

A bearing pressure of 12,000 lbs. per square inch is usually allowed on railroad work, and 15,000 lbs. for highway bridges where the metal is wrought-iron. These values would be increased to 15,000 and 18,000, respectively, if the material were steel.

As in the case of rivets in shear, the number of rivets required by the above tables would be increased by from 25 to 50 per cent if the rivets were to be "field" rivets.

Rivets will fail by flexure only in those cases of bad designing where the rivets are long, and it is impossible to drive them tight enough to have them upset and completely fill the holes, or possibly in those cases where an excessive thickness of pin plates is used at the ends of a compression member and the rivets are relied upon to transfer the bearing stress to the main section in a very short distance. The latter case will be understood from Fig. 288.

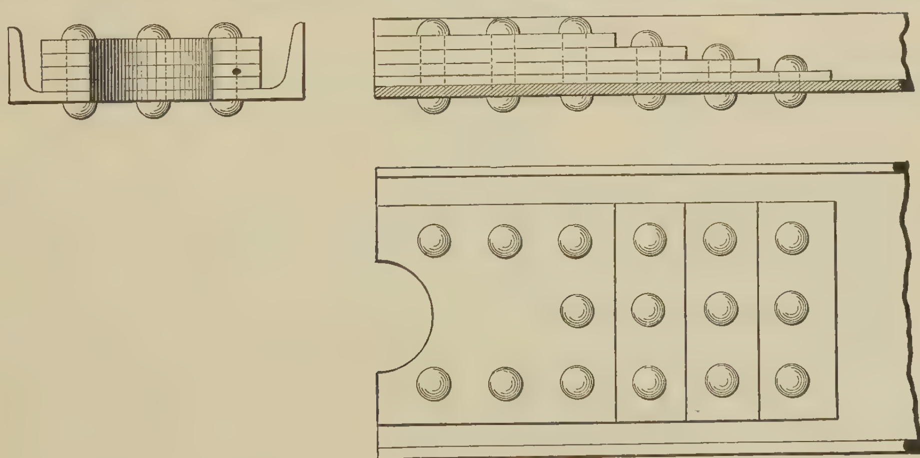


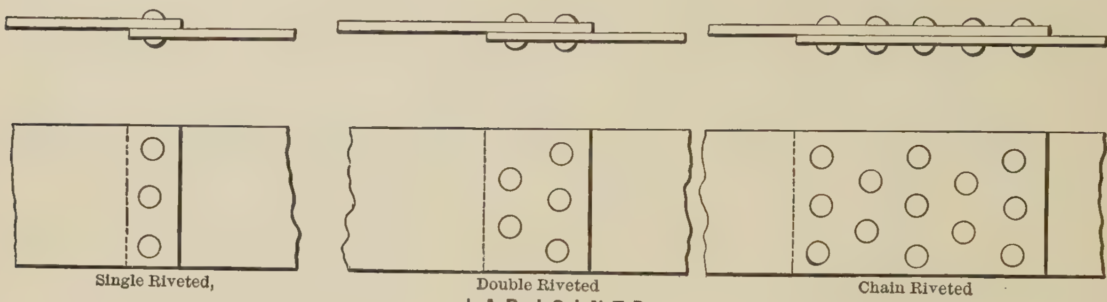
FIG. 288.

If the number of rivets in Fig. 288 were determined for single shear, and were $\frac{7}{8}$ in. in diameter and the plates $\frac{1}{2}$ in. thick, the bearing pressure which would be equivalent to this number of rivets, allowing 6000 lbs. per square inch shearing stress, would be 57,600 lbs. The bending moment produced by this force applied at the centre of the bearing plates would be $57,600 \times 1\frac{1}{4} = 72,000$ in.-lbs. (assuming the web of the channel to be $\frac{1}{2}$ in. thick). This moment is either resisted by flexure of the rivets or by direct tension on the rivets near the pin. No matter which way failure would occur, it can readily be seen that stresses are produced which ordinarily are not provided for, and which may prove to be important. Such details could be improved by lengthening the pin plates and increasing the number of rivets, as it would surely result in lessening the stress on the rivets from flexure and also lessen any direct tensile stress. Rivets are never proportioned for flexure.

242. Kinds of Riveted Joints.—Riveted joints may be designated as of two kinds: Lap joints, as shown in Fig. 289, and Butt joints, shown in Fig. 290.

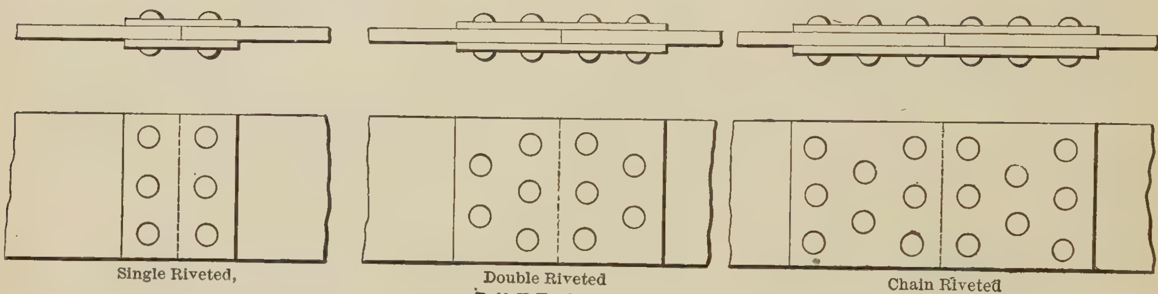
Either of these kinds of joints may be single-riveted (i.e., one line of rivets), double-riveted (i.e., two lines of rivets), or chain-riveted (i.e., more than two lines of rivets).

The butt joint is the one generally used, and is the more effective joint, owing to its symmetry and the absence of eccentric stresses. The lap joint is only used in unimportant details in structural work, and should always be discarded for the butt joint where possible.



LAP JOINTS.

FIG. 289.



BUTT JOINTS.

FIG. 290.

243. Strength of Riveted Joints.—A riveted joint may fail either by the rupture of the rivet or of the plate; hence the maximum efficiency is obtained where the strength of the rivets is just equal to the strength of the plate. As the shearing strength of wrought-iron is about three fourths of its tensile strength, it follows that the shearing area of the rivets should be one third greater than the net area of the plate which is in direct tension. Also, as the crushing strength of a rivet is about one and one half times the tensile strength of wrought iron, the bearing area of the rivet (i.e., diameter of rivet by the thickness of the plate on which it bears) must be two thirds of the net area of the plate.

The maximum economy in material is obtained where the net area of the plates or pieces joined is the greatest possible. Fig 291 shows how this is best accomplished. It also shows

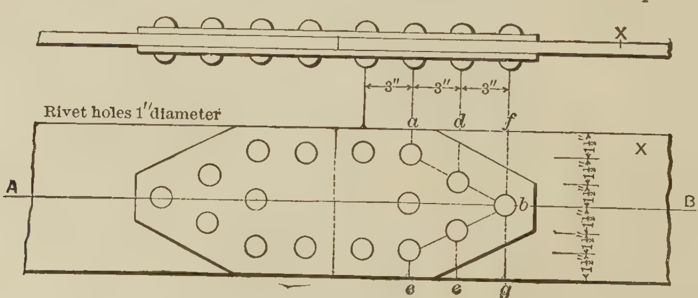


FIG. 291.

the best arrangement of rivets for a uniform distribution of the stress over the entire area of the plates joined. The rivets are also placed symmetrically about the axis AB of the plate. The plate X in Fig. 291 may fail by tearing apart on the broken lines abc, dbe, or on the line

fbg. A failure on either of the broken lines *abc* or *dbe* would be partly by direct tension on a section perpendicular to the fibre and partly by shearing parallel to the fibre. As the shearing strength of wrought-iron parallel to the fibre is only about one half of its tensile strength on a section at right angles to the fibre, the relative net area on the broken lines *abc* and *dbe* is less than the length of those lines diminished by the number of rivet-holes which are located on them. Thus, for the line *abc* there would be $9 - 5 = 4$ inches of width to be ruptured in direct tension, and $(6 - 2)2 = 8$ inches in length to fail by shearing parallel to the fibre. As the iron for this kind of shear is worth only one half of what it would be worth in direct tension on a section perpendicular to the fibre, the relative value of the 8 inches along the fibre is 4 inches across the grain; hence the relative *net* width of the plate is 8 inches. The relative net width of plate on the line *dbe* will also be found to be 8 inches, and the net width of plate on the line *fbg* is 8 inches. The net area of the cover plates should be equal to the net area of the plate spliced. This can be accomplished by arranging the rivets so as to get a maximum net width of plate, and then increasing the thickness of the cover plates if necessary to obtain the required net area.

What has been said regarding riveted joints refers particularly to joints in tension; but it applies fully as well to compression joints, excepting that in the latter no attention is paid to the net section, as it is generally assumed that rivets do not weaken compression members. Close spacing of the rivets would increase the danger of local failure in compression members, and should be avoided. In all joints care should be exercised to so arrange the rivets that the stress may be uniformly distributed over the area of the piece.

Watertown Arsenal Tests on Single-Riveted Double-Butt Joints.

I. OPEN-HEARTH STEEL PLATES $\frac{5}{8}$ INCH THICK, $\frac{1}{4}$-INCH COVER PLATES.

IRON RIVETS, MACHINE DRIVEN. HOLES DRILLED.

Material of Plates:
- Strength lengthwise = 56,500 lbs. per square inch.
- " crosswise = 56,500 " " " "
- Elastic limit = 33,000 " " " "
- Elongation in 10 in. = 26 per cent.
- Reduction of area = 55 " "

(Each result the mean of two tests. Method of failure indicated by bold-faced type.)

Size of Rivet and of Hole.	Pitch of Rivets.	Maximum Stress on Joint per Square Inch. Tension on Gross Section of Plate.	Tension on Net Section of Plate.	Compression on Bearing Surface of Rivet.	Shearing on Rivet.	Efficiency of Joint.
inch.	inches.					
$\frac{9}{16}$ and $\frac{5}{8}$	$1\frac{5}{8}$	39,940	**64,900**	103,800	38,550	73.6
	$1\frac{3}{4}$	39,420	**61,320**	110,300	**41,060**	72.6
$\frac{11}{16}$ and $\frac{3}{4}$	$1\frac{3}{4}$	37,000	**64,800**	86,400	26,900	68.2
	$1\frac{7}{8}$	37,740	**62,900**	94,400	29,500	69.5
	2	41,900	**67,050**	111,800	36,900	70.1
	$2\frac{1}{8}$	41,000	**63,300**	116,200	37,200	69.7
$\frac{13}{16}$ and $\frac{7}{8}$	$1\frac{7}{8}$	34,600	**64,900**	74,100	20,000	63.8
	2	37,900	**67,400**	86,600	24,500	63.4
	$2\frac{1}{8}$	38,900	**65,700**	94,400	25,800	66.1
	$2\frac{1}{4}$	38,900	**63,600**	100,000	27,500	69.4
	$2\frac{3}{8}$	39,400	**62,400**	107,000	29,300	71.4
	$2\frac{1}{2}$	40,600	**62,500**	116,000	32,400	75.6
$\frac{15}{16}$ and 1	2	34,000	**68,000**	68,000	16,600	57.6
	$2\frac{1}{8}$	35,100	**66,400**	74,700	18,300	59.6
	$2\frac{1}{4}$	35,600	**64,200**	80,200	19,400	63.4
	$2\frac{3}{8}$	36,900	**63,700**	87,600	20,400	66.8
	$2\frac{1}{2}$	37,400	**62,300**	93,500	22,400	69.6
	$2\frac{5}{8}$	39,300	**63,400**	103,300	24,300	67.8
	$2\frac{3}{4}$	40,100	**63,100**	110,000	26,200	69.4
	$2\frac{7}{8}$	39,800	**61,100**	114,400	28,000	74.1

II. OPEN-HEARTH STEEL PLATES $\frac{1}{2}$ INCH THICK, $\frac{5}{16}$-INCH COVER PLATES.

IRON RIVETS, MACHINE DRIVEN. HOLES DRILLED.

Material of Plates:
- Strength lengthwise = 58,560 lbs. per square inch.
- " crosswise = 58,380 " " " "
- Elastic limit = 31,600 " " " "
- Elongation in 10 in. = 24 per cent.
- Reduction of area = 50 " "

(Each result the mean of two tests. Method of failure indicated by bold-faced type.)

Size of Rivet and of Hole.	Pitch of Rivets.	Maximum Stress on Joint per Square Inch.				
		Tension on Gross Section of Plates.	Tension on Net Section of Plates.	Compression on Bearing Surface of Rivet.	Shearing on Rivet.	Efficiency of Joint.
inch.	inches.					
$\frac{11}{16}$ and $\frac{3}{4}$	$1\frac{3}{4}$	38,400	**67,100**	89,500	36,600	67.1
	$1\frac{7}{8}$	39,300	**65,400**	98,300	**40,300**	68.6
	2	37,000	**59,100**	98,800	40,800	64.7
$\frac{13}{16}$ and $\frac{7}{8}$	$1\frac{7}{8}$	36,900	**69,100**	79,100	27,700	64.5
	2	37,900	**67,400**	86,700	30,600	66.3
	$2\frac{1}{8}$	44,800	**76,100**	108,800	37,200	76.0
	$2\frac{1}{4}$	39,800	**65,200**	102,500	36,200	66.4
	$2\frac{3}{8}$	40,000	63,400	108,800	**38,500**	66.7
$\frac{15}{16}$ and 1	2	34,400	**68,800**	68,800	21,100	60.2
	$2\frac{1}{8}$	35,000	**66,000**	74,400	22,400	59.2
	$2\frac{1}{4}$	36,900	**66,500**	83,200	25,500	61.5
	$2\frac{3}{8}$	38,700	**66,800**	92,100	28,300	64.5
	$2\frac{1}{2}$	38,700	**64,600**	96,800	29,800	66.9
	$2\frac{5}{8}$	39,700	**64,700**	104,500	31,200	67.2
	$2\frac{3}{4}$	40,600	**63,900**	111,500	34,300	67.6
$1\frac{1}{16}$ and $1\frac{1}{8}$	$2\frac{1}{8}$	30,400	**64,800**	57,300	15,400	51.5
	$2\frac{1}{4}$	33,400	**66,800**	66,800	17,600	56.5
	$2\frac{3}{8}$	33,700	**63,900**	71,200	19,400	58.0
	$2\frac{1}{2}$	36,100	**65,000**	81,400	22,000	62.3
	$2\frac{5}{8}$	36,400	**63,700**	85,000	22,700	61.6
	$2\frac{3}{4}$	38,700	**65,400**	94,700	25,800	64.5
	$2\frac{7}{8}$	38,900	**63,900**	99,400	27,300	67.0
	3	39,900	**63,900**	106,300	29,100	68.8
	$3\frac{1}{8}$	38,900	60,800	**108,200**	29,600	67.7

III. OPEN-HEARTH STEEL PLATES $\frac{3}{8}$ INCH THICK, $\frac{3}{8}$-INCH COVER PLATES.

IRON RIVETS, MACHINE DRIVEN. HOLES DRILLED.

Material of Plates:
- Strength lengthwise = 56,100 lbs. per square inch.
- " crosswise = 56,700 " " " "
- Elastic limit = 28,200 " " " "
- Elongation in 10 in. = 27 per cent.
- Reduction of area = 47 " "

(Each result the mean of two tests. Method of failure indicated by bold-faced type.)

Size of Rivet and of Hole.	Pitch of Rivet.	Maximum Stress on Joint per Square Inch.				
		Tension on Gross Section of Plate.	Tension on Net Section of Plate.	Compression on Bearing Surface of Rivet.	Shearing on Rivet.	Efficiency of Joint.
inch.	inches.					
$\frac{13}{16}$ and $\frac{7}{8}$	$1\frac{7}{8}$	34,100	**64,000**	73,200	33,100	61.4
	2	35,900	**63,900**	82,100	**37,600**	64.9
	$2\frac{1}{8}$	36,800	**62,600**	89,400	40,600	66.9
$\frac{15}{16}$ and 1	2	32,100	**64,100**	64,100	25,400	58.3
	$2\frac{1}{8}$	34,100	**64,300**	72,400	28,600	60.9
	$2\frac{1}{4}$	36,300	**65,200**	81,600	31,800	63.2
	$2\frac{3}{8}$	37,600	**65,100**	89,200	36,500	65.7
	$2\frac{1}{2}$	37,100	**61,900**	92,900	**36,500**	66.3
	$2\frac{5}{8}$	36,300	58,800	95,300	**37,600**	64.7
$1\frac{1}{16}$ and $1\frac{1}{8}$	$2\frac{1}{8}$	31,100	**66,100**	58,700	20,700	56.5
	$2\frac{1}{4}$	33,200	**66,300**	66,200	22,800	57.9
	$2\frac{3}{8}$	34,500	**65,600**	72,800	25,200	60.2
	$2\frac{1}{2}$	34,900	**63,400**	77,500	27,100	62.3
	$2\frac{5}{8}$	36,700	**64,300**	85,600	29,900	65.4
	$2\frac{3}{4}$	38,100	**64,600**	93,100	32,100	66.5
	$2\frac{7}{8}$	38,400	**63,000**	98,400	33,900	67.0
	3	38,400	**61,500**	102,500	**35,900**	68.6
	$3\frac{1}{8}$	37,900	59,200	105,000	**37,100**	68.9

IV. OPEN-HEARTH STEEL PLATES $\frac{5}{8}$ INCH THICK, $\frac{7}{16}$-INCH COVER PLATES.

IRON RIVETS, MACHINE DRIVEN. HOLES DRILLED.

Material of Plates:
- Strength lengthwise = 58,300 lbs. per square inch.
- " crosswise = 60,200 " " " "
- Elastic limit = 28,300 " " " "
- Elongation in 10 in. = 27 per cent.
- Reduction of area = 48 " "

(Each result the mean of two tests. Method of failure indicated by bold-faced type.)

Size of Rivet and of Hole.	Pitch of Rivets.	Maximum Stress on Joint per Square Inch. Tension on Gross Section of Plate.	Tension on Net Section of Plate.	Compression on Bearing Surface of Rivet.	Shearing on Rivet.	Efficiency of Joint.
inch.	inches.					
$\frac{15}{16}$ and 1	2	32,000	**63,900**	64,100	30,400	54.2
	$2\frac{1}{8}$	34,600	66,100	73,500	**35,800**	58.6
	$2\frac{1}{4}$	35,900	**64,600**	80,800	38,900	60.8
	$2\frac{3}{8}$	36,100	**62,200**	85,700	40,400	61.1
$1\frac{1}{16}$ and $1\frac{1}{8}$	$2\frac{1}{8}$	31,500	**66,900**	62,000	24,900	53.4
	$2\frac{1}{4}$	34,000	**68,000**	68,100	28,700	57.6
	$2\frac{3}{8}$	35,300	**67,100**	74,700	32,100	59.8
	$2\frac{1}{2}$	38,100	**69,300**	84,200	35,200	63.0
	$2\frac{5}{8}$	35,700	62,600	83,300	**34,700**	59.3
	$2\frac{3}{4}$	37,700	63,800	92,300	**38,900**	64.0
	$2\frac{7}{8}$	39,300	**64,000**	93,800	41,300	66.5
$1\frac{3}{16}$ and $1\frac{1}{4}$	$2\frac{1}{4}$	29,500	**66,400**	53,200	20,100	50.0
	$2\frac{3}{8}$	33,600	**71,000**	63,900	23,800	55.6
	$2\frac{1}{2}$	34,700	**69,300**	69,500	26,400	57.5
	$2\frac{5}{8}$	36,500	**69,400**	77,200	29,500	60.4
	$2\frac{3}{4}$	34,500	**63,200**	75,900	29,200	58.3
	$2\frac{7}{8}$	38,300	**67,700**	88,200	35,000	63.4
	3	37,600	64,600	90,200	**35,800**	65.0
	$3\frac{1}{8}$	38,300	63,600	95,700	**36,500**	64.8
	$3\frac{1}{4}$	36,800	59,700	95,400	**36,200**	62.3
	$3\frac{3}{8}$	39,900	63,300	**107,800**	41,100	67.5

U. S. BUREAU OF STEAM ENGINEERING TESTS OF MULTIPLE-RIVETED BUTT JOINTS.

DOUBLE-RIVETED BUTT JOINTS 20 INCHES WIDE.

HOLES DRILLED. RIVETS STAGGERED AND MACHINE-DRIVEN.

(Method of failure indicated by bold-faced type.)

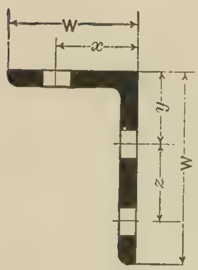
Size of Rivet and of Hole.	Pitch of Rivets.	Distance between Rows.	Kind of Rivet.	Thickness of Plate.	Strength of Plate in Pounds per Square Inch.	Maximum Stress on Joint per Square Inch. Tension on Gross Section of Plate.	Tension on Net Section of Plate.	Compression on Bearing Surface of Rivet.	Shearing on Rivets.	Efficiency of Joints.
inch.	inches.	inches.		inch.						
$\frac{3}{4}$ and $\frac{25}{32}$	3	$1\frac{1}{2}$	Steel	$\frac{5}{8}$	53,710	42,860	55,990	84,320	**45,470**	79.8
						40,960	**53,530**	80,690	43,000	76.2
						42,720	**55,800**	84,140	**42.540**	79.5
1 and $1\frac{1}{16}$	$3\frac{3}{4}$	2	Steel	$\frac{7}{8}$	51,190	35,180	**51,150**	62,560	32,340	68.7
						36,190	**52,640**	64,410	33,210	70.7
						35,780	**52,050**	63,630	32,970	69 9

TRIPLE-RIVETED BUTT JOINTS 20 INCHES WIDE.

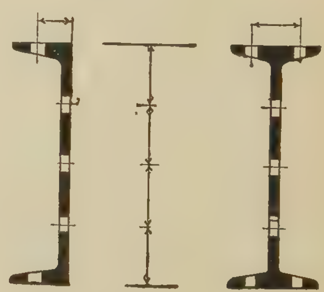
Size of Rivet and of Hole.	Pitch of Rivets.	Distance between Rows.	Kind of Rivet.	Thickness of Plate.	Strength of Plate in Pounds per Square Inch.	Tension on Gross Section of Plate.	Tension on Net Section of Plate.	Compression on Bearing Surface of Rivet.	Shearing on Rivets.	Efficiency of Joints.
$\frac{3}{4}$ and $\frac{25}{32}$	$3\frac{9}{16}$	$1\frac{5}{8}$	Steel	$\frac{5}{8}$	53,710	43,460	**54,040**	69,560	36,320	80.9
						40,390	50,200	64,630	33,410	75.2
						44,290	**55,050**	70,790	36,640	82.5
1 and $1\frac{1}{16}$	$4\frac{9}{16}$	$1\frac{15}{16}$	Steel	$\frac{7}{8}$	51,190	37,910	**51,080**	53,500	27,830	74.1
						38,400	**51,720**	54,190	28,420	75.0
						37,950	**51,130**	53,560	27,730	74.1
$\frac{3}{4}$ and $\frac{25}{32}$	$3\frac{9}{16}$	$1\frac{5}{8}$	Iron	$\frac{5}{8}$	53,710	43.600	**54,220**	69,720	**35,130**	81.2
						43,260	**53,780**	69,220	36,460	80.5
						43,000	53,460	68,820	**36,390**	80.1

244. Watertown Arsenal Tests of Riveted Joints.—In the preceding tables are given the results of some tests on riveted joints at the Watertown Arsenal. They agree practically with others made recently elsewhere and are worthy of study, as they show the value of close riveting in distributing the stress uniformly over the piece by the higher values of the net sections where the pitch is the least. They also show the very high crushing strength of rivets, their shearing strength, and seem to indicate that the real strength of a riveted joint is the friction between the plates caused by the tension on the rivets. Almost invariably it will be found that the tensile strength per square inch of net area of plate in the riveted joint is greater than the tensile strength of the specimen test of the same material.

245. The Designation and Location of Rivets on Working Drawings.—Working drawings, or those used in the manufacture of the work, should always show definitely and clearly the size, location, and kind of the rivets used. As it is customary to use only one size of rivet in one complete piece, a single note on each drawing giving the size of the rivet and of the hole is generally all that is necessary. This note should always be prominent enough to be easily found. The location of the rivets, inasmuch as this is a question which should be determined by the designing engineer and not by the shop workmen, can be given by dimensions on the drawing. It must be remembered that in the manufacture each component piece of a riveted member is punched separately, hence the dimensions must be given so that the proper location of the rivets on them will be understood. For plate iron the rivets are located from the end and edge of the piece as base lines. For angle and Z iron the base lines are the ends and backs. For channel iron the backs and end if the rivets are in the flanges, or from the sides and end if the rivets are in the web. In general, for any shape use as a base line an edge which is definitely finished in the rolling of the shape and never any filleted corner or rounded edge. In the following table is given the usual spacing of rivets for angle iron and the distances to be given in locating rivets for angles, channels, and I beams.



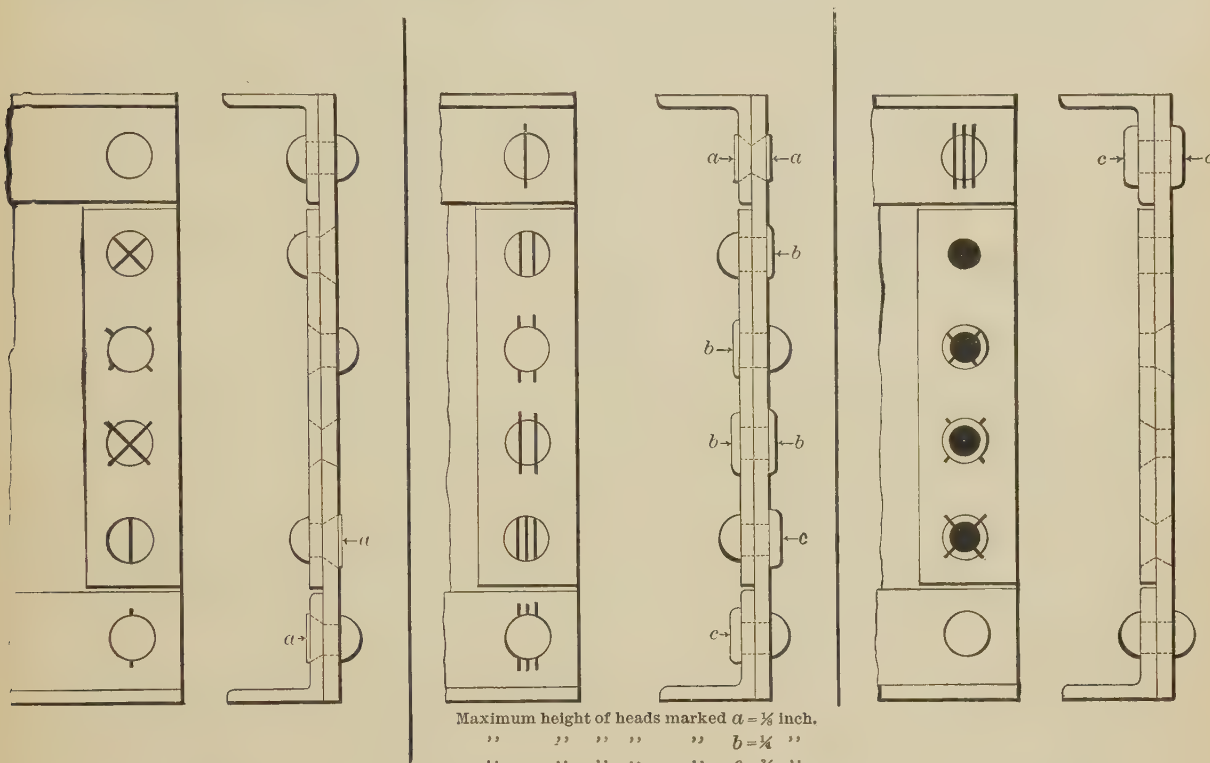
Width of Leg of Angle.	Pitch.		
	Double.		Single.
W	y	z	x
$1\frac{3}{4}$	Not	used	1
2	"	"	$1\frac{8}{16}$
$2\frac{1}{4}$	"	"	$1\frac{3}{8}$
$2\frac{1}{2}$	"	"	$1\frac{1}{2}$
$2\frac{3}{4}$	"	"	$1\frac{5}{8}$
3	"	"	$1\frac{3}{4}$
$3\frac{1}{4}$	"	"	$1\frac{7}{8}$
$3\frac{1}{2}$	"	"	2
4	"	"	$2\frac{1}{2}$
$4\frac{1}{2}$	"	"	$2\frac{3}{4}$
5	$1\frac{3}{4}$	2	3
6	$2\frac{1}{4}$	$2\frac{1}{2}$	4



Besides giving the size of the rivets and their location it is necessary to show on the drawing whether they are full-button-headed, flat headed, or countersunk; also, whether they are to be shop-driven or are field rivets. This may be done by adopting some conventional sign for each of the different kinds of rivets. A few years ago the conventional signs* shown on page 267 were generally adopted by the bridge companies and consulting bridge engineers, and it would save a great amount of annoyance if they were universally adopted. They have stood the test of continual use for two years, and are to be recommended for their simplicity and clearness.

* Originally devised by Mr. Frank C. Osborne, C.E.

The heads marked *a* are countersunk, but are not chipped so as to bring them flush with the surface of the plate. It is impossible to determine the exact length of material required for countersunk rivets which will result in tight rivets and also drive flush with the plate. The heads marked *b* and *c* are button-heads flattened.



Maximum height of heads marked $a = \frac{1}{8}$ inch.
" " " " " $b = \frac{1}{4}$ "
" " " " " $c = \frac{3}{8}$ "

CONVENTIONAL RIVET SIGNS.

246. Bolts are manufactured either "rough" or "finished." The finished bolt is the rough bolt finished to exact dimensions. Rough bolts are used in the temporary fitting up of work in the shops and during erection, and generally for all woodwork. Finished bolts are very expensive, and are only used in those cases where a close fit is absolutely essential. The latter are often used as a substitute for rivets, in which case they are proportioned for the same allowed stresses as rivets. In cases where rivets would be subjected to direct tension tending to pull off the rivet-heads, finished bolts are now generally used, as they are more reliable than rivets to resist such stresses. Where finished bolts are to be used it must also be borne in mind that the holes for them must be drilled to an exact fit with the bolts, and that this adds to the expense. An advantage of the use of bolts in place of field rivets is that the work can be done much quicker, and when it is desirable to erect a span as quickly as possible bolts may be used to advantage.

When ordering bolts give the diameter, length under the head, and length of thread necessary. Tables giving the standard dimensions of bolts may be found in various handbooks.

247. Anchor Bolts.—The ordinary anchor bolts which are used to hold the bed plates of a span in position are rough bolts and are made in various styles. The three styles shown in Fig. 292 are the ones most commonly used.

The wedge bolt, (*a*) Fig. 292, is split at the bottom and a small wedge inserted, which when the bolt is driven in expands the lower end of the bolt and prevents its being pulled out. The "rag" or "swedged" bolt (*b*) has indentations on its surface. The bolt (*c*) has a screw thread cut on its lower end. When the bolts are put in, the hole is filled with sulphur, lead, or cement, which sets and holds the bolt.

Where the anchor bolts are not relied on to resist any direct tension, they are put from six to nine inches in the masonry.

If the bolts are relied on to take a direct tensile stress—as, for instance, at the foot of a high trestle post or for elevated-railroad posts,—they are put in the masonry as far as it is necessary to insure a weight of masonry resting on them sufficient to resist the tension on

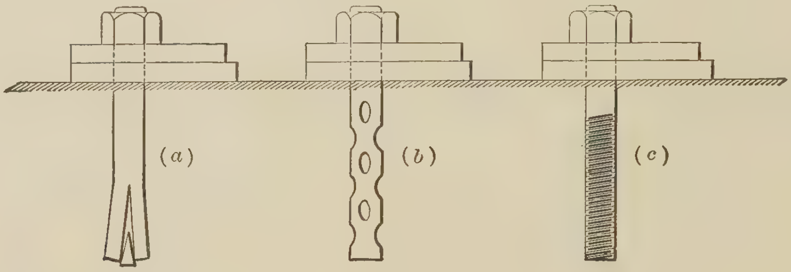
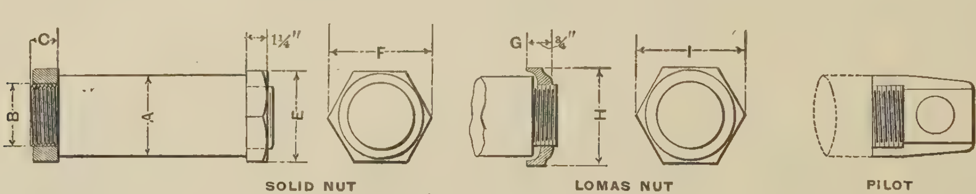


FIG. 292.

the bolt. These bolts are usually provided with a head on their lower end, which bears on an iron plate. If made in this way they are built into the masonry when the latter is laid. Bolts with screw-ends, as shown in (*c*), Fig. 292, have been set in masonry with cement, and when tested have developed the value of the bolt in tension.

248. Pins.—The following table gives the standard sizes of pins made by the Edge Moor Bridge Works. The diameter of the finished pin is given in sixteenths of an inch, as the material from which the pin is made is furnished in sizes measured in even eighths of an inch, and one sixteenth of an inch is taken off in turning or finishing the pin. The table also gives

STANDARD TRUSS PINS AND NUTS.



Diameter of Pin = A.		Thread.		Solid Nut.		Lomas Nut.		
Nominal.	Finished.	Diameter = B.	Length = C.	Short Diameter = E.	Long Diameter = F.	Depth of Recess = G.	Short Diameter = H.	Long Diameter = I.
in.	in.	in.	in.	in.	in.	in.	in.	in.
$2\frac{1}{2}$	$2\frac{7}{16}$	2	$1\frac{1}{2}$	3	$3\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{3}{4}$
$2\frac{3}{4}$	$2\frac{11}{16}$	2	"	$3\frac{1}{2}$	$4\frac{1}{16}$	"	$3\frac{1}{2}$	$4\frac{1}{16}$
3	$2\frac{15}{16}$	2	"	$3\frac{1}{2}$	$4\frac{1}{16}$	"	$3\frac{3}{4}$	$4\frac{3}{8}$
$3\frac{1}{4}$	$3\frac{3}{16}$	$2\frac{1}{2}$	"	4	$4\frac{11}{16}$	"	4	$4\frac{11}{16}$
$3\frac{1}{2}$	$3\frac{7}{16}$	$2\frac{1}{2}$	"	4	$4\frac{11}{16}$	"	$4\frac{1}{4}$	$4\frac{15}{16}$
$3\frac{3}{4}$	$3\frac{11}{16}$	$2\frac{3}{4}$	"	$4\frac{1}{2}$	$5\frac{3}{16}$	"	$4\frac{1}{2}$	$5\frac{1}{4}$
4	$3\frac{15}{16}$	3	"	$4\frac{1}{2}$	$5\frac{3}{16}$	$\frac{3}{4}$	$4\frac{3}{4}$	$5\frac{1}{2}$
$4\frac{1}{4}$	$4\frac{3}{16}$	$3\frac{1}{2}$	"	5	$5\frac{13}{16}$	"	5	$5\frac{13}{16}$
$4\frac{1}{2}$	$4\frac{7}{16}$	$3\frac{1}{2}$	"	5	$5\frac{13}{16}$	"	$5\frac{1}{4}$	$6\frac{1}{16}$
$4\frac{3}{4}$	$4\frac{11}{16}$	4	"	$5\frac{1}{2}$	$6\frac{3}{8}$	"	$5\frac{3}{4}$	$6\frac{11}{16}$
5	$4\frac{15}{16}$	4	"	$5\frac{1}{2}$	$6\frac{3}{8}$	"	$5\frac{3}{4}$	$6\frac{11}{16}$
$5\frac{1}{4}$	$5\frac{3}{16}$	4	"	6	$6\frac{15}{16}$	"	6	$6\frac{15}{16}$
$5\frac{1}{2}$	$5\frac{7}{16}$	4	"	6	$6\frac{15}{16}$	"	$6\frac{1}{4}$	$7\frac{1}{4}$
$5\frac{3}{4}$	$5\frac{11}{16}$	4	"	$6\frac{1}{2}$	$7\frac{1}{2}$	"	$6\frac{3}{4}$	$7\frac{13}{16}$
6	$5\frac{15}{16}$	4	"	$6\frac{1}{2}$	$7\frac{1}{2}$	"	$6\frac{3}{4}$	$7\frac{13}{16}$
$6\frac{1}{4}$	$6\frac{3}{16}$	4	"	7	$8\frac{1}{8}$	"	7	$8\frac{1}{8}$
$6\frac{1}{2}$	$6\frac{7}{16}$	4	"	7	$8\frac{1}{8}$	"	$7\frac{1}{4}$	$8\frac{3}{8}$
$6\frac{3}{4}$	$6\frac{11}{16}$	4	"	$7\frac{1}{2}$	$8\frac{11}{16}$	"	$7\frac{3}{4}$	$8\frac{15}{16}$
7	$6\frac{15}{16}$	4	"	$7\frac{1}{2}$	$8\frac{11}{16}$	"	$7\frac{3}{4}$	$8\frac{15}{16}$

NOTE.—The length of thread for pins with Lomas nuts is $1\frac{1}{4}$ inches for all pins less than $3\frac{15}{16}$ diameter.

the standard sizes for the solid pin nut and the Lomas pin nut. The Lomas nut is now the one most commonly used; the recess in the nut allowing the nut to be drawn up tight against the bars packed on the pin, as there are usually small inaccuracies in the thickness of the bars which prevent an exact calculation of the grip of the pin. Before the Lomas nut came into use a wrought-iron washer bored to fit the pin was used with the solid pin nut to accomplish the same purpose. The pilot nuts shown are put on the pin to protect the thread and assist in guiding the pin while it is being driven.

In calculating the grip of pins it is customary to increase the net grip $\frac{1}{16}$ of an inch for each eyebar, and one fourth of an inch for each riveted compound member. Thus in Fig. 293 the net grip would be 27 inches but to allow for inaccuracies in the thickness of the members, the grip of the pin would be made $27\frac{7}{8}$ iches. For the same reason, when there are several members packed inside of a built section, as for instance in a top chord, the clear width inside of the section must be enough to allow the eyebars to be $\frac{1}{16}$ of an inch thicker than their nominal thickness and the posts or built sections $\frac{1}{4}$ of an inch wider than their net width, and besides this the net clear width of the chord must be taken as $\frac{1}{4}$ of an inch less than the nominal net width.

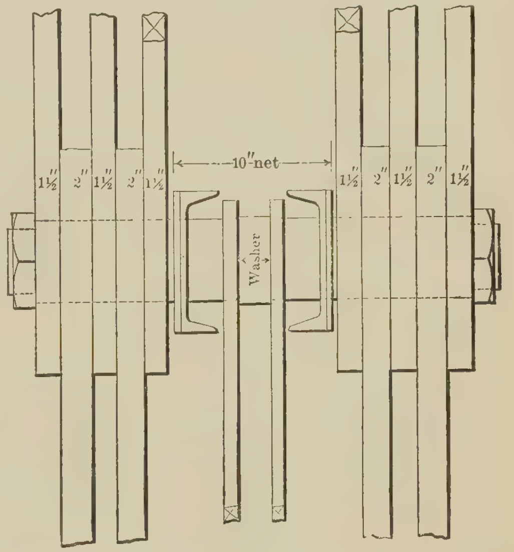


Fig. 293.

The members on a pin should be packed as closely as possible. When it is necessary, in order to reduce the bending moment on the pin, to pack eyebars in pairs, they should be separated at least one half of an inch, to prevent the collection of dirt and water and to allow the bars to be painted.

249. Lateral Pins.—The pins used in the lateral system, if large, are the same as those used for truss pins. If they are small or under $2\frac{1}{2}$ inches in diameter, the cotter pin shown in Fig. 294 is used. Sometimes both a nut and cotter are used on this style of pin. Lateral pins should always be packed so that they will be in double shear. This point is mentioned because it was formerly, and to a certain extent is now, the practice to use them in single shear.

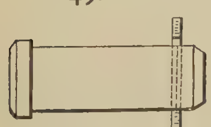


Fig. 294.

250. The Calculation of the Stresses on Pins.—A pin must be analyzed as a short beam which is subjected to excessive shears and bending moments. It must be dimensioned for three kinds of failure, viz.:

1. Shearing.
2. Cross-bending.
3. Crushing.

Dimensioning for Shear.—It is customary to regard the shearing stress as uniformly distributed over the cross-section, in which case the intensity of the shearing stress is

$$s = \frac{S}{\pi r^2}. \qquad (1)$$

But from eq. (10), page 134, we obtain for the intensity of the shearing stress on the neutral plane,

$$s_0 = \frac{4}{3}s = \frac{4}{3}\frac{S}{\pi r^2}; \qquad (2)$$

in other words, for a solid circular section the maximum shearing stress is four thirds the mean stress. A sufficiently low working intensity of the shearing stress is always taken to allow equation (1) to be used with safety, or to assume that the shearing stress is uniformly distributed.

For Wrought-iron Pins use for mean intensity of shearing stress in pins 8000 lbs. per square inch.

For Steel Pins use 10,000 lbs. per square inch for shear.

The maximum shearing stress at any section is found by making a continued sum of the horizontal and of the vertical components of the forces coming upon it. The maximum shear at any section is the square root of the sum of the squares of the horizontal and vertical shears at that section.

251. Bending Moment on Pins.—If a pin be regarded as a beam, the bending moment at any section may be found by assuming the loads to be concentrated at the centres of the bearings of the members meeting at that joint. So long as the pin remains unbent this is correct. When the pin bends appreciably the members no longer bear evenly upon it, and the above assumption may become very erroneous, since both the forces and the lever arms change as a result of the bending, and the bending moment becomes very much less than would appear from the computation.

The members should be packed on the pin in such a way as to produce the least moment.

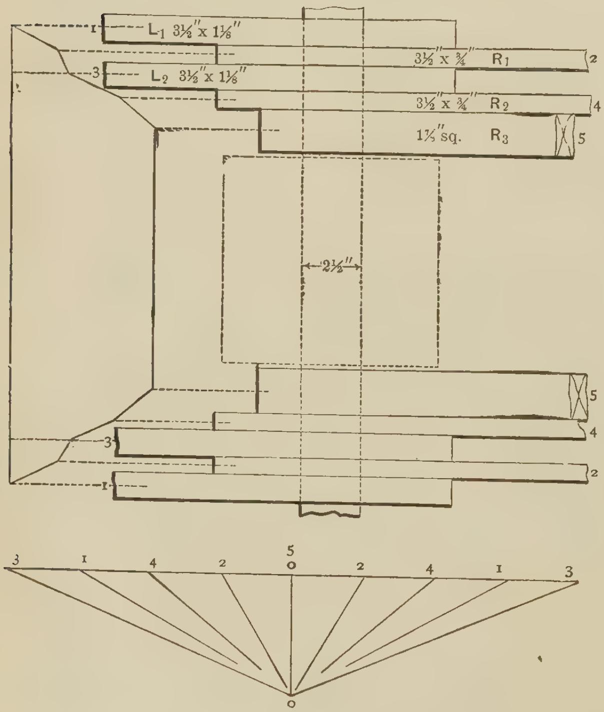


FIG. 295.

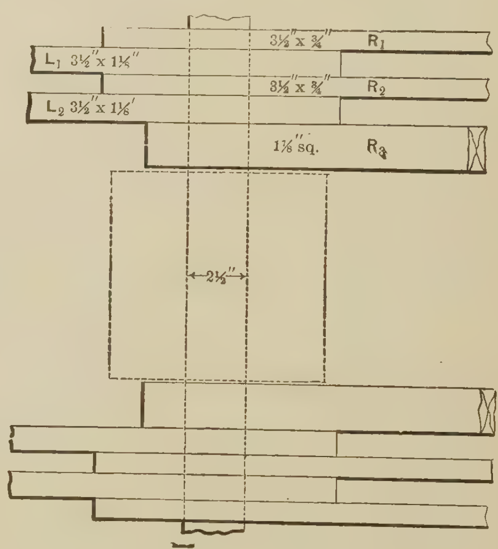


FIG. 296.

An apparently slight change in the arrangement produces astonishing changes in the maximum moment. Thus in Fig. 295, which is an actual case,* we have the following computation of the shears and bending moments:

*Reported by Mr. Frank C. Osborn, C.E., in *Engineering News*, Feb. 18, 1888.

HORIZONTAL MOMENTS AND SHEARS, FIG. 295

Member	Stress.	Shear.	Lever-arm.	Bending Moment. Increment.	Bending Moment. Total Moments in Inch-pounds.
L_1	+ 45,600				
		+ 45,600	1 inch	45,600	
R_1	− 30,200				45,600
		+ 15,400	1 "	15,400	
L_2	+ 45,600				61,000
		+ 61,000	1 "	61,000	
R_2	− 30,200				122,000
		+ 30,800	$1\frac{3}{8}$ "	42,300	
R_3	− 30,800				164,300
		0			

HORIZONTAL MOMENTS AND SHEARS, FIG. 296.

Member.	Stress.	Shear.	Lever-arm.	Bending Moment. Increment.	Bending Moment. Total Moments in Inch-pounds.
R_1	− 30,200				
		− 30,200	1 inch	− 30,200	
L_1	+ 45,600				− 30,200
		+ 15,400	1 "	+ 15,400	
R_2	− 30,200				− 14,800
		− 14,800	1 "	− 14,800	
L_2	+ 45,600				− 29,600
		+ 30,800	$1\frac{9}{16}$ "	+ 48,100	
R_3	− 30,800				+ 18,500
		0			

In this case the vertical components of the shears and moments would not materially add to those from the horizontal components and are not computed. It is evident at once that the second arrangement, shown in Fig. 296, gives less than one fifth as great a bending moment as the actual arrangement shown in Fig. 295.

The bending moment may also be found graphically, as shown in Fig. 295, by means of bending moment diagrams.*

252. Fibre Stress in Pins.—From the formula $M=\frac{fI}{y_1}$ or $f=\frac{My_1}{I}$, we have for a solid cylindrical beam

$$f=\frac{Mr}{\frac{\pi}{4}r^4}=10.2\frac{M}{d^3}. \quad \ldots \ldots \ldots \ldots \ldots \quad (3)$$

In the above example the pin was $2\frac{1}{2}$ inches in diameter; hence we have, as the stress on the extreme fibre with the first arrangement,

$$f=\frac{10.2\times 164{,}300}{15.6}=108{,}000 \text{ lbs. per square inch.}$$

Evidently there was no such fibre stress in the pin or it would have broken. It doubtless was considerably bent under this load, and this bending caused a redistribution of the eye-bar stresses in such a way as to greatly diminish the bending moment on the pin, but with the effect of overstraining some of the bars. The pin can only perform its function of transmitting the stresses and distributing them equitably amongst the members by remaining sensibly straight. In Chapter VIII it was shown that the computed stress on the extreme fibre of a bent beam always exceeds the actual stress on these fibres when bent beyond the elastic limit. In this case, although the pin did not break it was evidently greatly overstrained.

* By obtaining the vertical and horizontal forces acting on the pin, and constructing two moment diagrams on a line at 45° with these directions, the vertical forces forming a diagram with vertical ordinates, and the horizontal forces one with horizontal ordinates, the actual maximum moment may be taken off with a pair of dividers by setting on corresponding portions of these two diagrams. The authors prefer the tabular computations, however, as given above.

TABLE GIVING FACTORS WHICH IF MULTIPLIED BY THE BENDING MOMENT IN INCH-POUNDS ON A PIN WILL GIVE THE OUTER FIBRE STRESS ON THE PIN IN POUNDS PER SQUARE INCH.

Formula: $f = \frac{10.2}{d^3} M.$

	Diameter of Pin in Inches.														
	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$	4	$4\frac{1}{4}$	$4\frac{1}{2}$	$4\frac{3}{4}$	5	$5\frac{1}{2}$	6
Factor $\frac{10.2}{d^3}$	1.275	0.900	0.654	0.490	0.378	0.297	0.238	0.193	0.159	0.133	0.112	0.094	0.082	0.061	0.047

With these factors and with the assumed stresses on the extreme fibre as given in the following table we may find the permissible bending moments for pins of various diameters as follows:

MAXIMUM PERMISSIBLE BENDING MOMENTS ON PINS.

Diameter of Pin.	Extreme Fibre Stress per Square Inch.				
	15000	18000	20000	21000	22500
$1\frac{3}{16}$	2470	2960	3290	3450	3700
$1\frac{7}{16}$	4380	5250	5830	6130	6560
$1\frac{11}{16}$	7080	8500	9430	9910	10620
$1\frac{15}{16}$	10710	12860	14280	15000	16070
$2\frac{3}{16}$	15420	18500	20550	21580	23130
$2\frac{7}{16}$	21330	25600	28430	29850	32000
$2\frac{11}{16}$	28590	34310	38100	40050	42890
$2\frac{15}{16}$	37340	44800	49760	52250	56000
$3\frac{3}{16}$	47700	57240	63570	66750	71560
$3\frac{7}{16}$	59830	718cc	79740	83750	89750
$3\frac{11}{16}$	73860	88630	98430	103400	110800
$3\frac{15}{16}$	89920	107900	119800	125900	134900
$4\frac{3}{16}$	108200	129800	144100	151400	162200
$4\frac{7}{16}$	128700	154500	171500	180200	193100
$4\frac{11}{16}$	151700	182100	202200	212400	227600
$4\frac{15}{16}$	177300	212800	236300	248200	266000
$5\frac{3}{16}$	205600	246800	274000	287800	308400
$5\frac{7}{16}$	236800	284200	315600	331500	355200
$5\frac{11}{16}$	271000	325200	361100	379300	406500
$5\frac{15}{16}$	308300	370000	410900	431600	462500
$6\frac{3}{16}$	348900	418700	465000	488400	523400
$6\frac{7}{16}$	393000	471600	523700	550000	589400
$6\frac{11}{16}$	440500	528700	587100	616600	660800
$6\frac{15}{16}$	491800	590200	655400	688500	737700
$7\frac{3}{16}$	545800	655000	727800	764200	818700
$7\frac{7}{16}$	605900	727000	807800	848200	908800

In the case of suspension-bridge pins where nearly the same total stress is transmitted continuously in one plane, the transverse load being relatively very small, the stresses in these may be omitted in finding the stresses on the pins. There are then three methods of arranging a series of eyebars on a pin, as shown in Figs. 297, 298, and 299. In the first, Fig. 297, the bars are all the same size, and alternate as shown. In the second, Fig. 298, they are of the same size and are arranged in pairs. In the third, Fig. 299, the two outer bars are one half the width of the others, and are arranged alternately.

In the first case the bending moment increases regularly from the end to the centre, each pair of bars forming a couple the moments of which, on either half of the pin, are all of the same sign. The maximum bending moment on the pin is here nPt, where n is one half the number of bars coming to the pin from one direction, or $n =$ number of couples on one half of the pin; $P =$ stress in one bar; and $t =$ thickness of one bar plus $\frac{1}{16}$ in. for probable spacing.

In the second case, the bars being in pairs, the greatest moment is that due to but one couple, since this moment is at once reduced to zero by another couple of the opposite sign. Or the maximum moment here is Pt.

In the third case, the outer bars being half the thickness of the others, the moment is still less, as shown by the moment diagrams accompanying the several arrangements.

The arrangement in pairs, Fig. 298, can often be used in packing lower chord joints in truss bridges. A little intelligence and care in fixing the arrangement of these members on the pin may prevent very serious blunders when it comes to erection. The exact arrangement should always be clearly indicated.

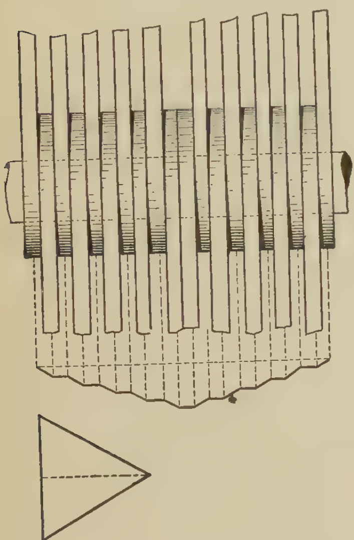


FIG. 297.

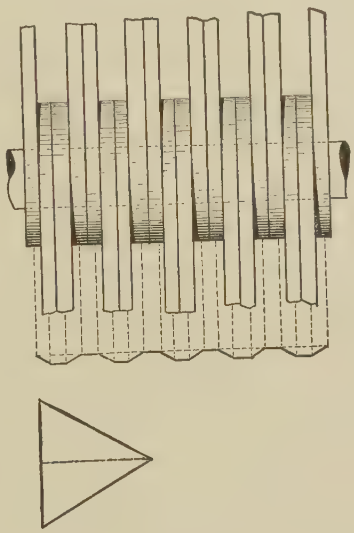


FIG. 298.

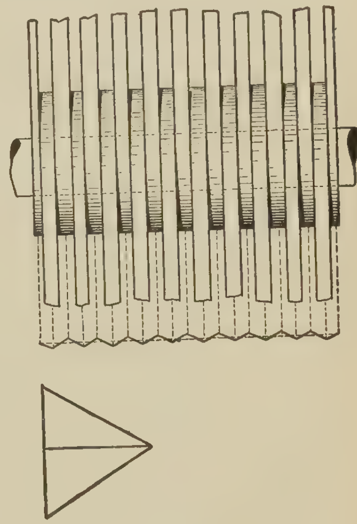


FIG. 299.

253. **The Bearing or Crushing Value of Pins** is taken as so many pounds per square inch on the bearing area of the pin on the bearing plates. This area is found by multiplying the diameter of the pin by the thickness of the bearing. The usual pressure allowed is 12,000 pounds per square inch for railway and 15,000 pounds per square inch for highway construction if either the plates or the pin or both are of iron. If both the bearing and the pin are of steel, 15,000 and 18,000 pounds per square inch are the corresponding values which are commonly used.

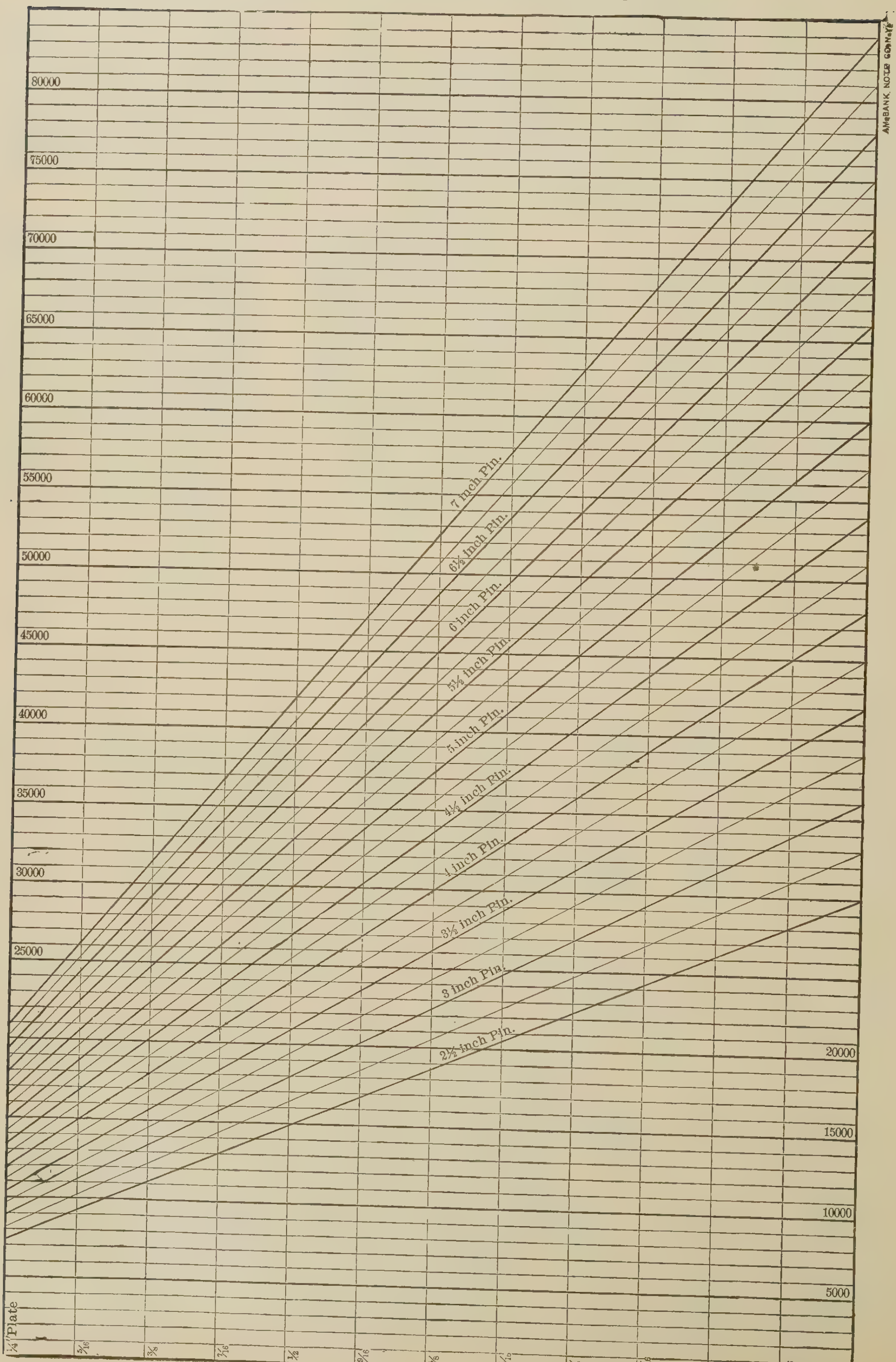
The pressure between the bearings of eyebars and pins is usually not limited except by a clause in the specifications which limits the *minimum* diameter of the pin to three fourths the width of the bar. When this minimum pin is used the bearing pressure per square inch is $33\frac{1}{3}$ per cent greater than the tensile stress per square inch on the bar, which is not an excessive ratio.

The diagram given on page 274 gives the bearing value of pins on plates of different thicknesses allowing a pressure of 12,000 pounds per square inch. Similar diagrams allowing 15,000 and 18,000 pounds per square inch should be made for practical use.

254. **The Bearing Strength of Rollers.***—When a cylindrical roller bears on a flat plate, or transmits a load when resting between two flat beds, the linear element in contact for a zero pressure becomes a surface of considerable width as the pressure increases, since the materials of both bed and roller are elastic. The law of the distribution of the stress from this small contact area over the cross-section of roller and plates is unknown. The

* The discussion contained in this and the following article is taken from a paper contributed by Prof. Johnson to the Engineers' Club of St. Louis, December, 1892.

BEARING VALUES OF PLATES ON PINS OF DIFFERENT DIAMETERS.
ALLOWED BEARING PRESSURE 12,000 POUNDS PER SQUARE INCH.



intensity of this stress, too, is always far beyond what is ordinarily known as the elastic limit. This limiting stress corresponds to the maximum distortion which will entirely disappear when the load is removed, or to the initial permanent set. This permanent set or fixed distortion is a kind of cold flowing of the material. When a plain cylindrical column is subjected to a uniform compressive stress over its entire cross-section, as in Fig. 300, it may be said to be in a condition of *free flow*, since it is free to spread in all directions

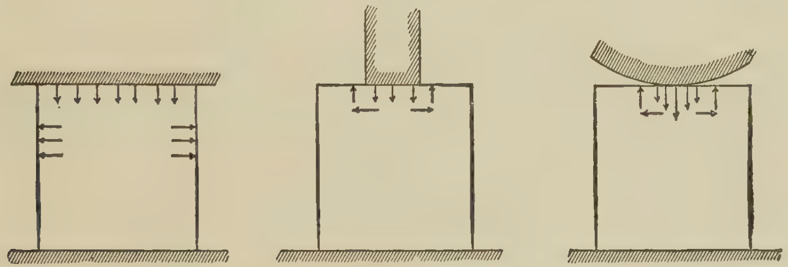


FIG. 300. FIG. 301. FIG. 302.

throughout the length of the column. In Fig. 301 the material is compressed uniformly over a small area, as with a die. Here there is a flowing of the metal laterally, and then vertically, finding escape around the edges of the die. This is a condition of confined or restricted flow, and evidently the elastic limit here will be much higher than with the simple column. In Fig. 302 the surface is compressed by a cylinder, or sphere, the greatest distortion being at the middle of the area of contact. When this metal is forced to move, or flow, it can find escape only out around the limits of the compressed area. But at these limits the metal is very little compressed, and hence must be moved from the centre. The confining ring of metal inside the limits of external flow is now much wider and hence the resistance to flow much greater, so that this condition will be found to have a higher elastic limit stress than that shown in Fig. 301, and very much above the ordinary "elastic limit in compression," which is found for the free-flow condition of Fig. 300.

What the relation between stress and strain is in the case of a roller on a plane surface is also unknown. It is evidently not that given by the ordinary modulus of elasticity, called Young's modulus, which gives the relation for uniform direct stress, as in Fig. 300.

Therefore, since we do not know the elastic limits for rollers on planes, and have no knowledge of the particular distribution of the stress, or of the modulus of elasticity, under such conditions, it is evident that we are not in a condition to make a theoretical analysis of the problem. All such analyses as have been made of this class of problems,* as of bearing rollers, of wheels on rails, which is the case of two cylinders crossing at right angles, and of spheres on planes, have been made on very violent assumptions, and the conclusions are far from agreeing with the facts as shown by experiment.

255. The Problem Solved Experimentally.—In order to obtain data for solving the problem of the bearing strength of rollers experimentally, Profs. Crandall and Wing of Cornell University made a careful and elaborate series of experiments on rollers 1, 2, 3, and 4 inches in diameter, on plates $1\frac{1}{2}$ inches thick, using cast-iron, wrought-iron, and steel in both rollers and plates, in all combinations. Thirty-three combinations of sizes and materials, and in all about two hundred observations of loads and corresponding areas of contact, were made. These experiments have never been published or discussed, but they were loaned to the authors of this work, and they have given them a very thorough study and analysis. Several assumptions were made as to the distribution of the stress, and all abandoned as inadequate. It was finally decided to make a purely empirical analysis of the results and obtain such equa-

* See a paper on the Strength of Metallic Rollers, by Prof. Johnson, in Journal Assoc. Eng. Soc., Vol. IV, p. 110. Also Prof. Burr's *Bridge and Roof Trusses*.

tions as they would give. The observations were made by using a coating of tallow to show the area of contact. It soon became evident that the widths of contact had usually been taken too great, especially for the lighter loads and for the larger diameters. When these were corrected (the corrections being from 0.01 to 0.03 inches, the observed widths being reduced by these amounts in several instances) the analysis showed the following relations:

Let p = load in pounds per linear inch of roller;
D = diameter of roller in inches;
A = area of contact per linear inch, in square inches,
= width of surface of contact in inches;
f_c = compressive stress in pounds per square inch at centre of area of contact;
f_e = compressive stress at the elastic limit;
f_w = working value of the compressive stress.
c, k, and K = constants determined from the observations.

It was found that

$$A = k\sqrt{pD}, \qquad (1)$$

where k has different values for the different materials.

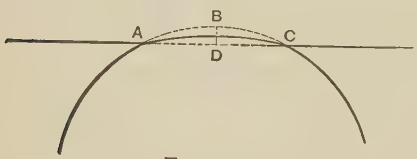


FIG. 303.

The distortion area $ABCD$, Fig. 303, may be considered as the segment of a parabola, in which case the ordinate BD is $\frac{3}{2}$ of the mean ordinate. If the stress be assumed to be proportional to the strains, or distortions, the maximum stress would also be $\frac{3}{2}$ of the mean stress. Or we could write

$$f_c = \frac{3}{2}\frac{p}{A}. \qquad (2)$$

From (1) and (2) we have

$$p = \left(\frac{2k}{3}\right)^2 f_c^2 D. \qquad (3)$$

The first indentation of the rollers and plates were not very closely observed in these experiments, but in a very careful set of experiments made by Prof. A. Marston, who, in connection with Prof. Crandall, has fully discussed this whole problem,* the elastic limits of steel rollers on steel plates were carefully determined. In these latter experiments eleven rollers were employed from 1 in. to 16 in. diameter. These results show that the elastic-limit load with soft-steel rollers on steel plates, per linear inch of roller, is

$$p_e = cD, \qquad (4)$$

where c is a constant, and in these experiments was found to be 880 for soft-steel roller and plate. Hence $p = 880D$ for soft steel. By combining eqs. (3) and (4), we find the elastic limit stress at the centre of the area of contact to be

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* See paper by Professors Crandall and Marston in *Trans. Am. Soc. C. E.*, Vol. XXXII., p. 99, and also discussion of same, p. 269.

$$f_e = \frac{3}{2}\frac{\sqrt{c}}{k}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Here both c and k are constants depending on the materials.

From both sets of experiments named above, we can derive the following approximate values of k, c, and p_e.

Combination of Materials.	k	c	f_e
Cast-iron roller on cast-iron plate	0.00050	1460	115,000
Cast-iron on wrought-iron or steel	0.00040	960	115,000
Soft-steel roller on soft-steel plate	0.00036	880	124,000

If the working stress be taken at about one fifth the elastic limit stress for cast-iron, and at about one third that stress for wrought-iron and soft steel, we have as *the working load per linear inch of roller for all combinations of cast-iron, wrought-iron, and soft steel, and for all diameters from* 1 *inch to* 16 *inches,*

$$p = 300D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If $P =$ total load to be carried, and

$l =$ total length of bearing rollers, we have

$$l = \frac{P}{p} = \frac{P}{300D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In case the load is liable to be unequally distributed over the rollers, it may be advisable to take smaller working stresses. As an extreme case, where the load may be concentrated on one half the rollers, but using in this case factors of safety of 3.5 and of 2 on the elastic limit strength, in place of 5 and 3 respectively, as before, we would have

$$p = 200D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or

$$l = \frac{P}{200D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

256. A Good Design for Heavy Movable Bearings.—Figs. 304 and 305 are taken from Mr. Geo. S. Morison's design for the movable bearings on the middle pier of the

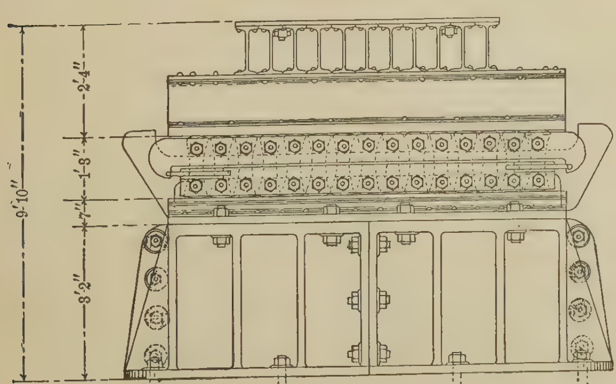

FIG. 304.

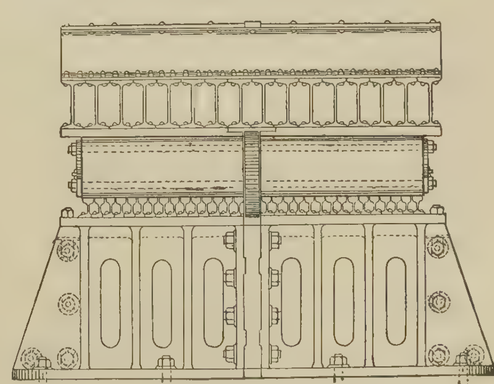

FIG. 305.

Memphis Bridge, there being a fixed span of 621 feet on one side and a cantilever span of 790 feet on the other. Thus this movable support sustains a length of span of over $705\frac{1}{2}$ feet, which has a dead load of some 5,000,000 pounds and a live load of some 3,000,000 pounds more, or some 4,000,000 pounds total load on each support. In order to increase the bearing surface without making the sole plate of the shoe too great, the sides of the cylindrical rollers are cut out and thus they may be made of large radius, as shown in the accompanying figures. These rollers are 15 inches in diameter and are spaced 8 inches apart. There are fifteen of them, each 112 inches long, or 1680 linear inches of roller under each truss bearing. This is nearly 2400 pounds per linear inch. By our formula $p = 300D$ per lineal inch. Since D here is 15 inches, our formula would allow 4500 pounds per linear inch, or nearly twice the actual load on these rollers. The distribution of the loads evenly over the rollers from the pin is a very important matter, and the method here adopted of using two courses of I-beams above and a course of railroad rails, with one flange removed so as to pack solidly together, for a base, and below all a high casting for distributing evenly upon the pier, is to be commended. An objection which can be made to the scheme is that it is very expensive.

257. Provision for Expansion.—All spans under 75 feet usually rest on planed bed-plates and expand or contract by sliding on the surface between the sole and bed-plate. Rollers are used under the sole-plates or shoes of all spans over 75 feet. When rollers are used the span should be supported on an end pin so as to insure the pressure coming on the rollers centrally. This applies to plate and lattice girder spans as well as to pin-connected truss spans, because otherwise the deflection of the span would necessarily transfer the pressure to the inner edge of the sole plate and to a few rollers at that end of the nest of rollers. Owing to the expense of boring the pin-holes in a long plate girder, the plan shown in Fig. 306 is sometimes used. In using this plan care must be taken to make the plates stiff enough to distribute the pressure over the rollers or the masonry. For plate girders and single-web lattice girders (as distinguished from girders with box chords) the plan Fig. 306 is preferable to a shoe and pin owing to its greater lateral stiffness.

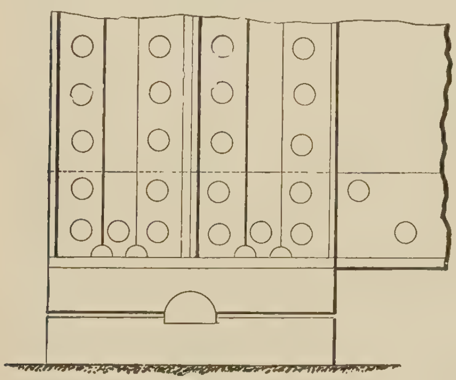

Fig. 306.

The expansion may be provided for by suspending the end of the span on links. This

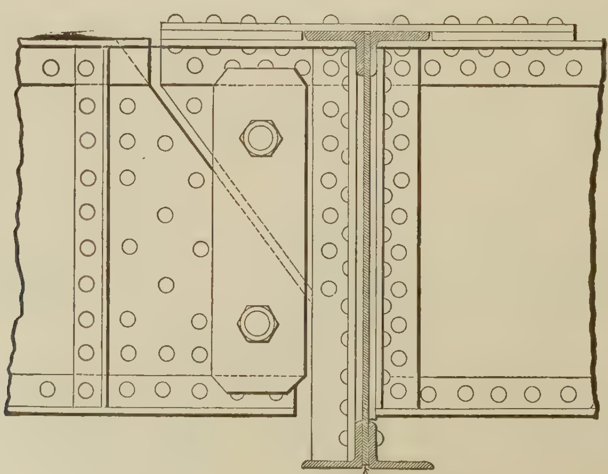

Fig. 307.—Expansion Joint, Elevated Railroad.

is sometimes done in elevated-railroad work, and is always the case with the suspended span of a cantilever bridge.

When the plan Fig. 307 is used, the allowed bearing pressure between the pins and the links should be reduced from that ordinarily allowed on pins in order to prevent the continual motion from wearing the pin.

In viaducts where the girders rest on the caps of the columns the girders expand by sliding on the column caps. Usually the longitudinal girders of elevated-railroad structures are allowed to expand on brackets built out from the cross girders.

The towers of viaducts are allowed to expand by sliding on their bed-plates. There are two ways for providing for this expansion, as shown in Fig. 308. In plan (1) the base of the

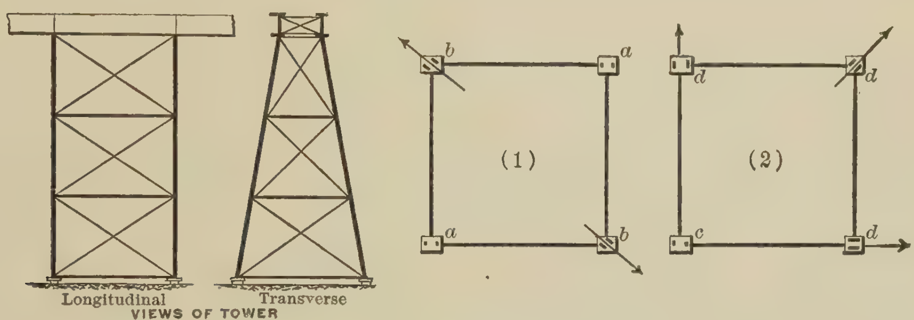

FIG. 308.

columns *a* are fixed or anchored fast to the masonry while the bases of columns *b* are allowed to move in the direction of the arrows. In (2) the base of *c* is fixed and the bases of columns *d* are allowed to expand in the direction of the arrows. In each of these cases the bottom struts of the tower must be made strong enough to overcome the friction of the column on the bed-plates considering the tower unloaded.

In making allowance for expansion a variation of temperature of 150 degrees is usually provided for. The change in length is generally assumed to be one inch in eighty feet.

258. The Design of the Truss.—For through spans the Pratt truss with inclined end posts has been found to be the most economical for spans under 180 feet; from 180 to 225 feet the curved chord single intersection truss is generally used, and above that length the curved chord single intersection truss with sub-panels. For skew through spans the straight chord must be used for all lengths, or a curved chord truss with one inclined and one vertical end post may be used. In the latter case the portal strut would connect the vertical end post of one truss with the vertical hanger from the top pin of the inclined post of the other truss. When inclined end posts are used in a skew bridge the two end posts that the portal connects must have the same inclination in order that the portal may not be a warped surface.

For deck spans the triangular truss will generally be found to be economical. For long spans sub-panels would be used. In general deck spans are more unstable, unless made very wide centre to centre of trusses, than through spans, as the floor and the train surface exposed to the wind are further from the shoe or point of support. However, if the deck span is supported on the end top chord pin it is *more* stable than a through span, as the wind force on the bottom chord produces a negative moment to that of the wind force on the floor and train surface. Deck *skew* spans supported on the end bottom chord pin are to be avoided when possible owing to difficulty of designing efficient end sway bracing.

259. Various Methods of Sub-panel Trussing.—In Fig. 309 are shown three methods in use for supporting the floor-beam at the intermediate panel point in a sub-panelled truss.

As a question of economy there is practically no difference between them, (*c*) being a little more expensive than the others. Design (*b*) is to be preferred because the primary concep-

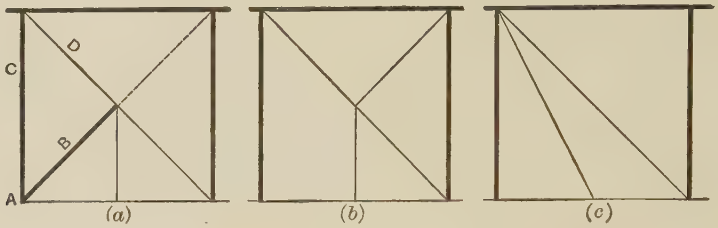

FIG. 309.

tion of the truss is to have all diagonals tension members and in all cases there is no ambiguity of stresses.

260. Stiff End Bottom Chords.—A great many engineers require that the end bottom chord members of a through span be made stiff. It has been found that this adds considerably to the rigidity of the span, but no definite method of proportioning these members has been advanced. The wind causes a compressive stress in these members, but rarely enough to overcome the tension from the vertical load. As a matter of safety the wind stress in the bottom chord should always be computed and the most unfavorable assumptions possible made, and if the tension is overcome, or nearly so, the chord should be made stiff.

These stiff chords are made by stiffening the eyebars as shown in Fig. 310, or by using a

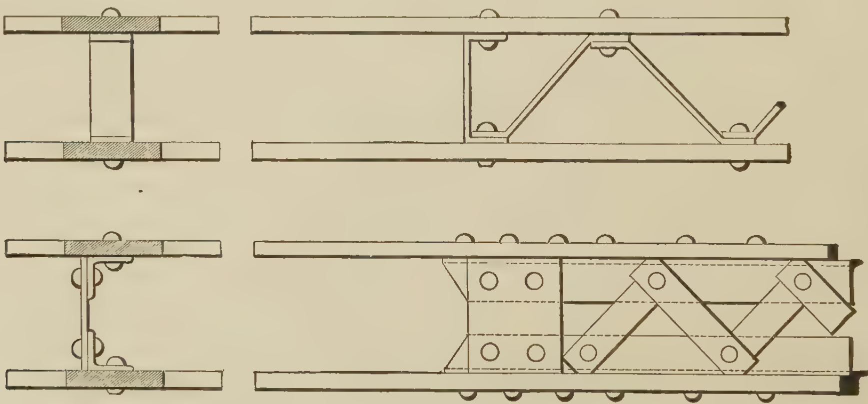

FIG. 310.

compound section of four angles or of two channels latticed. In either case it must be noted that the section is reduced by the rivet-holes, and in proportioning for the tensile stress the net area must be used.

261. Stiff Floor-beam Hangers.—When a floor-beam is suspended from a pin some distance above the beam the hanger or tie supporting the beam is often made stiff. The advantage of a rigid member for such cases is that greater stiffness of the entire structure is obtained. The hanger may be made of any of the various forms mentioned for stiff bottom chords.

Engineers differ as to the form of the member to use in the case of a stiffened tie, some holding that it should simply be that of a tension member braced as shown in Fig. 310, while others prefer that it should be made of the ordinary post section. The form shown in Fig. 310 no doubt looks better, but as the post section is the stiffer form and can be made amply strong it will make a more rigid member.

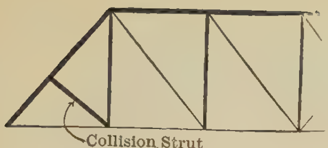

FIG. 311.

262. Collision Struts are sometimes used to brace the end post against a horizontal force, generally assumed to be that of a train off the track, striking the end post, in the plane of the truss. They are redundant members of doubtful utility and are not generally used. Fig. 311 shows the usual position of this strut.

263. Sub-struts are those struts which are used to stiffen compression members of a truss by holding them against flexure. Thus, in Fig. 312 the

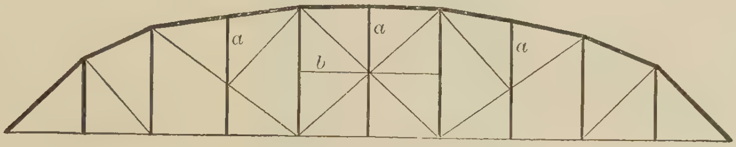

FIG. 312.

sub-struts *a* support the top chord sections in the middle and reduce their unsupported length as a long column one half. The strut *b* similarly braces the main vertical posts. A saving in material is often accomplished by the use of these struts, as the area required in the top chords or posts to resist their compressive stress is less owing to the shorter unsupported lengths.

264. The Design of the Floor System.—The main objects to be sought after in the design of the floor system are strength and stiffness of the floor-beams and stringers and rigidity in their attachments to the floor-beams and to the trusses. As the floor-beam is usually utilized as a strut of the lateral system, it must be located as near as practicable to the chords, which also serve as chords for the lateral system, and further, the beams must be in such a position that the lateral rods may be put in below the cross ties and be in the plane, or nearly so, of the chords.

To secure beams and stringers of sufficient strength it is only necessary to use sufficiently low unit stresses in proportioning them. The usual permissible stresses in practice fulfil this condition. To secure ample stiffness the deflection of the beams and stringers under load must be considered. In general, it may be said that the deflection of a beam varies directly as the stress per square inch on the flanges and inversely as the depth.* Hence, an increase in stiffness may be obtained by increasing the depth of the beams and stringers or by decreasing the permissible unit stresses on the flanges. As a reduction of the allowed unit stresses would result in increasing the amount of material necessary in the beams and stringers, it is economy to obtain the required stiffness by increasing their depth. For the usual unit stresses the stringers should have a depth *not less than one twelfth of their length.* Stringers having less depth than this would have a perceptible deflection which would tend to loosen the rivets attaching the stringer to the floor-beam.† The tendency at present is to the use of deep floor-beams and stringers. Stringers having a depth of one sixth to one tenth of their length are commonly used and are generally the most economical in material.

To secure the greatest amount of rigidity in the floor system the stringers should be riveted between the floor-beams and the beams riveted to a stiff member of the truss. In order to obtain a simple detail for the floor-beam in such a case it is necessary that the post be vertical. This makes it necessary to use redundant members in a Warren truss, and has

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* See column 4 of tabular form, p. 132.

† In many of the old iron bridges now in use the stringers are too shallow and their end connections are continually giving way due to the excessive deflection bringing a tensile stress on the rivets. This weakness of the riveted connection of the stringer to the beam is often cited as a fault of this kind of connection, but it is really due to the fact that the stringer is too shallow. Shallow stringers sometimes fail by the splitting of the web plate at the ends, a failure which would only occur in the case of a faulty detail of the ends.

FIG. 313.

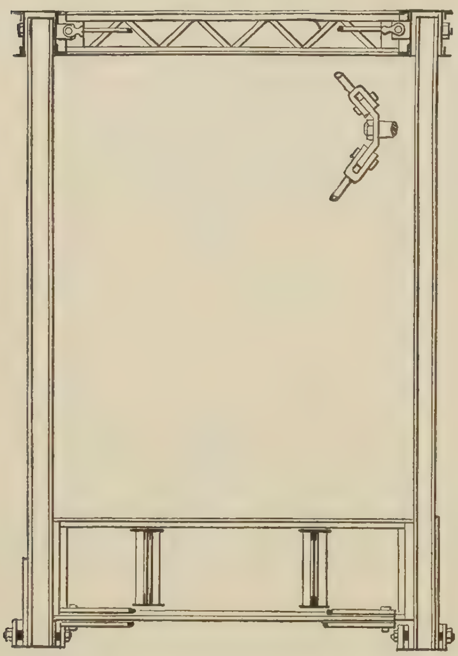
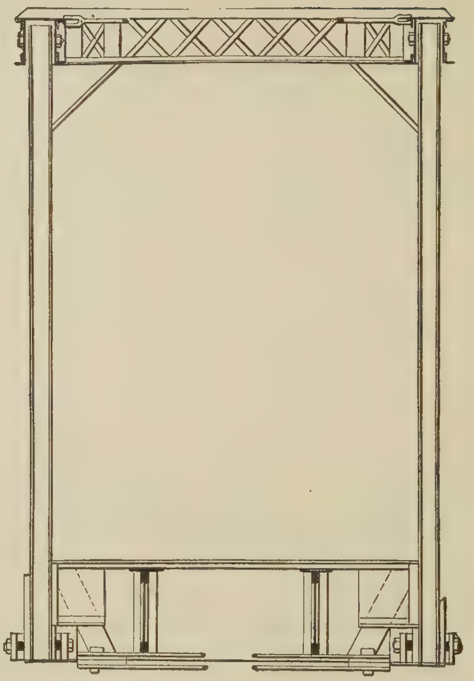


FIG. 314.



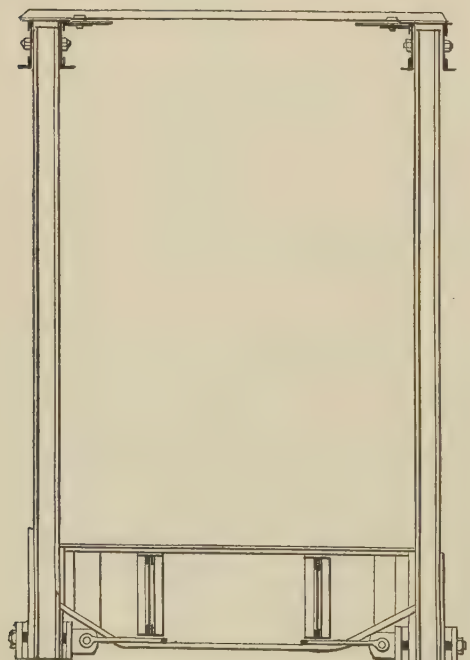


FIG. 315.

FIG. 316.

FIG. 317.

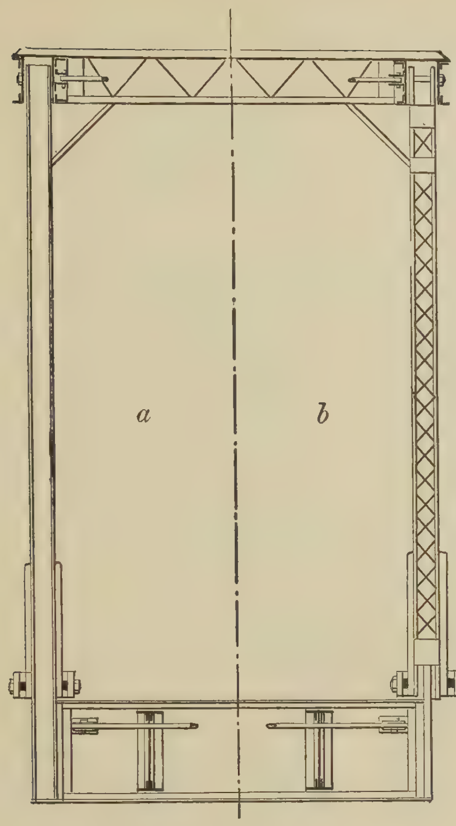
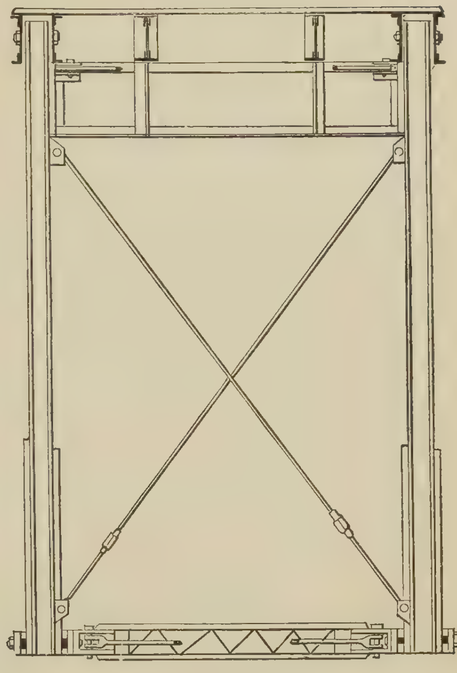


FIG. 318.



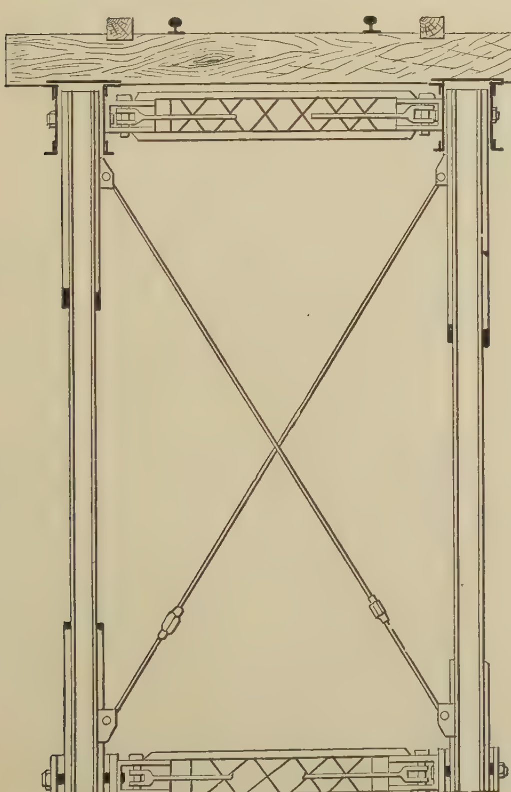


FIG. 319.

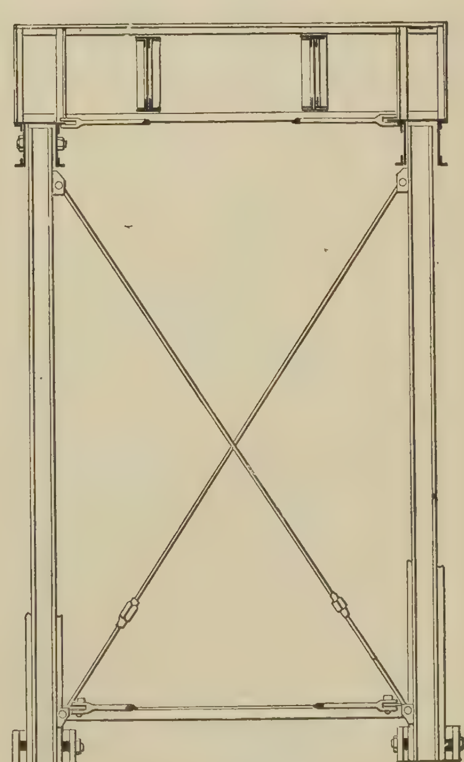


FIG. 320.

led to the adoption of the plate hanger (Fig. 317*b*) for the late designs of the Pegram truss. In the latter case the lateral connection is made as shown in Fig. 316 or 328.

The plate hanger detail is one in which the floor-beam is riveted to a vertical plate which has a pin-hole in its upper end to receive the truss pin. The great advantage of this detail is that the load from the beam is applied centrally to each truss, thus insuring equal stress in the tie-rods. In some of the older bridges in which the floor-beams are riveted to the posts the inside rods of each truss receive a greater stress than the outer rods—a fact made evident by inspection after years of service.

Figures 313, 314, 315, 316, 317, 318, 319, and 320 illustrate most of the many designs for the floor system now in common use. The designs shown in Figs. 313, 317, 318, and 320 are to be preferred as they are cheaper, simpler in construction, and are for any but the most careful workmanship and best material much safer than the others. The bending of the flange angles of the floor-beam or of any girder should be avoided as expensive and dangerous. A sharp bend in an angle iron is made by cutting out a V-shaped piece, then bending the angle and afterwards welding the parts together. It is very doubtful whether a perfect weld can be made, and if it is not the value of the angle is destroyed.

The floor system is subjected to severer stresses than the trusses, and the design should always be such as will insure the maximum of safety.

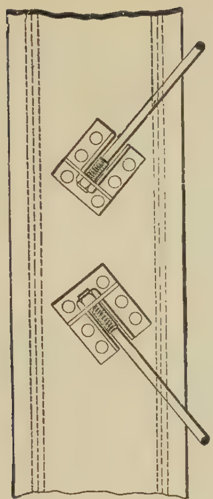
It is customary to use a system of lateral bracing between the top flanges of the stringers for panels the length of which is over twenty-five times the width of the stringer flange. The purpose of the bracing is to stiffen the stringer flanges, and it is usually made of angle iron with riveted connections to the stringers.

265. The Attachment of the Lateral Systems.—Figs. 321, 322, 323, 324, and 325 illustrate the usual detail for the connection of the top lateral rods of through bridges. Figs. 326, 327, 328, 329, 330, and 331 show the usual details for the attachment of the lower lateral rods of through bridges. In all of these connections the connection is eccentric, i.e., the rods do not intersect on the centre line of the chord and they are so far imperfect, but they are illustrations of the various methods employed to so connect the rods as to produce as little eccentricity as possible. The details Figs. 330 and 331 were formerly used quite extensively, but are now seldom employed. In selecting the style of detail to use, the designer must be governed by the special conditions of the span he has in hand. The connection must be so made that the stress from the rod can be transferred to the chord *and* to the lateral strut without overstressing any part.

The lateral strut has been omitted from the sketches of top lateral connections, as the detail of its attachment depends on the form of the strut. The strut is usually riveted between the chords. From Figs. 313–320 a fair idea of the connection of the struts may be obtained.

For short spans where the wind or lateral stress is comparatively small any of the designs shown will be satisfactory if care is taken to get the laterals as nearly in the plane of the chords as practicable.

For long spans where the lateral stress is great it becomes necessary to design a detail which will avoid all eccentricity and the consequent overstressing of some member. No satisfactory detail for this case has yet been universally adopted. The detail shown in Fig. 332 was used in some long truss spans recently built over the Ohio River and satisfies all the important conditions. The lower chord is built in two lines placed far enough apart to allow the floor-beam to be riveted between them, the line of the diagonal ties of the truss intersecting the centre line of the post at a point midway between the two lines of chords. The lateral rods also intersect at the same point. The chord components of the truss ties and the lateral rods are transferred to the chords by the hanger plate to which the floor-beam is riveted. This hanger plate is made stiff enough to resist the forces acting upon it. This detail was designed by Mr. H. G. Morse, President of the Edge Moor Bridge Works.



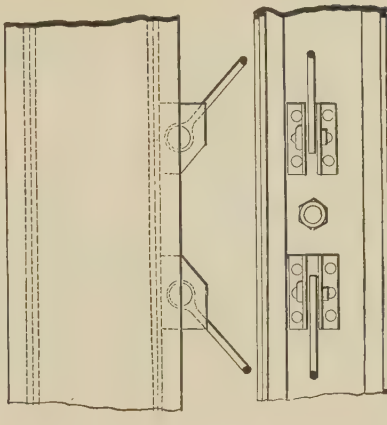


FIG. 324

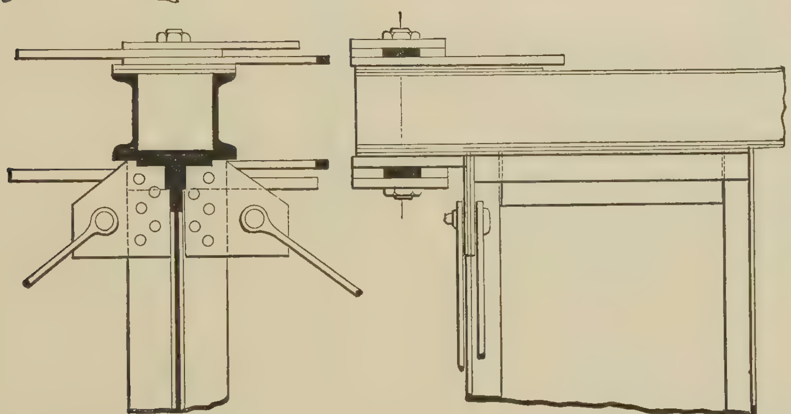


FIG. 326.

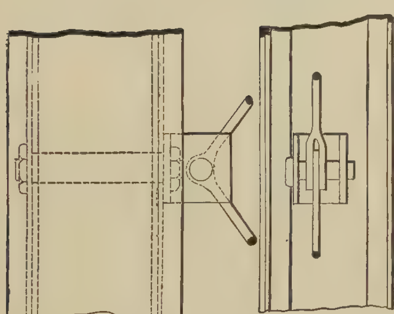


FIG. 322.

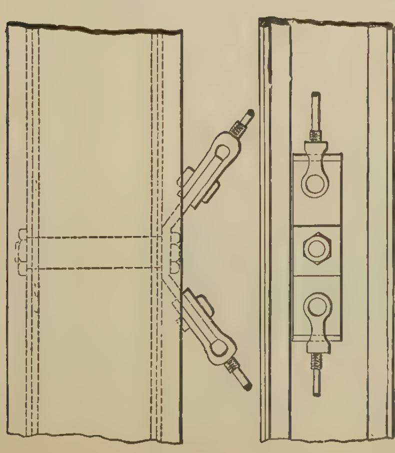


FIG. 321.

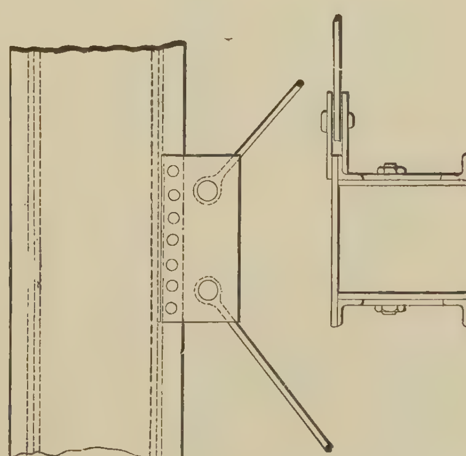


FIG. 325.

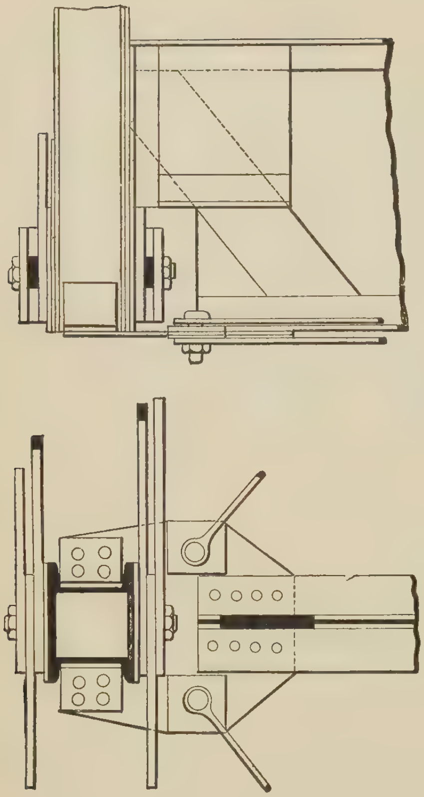


FIG. 327.

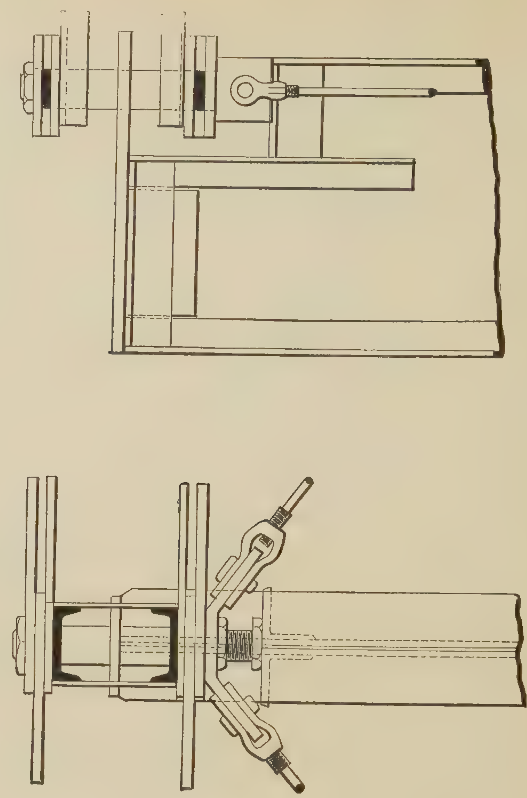


FIG. 328.

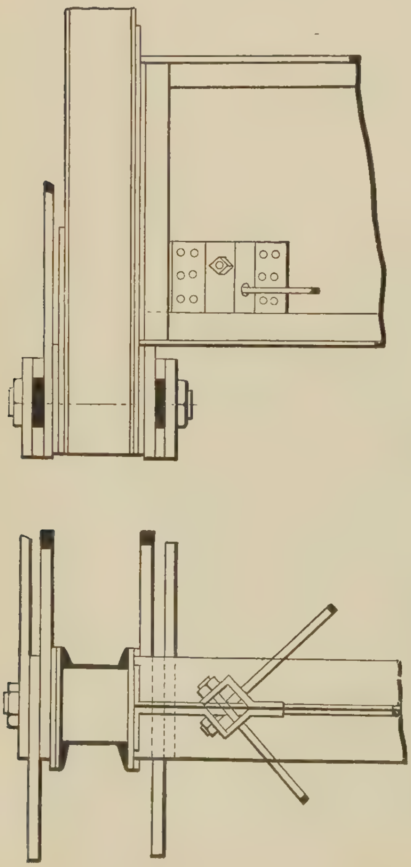


FIG. 330.

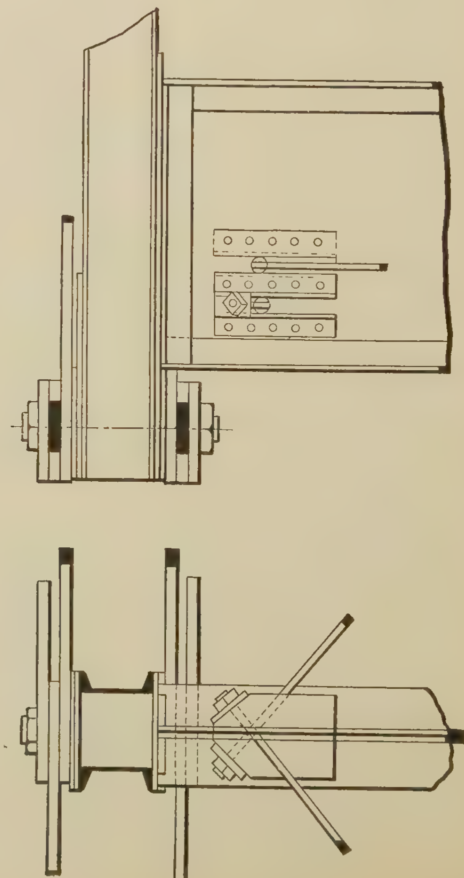


FIG. 331.

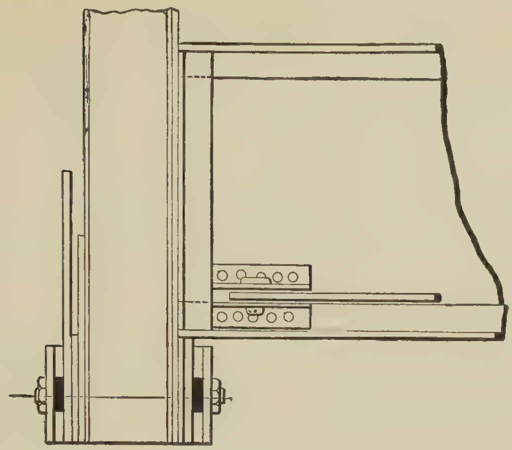
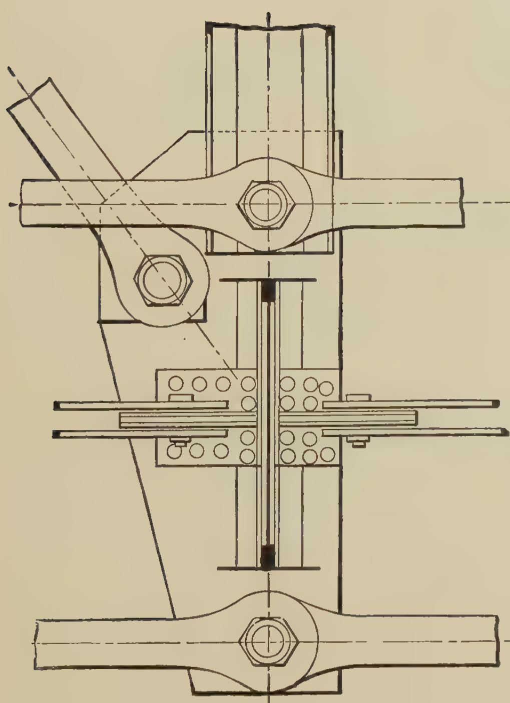


FIG. 329.



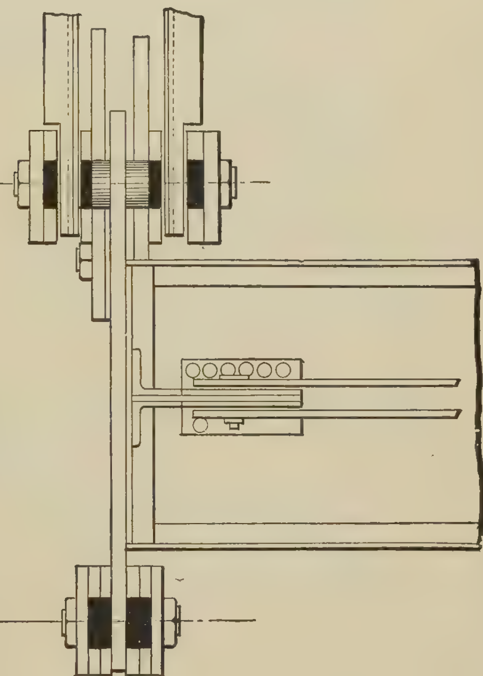
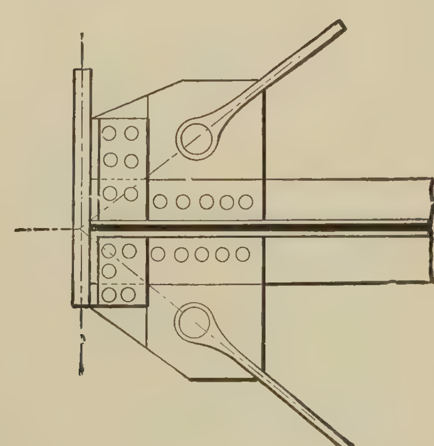


FIG. 332.



266. Portal and Sway Bracing.*—Considerable attention has been paid of late years to the design of efficient portal bracing. The fact that the connections for the portal are subjected to severe reverse stresees and that the stress in the end post is usually of considerable magnitude makes the problem very difficult and the necessity for correct designing very apparent. The connection with the end post should be such that it will withstand both tension and compression, and the end post should receive its stress centrally. Practically these conditions cannot readily be fulfilled, as it is probably impossible to make a central attachment to the end post. The usual plan is to rivet the portal strut between the end posts and rely on the rivets in the connection for the transfer of the stress. For the shorter spans this has proved by experience to be sufficient, but nevertheless there are parts which are stressed beyond what would ordinarily be allowed. For long spans the portal strut is now usually made a box girder with the webs in the planes of the top plates and of the under sides or tie plates of the two end posts. The connecting plates extend across the entire width of the end post in both planes. This avoids direct tension on rivets and is as nearly central as it is possible to make the joint.

Skew portals increase the difficulties in the design, as they bring in a very serious question as to the manufacture. They are difficult to make fit, requiring very careful and accurate work, and, as in all similar cases requiring close work, the designer is necessarily limited in the style of his detail or connection. Skew bridges should always be designed with *parallel* end posts.

The portal if necessarily shallow is composed of one strut the cross-section of which is similar to a plate or single-web lattice girder. If the depth of truss admits of it, a strut at the top of the end post and one as far down as the required headroom will allow, with diagonal rods or braces between, is used. Figs. 333, 334, and 335 illustrate the common forms used for portals and the stresses in the various parts. Fig. 335 would be a portal composed of one

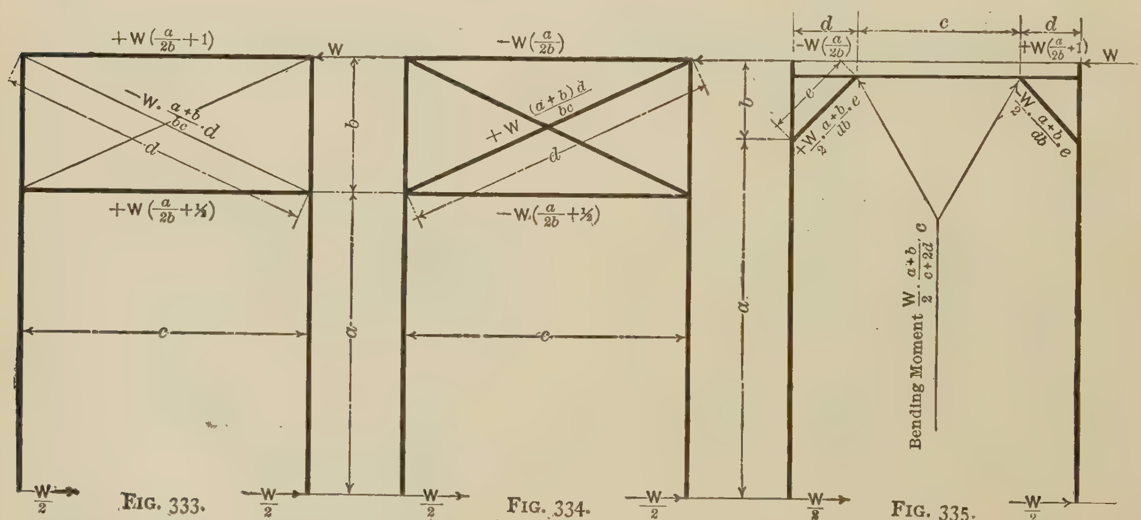


FIG. 333. FIG. 334. FIG. 335.

+ denotes compression. — denotes tension.

The vertical reactions in all these cases are upward on the left post and downward on the right post, and in all cases equal to $W\left(\frac{a+b}{c}\right)$.

strut with knee braces, and is the common form for short spans. The stresses† are given in the most convenient terms to use.

For through bridges in which the height of truss will admit of it intermediate sway bracing is put in between the posts at each panel. This sway bracing usually consists of a strut

* See also pages 110, 159, and 329.

† To Mr. W. F. Gronau of Pittsburg, Pa., belongs the credit of having computed the stresses using the easily determined dimensions given in the formulæ.

connecting the two opposite posts of the two trusses, and diagonal rods in the plane of and connecting the top strut and this sway strut. The rods are usually of the minimum size used, as their function is merely to prevent vibration.

Where the height of truss will not permit of sway bracing like the above, knee braces, see Fig. 314, are often used.

For deck bridges intermediate sway bracing, see Figs. 318–320, is put in at each panel. The rods in this case also need only be of the smallest size used unless it be a special case where extra stiffness is required. This would be in the case of a curve on the bridge or when the rods are unusually long.

Sway bracing should be used at the ends of all deck spans of sufficient strength to transfer the wind force from the top lateral system to the shoe. This end bracing will be the most efficient if put in the plane of the end posts which carry the load to the shoe. These posts, having the largest sectional area, are always stressed and are consequently stiff.

The present tendency seems to be toward the use of angle-iron diagonals with riveted connections for sway bracing instead of the adjustable rod with the pin connection. It adds very little if anything to the cost to use the angle iron or "stiff" bracing, as it is termed, instead of rods and adds very materially to the rigidity of the structure.

267. Top Chord Joints.—The splices or joints of the top chord are made as shown in Figs. 336, 337, 338, 339, and 340, the most common ones being that shown by Fig. 336 for an intermediate top chord joint and that shown by Fig. 340 for the joint at the top of the inclined end post. For the intermediate joint the two chord sections are planed off to a true surface and the bearing of the sections on each other is relied on to transfer the stress. Side and top splice plates are riveted to each section to hold them in position. The field rivets in these splices must be so located that they will be accessible for driving after the chords posts and ties are in place. The splice is usually located on the side of the pin furthest from the centre of the span. The hip joint or that at the top of the end post (see Fig. 340) is made in a different manner; the two sections do not bear on each other, but an opening of from one quarter to three eighths of an inch is left between them, and the bearing plates on the pin for each section are made thick enough to transfer the stress. This style of joint is often used for those intermediate top chord joints in curved chord bridges where the two sections of chord make an angle with each other, instead of the joint shown in Fig. 339.

268. The Shoe.—The reason for the use of a shoe at the end of a span is to insure a uniform pressure on the rollers or to secure an evenly distributed pressure on the masonry. The usual limits for the bearing of the shoe on the masonry are from 200 to 300 pounds per square inch. In order that the pressure may be uniform on the rollers or the masonry it is necessary that the shoe be stiff enough to so distribute it. This requires that the ribs of the shoe, if they are made of the thickness required for the bearing of the end pin, must also be made *deep* enough to give the required stiffness. Very deep shoes are however to be avoided or else they must be given ample lateral stiffness, as the ribs of the shoe are relied on to transfer all the wind force from both the top and bottom lateral systems to the masonry.

When two shoes rests on one pier it is always better to have them rest on one continuous bed-plate under both shoes. This plate acts as a tie to bind the pier together and to prevent the friction of the shoes from cracking the masonry.

269. The Packing of Joints.—The members connecting on a pin should always be so arranged as to produce as small a bending moment as practicable, and they should always be arranged symmetrically. Ample clearances for inaccuracies such as are liable to occur in manufacture should be allowed. The pieces should be so placed that no cutting, or as little as possible, of the flanges of compression members is necessary for fits. Always keep in mind that the lateral rods must be located as nearly as possible in the plane of the chords. Eyebars should never be bent in order to pack nicely on the pins they connect, but it should

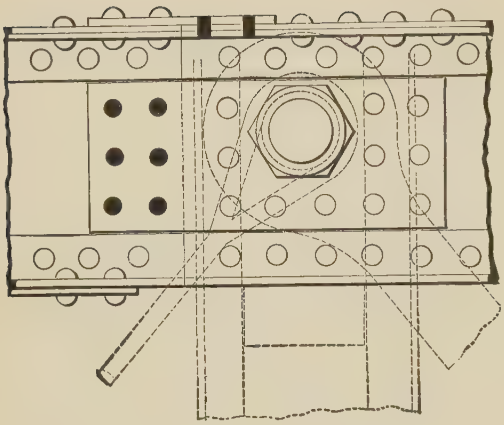


Fig. 336.

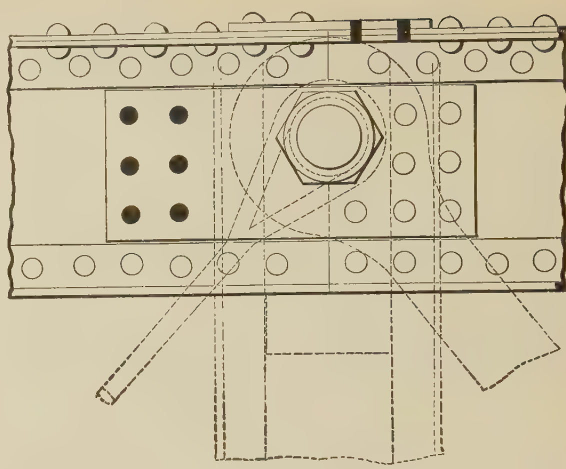


Fig. 337.

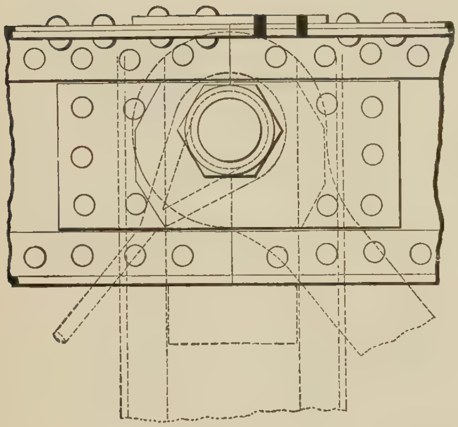


Fig. 338.

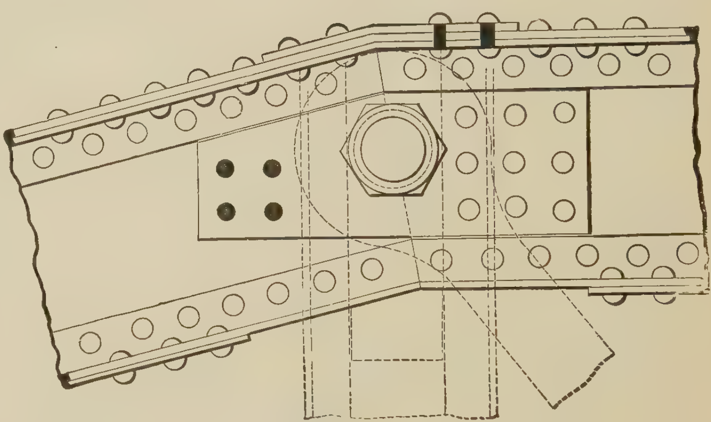


Fig. 339.

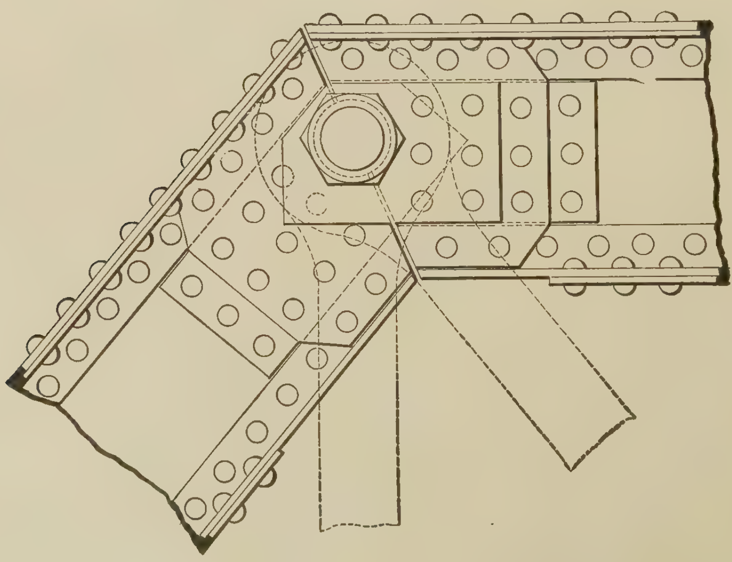


Fig. 340.

be an invariable rule to have them as nearly parallel to the plane of the truss as possible. Details which require accurate work or fitting in the field should be avoided, as the chances are that in the hurry to "swing" the span it will be neglected. Details should be used which will facilitate the work of erecting if at no sacrifice of strength.

270. **Camber.***—Bridges are so constructed that they will when loaded to their capacity take the form which was assumed in the calculation of the stresses. This is accomplished by curving the trusses upward, i.e., giving them a camber. This is done by increasing the length of the top chord, decreasing the length of the lower chord, and making the corresponding necessary changes in the length of the diagonals. Formerly the rule was to give all trusses a camber or raise the centre of the span one twelve-hundredth of the length of the span. This rule has now been very generally superseded by one which more nearly satisfies the theoretical conditions, and that is to make the top chord one eighth of an inch longer than the bottom chord for every ten feet in length of span. The old method would make the camber the same for all depths if the span length were constant, while the new recognizes almost perfectly the fact that the deeper the truss the less the deflection under load will be and hence the less the camber should be. The following formulæ will be found to be serviceable in finding the camber when the increase in length of the top chord over the bottom chord is known, or to find the necessary increase in length of top chord when the camber is known.

Let $c =$ camber in inches;
$i =$ increase in length of top chord over the bottom chord in inches;
$h =$ height of truss in feet;
$l =$ length of span in feet.

Then $$c = \frac{il}{8h}, \quad \text{and} \quad i = \frac{8ch}{l}.$$

In some bridge works it is customary to increase or diminish all truss members by the amount of their estimated distortion under their maximum loading. This gives a camber equal to the deflection as computed by the method given in Chapter XV.

271. **Sizes of Lattice Bars.**—The following table gives the ordinary sizes of lattice bars used on compression members of railroad bridges. By the depth of the member is meant the size of the channels, rolled or compounded of plate and angles, which are used in the member.

Depth.	Lattice.	Depth.	Lattice.
in.	in.	in.	in.
7	$1\frac{3}{4} \times \frac{1}{4}$	14	$2\frac{1}{4} \times \frac{3}{8}$
8	$2 \times \frac{5}{16}$	15	$2\frac{1}{2} \times \frac{3}{8}$
9	$2 \times \frac{6}{16}$	16	$2\frac{1}{2} \times \frac{3}{8}$
10	$2 \times \frac{3}{8}$	18	$4 \times \frac{3}{8}$ single $2\frac{1}{2} \times \frac{3}{8}$ double
12	$2\frac{1}{4} \times \frac{3}{8}$	over 18	$4 \times \frac{7}{16}$ single $2\frac{1}{2} \times \frac{3}{8}$ double $3 \times 2 \times \frac{1}{4}$ L single

* See also Chapter XV, p. 219.

CHAPTER XIX.

THE PLATE GIRDER.

THEORETICAL TREATMENT.

272. **The Moments and Shears** in a plate girder at any section are found from the outer forces in the same manner as for a section of a truss, and as explained in Chapter II. The relations of moments and shears in a solid beam and also in an I beam or in a plate girder are explained in Chapter VIII. It is there shown that the vertical shearing stress in the web of a plate girder is nearly uniform throughout its depth, at any vertical section, and in the following analysis it will be assumed to be uniformly distributed. The bending moment comes from the shear acting with lever arms measured longitudinally along the girder. The resisting moment is developed first in the web and is transferred to the flanges through the rivets as the web distorts under the moment which is developed in it. The moment, therefore, is primarly in the web, and the amount of bending moment which is resisted by the web is necessarily such a proportion of the bending moment at a given vertical section as the moment of resistance of the web is to the total moment of resistance of the girder at that section. Since the web and flanges distort together as a solid beam, there is no reason why the web should not be assumed to resist its due proportion of the bending moment. There can be no question but that it does actually perform this service.

Let F = area of one flange section, not counting the included portion of the web;
h = height of girder between centres of gravity of flanges;
t = thickness of plate in web;
f = stress allowed per square inch in flanges.

Then the total moment of resistance of the girder is

$$M_0 = Ffh + \frac{fth^2}{6}. \qquad (1)$$

Calling $th = A$ = area of web, we have, as the moment of resistance of the web,

$$\frac{fth^2}{6} = \frac{Afh}{6} = \frac{A}{6}fh.$$

Hence

$$M_0 = \left(F + \frac{A}{6}\right)fh. \qquad (2)$$

This shows that the influence of the web in resisting bending moment is fully provided for when one sixth of its area is added to each flange area, as indicated in equation (2).

Where there is a vertical line of rivet-holes in the web plate the influence which these holes have in reducing the moment of resistance of the web must be taken into account. The assumption that the net moment of resistance of the web is equivalent to a flange area of one eighth of the gross area of the web plate is not far in error in any practical case. It must

also be borne in mind that the web has the same effect on both tension and compression flanges, and that it is *not* correct to use one sixth of the *gross* area of web for the web equivalent in the compression flange and to use one sixth of the *net* area of the web as the corresponding equivalent for the tension flange, as is sometimes done. The web resists a certain amount of the bending moment, and the flanges must be proportioned for the remainder.

Making this change in equation, (1) and (2), we have, for practice,

$$M_0 = Ffh + \frac{fth^2}{8} \quad \text{(1A)}$$

and

$$M_0 = \left(F + \frac{A}{8}\right)fh. \quad \text{(2A)}$$

273. **The Web** of a plate girder, or of a floor-beam or stringer, is made of a single plate if possible. In general, web plates are limited by the conditions of manufacture to a net weight of about 1600 pounds for standard prices. If more than one plate is required, it is customary to make up the web symmetrically as to the splices in it.

In light work, as for highways, a minimum thickness of one fourth inch may be used up to a width of 5 or 6 feet. Such plates are apt to be more or less buckled, however. For railway work a minimum thickness of three eighths of an inch should be used for all depths; this thickness to be increased if necessary to give sufficient bearing area on the rivets at the flanges.

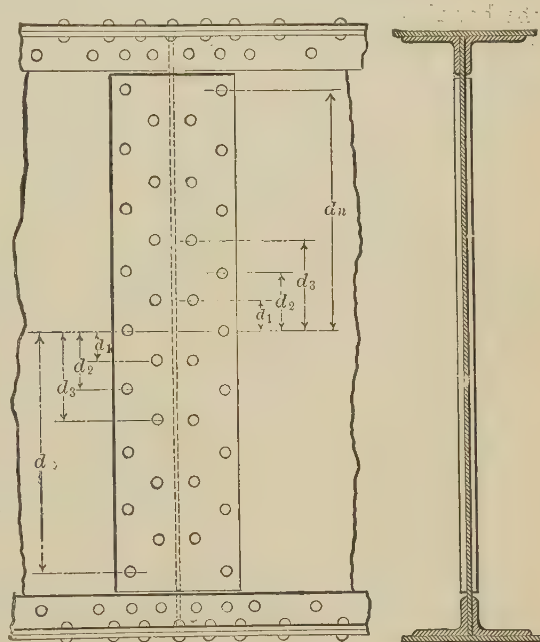


FIG. 341.

274. **Plain Web Splices.**—The web carries all the shear (or nearly all, and is assumed to carry all, see Art. 130) and its due proportion of the bending moment.* When it has to be spliced, the shearing and bending stresses at that section must be provided for by double splice plates with a sufficient number of rivets. With a three-eighths web, five-sixteenths splice plates would be used. To find the proper number of rivets to use in such a splice,

Let S = shear on the section;
M_1 = moment at the section;
F = area at one flange;
th = area of the web;
f_1 = fibre stress per square inch in the flange at that section;
r = resistance of one rivet;
$2n$ = number of rivets on one side of splice;

Then we have, from eq. (2A),

$$f_1 = \frac{M_1}{\left(F + \frac{th}{8}\right)h}. \quad \text{(3)}$$

* When the flanges are proportioned for carrying all the bending moment, the web-splice may be proportioned for carrying the shear only.

This is the flange stress at that section. Having this, we find the total moment carried by the web from the usual formula,

$$M_w = \frac{f_1 t h^2}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The splice must provide, therefore, for the shear S and the moment M_w. Since the shear is supposed to be uniformly distributed over the web (see Art. 272), the shearing stress on each rivet is

$$r_s = \frac{S}{2n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For resisting the bending moment on the web the rivets are not equally stressed. Since the bending stresses are zero at the neutral axis and increase uniformly to the maximum value f_1 at the extreme rivet, the moment stress in each rivet will be as its distance from the neutral axis. Its arm is also as this distance; hence the moment of resistance of the rivets are as the squares of their distances from the neutral axis.

If d_1, d_2, d_3, etc., are the distances of the rivets from the neutral axis, then the moments of resistance of the several rivets will be ad_1^2, ad_2^2, ad_3^2, etc., on one side, and ad_1^2, ad_2^2, ad_3^2, etc., on the other. If the rivets on one side of the joint be taken in pairs, symmetrical about the neutral axis, the moments of resistance of the several pairs are $2ad_1^2$, $2ad_2^2$, $2ad_3^2$, etc., where a is the resistance of one rivet at a unit's distance from the neutral axis. The sum of the moments of the several pairs of rivets must equal the total moment of resistance of the web at this section, as given by eq. (4). Therefore we have

$$2a(d_1^2 + d_2^2 + d_3^2 \ldots + d_n^2) = M_w, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or

$$a = \frac{M_w}{2(d_1^2 + d_2^2 + d_3^2 \ldots + d_n^2)}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since a is the rate of increase in the stress on the rivets out from the neutral axis, it is equal to the bending stress on the extreme rivet divided by its distance out, d_n. The allowable stress from cross-bending on this rivet is equal to $\sqrt{r^2 - r_s^2} = r_m$, since the shearing stress, r_s, and the stress from bending, r_m, act at right angles to each other and both combine to produce the allowable stress r. We have, therefore,

$$a = \frac{r_m}{d_n} = \frac{M_w}{2(d_1^2 + d_2^2 + d_3^2 \ldots + d_n^2)}, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or

$$\Sigma(d^2) = \frac{M_w d_n}{2r_m}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But $\Sigma(d)^2 = (1^2 + 2^2 + 3^2 + \text{etc.})(\text{pitch})^2$, and the sum of the squares of the serial numbers $1, 2, 3, \ldots n$ is equal to $\frac{n(n+1)(2n+1)}{6}$. Therefore we may write

$$6\Sigma(d^2) = n(n+1)(2n+1)(\text{pitch})^2 = \frac{3M_w d_n}{r_m}. \quad . \quad . \quad . \quad . \quad (10)$$

But pitch $= \frac{d_n}{n}$; hence we may write

$$\frac{(n+1)(2n+1)}{n} = \frac{3M_w}{r_m d_n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where $n =$ one half the number of rivets in a web splice on one side of the joint;

$M_w =$ the moment carried by the web,

$= \frac{f_1 bh^2}{8}$, where f_1 is given by eq. (3),

$d_n =$ distance out from neutral axis to extreme rivet in splice;

$r_m =$ working resistance of extreme rivet to bending stress $= \sqrt{r^2 - r_s^2}$, where $r =$ total resistance of rivet and $r_s =$ shearing stress on rivet $= \frac{S}{2n}$.

Equation (11) would be solved by trial. The splice plate may have to be made so wide as to admit of two or more rows of rivets, when they should be staggered.

From these three equations,

$$r = \sqrt{r_s^2 + r_m^2},$$

$$r_s = \frac{S}{2n},$$

$$\frac{(n+1)(2n+1)}{n} = \frac{3M_w}{r_m d_n},$$

we may find the three unknown quantities r_s, r_m, and n.

After eliminating r_s and r_m, we obtain

$$\frac{(n+1)(2n+1)}{n^2}\sqrt{4n^2r^2 - S^2} = \frac{6M_w}{d_n}. \quad \quad (12)$$

This equation can best be solved by trial.

EXAMPLE.—Design a splice joint for the web of a 50-foot plate girder railway bridge, 60 inches deep, the splice occurring 12½ feet from the end. Let the flanges at this section contain 13.5 square inches each ($= F$) and the web 22.5 square inches. The live load bending moment here from a 100-ton engine would be about 530,000 foot-pounds and from dead load 235,000 foot-pounds or a total moment of 9,180,000 inch-pounds $= M_1$. The total live and dead load shear would be 53,500 lbs. $= S$.

From eq. (3) we have, as the unit chord stress,

$$f_1 = \frac{M_1}{\left(F + \frac{th}{8}\right)h} = \frac{9,180,000}{\left(13.5 + \frac{22.5}{8}\right)59} = 9550 \text{ lbs. per square inch.}$$

From eq. (4) we have

$$M_w = \frac{f_1 th^2}{8} = \frac{9550 \times 22.5 \times 60}{8} = 1,612,000 \text{ inch-pounds.}$$

This is the total bending moment to be resisted by the web plate splice.

From eq. (12) we have, for $r = 4000$ lbs., $S = 53,500$, and $d_n = 24$,

$$\frac{(n+1)(2n+1)}{n^2}\sqrt{64,000,000n^2 - 2,860,000,000} = \frac{6 \times 1,612,000}{24},$$

or

$$8000\frac{(n+1)(2n+1)}{n^2}\sqrt{n^2 - 45} = 403,000,$$

or

$$\frac{(n+1)(2n+1)}{n^2}\sqrt{n^2 - 45} = 50,$$

from which we find, by trial,

$$n = 25.$$

That is to say, if both the shear and the moment credited to the web plate at this section are to be transmitted through the rivets of the splice plates, with a maximum stress of 4000 lbs. on one rivet, it will require 50 rivets on each side of the joint, or 100 rivets all told, in this splice plate. This would require three rows

of $\frac{7}{8}$-in. rivets each side of the splice, 16 rivets in the two outside rows and 17 rivets in the middle row, all with 3-inch pitch.

It is evident at once that such a splice is heavy and expensive, and that some other means should be sought for transmitting the bending stresses across the joint. The rivets near the neutral axis are of no appreciable assistance for this purpose.

Such a splice as that shown in Fig. 341 is therefore wholly incompetent to transmit the web bending stresses. In such a case these stresses pass through both the splice plate and the flange, thus producing very much larger rivet and flange stresses than they were designed to carry. When such splices as shown in this figure are used the flanges should be designed for carrying all the bending moment.

275. An Efficient Web Splice.—If the rivets in the flange angles are already stressed up to their working limits to transmit the flange stress, the bending stress in the web should be carried directly across the joint through splice plates and into the web again on the other side, without going through the flange angles, plates, or rivets. These direct stresses (compression at top and tension at bottom) are most efficiently transmitted through long splice-plates placed just inside the angles, as shown at AB and $A'B'$ in Fig. 342. Let the distance between centres of these plates be d_m. Then we have, as the total stress transmitted through one pair of plates,

$$\textit{Stress in splice plates } AB = \frac{M_w}{d_m}, \quad \ldots \ldots \quad (13)$$

and also

$$\textit{Number of rivets in one end of splice } AB = \frac{M_w}{rd_m}. \quad \ldots \ldots \quad (14)$$

From these two equations the net areas of the plates and the numbers of rivets required to transmit the bending stresses are found.

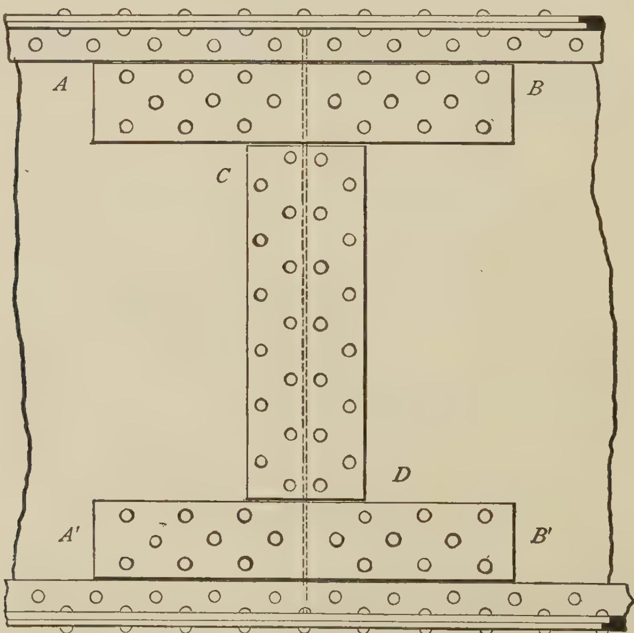


FIG. 342.

Thus with the previous example we have from eq. (4),

$$M_w = 1,612,000 \text{ inch-pounds,}$$

$$r = 4000 \text{ lbs. per rivet.}$$

If the vertical leg of the flange angle be 4 inches, and the splice plates AB be 8 inches wide, the distance between centres, d_m, will be $60 - 16 = 44$ inches. The stress in the plate will be, therefore, 37,000 lbs., requiring 9 rivets on each end of each pair of plates, as shown in Fig. 342.

The shear will be carried by the splice plate CD with $\frac{53,500}{4000} = 13$ rivets on each side.

This joint has now 62 rivets in place of 100 required for a uniform plate and rivet distribution as computed in the previous article.

The splice plates AB should not extend over the vertical legs of the angles, since this would give double duty to the rivets in the angle. With a web spliced as here designed there is no objection to designing the girder flanges on the common assumption that one eighth of the area of the web is added to each flange area to resist bending moment.

276. Distribution of Concentrated Loads Over the Web.—Since the flange stresses are first developed in the web from the shear, any external force, whether coming on the top of the girder as a load, or on the bottom as the end support, must be distributed through the web by means of vertical stiffeners, which are usually angle irons. The number of rivets in these is equal to the total external load divided by the resistance of one rivet. These

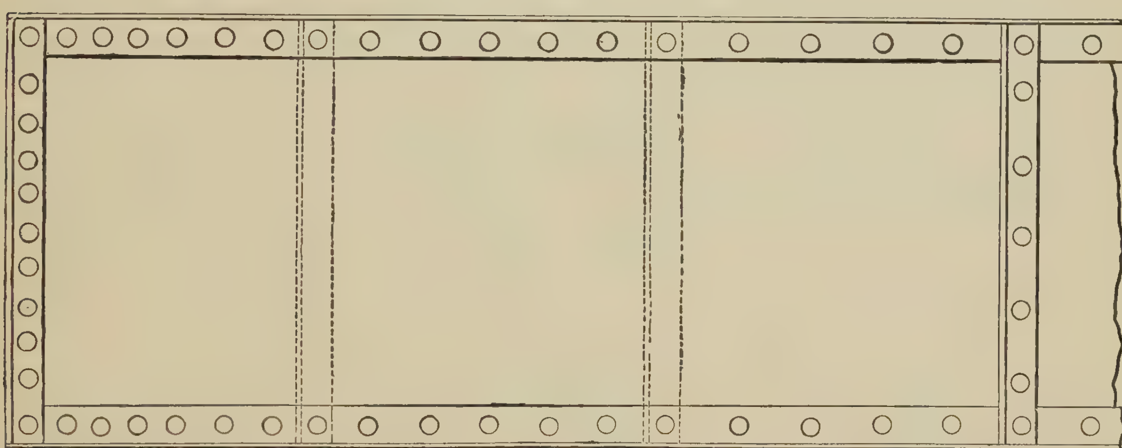


FIG. 343.

stiffeners should extend over the vertical legs of the flange angles and abut against the inside surfaces of the outer legs of these angles, being either bent over the vertical leg or supported by a filler-plate of the same thickness as the flange angle.

277. Web Stiffeners.—To assist the web plate to resist the compressive stresses acting at 45° with the axis of the girder, as explained in Art. 130, and to insure against the buckling of the web under this stress, angle irons are riveted to the web, usually in a vertical position. They would be much more efficient if put in an inclined position, extending outward and downwards towards the abutment. The vertical stiffeners are sufficient to do the work if placed somewhat less than the depth of the girder apart. If placed farther apart than this they do very little good, since the buckling tendency can have a free development between the adjacent stiffeners. It is not possible to rationally design these stiffeners. The equal tensile stresses at right angles to the compressive stresses in the web tend to prevent the buckling and cause it to develop in short curves, if it occurs at all. Almost any angles, however light, placed as here described (shown in dotted lines in Fig. 343), will serve to prevent buckling in a $\frac{3}{8}$-inch web of the ordinary depths. No column formula can be made to apply to the web of a plate girder.

278. Flange Areas.—If the web is designed to carry its due proportion of the bending

moment, as given by eq. (4), then the remainder only is to be carried by the flanges. Thus from eq. (3) we obtain, as the moment carried by the flange,

$$M_f = f_1 F h = M_1 - \frac{f_1 t h^2}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The flange area is, therefore,

$$F = \frac{M_f}{f_1 h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

This area is made up of two angles and one or more cover plates. The area of the web included between the angles is considered a part of the web and not a part of the flange. The rivet-holes are to be deducted from the total area of the tension flange. Unequal-legged angles are commonly used so as to throw the centre of gravity of the angles as high as possible and also to make the flange wide, and therefore rigid against lateral deflection or buckling.

279. **Distribution of Rivets in the Flanges.**—The pitch of the rivets in the flange angles and web plate is usually determined by computing total flange stress at intervals along the beam and dividing the stress increments in the flanges by the resistance of one rivet. The rivets being in double shear and the web plate thin, this resistance is always determined by the bearing area on the web plate. A better solution is as follows, by which the proper pitch to give to these rivets at any section is at once found.

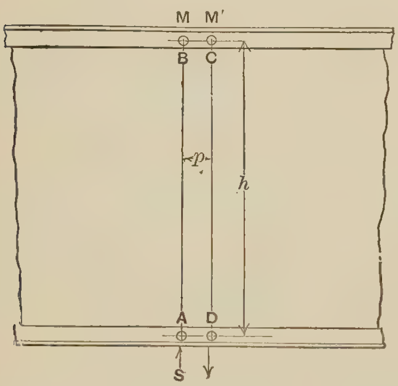


FIG. 344.

Let Fig. 344 represent any portion of a plate girder, the shear on the section through AB being S. Taking moments about D, and assuming for the present that all the moment is resisted by the flanges, we have

$$Sp = rh, \quad \text{or} \quad p = \frac{rh}{S}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where S = shear on the section;
p = pitch of rivets in flange angles;
r = resistance of one rivet;
h = distance between rivet lines in top and bottom flanges.

If the flanges are designed to carry all the bending moment, this equation would give the pitch of the rivets at any section. From eq. (2), however, we see that one eighth of the area of the web is equally effective with that of the flange in resisting the moment. Therefore the portion of the moment developed at this section arising between rivets, which is equal to Sp, is only partly resisted by the flange rivet. The portion going to the flange is always equal to

$$\Delta M_f = \frac{\Delta M F}{F + \frac{A}{8}} = \frac{SpF}{F + \frac{A}{8}} = rh, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and therefore

$$p = \frac{F + \frac{A}{8}}{F} \cdot \frac{rh}{S}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where F = area of one flange;
A = area of web.

PRACTICAL DESIGNING.

280. The Detail Design of a Deck Plate-girder Span.—Let the span be assumed to be 47 feet 6 inches in the clear. The distance from the base of rail to masonry to be made to suit the girders as designed. The live load, or capacity, to be Cooper's "Class Extra Heavy A" loading. (A table of the bending moments and shears for this load is given on page 329). The floor is assumed to weigh 400 lbs. per linear foot of track. The allowed stress per square inch on the net area of lower flanges to be 15,000 lbs. for the dead load stress and 7500 lbs. for the live load stress. The allowed shearing stress on rivets to be 6000 lbs. per square inch and the allowed bearing pressure of rivets on the web plate to be 12,000 lbs. per square inch. The top flanges to be of the same gross area as the bottom flange, and at least one-half the area of the flanges to be in the angles. The pressure on the masonry to be limited to 250 lbs. per square inch.

Assuming a length of fifty feet centre to centre of end bearings, and that the iron weight is given by the formula $9 \times l + 110$, we get an end reaction or total pressure on the masonry of $(9 \times 50 + 110 + 400)\frac{50}{4} = 12{,}000$ lbs. for the dead load. From the table we find the live load end reaction to be 66,500 lbs., making a grand total of 78,500 lbs. At 250 lbs. per square inch this requires 314 square inches of bearing, requiring a bed-plate 18 inches square. These bed-plates when placed on the masonry within 6 inches of the face of the masonry under the coping will be 50 feet centre to centre, and the over-all length of the girder will be 51 feet 6 inches. Our assumption of 50 feet centre to centre of end bearings is therefore correct. The weight of iron will be changed slightly, as l in the formula is the length over all and we have used it as the centre to centre length for convenience in the preliminary calculation of the end shear. The correct end shear will be as follows:

From dead load $(9 \times 51.5 + 110 + 400)\frac{50}{4} = 12{,}200$ lbs.
" live " $= 66{,}500$ "

281. The Economical Depth of a plate girder may be closely approximated by the following formulæ;

1st. *If the moment of resistance of the web is neglected.*

Let m = centre moment in inch-pounds from dead and live loads;
x = depth of girder in inches;
y = weight of girder in pounds;
f = allowable fibre stress = 7200 lbs. per square inch on gross area;
t = thickness of web = $\frac{3}{8}$ inch;
l = length of girder in feet.

Then, as the weight of the flanges when the flange plates are cut off at the theoretical points may be assumed as eight tenths of their weight in case they were the same area end to end of girder, we have

$$y = \frac{10}{3}ltx + 1.6\frac{m}{xf}\cdot\frac{10}{3}\cdot l,*$$

and

$$\frac{dy}{dx} = \frac{10}{3}lt - 1.6\frac{m}{f}\cdot\frac{10}{3}l\cdot\frac{1}{x^2} = 0,$$

* The term $1.6\frac{m}{xf}\cdot\frac{10}{3}\cdot l$ would be $2\cdot\frac{m}{xf}\cdot\frac{10}{3}l$ if the flanges were of constant area end to end.

whence

$$x = 1.27\sqrt{\frac{m}{ft}};$$

which would give for the girder assumed 79 inches as the economical depth.

2d. *If the moment of resistance of the web is not neglected.*

Using the same notation as before, and also assuming that the *net* moment of resistance of the web results in an allowable reduction of each flange area by an amount equal to *one eighth* of the gross area of the web, we have

$$y = tx \cdot \frac{10}{3} \cdot l + 1.6\frac{m}{fx} \cdot \frac{10}{3} \cdot l - \frac{2}{8} \cdot tx \cdot \frac{10}{3} \cdot l,^*$$

$$y = \frac{6}{8}tx \cdot \frac{10}{3} \cdot l + 1.6\frac{m}{fx} \cdot \frac{10}{3} \cdot l,$$

and

$$\frac{dy}{dx} = \frac{3}{4}t \cdot \frac{10}{3}l - 1.6\frac{m}{fx^2} \cdot \frac{10}{3}l = 0;$$

or

$$\frac{3}{4}t = 1.6\frac{m}{fx^2},$$

and

$$x = \sqrt{\frac{6.4}{3}\frac{m}{ft}} = 1.46\sqrt{\frac{m}{ft}};$$

which would give 92 inches as the economical depth.

The quantities which may vary with the depth and are neglected in the above formulæ are the weights of stiffeners, sway bracing between girders, and the web splice plates. These would have little effect if the stiffeners were of a constant size and placed a constant proportion of the depth apart. Whatever effect these neglected quantities would have would tend to reduce the depth.

282. **Working Rule for Economic Depth.**—A very convenient rule for determining approximately the economical depth of girder in the case when the moment of resistance of the web is neglected is to select the depth at which the area of the web plate is equal to the combined area of the two flanges if the flanges are of constant area end to end, or eight tenths of the combined area of the two flanges at the centre of the girder if flange plates are used and stopped off at the theoretical lengths. If the web is taken into account at its full value in resisting the bending moment, select the depth at which *three quarters of the area of the web plate* is equal to the combined area of the two flanges, if the flanges are of constant area end to end of girder, or eight tenths of the combined area of the two flanges if flange plates are used and stopped off at the theoretical lengths. In the latter case the area of the flanges *includes* the web equivalent, or is equal to the bending moment divided by the product of the depth and unit stress.

283. **Determination of the Flange Areas.**—For the span under consideration the dead load centre moment for *one* girder is $\frac{974 \times \overline{50}^2}{16} = 152,200$ ft.-lbs., and the live load centre

* The term $1.6\frac{m}{xf} \cdot \frac{10}{3} \cdot l$ would be $2 \cdot \frac{m}{xf} \cdot \frac{10}{3} l$ if the flanges were of constant area end to end.

moment taken from the table is 744,400 ft.-lbs. As the usual practice is to neglect the effect of the web in resisting the bending moment, and as shown previously in this chapter it being very difficult to make a web splice which will effectively resist bending, the influence of the web will be neglected in the design of this girder. The formulæ for economical depth give 79 inches, but in order to make allowance for the possible errors in the assumptions 6 feet will be taken for the depth. Assuming the depth centre to centre of gravity of the flanges as 71 inches, we get 25,700 lbs. as the dead load flange stress and 125,800 lbs. as the live load flange stress. The required net area of the bottom flange is then found as follows:

$$\left.\begin{aligned} 25,700 \div 15,000 &= 1.7 \\ 125,800 \div 7,500 &= 16.8 \end{aligned}\right\} = 18.5 \text{ square inches.}$$

The flange will be made of

Two $6'' \times 4'' \times \frac{9}{16}''$ angles = 9.5 square inches net area;
" $14'' \times \frac{3}{8}''$ plates = 9.0 square inches net area.

The centre of gravity of this flange section is .33 inch from the backs of the angles, making the depth centre to centre of gravity of the flanges 71.34 inches. This is near enough to the assumed depth to require no change in the flange area.

The rivets are seven eighths of an inch in diameter, and the net area is found by deducting one hole one inch in diameter from each angle and two holes from each plate, it being assumed that the rivets are staggered in the angles.

284. The Lengths of Flange Plates are usually determined as follows:

The curve of maximum bending moments is assumed to be a parabola as shown in Fig. 345, an assumption which is in error about three per cent in maximum actual effect, and the curve plotted with a middle ordinate representing the maximum bending moment.

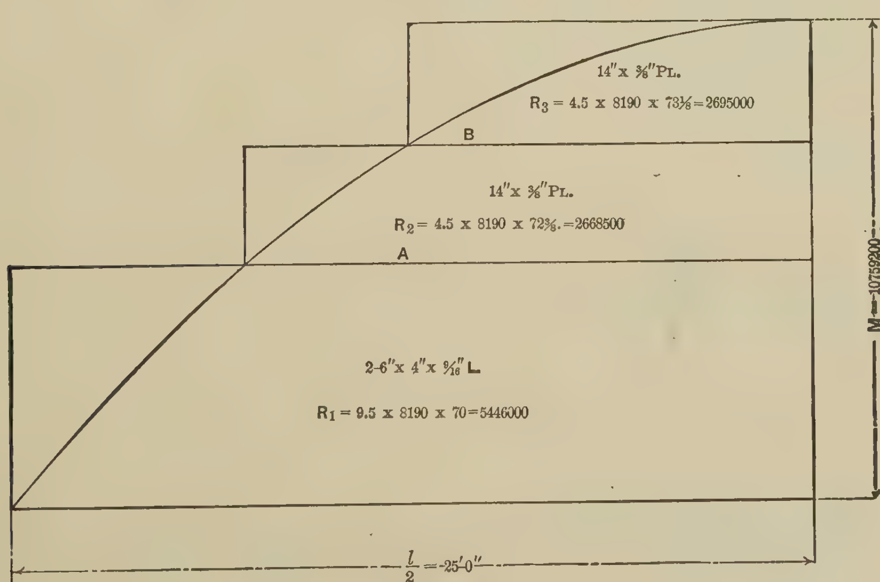


FIG. 345.

The corresponding moments of resistance of the plates and angles (R_1, R_2, and R_3) are then laid off on the figure to the same scale, and the lengths of the plates determined by the intersection of the limiting lines A and B with the curve. Or an easy way of determining

these lengths, assuming the curve to be a parabola and also that the areas of all the members are at the centre of gravity of the flange as a whole, is as follows, and is based on the law of the parabola:

Let $A =$ total flange area;
$a_1, a_2, \ldots a_n =$ areas of the plates, the subscript denoting the number of the plate from the outside;
$x_1 \ldots x_n =$ length of the plate;
$l =$ length of girder centre to centre bearings.

Then

$$x_n = l\sqrt{\frac{a_1 + a_2 + \ldots a_n}{A}}.$$

Example.—In the above girder the length of the first flange plate would be

$$50\sqrt{\frac{4.5}{18.5}} = 24.66 \text{ ft.},$$

and the length of the second flange plate would be

$$50\sqrt{\frac{9.0}{18.5}} = 34.88 \text{ ft.},$$

Whichever of the above methods is used, it is customary to add two feet to the theoretical length of the plate which lies next to the angles, and one foot to all others, in order to get some rivets beyond the point where the plate is necessary.

285. The Thickness of the Web and Size of Flange Angles.—It is customary in railroad work to make the least allowable thickness of web three eighths of an inch, and to limit the vertical shearing stress on the vertical section of the web plate to 5000 lbs. per square inch. Thus in the girder assumed the maximum shear is 78,700 lbs., and the area of a web plate three eighths of an inch thick is 27.0 square inches, giving a shearing stress of 2900 lbs. per square inch. A web plate three eighths of an inch thick will, therefore, satisfy this requirement. There, is, however, another question involved, and that is that there must be enough bearing area provided for the rivets through the flange angles and web plate to enable them to transfer the flange stress to the flanges without exceeding the allowed pressure per square inch or spacing the rivets less than the minimum limit of three diameters centre to centre. In the girder in question the maximum shear is 78,700 lbs., and from eq. (17) the rivet spacing is

$$p = \frac{r \times h}{S};$$

where $p =$ distance centre to centre of rivets; $r =$ the bearing value of one rivet on the web plate, in this case 3940 lbs. ($= 12{,}000 \times \frac{7}{8} \times \frac{3}{8}$); $h =$ the distance from the rivet lines in bottom flange angles to the rivet lines in top flange angles, which in the girder under consideration equals 68 inches; and $S =$ the shear. Introducing the proper values into the formula,

$$p = \frac{3940 \times 68}{78{,}700} = 3.4 \text{ in.}$$

If p had been less than $2\frac{5}{8}$ inches, or three times $\frac{7}{8}$, it would have been necessary to thicken the web plate, or use flange angles in which two rows of rivets could be used as shown in Fig. 346.

Thus, suppose the shear had been, for the case under consideration, 130,000 lbs. This would still have been within the limits of the allowed shearing stress on the web plate, but the pitch of the rivets for a three-eighths plate would be

$$p = \frac{3940 \times 68}{130,000} = 2.06 \text{ in.},$$

which is less than is allowable. To find the thickness of web required for flange angles in which only one

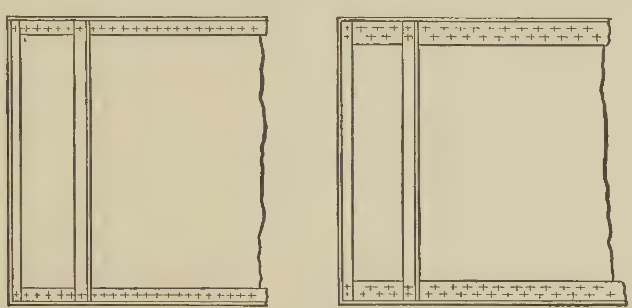


Fig. 346.

line of rivets can be used, we must solve the general formula $p = \dfrac{r \times h}{S}$ for r. The value of r would be

$$r = \frac{pS}{h}.$$

Introducing the proper values, using $2\frac{5}{8}$ as the value of p, we get

$$r = \frac{2\frac{5}{8} \times 130,000}{68} = 5019 = 12,000 \times \tfrac{7}{8} \times t, \qquad \text{and} \qquad t = .478 \text{ in. or } \tfrac{1}{2} \text{ in.}$$

If we used $6'' \times 6''$ angles in which two rows of rivets could be put through each of the flange angles, we would get, bearing in mind that h for this case is 65.5 in.,

$$p = \frac{7880 \times 65.5}{130,000} = 3.97 \text{ in.}$$

The use of the larger angles would save some material, but would increase the cost of manufacture owing to the increased number of rivets to be driven.

286. The Riveting of Plate Girders.—It is necessary in designing girders to provide sufficient rivets for the transfer of the external loads or forces to the web plate, and to so rivet the flanges to the web and the various sections of the web to each other that the rivets in no case will be subjected to excessive stresses. When the flange plates and angles are over 50 ft. long, which is about the limiting lengths of material now obtainable at standard prices, it also becomes necessary to splice them and the number of rivets required must also be determined for this case.

287. The Transfer of Concentrated External Loads or Forces to the Web.—The usual method of transferring an external load to the web is to rivet distributing angles, which fit tight against the flange angles at the point of application of the load, to the web, using enough rivets to resist the load without exceeding the limiting values of shear or bearing. In

the case of the ordinary wheel load of a locomotive it is considered to be distributed over a length of flange of about 3 ft. by the rails, and no provision need necessarily be made for such a case further than using sufficient rivets through the flange angles to transfer it directly to the web without distributing angles. In the usual case of a deck plate girder the only concentrated load which need be considered worthy of distributing angles are the abutment reactions for which distributing angles are necessary. The usual method is as shown in Fig. 346, in which a pair of angles is riveted to the web over each end of the bed-plates. These angles should have a tight fit against the horizontal legs of the *lower* flange angles. The number of rivets required for the end shear in the girder here considered would be $78,700 \div 3940 = 20$. It is customary to divide this number equally between the two pairs of angles, but, owing to the deflection of the girder bringing the greater load on the inside pair, it is better to provide more than this in the inner pair, putting, however, at least one half of the total number required in the outside pair or those at the extreme end of the girder. The size of these angles need not be greater than is required to insure no greater bearing pressure between them and the flange angles than 12,000 lbs. per square inch. The usual practice is to make the outstanding leg about 1 inch less than the horizontal leg of the flange angles and the leg against the web 3 or $3\frac{1}{2}$ in., and the minimum thickness $\frac{3}{8}$ in.; the bearing requirement determining the thickness.

288. The Transfer of the Flange Stresses to the Flanges.—There are two cases to be considered, as follows: 1st. When the web plate is assumed to have no moment of resistance and to transfer shear only; 2d. The *correct* assumption where the web is considered at its actual value in resisting both moments and shears.

1st. *The Transfer of the Flange Stresses to the Flanges, neglecting the Moment of Resistance of the Web Plate.*—From Art. 279 we get the formula $p = \frac{rh}{S}$, when p = pitch or the horizontal distance centre to centre of rivets through the flange; r, the value of the rivet in double shear or bearing value on the web plate at the limiting allowed stresses; h, the distance from the line of rivets through the web in the top flange angles to the line of rivets through the bottom flange angles; and S, the vertical shear at any point in the girder. A brief explanation of the formula will be given. The increment of flange stress or the amount of flange stress which must be transferred to the flange by one rivet at any point in the girder is dependent on the shear at that point and the horizontal distance centre to centre of rivets. The shear acting with a lever arm p produces an increment of bending moment which must be resisted by a moment equal to it and equal to the increment of flange stress acting with a lever arm of h. This increment of flange stress is transferred from the web to the flange by the rivet, and therefore the stress on the rivet must be equal to this increment. We may then write

$$Sp = rh, \qquad \text{or} \qquad p = \frac{rh}{S}.$$

Now as the total shear practically increases uniformly from the centre to the end of the girder, we can easily find the spacing required at any point. By finding the spacing required at the end, at one or two intermediate points, and at the centre, we can readily sketch in a curve as shown in Fig. 347, from which the required spacing at any point may be found graphically. For the girder we have under consideration the spacing required at the end is

$$p = \frac{3940 \times 68}{78,700} = 3.4 \text{ ins.},$$

the spacing required at the quarter point is

$$p = \frac{3940 \times 68}{48,600} = 5.5 \text{ ins.},$$

and the spacing required at the centre is

$$p = \frac{3940 \times 68}{18,600} = 14.4 \text{ ins.}$$

Plotting these results and sketching a curve, as in Fig. 347, we are able to find the spacing required at intermediate points. The usual spacing is in even inches or half inches, and

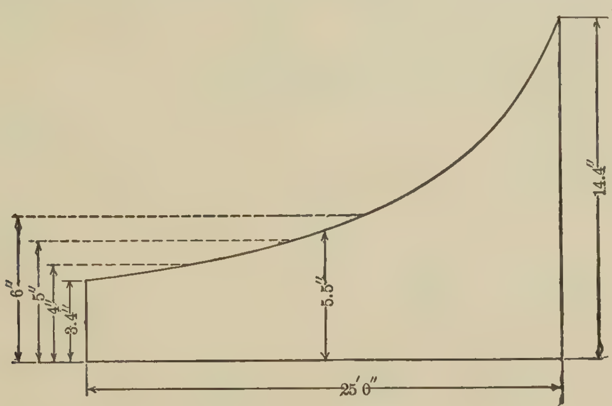


FIG. 347.

the maximum allowable space is 6 ins. for practical reasons. From the diagram it is easy to determine the points where the spacing may be changed without exceeding the allowed stress on the rivets.

2d. *The Transfer of the Flange Stresses to the Flanges using the Web Plate at its True Value.*—The difference between this case and the one just considered is that in the former case the *entire* bending moment is assumed to be resisted by stresses in the flanges alone, and therefore requiring rivets enough to transfer this stress from web to flange, while in the present case only such an amount of stress is transferred as actually goes to the flanges. The amount of bending moment resisted by the web is such a proportion of the total bending moment at any given section as the moment of resistance of the web is to the total moment of resistance of the girder at that section, and hence the amount of stress to be transferred to the flanges is less than in the former case by the amount of flange-stress equivalent that the web takes. It has been shown in Art. 272 that

$$M = Ffh + f\frac{th^2}{8} = \left(F + \frac{A}{8}\right)fh;$$

$$\therefore \frac{M}{h} = \text{equivalent flange stress} \doteq \left(F + \frac{A}{8}\right)f.$$

Of this total equivalent flange stress the part Ff only is actually transferred to the flanges by the rivets; hence the rivet spaces in this case would be $\left(\frac{F + \frac{A}{8}}{F}\right)p$, where p is the pitch, or distance from centre to centre, of rivets determined by neglecting the effect of the web plate. When there is a series of flange plates stopped off at the theoretical points, a section of the girder near the end of a plate would show a larger flange area than was really effective, and the corresponding rivet spacing would be less than is actually required. This would be

an error on the side of safety. The correct way to determine the rivets for this case would be to assume for $F+\frac{A}{8}$ the theoretical flange area required at the section, and for F the difference between this theoretical area and $\frac{A}{8}$.

289. **The Combination of the Vertical Stress** on the rivets from the external loads and the horizontal stress from the bending moment will now be considered. The maximum wheel load is 15,000 lbs., and this is assumed to be uniformly distributed by the rail over the space occupied by three cross-ties which is about 42 inches. Then the vertical stress on each rivet would be $\frac{15,000}{42} \cdot p = 357p$, approximately.

The horizontal stress would be as found before, $r = \frac{s \cdot p}{h}$, and the resultant stress from these two would be

$$r_r = \sqrt{(357 \cdot p)^2 + \left(\frac{s \cdot p}{h}\right)^2}, \quad \text{from which} \quad p = \sqrt{\frac{r_r^2}{127,400 + \left(\frac{s}{h}\right)^2}}.$$

In the case of the girder under consideration $r_r = 3940$ and $h = 68$, so that the formula reduces to

$$p = \sqrt{\frac{15,523,600}{127,400 + \left(\frac{s}{68}\right)^2}}.$$

For sections taken five feet apart we have

$s = 78,700$,	$p = 3.25$,	spacing at end;
$s = 66,700$,	$p = 3.75$,	spacing 5 ft. from end;
$s = 54,600$,	$p = 4.5$,	spacing 10 ft. from end;
$s = 42,700$,	$p = 5.5$,	spacing 15 ft. from end;
$s = 30,700$,	$p = 6.9$,	spacing 20 ft. from end;
$s = 18,600$,	$p = 8.75$,	spacing 25 ft. from end.

In the above the live load shear is assumed to increase uniformly from centre to end, which is a small error on the side of safety. When the stiffener angles are spaced greater distances apart than three feet, the rivet spacing should be determined by this method. By the correct method of determining rivet spacing, or that method which includes the web in the calculations, the term $\left(\frac{s}{h}\right)^2$ which appears in the denominator would be $\left\{\left(\frac{F}{F+\frac{A}{8}}\right)\frac{s}{h}\right\}^2$.

In the fifty-foot girder the spacing of the rivets through the top flange angles and web plate would be three inches until six-inch spaces are sufficient, and then six inches the remainder of the distance. In the bottom flange the same rule would be followed, but in this case it will be noted that the six-inch spaces will begin nearer the end of the girder. The point when six-inch spaces are sufficient in the lower flange would be determined by Fig. 347, and where they may begin in the top flange by Fig. 348.

This spacing is taken because it will enable us to put the rivets in the bottom flange angles on the same vertical lines as those in the top flange angles, and thus simplifies the manufacture, at the same time giving us a larger margin of safety in the top rivets, and also avoids unnecessary punching and weakening of the lower flanges. The number of rivets will

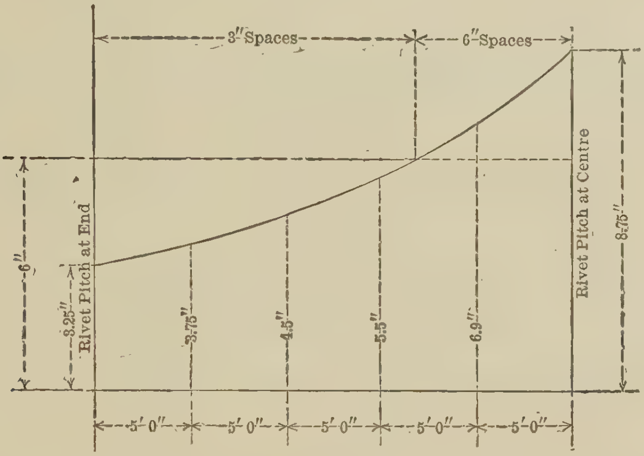


Fig. 348.

be about the same as if we had used the spacing indicated by Fig. 348 for both flanges, which is the practical alternative. The rivets through the flange plates and angles are always staggered as much as possible with those through the web and flange angles. The pitch is made six inches everywhere except near the ends of the plates, where a few spaces of three inches are used.

290. Rivets in Web Splices.—When the web plate is not calculated as resisting any of the bending moment, the web need be spliced for vertical shear only. Any assumption which may be made does not relieve the web of its true stress from the bending moment, so that it is only at the web splices that the flanges are subject to the stress that this assumption indicates. At this point the excess of material which is put in the flanges by this method acts as a web splice, transferring the part of the bending moment which the web plate does resist across the spliced section of the web plate. The ordinary web plate splices consist of a pair of plates $\frac{5}{16}$ to $\frac{3}{8}$ of an inch thick for a $\frac{3}{8}$-inch web with a single vertical line of rivets on each side of the joint. The rivets in these plates are figured to transfer the maximum shear at this point without exceeding the allowed shearing stress per square inch on rivets or the allowed bearing stress per square inch on the web plate. In the fifty-foot girder the web is spliced about 8 feet 4 inches from the centre, where the shear is 39,000 lbs. The bearing value of one rivet being the limiting value and equal to 3940 lbs., ten rivets on each side of the splice are required. In order not to exceed six-inch pitch, eleven rivets on each side would be used.

An example of the proper method of splicing webs when the web is taken into consideration in determining the flange areas is given in Art. 275.

291. Rivets in Flange Splices.—It is customary, and also economical, when the flange section must be spliced, to splice only one piece at a time. For example, suppose the plates and angles in the flanges of the fifty-foot girder were longer than they could be rolled, the splice would be made as shown in Fig. 349. The flange plates are cut on the lines *BB* and *CC* respectively, and one flange angle is cut at *AA* and the other at *EE*. The rivets through the splice angles and plate between *FF* and *AA*, *AA* and *BB*, *BB* and *CC*, *CC* and *EE*, and *EE* and *GG* must in each case equal in value the largest piece cut, which in this case is one angle.

The value of this angle is $4.75 \times 8200 = 39{,}000$ lbs., which requires eleven $\frac{7}{8}$ rivets in single shear. The value of one $\frac{7}{8}$ rivet is 3660 lbs. ($= 6000 \times .61$). The sketch shows at least twelve rivets in each case in single shear. The net area of the splice plate or of the two splice angles should be equal to the net area of the largest piece cut. Two splicing

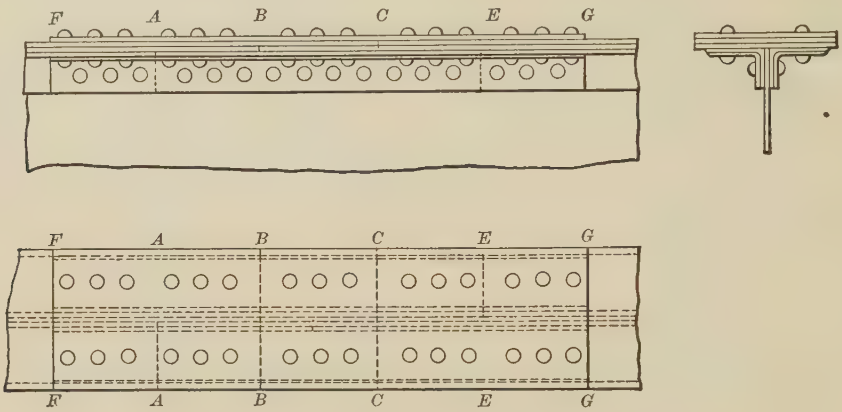


FIG. 349.

angles and one plate are generally used. Owing to the three eighths limit of thickness generally specified, there may be used in this case two $5 \times 3\frac{1}{2} \times \frac{3}{8}$ angles and one $14 \times \frac{3}{8}$ plate.

It is better to use six-inch pitch for rivets in the tension flange splice in order that the staggering of the rivets may be as effective as possible. In the compression flange the pitch may be made as small as three diameters of the rivet.

292. Stiffeners.—No rational method has yet been discovered by which stiffeners, or the angles which are riveted to the web to prevent its buckling under stress, can be proportioned. It is believed, however, that if they are spaced at distances apart a little less than the depth of the web plate, and made to consist of a pair of angles the outstanding legs of which are one thirtieth of the depth of the web plate, an excessive provision has been made. The stiffeners under this rule would be spaced close enough to resist the tendency to buckle on a line making forty-five degrees with the vertical, and the combined width of the outstanding legs would make a column fifteen diameters long. This is below the limit at which practical tests have shown that columns begin to fail by flexure. This rule would also agree closely with present practice. The thickness of the angles should not be less than five sixteenths of an inch.

Cooper's specifications require that stiffeners must be used at distances apart about equal to the depth of the web when the vertical shearing stress per square inch on the web plate exceeds the allowed stress found from the following formula: $s = \dfrac{12{,}000}{1 + \dfrac{H^2}{3000}}$, where s = allowed stress and H is the ratio of the depth of the web plate to its thickness, or $\dfrac{h}{t}$, where h is the depth of the web plate and t the thickness of the web.

Another very general specification is that stiffeners must be used whenever the vertical shearing stress per square inch on the web plate exceeds that given by the formula $s = \dfrac{10{,}000}{1 + \dfrac{l^2}{3000t^2}}$, where s = the allowed stress, l = depth of web plate or distance centre to centre of stiffeners, and t = the thickness of the web. From this formula the distance centre to centre of stiffeners is found. In the standard specifications of the

Pennsylvania Lines another good method is specified. It is that stiffeners must be used whenever the thickness of the web plate is less than one fiftieth of the clear distance between the vertical legs of the flange angles. They must be spaced at distances apart at the ends of the span not greater than one half the depth of the web plate, and may be gradually placed further apart until at the middle of the span they are not greater than the depth of the web plate centre to centre.

It is not customary to specify the sizes to be used, except to give a minimum size or in the general clause which limits the sizes of material to be used in any part of the structure.

293. **Lateral Bracing.**—The lateral bracing of a plate girder bridge is now usually made of angles with riveted connections. It is sufficient to use only one system of bracing, and that in the plane of the loaded chord. The wind pressure on the bridge is taken at 30 lbs. per square foot of the bridge as seen in elevation, and this is combined with a moving load of 300 lbs. per linear foot of track. The stresses for this bracing used in the fifty-foot span are given on the general drawing of the completed design shown on page 312, and are calculated by the methods explained in Part I. The bracing in the horizontal plane is proportioned for compression only by the formula $13{,}500 - 60\frac{l}{r}$, which allows fifty per cent greater stresses than that allowed by the usual "straight-line" formula substitute for Gordon's formula, $\dfrac{8000}{1 + \dfrac{l^2}{8000r^2}}$. It is the usual practice to increase the allowed stresses in wind bracing fifty per cent over those allowed in the main trusses.

294. **The Frames** at the ends of the span must be proportioned to resist the wind stresses, assuming *all* the forces to come to the end through the top bracing. Intermediate frames should be used at distances apart not over sixteen times the width of flange. This rule agrees with the general practice.

295. **Bed-plates** should always be made thick enough to insure uniform pressure over the masonry. They are made $\frac{3}{4}$ of an inch thick for the fifty-foot span under consideration, which will be sufficient as the pressure of the girder on the bed-plates is distributed over an area of $18'' \times 12\frac{3}{8}''$, leaving less than 3 inches projection unsupported.

296. **There are Sole Plates,** generally of the same thickness as the bed-plates, riveted to the bottom flanges of the girders, and at the expansion ends the surfaces of contact between bed and sole plates are planed to insure a minimum amount of friction.

297. **Expansion and Contraction** in a plate girder bridge of a span less than 75 feet is usually provided for by allowing one end to move on the planed surfaces between the sole and bed plates mentioned in the previous article, the other end being fixed. For lengths over 75 feet, rollers not less than two inches in diameter are put between the bed and sole plates at the expansion end. When any bridge rests on rollers, it is important that the pressure should be distributed as uniformly as possible over the rollers. The present practice of merely putting the rollers between the sole and bed plates without any attempt being made to make the pressure equal on the rollers is not correct, as has been explained in Chapter XVIII.

298. **Anchor Bolts,** usually one inch in diameter for all plate-girder spans, are put through the bed and sole plates and run about six inches into the masonry. They are either "rag" or "wedge" bolts, and after being put in place the hole in the stone-work is packed with cement or sulphur.* Slotted holes for these bolts are made in the *sole*-plate only, at the expansion end, to allow for the movement of the girder on the bed-plate. There are usually two of these bolts through each bed or sole plate, or eight to each complete single track span.

299. **The Width** centre to centre of the girders of deck plate-girder spans varies from 5 feet to 10 or 12 feet on straight track. Cooper specifies a minimum width of 6 feet 6 inches on straight track and that the girders in no case shall be less than 3 feet 3 inches from the

* See Fig. 292.

centre of the track. A good rule to follow is to space the girders as far apart as the depth of the girder on straight track and increase this distance by the middle ordinate of the curve on the bridge for curves, with a minimum distance of 3 feet 3 inches from centre of track to the nearer girder for all cases.

300. The Complete Design of the girder which has been referred to all through this discussion is given on page 312.

301. The Design of Through Plate Girder Spans.—If the cross-ties are supported on the inside bottom flange angles or on a shelf angle as shown in Fig. 268, Chapter XVI, the girders are dimensioned in the same manner as for a deck span. If shelf angles are used, they should be not less than 4 inches by 3 inches, and $\frac{5}{8}$ thick, with the 4-inch leg horizontal. It is better to use distributing angles under the shelf angle about 3 feet apart, to transfer the load to the web, than to depend on the rivets attaching the shelf to the web, owing to the necessarily eccentric loading of the shelf angles producing a bending moment which would bring a tensile stress on the rivets. The rivets in the flanges for this case would be spaced by the diagram Fig. 347, as there are no external loads acting on the flanges to be transferred to the web. If the cross-ties are supported on the inside bottom flange angles, these angles should not be less than $\frac{5}{8}$ of an inch thick and the rivets through the web and these angles should be spaced the minimum allowable pitch, as there is necessarily a tensile stress on them from the eccentric application of the bearing of the cross-ties on the angles. The rivets through the top flange may be spaced by the diagram Fig. 347. It must be noted that there is no necessity for increasing the area of the lower flange, except when necessary in order to reach the minimum allowed limit of thickness for the inside angles of $\frac{5}{8}$ of an inch, as the stresses are increased a very small amount.

If an iron floor system of floor-beams and stringers is used, the live load and the weight of the floor and floor system are concentrated at the points of attachment of the floor-beams or panel points, and are no longer distributed over the length of the girder as in the case of the deck bridge. The panel lengths used range from 8 to 15 feet, varying with the length of the girder, but are usually about 10 feet. In Fig. 350 is shown a longitudinal sectional view

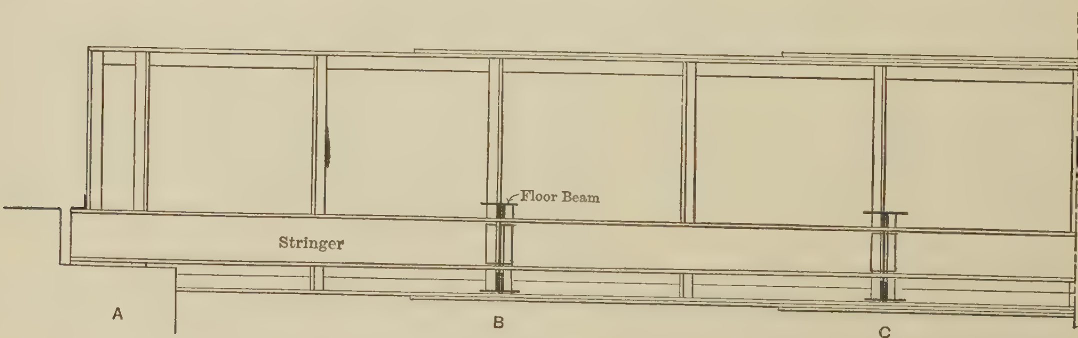


FIG. 350.

on the centre line of a through plate-girder span with an iron floor system. The bending moment and shears are found at the critical points A, B, and C, and the flanges dimensioned as illustrated for the deck span. The bending moment varies uniformly from A to B, and from B to C, and is nearly uniform from C to the centre. The shear is constant from A to B, B to C, and from C to the centre.*

* This is not strictly true, as the weight of the girder, which is a small proportion of the load, is distributed over the entire length of the girder. It is the usual practice to consider the weight of the girder concentrated at the panel points in order to simplify the work of calculation and as the error is insignificant.

The rivet spacing is easily determined after the critical moments and shears are found, by the same formula as was used for the deck girder, remembering that there are no concentrated external loads on the flanges.

302. Attachments of the Floor-beams and Stringers.—There must be enough rivets in the connection between the floor-beam and the girder to transfer the end shear of the floor-beam to the girder. The rivets are in single shear. If they are "field" rivets, it is customary to put 25 to 50 per cent more rivets than would be used if the rivets were to be shop-driven. The floor-beam is often attached as shown in Fig. 351, in which case there must be the same number of rivets in the splice plate *B* to splice the web plate to the gusset plate as though it were simply a web splice. The number of rivets connecting the gusset plate to the stiffener angle and the stiffener angle to the web must be sufficient to transfer the total end shear of the floor-beam. In the attachment of the stringers to the floor-beam as shown in Fig. 350 there must be enough rivets to transfer the total end shear of one stringer, counting the rivets in single shear; and if there are stringers attached on each side of the beam at the same point, there must be enough rivets to transfer the maximum combined end shear, or total concentration on the beam at that point, to the floor-beam, counting the rivets in double shear or at the allowed bearing value on the web of the beam.

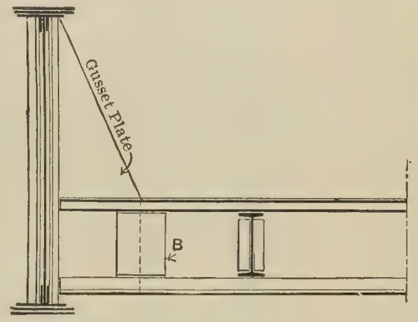


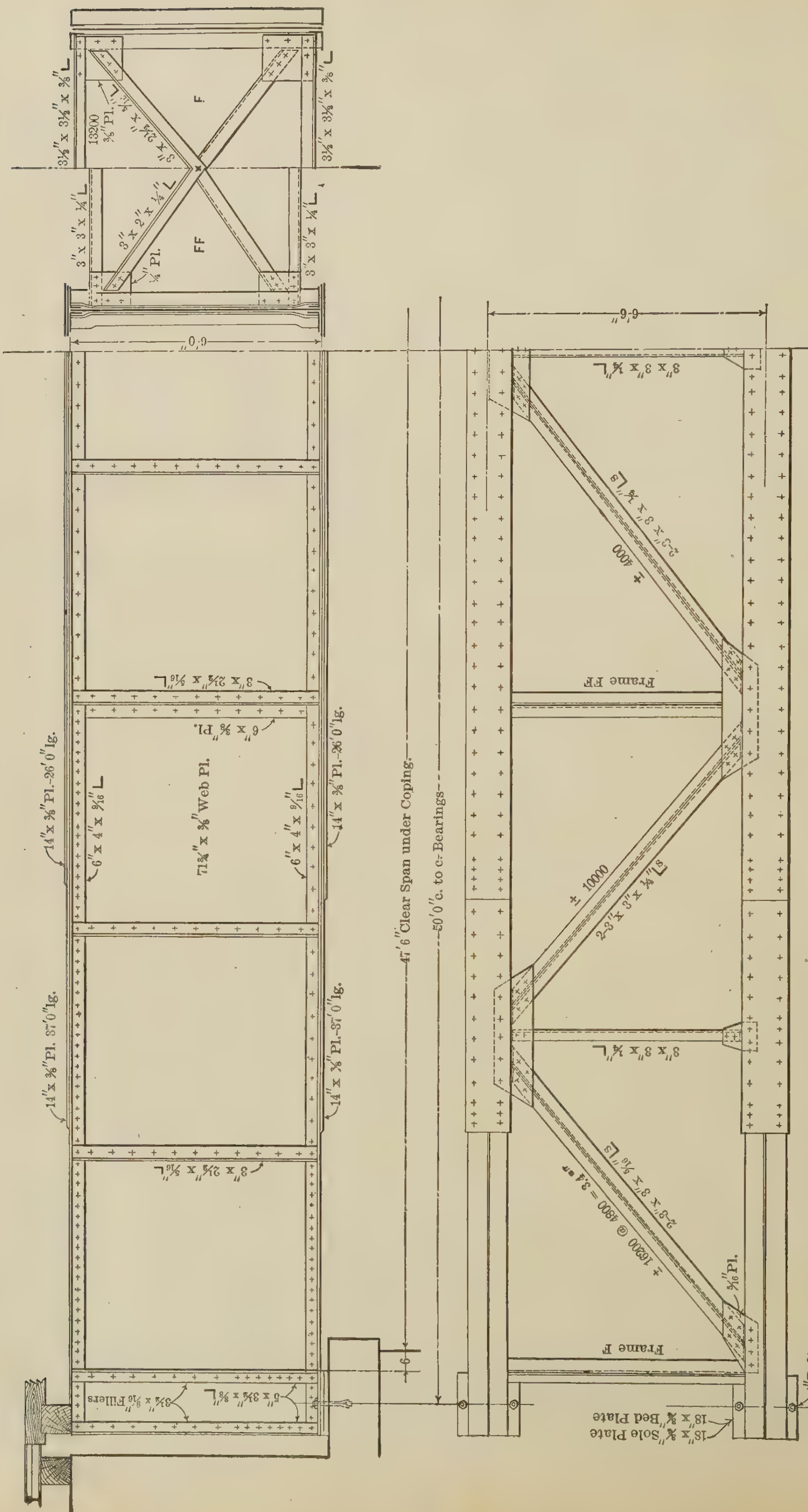
FIG. 351.

There is usually no end floor-beam in through bridges of this kind, the stringers resting on the masonry direct and held in line by an end strut which runs from girder to girder and which also attaches to the end gusset plates.

The floor-beams for through girders of this kind are calculated for a length centre to centre of end supports equal to the width centre to centre of main girders. The stringers are calculated for the length centre to centre of floor-beams, or in case of the end stringers where no end floor-beam is used the length assumed in the calculation would be from the centre of the end bearing plate to the centre of the first beam. For panel lengths less than 15 feet the stringers are usually rolled I beams and are always proportioned by their moment of resistance.

303. The Bracing of a Through Plate-girder Span differs slightly from that of a deck span, inasmuch as in this case provision has to be made for supporting the top or compression flange against side deflection. This is usually done by means of gusset plates from the floor-beam, or cross-strut, in the case where no iron floor system is used, similar to the method shown in Fig. 351. Often there is only an angle brace riveted to the vertical leg of the top flange angle and bent out so as to attach to the floor-beam or cross-strut two or three feet from the girder. These gussets or braces and also the floor-beams or cross-struts should be spaced not over 15 feet apart. The lateral bracing proper may consist either of adjustable rod or of angle-iron bracing attached to the main girders. It is customary to use angle bracing with riveted connections, and owing to their being of greater length than for a deck span they are usually made to cross the panel on both diagonals, each piece being proportioned to take the total stress in tension.

304. The Width Centre to Centre of Girders for a through plate-girder span is never less than 12 feet on straight track. The girders should be spaced so as to give at least 7 feet clear distance from the centre of the track to the inside edge of the flange plates if they extend more than one foot above the rails, in order to give ample clearance for the passage of trains. This is the usual requirement now, although some of the Western roads require 7 feet 6 inches from the inside of the flange plates to the centre of the track.



DETAIL SKETCH OF A DECK PLATE GIRDER SPAN

50 ft. c. to c. of Bearings. 51 ft. 6 in. Total Length   Live load Cooper's Class Extra Heavy A.

FIG. 352.

CHAPTER XX.

ROOF TRUSSES.

305. **The Type of Truss** most commonly used for shop, warehouse, and small train-shed iron roofs is what is known as the Fink truss, shown in Fig. 353. It has proved to be a very

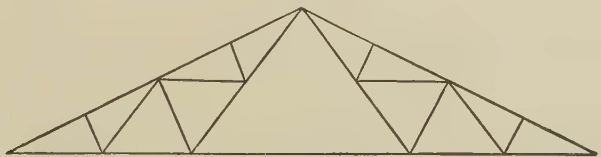


FIG. 353.

satisfactory and economical type for the ordinary lengths, which are under 100 feet. There are many conditions which may, however, affect the design of a roof; and as there are very few reasons why any special type of truss should be adopted, it may be generally accepted that the best truss to select is the one which fulfils the special conditions the best and at the same time is economical in material. It would be very creditable designing to select a truss in which the lines present the most pleasing appearance if the fitness of the truss for the purposes for which it is to be used is not impaired thereby.

For the longer spans the sickle roof truss and the three-hinged arch are generally used wherever the conditions permit. Either of these designs will make creditable trusses. For train sheds the three-hinged arch is preferred before all others. The recently constructed train sheds of the Pennsylvania and the Philadelphia and Reading railroads at Philadelphia and Jersey City are very fine examples of the three-hinged arch construction. (See Fig. 65.)

For shop construction one of the usual requirements is a horizontal lower chord for the purpose of supporting shafting or trolley runways, and in these cases the Fink truss is usually employed; but any other form with a horizontal lower chord will suffice.

If the trusses are supported on iron columns instead of walls, the wind force is transferred to the foundations through the columns. This produces a bending moment in the columns which is a maximum at the top and must be transferred to the truss, a requirement which often modifies the form of the latter if economy is an object.* The stresses in the truss are less if the truss is deep at the ends; and if the Fink truss is used, knee braces from the column to the first joint of the lower chord are necessary. If an ordinary triangular truss is used with some depth at the ends, knee braces are not used.

The slope of the top chord is usually determined by the kind of roof covering used. This slope must be steep enough to allow the covering used to be of the best service; thus, slate should not be used on a roof having a slope less than one to three and preferably one to two. Tin or gravel may be used on a slope of one to twelve. The greater the pitch the greater is the area of the roof covering required.

306. **Riveted or Pin Connected Roof Trusses.**—Roof trusses are usually made with riveted connections, this being the cheaper kind of construction for the usual short spans and small truss members. There are cases, however, when the pin connection may be the cheaper

* For a full discussion of these stresses, see Chap. XXIX, on Iron and Steel Mill Building Construction.

or more advisable construction. When the span is long or very heavy, requiring comparatively large members in the trusses, or when the material must be transported a great distance from the shops, there may be a saving in manufacture or in transportation. The pin connection may also be used to avoid any field riveting. In the case of a roof over a building in which there are gases which have a very marked corrosive effect on iron, it is generally advisable to use pin connections, because the members of this kind of a truss expose a less surface to corrosion than do the thin angle irons which would be used in a riveted truss.

Sometimes, in order to provide for corrosion and at the same time get the benefit of the cheaper construction of the riveted truss, a minimum thickness is specified which must be added to the thicknesses of all material as required by the stresses. Thus, if $\frac{1}{32}$ of an inch was to be added on all sides, making a total addition in thickness of $\frac{1}{16}$ of an inch, an angle iron $3'' \times 2'' \times \frac{1}{4}''$ would be increased $\frac{5}{16}$ of a square inch or 25 per cent in area, while a round rod $1\frac{1}{4}$ inches in diameter would be increased only $\frac{1}{8}$ of a square inch or 10 per cent in area.

307. **Ordinary Roof Coverings.**—For buildings with iron roof trusses the coverings most commonly used are corrugated iron, tin, and slate. Corrugated iron and slate are used when the slope of the roof is greater than three horizontal to one vertical. For flatter roofs than this they are liable to leak, as the usual joints are not tight. Tin may be used on any slope or on a flat roof. Corrugated iron is usually laid directly on the purlins, to which it is attached by means of clips. Owing to its being stiff enough to span a distance of about six feet, sheathing boards are not necessary. Tin and slate are usually laid on sheathing, a layer of roofing felt being put between the tin or slate and the sheathing. A very expensive construction is that in which the slate is supported directly on iron purlins, which must generally be about $10\frac{1}{2}$ inches apart. The expensiveness of this plan is due to the weight of the purlins.

The weight of any kind of roof covering may be obtained from various handbooks. The sheathing is usually assumed to weigh four pounds per foot, board measure. The thickness of the sheathing is determined by the distance apart of the purlins, between which it must be able to carry the weight of the roof covering and the wind or snow load, usually allowing a fibre stress of 1500 lbs. per square inch. The usual thicknesses are $\frac{7}{8}$, $1\frac{1}{4}$, and 2 inches.

308. **The Loads** for which roofs are usually proportioned are the weight of the roof covering, the sheathing if any, the iron weight in trusses and purlins and the snow and wind loads. The weight of the covering which it is proposed to use may be obtained from any good handbook. The weight of the sheathing is dependent upon its thickness, and that may be determined from the distance centre to centre of purlins. The weight of the iron may be closely obtained from the formula $w = \frac{l}{25} + 4.0$, where w = weight per square foot of covered horizontal area and l = length of span. This weight of iron is only for the case of the roof which carries no local concentrated loads. The wind load is usually assumed at 30 lbs. per square foot of roof surface. The snow load is usually taken at 20 lbs. per square foot of horizontal surface.

In the calculation of trusses with curved chords it is the usual practice to find the stresses for all the different loadings separately. Thus, the stresses would be calculated for the wind on the side of the truss nearer the expansion end, and for the wind on the side of the truss nearer the fixed end of the truss. The snow may be figured as covering the entire roof or only one half, and even in special cases only a small area on one side. It is not generally assumed that the maximum wind pressure and the snow load can act on the same half of the truss at the same time. In the Fink truss a partial load like any of the above never causes any maximum stresses, so that it is customary to calculate these trusses for a uniform load over the entire truss, the wind and snow loads combined being usually assumed at 30 lbs. per square foot of covered area. This simplifies the calculations very much, and the results are as nearly correct as we can hope to arrive at by any assumed exact loading.

309. The Allowed Stresses per square inch which have been very generally adopted correspond to what is termed a *factor of safety* of four. The usual allowed stresses per square inch on iron and steel in roof construction are given in the following table:

Wrought-iron.

Tension shape-iron	12,000 lbs.
" rods and eyebars	15,000 "
Maximum fibre stress on I beams	12,000 "
Compression	$\begin{cases} \dfrac{10{,}000}{1+\dfrac{l^2}{36{,}000r^2}} & \text{flat ends,} \\ \dfrac{10{,}000}{1+\dfrac{l^2}{18{,}000r^2}} & \text{pin ends.} \end{cases}$
Shear on rivets and pins	7500 lbs.
Bearing of rivets and pins	15,000 "
Bending fibre stress on pins	18,000 "

Steel.

Tension (shapes)	15,000 "
" rods and eyebars	18,000 "
Maximum fibre stress on I beams	16,000 "
Compression 20 per cent more than that allowed on wrought-iron.	
Shear on rivets and pins	9000 lbs.
Bearing on rivets and pins	18,000 "
Bending fibre stress on pins	22,500 "

In case the roof was subjected to a load from cranes, shafting, etc., the above allowed stresses would be reduced. In the riveted trusses there are secondary stresses arising from the eccentric application of the stresses on the various members which are not provided for further than to reduce them to the minimum possible amount. This is a point in the design of riveted trusses which should always have careful consideration.

310. The Economical Distance Centre to Centre of Roof Trusses is dependent principally upon the relative unit stresses in the purlins and the main trusses. The weight of the purlins *per square foot of covered area* varies almost proportionally to the distance centre to centre of trusses. The trusses for a constant length of span would, if it were possible to realize the small areas required for the lighter trusses, weigh the same per square foot of covered area for all distances centre to centre of trusses, as the weight of the trusses would in this case vary as the load on them, which is a fixed number of pounds per square foot of covered area. The total weight per square foot of covered area for the purlins and trusses would, if the above were true, be a minimum where the distance centre to centre of trusses is a minimum or zero. The trusses for a given span, however, do not vary directly as the load, but they are found to vary in weight approximately as follows:

$$w = a + \frac{b}{x},$$

where w = weight of trusses per square foot of horizontal surface covered, a and b are constants, and x = distance centre to centre of trusses. That is, a part of the weight of

trusses per square foot of covered area remains constant for any distance centre to centre of trusses, and the rest varies inversely as the distance centre to centre of trusses or *directly as the number of trusses.* The constants a and b may be found for any length of span by assuming various widths centre to centre of trusses and plotting the results, using w and x as the variables. Or it may be generally assumed that the width centre to centre of trusses for maximum *economy in material* is about one fifth of the span. For economy in manufacture a greater distance between trusses is necessary, as the cost of manufacture varies almost directly as the *number* of trusses.

311. The Detail Design of the Purlins and Trusses for a Roof.—Assume the trusses to be 60 feet centre to centre of end bearings and spaced 20 feet centre to centre. The allowed stresses to be as given in Art. 309. The covering to be slate on two-inch sheathing. The truss will be the ordinary Fink truss shown in Fig. 353, with a depth at the centre of 15 feet.

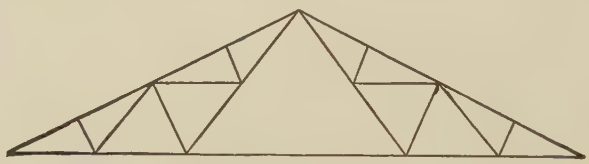


FIG. 353.

312. Calculation of the Purlins.—The purlins or stringers rest on the trusses at the joints of the upper chord and serve to support the loads on the roof between the trusses. As the trusses in our case are 60 feet centre to centre of end bearings, and as there is a purlin at each joint of the upper chord of the truss, the distance between purlins is $\frac{1}{4}(\sqrt{\overline{15}^2 + \overline{30}^2}) = 8.4$ feet, nearly. The slate covering, felt, and sheathing will weigh 13 lbs. per square foot. The purlin itself we will assume to weigh 2 lbs. per square foot, and the wind or snow load is taken at 30 lbs. per square foot, making a total load of $(13 + 2 + 30)8.4 = 378$ lbs. per linear foot of purlin. The maximum moment on the purlin, assuming an I beam, will be

$$378 \times \frac{\overline{20}^2}{8} = 18{,}900 \text{ foot-lbs.,} \qquad \text{or} \qquad 226{,}800 \text{ inch-lbs.}$$

From the formula $M = \frac{fI}{y_1}$ we get $\frac{M}{f} = \frac{I}{y_1}$. From any of the rolling-mill handbooks we can get the value $\frac{I}{y_1}$ directly from the tables giving the properties of I-beam sections. This term $\frac{I}{y_1}$ is called the Section Modulus of the rolled form. Dividing the moment 226,800 by 16,000, the allowed stress in extreme fibres for *steel* I beams, we get 14.2 for the section modulus of the required steel I beam. Using the sections and handbook of the Carnegie Steel Co., we find the lightest beam which has the required section modulus to be an eight-inch beam weighing 18 lbs. per foot.

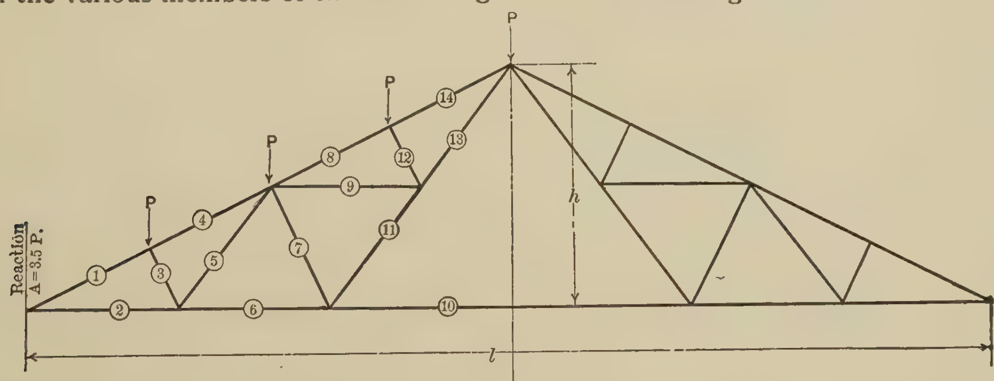
When the exact section modulus is not found in the handbook take the one next higher. Only the particular weights given in the handbooks of 1896 and subsequently can be purchased, as these have been made to conform with the American standards. If the section reduces to that of a six-inch I beam, then six-inch Z bars may be used as purlins. Since the static load on the purlins is vertical, the I-beam form is not as well adapted for use as purlins as is the Z-bar form. This latter, however, is not rolled in larger sizes than six-inch webs. A Z bar placed with its bottom flange pointing down the truss has the line

of its maximum moment of inertia nearly vertical, and this maximum moment of inertia is greater than that given in the handbook. The reverse is, of course, true of the I beam when so placed.

313. **The Detail Design of the Sixty-foot Roof Truss** can now be made. The 'ead weight of the covering, sheathing, purlins, and trusses will be assumed as follows:

Slate and felt	5	lbs. per square foot;
Sheathing (2-inch)	8	" " " "
Iron $\left(= \frac{\text{span}}{25} + 4\right)$	6.4	" " " "

making a total dead weight of 19.4 lbs. per square foot. For convenience it will be assumed 20 lbs. per square foot of covered area. The snow and wind loads will be assumed as usual to be 30 lbs. per square foot of horizontal area covered. The load at each truss joint of the top chord (assuming the iron weight of the truss to be concentrated there also) will be, combining all the loads, $50 \times 7\frac{1}{2} \times 20 = 7500$ lbs., where 50 equals the total load per square foot of covered area, $7\frac{1}{2}$ the horizontal distance in feet between purlins, and 20 the distance in feet centre to centre of trusses. The stresses in the truss may then be found by any of the methods given in Part I. Owing to the similarity of trusses of this kind it is more convenient to work out the stresses in each member for a panel load of one pound and tabulate the results as shown in the table of coefficients on the following page, from which the stresses may be derived quickly by the use of the slide-rule. The total stresses, allowed stresses per square inch, the length centre to centre of support of compression members, the least radius of gyration of compression members, the required area in square inches, and the make-up and area of the various members of the truss are given in the following table.



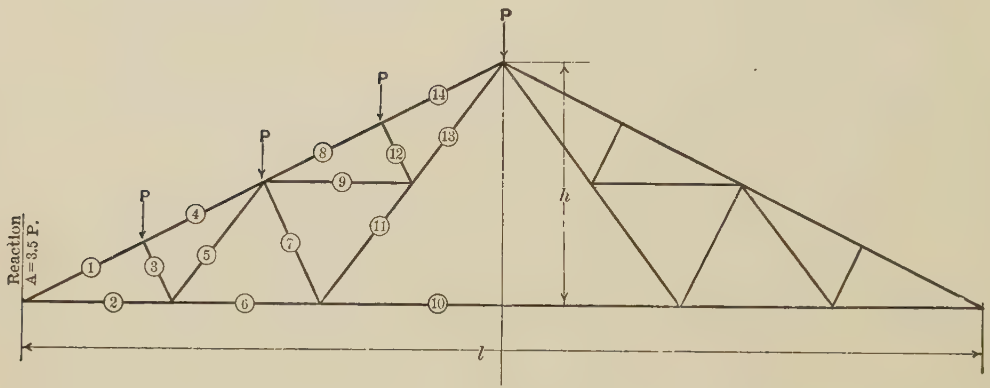
Member.	Total Stress.	Length.	Radius of Gyration.	Unit Stress Allowed.	Area Required.	Make-up of Members.	Area Used.
1	+ 58,700	101″	1.25	8,500	6.91	Two 5″ × 3″ ∠s, 34.5 lbs. per yard	6.9
4	+ 55,300	101″	"	8,500	6.51	" 5″ × 3″ ∠s, 34.5 " "	6.9
8	+ 52,000	101″	"	8,500	6.12	" 5″ × 3″ ∠s, 34.5 " "	6.9
14	+ 48,600	101″	"	8,500	5.72	" 5″ × 3″ ∠s, 34.5 " "	6.9
3—12	+ 6,700	50 5″	.78	9,000	.75	" 2″ × 2½″ × ¼″ ∠s	2.16
5—9	− 7,500	....	....	12,000	.63	" 2″ × 2½″ × ¼″ ∠s	1.7 net
7	+ 13,400	101″	.78	6,800	1.97	" 2″ × 2½″ × ¼″ ∠s	2.16
11	− 15,000	....	....	12,000	1.25	" 2″ × 3″ × ¼″ ∠s	1.9 net
13	− 22,500	....	....	12,000	1.88	" 2″ × 3″ × ¼″ ∠s	1.9 net
2	− 52,500	....	....	12,000	4.38	" 4″ × 3″ ∠s, 26 lbs. per yard	4.5 net
6	− 45,000	...	....	12,000	3.75	" 4″ × 3″ ∠s, 26 " "	4.5 net
10	− 30,000	....	....	12,000	2.50	" 3″ × 3″ ∠s, 16 " "	2.6 net

+ denotes compression; − denotes tension.

TABLE OF COEFFICIENTS FOR CALCULATING STRESSES IN FINK ROOF TRUSSES.

P = load at each top chord joint ;
$P \times$ coefficient = stress in member ;

$$n = \frac{l}{h}.$$



No.	$n = 3.$	$n = 4.$	$n = 5.$	General Formulæ.	
1	6.310	7.826	9.4247	$+\frac{7}{4}\sqrt{n^2+4}$	$\times P$
2	5.25	7.0	8.75	$-\frac{7}{4}n$	$\times P$
3	0.832	0.8945	0.9285	$+\frac{n}{\sqrt{n^2+4}}$	$\times P$
4	5.755	7.379	9.053	$+\frac{1}{\sqrt{n^2+4}}\left(\frac{7}{4}n^2+5\right)$	$\times P$
5	0.75	1.0	1.25	$-\frac{n}{4}$	$\times P$
6	4.5	6.0	7.5	$-\frac{3}{2}n$	$\times P$
7	1.664	1.789	1.857	$+\frac{2n}{\sqrt{n^2+4}}$	$\times P$
8	5.200	6.932	8.681	$+\frac{1}{\sqrt{n^2+4}}\left(\frac{7}{4}n^2+3\right)$	$\times P$
9	0.75	1.0	1.25	$-\frac{n}{4}$	$\times P$
10	3.0	4.0	5.0	$-n$	$\times P$
11	1.5	2.0	2.5	$-\frac{1}{2}n$	$\times P$
12	0.832	0.8945	0.9285	$+\frac{n}{\sqrt{n^2+4}}$	$\times P$
13	2.25	3.0	3.75	$-\frac{3}{4}n$	$\times P$
14	4.646	6.485	8.310	$+\frac{1}{\sqrt{n^2+4}}\left(\frac{7}{4}n^2+1\right)$	$\times P$

NOTE.—For coefficients for a similar truss except that two struts are used in place of one at 3 and 12, in the above figure, see *Engineering News*, Oct. 31, 1895.

It will be noticed that the top chord is made the same area throughout, as it is better construction and costs very little more than if the chord were spliced. The members 11 and 13 and 2 and 6 are made of the same area for the same reasons.

The detail sketch of this truss is shown in Fig. 354, page 320. The plates at the joints are all three eighths of an inch thick except those at the ends, which are one half of an inch thick. The rivets are all three quarters of an inch in diameter. The required number of rivets through a member connecting it at its ends with a joint plate is equal to the stress in the member divided by the value of one rivet in bearing on the joint plate. The number of rivets connecting the bottom chord to the half-inch joint plate at the ends is increased beyond what the stress in the chord would require owing to the fact that the end reaction on the masonry produces a vertical stress on the rivets in addition to the horizontal stress from the chord. The required number of rivets for this case is found as follows, assuming the end reaction to be resisted by the rivets directly over the bearing plate: The end reaction is 26,250 lbs., and as at the ordinary three-inch pitch there are six rivets resisting this end reaction, we have a vertical stress on each rivet of $26,250 \div 6 = 4375$ lbs. As the value of one three-quarter-inch rivet in bearing on the half-inch joint plate is $\frac{1}{2} \times \frac{3}{4} \times 15,000 = 5625$ lbs., each rivet will resist $\sqrt{5625^2 - 4375^2} = 3535$ lbs. horizontal stress before the limiting bearing pressure is exceeded. The six rivets are then able to resist 21,210 lbs. of the chord stress, leaving 31,040 lbs. to be resisted by the remaining rivets through the chord angles. This requires six more rivets, or twelve in all. The splice at the end of the lower chord piece 10 requires consideration. There must, in addition to sufficient rivets, be enough area in the splice plates to equal the area required in this chord. Of the joint plate, only a width of plate which is symmetrical about the rivet lines in the chord angles is available in the splice, and also the centre of gravity of the splicing plates should coincide with the centre of gravity of the chord angles. This latter condition, counting $3\frac{1}{2}$ inches as the available width of the joint plate, requires $6\frac{1}{2}$ inches as the width of the bottom splice plate, assuming all plates to be three eighths of an inch thick. The splicing plates have therefore an area equal to $(3\frac{1}{2} + 6\frac{1}{2} - 3 \times \frac{7}{8})\frac{3}{8} = 2.77$ square inches net area, which is sufficient. If this area had been less than the net area required in the chord 10, the joint plate would have to be made thicker.

It will be noted that the neutral axes of all the members meeting at a joint intersect at a point. It is not more expensive to do this in practice than it is to neglect it, although it is not generally done. The secondary stresses in the members due to the fact that the connecting rivets at the ends of the members are not on the neutral axis, or symmetrical about it, still remain. These stresses should be looked into in order to see whether there are any probably dangerous stresses. It is not practicable to make a rigid analysis of these stresses, owing to the rigid joints and details, in the manufacture of which we cannot estimate the effect.

314. **The Bracing of Roofs** is usually put in the plane of the top chord, the purlins being used as struts for the lateral system. When a line of shafting is supported near the lower chord a system of lateral bracing should be introduced there also, to resist any vibration from the shafting. For a building with a number of trusses, bracing is not needed between each pair of trusses, but may be put in at distances of about 100 feet between the bays which have complete systems, the purlins being relied upon to carry any side stress which calls for bracing between the necessary expansion joints in the purlins. If the end of the building is covered with glass, as the end of a train-shed for instance, it must be stiffened against bending and the consequent destruction of the glass. This is usually accomplished by using a horizontal truss in the plane of the lower edge of the glass end and another complete system in the plane of the top chords of the truss. The glass between these two lateral systems would then be stiffened by vertical girders attached at their ends to these lateral systems.

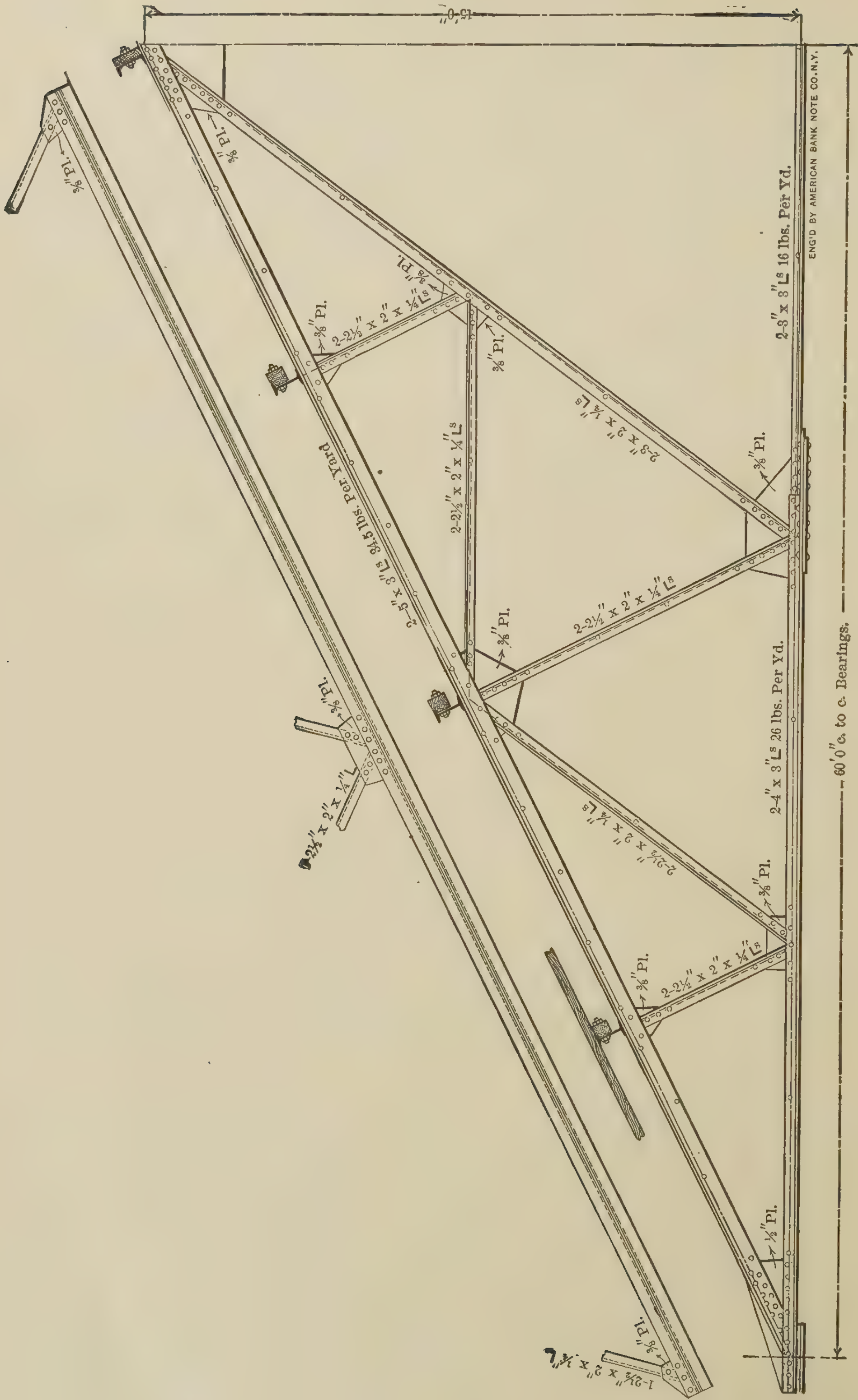

15'0"
2-3" x 3" Ls 16 lbs. Per Yd.
ENG'D BY AMERICAN BANK NOTE CO. N.Y.
2-4" x 3" Ls 26 lbs. Per Yd.
2-5" x 3" Ls 34.5 lbs. Per Yard
2-2½" x 2" x ¼" Ls
2-3" x 2" x ¼" Ls
2-2½" x 2" x ¼" L
⅜" Pl.
½" Pl.
1-2½" x 2" x ¼" L
60'0" c. to c. Bearings

FIG. 354.

CHAPTER XXI.

THE COMPLETE DESIGN OF A SINGLE TRACK RAILWAY BRIDGE.

315. **Data.**—The span length will be 150 feet centre to centre of end pins. Bridge square ended, not skew. Straight track. The capacity or live load to be Cooper's Class "Extra Heavy A" loading (Fig. 129). The specifications for allowed stresses, details of construction, and material will be the standard specifications of the Pennsylvania Railroad dated 1887, modified so as to allow the use of steel eyebars and pins at allowed stresses 20 per cent above those specified for wrought-iron. The panels may be of any length if the depth is always made equal to or greater than the panel length, and the stringers may be spaced any distance centre to centre if the size of cross-tie is made to correspond.

316. **Dimensions of Truss.**—The proper depth for economy in material of a single track through span varies from one fifth to one sixth of the span. The depth is dependent to a small extent on the length of panel, the economical depth being greater if the panel is longer and less if the panel length is made less. The *weight* of the bridge varies very little for a change in the number of panels, but the cost of manufacture changes rapidly, being less per pound for the longer panels. There are two good reasons for selecting long panels, one being a reduction in the cost of the bridge, and the other the concentration of a greater mass in the floor system to resist impact and vibration from the train. The objection which can be urged against long panels is that, owing to the fact that the trusses for spans under 300 feet are not usually spaced much over 16 feet centre to centre, the angle made by diagonals of the lateral systems with the centre line of truss is so small as to make this bracing less effective than if the panels were shorter. This objection deserves consideration, although it is usually not given much weight. The greater number of joints required for the shorter panel, and the necessary play, or required looseness of fit, at the joints, tend to counterbalance this advantage. A panel length of 25 feet and a depth of 28 feet will be assumed, as these dimensions are probably very near the correct ones for economy in material and cost. Five panels of 30 feet and a depth of 30 feet may possibly be cheaper, but the panel length is too long for good lateral bracing.

317. **Dead Load.**—For the weight of floor the specifications require that this shall be arrived at by adding to the weight of the cross-ties used 165 lbs. per linear foot of track, which includes the weight of the rails, guard rails, spikes, bolts, etc. The cross-tie will be taken as 8 inches by 10 inches, laid flat, and 12 feet long, spacing the stringers 7 feet centre to centre. The clear opening allowed between cross-ties is 6 inches, and the cross-ties are therefore 16 inches centre to centre. The weight of the cross-ties per linear foot of track will be, assuming the wood to weigh $4\frac{1}{2}$ lbs. per foot board measure as specified, $8 \times 10 \times 12 \times \frac{1}{12} \times \frac{3}{4} \times 4\frac{1}{2} = 270$ lbs. The total weight of the floor will therefore be $270 + 165 = 435$ lbs. per linear foot. The weight per linear foot of the iron and steel in the bridge will be obtained from the formula $w = 5l + 350$,* when w = weight per linear foot and l = length of span. The total dead load on the bridge will therefore be $435 + 1100 = 1535$ lbs. per linear foot. This gives a panel load *per truss* of 19,188 lbs., or for convenience 19,200 lbs. We will consider one third of this concentrated at the top chord joint and two thirds at the bottom chord joint. This is the usual division of the panel load, but in long spans it is always best to get the correct division of load from a trial span worked out on the basis of the above division.

* For a double track bridge add 90% of this, or for double track $w = 9.5l + 665$.

318. Allowed Stresses per Square Inch.—We give below an abstract of that part of the specifications relating to the allowed stresses.*

The maximum and minimum stresses in compression and tension for the specified loads are to be used in determining the permissible working stress in each piece of the structure, according to the following formulæ:

For pieces subject to one kind of stress only (all compression or all tension),

$$a = u(1 + r). \qquad \text{[Same as eq. (5), p. 244.]}$$

For pieces subject to stresses acting in opposite directions,

$$a = u(1 - r_1). \qquad \text{[Same as eq. (6), p. 244.]}$$

In the above formulæ,

$a =$ permissible stress per square inch, either tension or compression;

$u = \begin{cases} 7500 \text{ lbs. per square inch for double-rolled iron in tension (eyebars or rods),} \\ 7000 \text{ " " " " rolled iron in tension (plates or shapes),} \\ 6500 \text{ " " " " rolled iron in compression;} \end{cases}$

$$r = \frac{\text{minimum stress in piece}}{\text{maximum stress in piece}};$$

$$r_1 = \frac{\text{maximum stress of lesser kind}}{2 \times \text{maximum stress of greater kind}}.$$

The permissible stress a for members in compression is to be reduced in proportion to the ratio of the length to the least radius of gyration of the section, by the following formula:

$$b = \frac{a}{1 + \dfrac{l^2}{18,000g^2}};$$

where $a =$ permissible stress previously found;
$b =$ allowable working stress per square inch;
$l =$ length of piece in inches, centre to centre of connections;
$g =$ least radius of gyration of section in inches.

The permissible stress in *long vertical suspenders* in through bridges to be 10 per cent less than that given by above formulæ.

The compression flanges of plate girders must not have a greater stress than that given bv the following formula:

$$c = \frac{a}{1 + \dfrac{l^2}{5000w^2}};$$

where $a =$ permissible stress previously found;
$c =$ allowable working stress per square inch;
$l =$ unsupported length of flange in inches;
$w =$ width of flange in inches.

Iron pins are to be so proportioned that the maximum fibre stress shall not exceed 15,000 lbs. per square inch.

The bearing stress on pins, on a diametrical section of pin-holes, shall not be greater than $1\frac{1}{2}$ times the compression unit stress a in the piece considered.

The shear on the net section of any member shall not exceed the compression unit stress a for that member, and in case of rivets at least 20 per cent extra section must be allowed.

Rivets must not have a bearing pressure per square inch against the web plates of more than twice the compressive unit stress a used in the upper flange of the girder.

The tension on laterals shall not exceed 15,000 lbs. per square inch for double rolled iron or 12,000 lbs. per square inch for plates or shapes.

* These specifications are more complicated in the matter of allowed stresses than some of those now in common use. Seventh Edition, 1898.

The compressive stress on lateral struts shall not exceed that given by the following formula:

$$d = \frac{12{,}000}{1 + \frac{l^2}{eg^2}};$$

where d = permissible stress per square inch;
l = length of piece centre to centre of connections in inches;
g = least radius of gyration of the piece in inches;

$$e = \begin{cases} 36{,}000 \text{ for both ends fixed;} \\ 24{,}000 \text{ for one end hinged, the other fixed;} \\ 18{,}000 \text{ for both ends hinged.} \end{cases}$$

[The student will bear in mind that for steel eyebars and pins the above allowed stresses will be increased 20 per cent.]

Minimum Sections.—No iron shall be used of less thickness than three eighths of an inch, except in lateral struts, lattice straps, pin-plates, and similar details, and in special cases of girder flanges, where a minimum thickness of five sixteenths will be allowed.

No iron used in compression members shall have an unsupported width of more than forty times its thickness.

No rod shall be used having a less sectional area than one square inch.

319. Tabulation of the Stresses and the Determination of the Sectional Areas of the Members of the Truss.—It is assumed that the student can calculate the stresses in the truss members from the dead and live loads by the methods explained in Part I. In the following table are given the stresses on the various pieces, with the factors which determine the

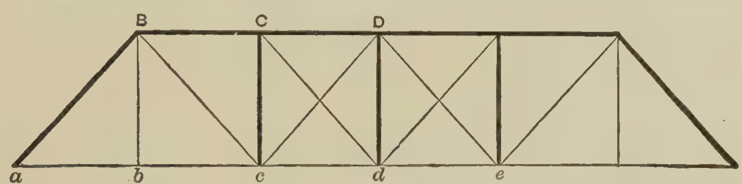


FIG. 355.

Member.	Stresses. Dead.	Live.	Minimum.	Maximum.	Fibre Stress. a	Length of Member in Inches. l	Radius of Gyration. g	Unit Stress.	Area required.	Make-up of Section.	Area Used.	Material.
ab	− 42.9	−111.0	42.9	153.9	11.5			11.5	13.4	Two 6″ × 1⅛″ bars	13.5	Steel
bc	− 42.9	−111.0	42.9	153.9	11.5			11.5	13.4	Two 6″ × 1⅛″ bars	13.5	"
cd	− 68.6	−169.5	68.6	238.1	11.6			11.6	20.5	Two 6″ × 1¾″ bars	21.0	"
Bc	− 38.6	+ 8.4 −110.8	30.2	149.4	10.8			10.8	13.8	Two 5″ × 1⅜″ bars	13.75	"
Cd	− 12.9	− 64.9	0.0	77.8	9.0			9.0	8.7	Two 5″ × ⅞″ bars	8.75	"
De		− 17.5	0.0	17.5	7.5			7.5	2.3	One 1½″ square rod	2.25	Iron
Bb	− 12.8	− 59.4	12.8	72.2	10.6			9.5	7.6	Two 4″ × $\frac{15}{16}$″ bars	7.5	Steel
Cc	+ 16.0	+ 48.4	16.0	64.4	8.1	336	3.8	5.64	11.4	Two 10″ ⊔s, 58 lbs. per yd.	11.6	Iron
Dd	+ 6.4	+ 13.1	6.4	19.5	8.6	336	2.54	4.360	4.5	Two 7″ ⊔s, 38 lbs. per yd.	7.6*	"
aB	+ 64.4	+166.6	64.4	231.0	8.3	450.5	6.34	6.460	35.5	One 22″ × $\frac{7}{16}$″ top plate Two 3″ × 3″ ∠s, 30 lbs. per yd. Two 16″ × $\frac{7}{16}$″ side plates Two 3″ × 4″ ∠s, 30 lbs. per yd.	35.6	Iron
BC	+ 68.6	+169.5	68.6	238.1	8.4	300	6.44	7.470	31.9	One 22″ × $\frac{7}{16}$″ top plate Two 3″ × 3″ × ⅜″ ∠s Two 16″ × ⅜″ side plates Two 3″ × 4″ ∠s, 30 lbs. per yd.	31.9	Iron
CD	+ 77.1	+192.9	77.1	270.0	8.4	300	6.44	7.470	36.2	One 22″ × $\frac{7}{16}$″ top plate Two 3″ × 3″ ∠s, 30 lbs. per yd. Two 16″ × $\frac{7}{16}$″ side plates Two 3″ × 4″ ∠s, 33 lbs. per yd.	36.2	Iron

Areas are given in square inches. Stresses are given in thousand-pound units.
+ denotes compression. — denotes tension.

* The area given to this post by the formulæ is altogether too small for lateral stability.

allowed stresses, the required areas, the make-up of the section, and the area used, from which the student can readily understand the methods employed. The members are designated by the letters of the joints at their ends as indicated in Fig. 355.

The order in which the members are arranged in this table is the most convenient one to use when making up the sections. The sizes of the tension members are fixed first, then the sizes of the posts, and lastly the inclined end posts and top chords, which are usually made with the same general dimensions. The top chords must be made wide enough to allow the posts and diagonals to be packed inside of them, and they must also be made deep so that the neutral axis or pin centre may be far enough from the top plate to allow room enough for the eyebar head.

Owing to the requirement of established practice that posts must never have a ratio of length to least width of over 48, the posts *Dd* are made heavier than the stresses would require. A lighter 7-inch channel could have been used except for the clause in the specifications allowing no metal less than three eighths of an inch thick.

The external dimensions of the top chord section were determined as follows: The minimum width of top plate is fixed by the members which have to be packed inside at joint *C*, where the largest post is. The side plate is determined by the piece *BC*, where the minimum section is required, as this area must be made up without using material of less thickness than the specifications allow. By referring to Fig. 356, it will be seen that 22 inches is the least allowable width for the top plate, requiring a plate at least seven sixteenths of an inch thick in order that the ratio of the unsupported width to the thickness of the plate may not exceed 40. The unsupported width is the distance between the rivets which attach the plate to the top angles, and is usually 4 inches less than the width of the plate. The depth of the chord or the width of the side plates must be determined by trial. The least radius of gyration of a top chord section does not vary much from four tenths of the width of the side plate when the area of the side plates is one half, or less than one half, of the total area of the section. Using this as a basis for an approximate determination of the area in *BC* for various widths of side plate, we select the section with a width of side plate which will give us the least metal in the top chords and end posts. For short spans this is generally the one which will give the required area in *BC* when the minimum allowed sections are used in its make-up. The angles used in these sections are as small as should be used for convenience in driving rivets. Care should be taken to select a chord section deep enough to allow the pin to be placed at such a distance from the top plate that the eyebar head will not strike the top plate. The pin is usually placed below the neutral axis of the section in order that the total direct stress acting with a lever arm equal to the distance of the centre of the pin from the neutral axis may produce a moment equal to the centre bending moment in the piece from its weight. As the pins are at the same distance from the top plate in all the sections of top chord and end post, this requirement must be observed in the design of the sections in order to avoid any unnecessary bending stresses. The pin should never be placed at such a distance from the neutral axis that the algebraic sum of the bending moments produced by the maximum direct stress and the weight of the member would cause an extreme fibre stress per square inch greater than 10 per cent of the stress per square inch allowed by the formula for direct stress alone.

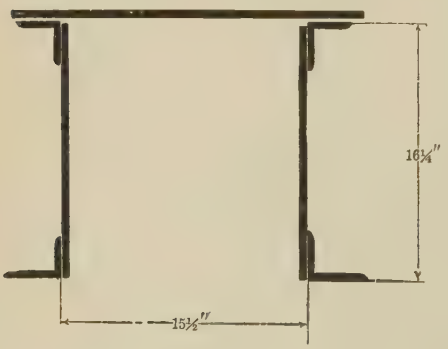


FIG. 356.

The top chords and end posts for this span are made of the general dimensions shown in Fig. 356. It will be noted that the vertical distance back to back of angles is one quarter

of an inch more than the width of the side plate. This is required in manufacture. The pin centre will be placed 1⅝ inches above the centre of the web plate. In the following table are given the distances of the neutral axes from the centre of the web plates, the weights of the pieces, the maximum bending moments produced by the weights of the pieces, considering each piece as a beam of a span equal to the length of the piece, the maximum direct stresses in each piece from the dead and live loads, and the distance the pin must be placed from the centre of the plate in order that the direct stress may produce a reverse bending moment which will counteract the moment of the weight.*

Piece.	e	W	M_w	S	$\frac{M_w}{S}$	$e_1 = e - \frac{M_w}{S}$
aB	2.23	5,100	191,200	231,000	0.83	1.40
BC	2.11	3,000	112,500	238,100	.48	1.63
CD	2.08	3,400	127,500	270,000	.47	1.51

In this table e is the eccentricity or distance from the centre of the web plate to the neutral axis; W is the weight of the member, allowing fifteen per cent of the area as the weight of the rivet-heads, lattice bars, etc.; M_w is the centre bending moment produced by the weight W; S is the direct stress in the member assumed to be applied at the centre of the end pins of the piece; and e_1 is the distance the pin centre should be placed from the centre of the web plate in order that the bending moment produced by the weight of the piece may be counteracted by the moment of the direct stress acting about the neutral axis of the piece.

As any future increase of the live load would cause greater direct stress in the piece, and consequently e_1 would be larger, it is better to use one of the larger values of e_1.

The method of computing the eccentricity e and the radius of gyration is given in the following tables:

ECCENTRICITY FOR *aB*.

Piece.	Area, A	Distance, d	d_1	$A \times d$	$e = 8.34 - d_1$
Top plate......	9.625	0.0			
Top angles....	6.0	1.16		6.96	
Web plates....	14.0	8.34		116.76	
Bottom angles.	6.0	15.65		93.90	
Total section..	35.625		6.11	217.62	2.23

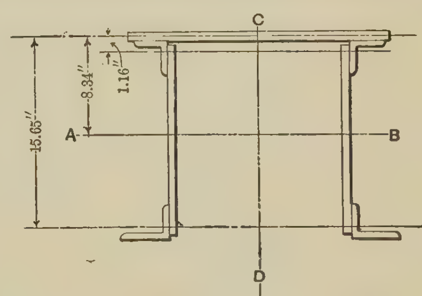


FIG. 357.

$d =$ distance of the centre of gravity of the piece from the centre of the top plate; $d_1 =$ distance of the centre of gravity of the total section from the centre of the top plate; $e =$ the eccentricity, or the distance of the centre of gravity of the total section from the centre of the web plate; $8.34 =$ the distance from the centre of the web plate to the centre of the top plate. All distances in inches, and all areas in square inches.

THE RADIUS OF GYRATION FOR *aB*.†

Piece.	Area, A	c	c^2	g^2	$A \times c^2$	I	$A \times c^2 + I$	g
Top plate.......	9.625	6.11	37.33		359.30	0.15	359.45	
Top angles.....	6.00	4.95	24.50		147.00	4.72	151.72	
Web plates.....	14.00	2.23	4.97		69.58	298.67	368.25	
Bottom angles..	6.00	9.54	91.01		546.06	4.44	550.50	
Total section...	35.625	0		40.14			1429.92	6.34

* It is here assumed that the total chord stress passes through the pin. Since all the pins are eccentrically placed, the total horizontal stress in any panel is applied in the line of these pins, notwithstanding the continuity of the chords at the splices.

† See Art. 126 for another form of tabular computation of I and r.

A is the area of each piece in square inches; c is the distance of the centre of gravity from the neutral axis of the full section; I is the moment of inertia of each piece with reference to the neutral axis of the piece parallel to AB (see Fig. 357); and g is the radius of gyration of the full section of aB. All areas are in square inches, and all distances or dimensions in inches.

The position of the neutral axes and the moments of inertia for the angles are obtained from any of the handbooks issued by the rolling mills.

The above computations are made with reference to an axis parallel to AB (Fig. 357). The radius of gyration with reference to the axis CD should always be equal to or greater than that found above. This is true whenever the width between the outer sides of the side plates is nine tenths of the breadth of the side plates.

320. The Design of the Iron Floor System.—Having fixed the size of the truss members, we will now take up the floor-beams and stringers. The stringers are plate girders of a span length equal to the panel length. The dead load on a pair of stringers consists of the weight of the stringer plus the weight of the floor. The floor we have found weighs 435 lbs. per foot of track. The iron weight may be approximated by the formula $9 \times l + 55$, where $l =$ length of panel. This is the formula previously used for deck plate-girder spans with the constant 110 reduced one half to allow for the weight of the bracing, which is not generally used for stringers under 30 feet long. The total dead load per linear foot on a pair of stringers is, therefore, $435 + 280 (= 9 \times 25 + 55) = 715$ lbs. The maximum bending moments for *one* stringer are $\dfrac{715 \times \overline{25}^2}{2 \times 8} = 27{,}900$ ft.-lbs. from the dead load and 234,700 ft.-lbs. for the live load. The live load moment is taken from the table on page 329. The total or maximum bending moment is 262,600 ft.-lbs. The economical depth for the stringers will be found by the formula given on page 300 for a girder span whose flanges are of constant area end to end. In this case $h = 1.46\sqrt{\dfrac{m}{ft}}$, where $h =$ the depth of girder, $m =$ the bending moment in inch-pounds, $f =$ the average flange stress per square inch on the *gross* area of flange, and $t =$ the thickness of web plate in inches. In the case of the stringers under consideration $t =$ three eighths, $f = 6600$, and $m = 3{,}151{,}200$; from which $h = 50$. The specifications require a cover plate on the top flange from end to end of the stringer, and as we will assume our stringers to be 48 inches back to back of flange angles, the distance centre to centre of gravity of the flanges will be taken as $46\frac{1}{2}$ inches. The maximum flange stress will be $3{,}151{,}200 \div 46\frac{1}{2} = 67{,}800$ lbs. The allowed stress per square inch on the bottom is $7000\left(1 + \dfrac{27{,}900}{262{,}600}\right) = 7750$ lbs. The required net area of bottom flange is $\dfrac{67{,}800}{7750} = 8.77$ square inches. Two $6'' \times 4''$ angle irons, weighing 49 lbs. per yard each, will be used for this flange. The allowed stress per square inch for the top flange is $6500\left(1 + \dfrac{27{,}900}{262{,}600}\right) = 7200$ lbs., reduced by the formula $\dfrac{7200}{1 + \dfrac{l^2}{5000b^2}}$, where $l = 25$ and $b = 1$. From this we get 6400 as the allowed stress per square inch in the top flange. The required area of top flange is $67{,}800 \div 6400 = 10.6$. For this flange use two $5'' \times 3\frac{1}{2}''$ angle irons, weighing 31 lbs. per yard, and one $12 \times \frac{3}{8}$ inch plate, making a gross area of 10.7 square inches. For stiffeners we will use $3'' \times 2'' \times \frac{3}{8}''$ angles except at the ends. The end stiffeners have to be large enough to allow $\frac{7}{8}''$ rivets to be driven through each leg of the angle. This limits them to $3'' \times 3''$ angles as a minimum, and in order to make the driving of the field rivets less confined we will use $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angles for the end stiffeners. The end shear is 4500 lbs. from the dead load and 43,700 lbs. from the live load, making a total of 48,200 lbs. This requires 11 rivets through the end stiffeners and the web of the stringer. The value of a $\frac{7}{8}''$ rivet in bearing on a $\frac{3}{8}$-inch web plate is, as the allowed

bearing pressure per square inch is twice the allowed stress per square inch in the top flange before reduction for the length of flange, $2 \times 7200 \times \frac{7}{8} \times \frac{3}{8} = 4700$ lbs. The number of rivets through these end stiffeners and the web of the floor-beam is determined as follows: the maximum end shear of the stringer, in this case 48,200 lbs., must be transferred to the floor-beam by the rivets in single shear, or the maximum concentration on the floor-beam from the two stringers which attach on opposite sides of the beam must be transferred to the floor-beam by the rivets without exceeding the limiting *bearing* pressure allowed between the rivets and the web of the beam. The allowed bearing pressure per square inch for the rivets on the web of the beam is, as the dead load concentration is 9000 lbs. and the maximum live load concentration is, from the table page 329, 59,500 lbs.,* $2 \times 6500\left(1 + \frac{9000}{68.500}\right) = 14{,}700$ lbs., and the bearing value of one $\frac{7}{8}$-inch rivet on a $\frac{3}{8}$-inch plate is $\frac{3}{8} \times \frac{7}{8} \times 14{,}700 = 4800$ lbs. The maximum end shear of one stringer requires, as the value of a $\frac{7}{8}$ rivet in single shear is 3660 lbs., $48{,}200 \div 3660 = 13$ rivets. The maximum concentration on the floor-beam requires, as the value of one $\frac{7}{8}$ rivet on the $\frac{3}{8}$-inch web of the beam is 4800 lbs., $68{,}500 \div 4800 = 15$ rivets. The rivets through the flange angles and web plate would be spaced by the methods explained in Chapter XIX.

321. Floor-beams.—The length, centre to centre, of end supports of a floor-beam should be taken as the distance centre to centre of trusses, and provision should be made for connecting the beam so that its end reaction will be transferred to the truss as nearly central as practicable.

The weight of the beam can be arrived at by trial. The depth of the beam must usually be great enough to allow the full depth of the stringer and about one half inch of clearance between the vertical legs of the beam flange angles, or in this case $48'' + \frac{1}{2}'' + 7'' = 55\frac{1}{2}''$. If the stringers are made the economical depth, this usually requires the beams to be deeper than would be economical.

The above applies to stringers framed in between the floor-beams. When the stringers are supported on top of the beams both may be made the economical depth. The depths required by the formulæ for economy, like the results of all formulæ for maximum or minimum, may be deviated from considerably without much loss of weight.

Assuming the weight of the beam as 2400 lbs., we get, remembering that the stringers are 7 feet and the trusses 16 feet centre to centre, $\frac{2400 \times 16}{8} = 4800$ ft.-lbs. centre moment from the weight of the beam, $9000 \times 4\frac{1}{2} = 40{,}500$ ft.-lbs. centre moment from the concentrated dead load of the stringers on the beam, and $59{,}500^* \times 4\frac{1}{2} = 267{,}750$ ft.-lbs. centre moment from the live load concentrated at the stringer points. It will be noted that the moments from the stringer concentrations are constant between stringer points. The total centre moment is therefore 45,300 ft.-lbs. from the dead load and 267,750 ft.-lbs. from live load, or, combining them, 313,050 ft.-lbs. As the beam must be $55\frac{1}{2}''$ deep and the specifications require a top flange plate, we will assume 54 inches or 4.5 feet as the effective depth. The flange stress is therefore $313{,}050 \div 4.5 = 69{,}600$ lbs. The allowed stress per square inch on the bottom flange is $7000\left(1 + \frac{45{,}300}{313{,}050}\right) = 8000$ lbs., requiring a net area of bottom flange of $69{,}600 \div 8000 = 8.7$ square inches. Use two $6'' \times 4''$ angles weighing 49 lbs. per yard. The allowed stress per square inch on the top flange area is $6500\left(1 + \frac{45{,}300}{313{,}050}\right) = 7440$ lbs., reduced by the formula $\frac{7440}{1 + \frac{l^2}{5000 b^2}} = 7366$ lbs., as $l = 7$ feet and $b = 1$ foot. The required area of top flange is $69{,}600 \div 7366 = 9.45$ square inches. Use two $5'' \times 3'' \times \frac{3}{8}''$ angles and one $12 \times \frac{5}{16}$ plate.

* See Art. 92, p. 76.

The allowed shearing stress per square inch on rivets in the floor-beam is, according to the specifications, $7440 \div 1.2 = 6200$ lbs. The value of one $\frac{7}{8}$ rivet in single shear is therefore $.6 \times 6200 = 3720$ lbs. The allowed bearing stress per square inch for rivets on the web of the floor-beam is $2 \times 7440 = 14,880$. The value of one $\frac{7}{8}$ rivet bearing against a $\frac{3}{8}$ plate is $\frac{7}{8} \times \frac{3}{8} \times 14,880 = 4880$.

The number of rivets required through the web of the beam and the end angles which rivet to the post of the truss is $69,700 \div 4880 = 15$. The number of rivets needed through these end angles and the post is $69,700 \div 3720 = 19$. The pitch of the rivets through the flanges and web plate is constant between the stringer point and the end of the beam and is $\frac{4880 \times 52}{69,700} = 3.64$ inches. Between the stringer points the flange stress is practically constant, so that 6-inch spacing, the maximum allowable, will be used.

322. The Design of the Lateral Bracing.—The wind pressure specified is 30 lbs. per square foot, counting as the exposed surface *twice the area of one truss* as seen in elevation and the surface of the floor. This gives 130 lbs. per linear foot as the wind force for the top lateral bracing, and 230 lbs. per linear foot for the wind force for the bottom lateral bracing. The train is assumed to expose a surface 10 feet in height, or, as it is usually taken, the moving wind load is 300 lbs. per linear foot. From the foregoing we get 3250 lbs. as the panel load for the upper lateral system, and 5750 lbs. as the static and 7500 lbs. the moving panel loads for the lower lateral system. The stresses in the lateral systems with the sections used are given in the following tables.

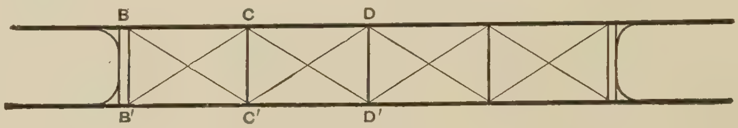


FIG. 358.

Piece.	Total Stress.	l	g	f	A_r	Make-up of Section.	Area.	Material.
DD'	1,625	192"	1.19	7.000	0.23	Four $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{5}{16}''$ ∠s	6.0	Iron
CC'	3,250	192	1.19	7.000	0.47	Four $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{5}{16}''$ ∠s	6.0	"
CD'	3,014			15.000	0.20	One 1" sq. rod	1.0	"
BC'	9,043			15.000	0.60	One 1" sq. rod	1.0	"

l = length of piece in inches; g = least radius of gyration of piece in inches; f = allowed stress per square inch; A_r = area required.

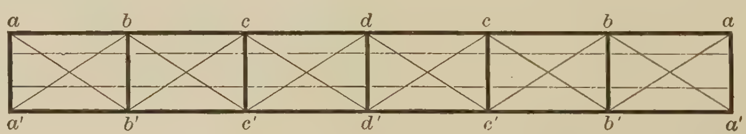
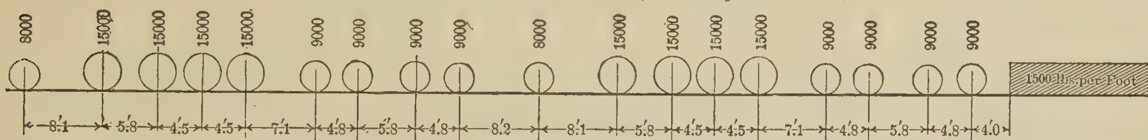


FIG. 359.

Piece.	Total Stress.	l	g	f	A_r	Make-up of Section.	Area.	Material.
aa'	33,100	84	0.94	9,900	3.4	Two $3'' \times 3'' \times \frac{5}{16}''$ ∠s	3.6	Iron
bb'	19,600	....	....	....	....	Lower flange of the floor-beam used for these struts		
cc'	8,600	....	....	....	....			
dd'	4,750	....	....	....	....			
ab'	61,400			15,000	4.1	Two $1\frac{7}{16}''$ sq. rods	4.12	Iron
bc'	38,300			15,000	2.55	Two $1\frac{1}{8}''$ sq. rods	2.53	"
cd'	17,900			15,000	1.2	One $1\frac{1}{8}''$ sq. rod	1.27	"

The strut *aa'* is attached to the end post and to the stringers; the greatest *unsupported* length is between the stringers and is seven feet.

MAXIMUM SHEARS AND BENDING MOMENTS ON GIRDERS FOR COOPER'S CLASS EXTRA HEAVY "A" LOADING. (In 1000-pound units.)



Weights given are for one Rail.

Length of Girder.	Shear at End.	Shear at Quarter Point.	Shear at Centre.	Maximum Moment near Centre.	Length of Girder.	Shear at End.	Shear at Quarter Point.	Shear at Centre.	Maximum Moment near Centre.
10′ 0″	24.8	15.8	8.3	45.0	43′ 0″	60.4	37.1	16.7	580.3
11′ 0″	26.6	16.4	8.9	56.3	44′ 0″	61.4	37.6	17.0	602.1
12′ 0″	28.1	16.9	9.4	67.5	45′ 0″	62.2	38.2	17.3	625.8
13′ 0″	29.4	18.2	9.8	78.8	46′ 0″	63.1	38.7	17.6	649.5
14′ 0″	30.5	19.3	10.2	90.0	47′ 0″	64.0	39.2	17.8	673.3
15′ 0″	31.7	20.2	10.3	101.3	48′ 0″	64.8	39.7	18.1	697.0
16′ 0″	33.5	21.1	10.3	112.5	49′ 0″	65.7	40.2	18.3	720.7
17′ 0″	35.0	21.8	10.3	123.8	50′ 0″	66.5	40.7	18.6	744.4
18′ 0″	36.4	22.5	10.3	135.0	51′ 0″	67.4	41.3	18.8	768.2
19′ 0″	37.7	23.1	10.7	146.3	52′ 0″	68.2	41.8	19.0	793.4
20′ 0″	38.8	23.8	11.1	160.4	53′ 0″	69.1	42.4	19.3	819.3
21′ 0″	39.8	24.8	11.4	175.2	54′ 0″	69.9	42.9	19.5	845.2
22′ 0″	40.7	25.7	11.7	189.9	55′ 0″	70.8	43.4	19.8	871.1
23′ 0″	41.6	26.5	12.0	204.7	56′ 0″	71.6	43.9	20.1	897.4
24′ 0″	42.7	27.3	12.2	219.6	57′ 0″	72.4	44.3	20.3	925.6
25′ 0″	43.7	28.0	12.2	234.7	58′ 0″	73.2	44.8	20.6	953.9
26′ 0″	44.6	28.7	12.3	251.9	59′ 0″	74.1	45.2	20.8	982.1
27′ 0″	45.5	29.3	12.3	269.1	60′ 0″	74.9	45.8	21.1	1010.3
28′ 0″	46.3	29.8	12.4	286.4	61′ 0″	75.7	46.2	21.3	1038.6
29′ 0″	47.0	30.3	12.4	303.6	62′ 0″	76.5	46.7	21.5	1066.8
30′ 0″	48.0	30.7	12.7	320.9	63′ 0″	77.3	47.2	21.7	1095.2
31′ 0″	48.9	31.2	13.1	338.1	64′ 0″	78.0	47.6	21.9	1125.7
32′ 0″	49.8	31.8	13.5	355.3	65′ 0″	79.1	48.1	22.1	1156.2
33′ 0″	50.8	32.3	13.9	373.7	66′ 0″	80.1	48.6	22.4	1187.7
34′ 0″	51.9	32.8	14.3	393.1	67′ 0″	81.1	49.0	22.7	1219.1
35′ 0″	52.9	33.2	14.6	412.6	68′ 0″	82.1	49.4	22.9	1250.6
36′ 0″	53.8	33.6	14.9	432.1	69′ 0″	83.2	49.8	23.1	1282.4
37′ 0″	54.7	34.1	15.2	451.6	70′ 0″	84.4	50.3	23.4	1314.3
38′ 0″	55.8	34.4	15.5	472.7	71′ 0″	85.5	50.7	23.6	1346.4
39′ 0″	56.8	35.1	15.8	494.3	72′ 0″	86.6	51.1	23.8	1378.6
40′ 0″	57.8	35.6	16.0	515.8	73′ 0″	87.7	52.0	24.0	1411.0
41′ 0″	58.7	36.1	16.3	537.3	74′ 0″	88.7	52.1	24.2	1443.8
42′ 0″	59.6	36.6	16.5	558.8	75′ 0″	89.7	52.4	24.5	1476.7

323. The Design of the Portal Strut.*—The wind force concentrated at the hip joint is assumed to be carried to the abutments by the inclined end posts. These posts act as beams fixed at both ends, and the distribution of external forces will be as shown in Fig. 360. The maximum bending moment in the portal strut is $\frac{Ph}{4}$, and this is also the maximum bending moment in the end posts. The assumption that the end posts are *fixed* at their lower ends is dependent upon the amount of direct stress in the end post and the width of the end posts. It is true whenever the total stress in the end post multiplied by one half the distance centre to centre of bearings on the end pin is equal to or greater than $\frac{Ph}{4}$.

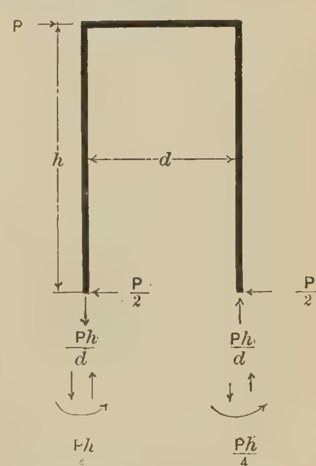


FIG. 360.

The force P, in the case of the 150-foot span under consideration, is equal to $2\frac{1}{2} \times 3250 = 8125$ lbs.; $h = 450.5$ inches and $d = 192$ inches. The portal strut, in order to give the specified clearance from the rail to the under side of the strut of 20 feet, is about 30 inches deep. It will be assumed to be 29 inches centre to centre of gravity of its

* See also pages 110, 159, and 288.

flanges. The maximum stress in each flange of the portal in order to fix the posts in direction at the top, will then be $\frac{8125 \times 450.5}{4} \times \frac{1}{29} = 31{,}600$ lbs. To this stress must be added $\frac{P}{4}$, a direct compression due to the direct application of the wind force at the end of the strut, one half of the force P being assumed as resisted by each end post. The total stress on *each* flange of the portal strut is, therefore, $31{,}600 + 2000 = 33{,}600$. Assuming each flange to be composed of two $3'' \times 3\frac{1}{2}''$ angles, the allowed stress in compression is found to be 8700 lbs. per square inch, requiring $3\frac{9}{10}$ square inches area. Two $3'' \times 3\frac{1}{2}'' \times \frac{5}{16}''$ angles are therefore sufficient. The portal strut is usually made in the form of a girder, the web being either lattice work or a solid plate. The latter is preferable and generally no more expensive. It will be noted that the maximum bending stresses in the end post from the wind forces occur at the shoe and at the point of the attachment of the portal strut, and also that the portal strut connections must be designed to resist this maximum moment.

324. The Stress Sheet.—The sectional areas of all the members have now been determined and the stress sheet can be made. This sheet should show the dead and live load stresses on each member, the allowed stresses per square inch, the make-up of the members, the material of which they are made, whether wrought-iron or steel, the live and dead loads used, the specifications under which the design was made, all the general dimensions—such as length centre to centre of end pins, number of panels and length of each, depth centre to centre of chords, width centre to centre of trusses and stringers, the skew, if any, and the alignment of the track over the bridge. The stress sheet for the 150-foot span is shown in Fig. 361.

325. Details at the Joints.—The determination of the sizes of pins to use and the arrangement of the packing on a pin at a joint is most conveniently made at the same time the sizes of the pin plates or bearings are proportioned. Each joint will be designed completely as we proceed, in order to avoid as much repetition as possible.

326. Joint *a*.—The pieces meeting at this joint are the inclined end post, aB; the bottom chord, ab; the shoe; the lateral strut, aa'; and the lateral diagonal, ab'. The members of the lateral system, aa' and ab', will be attached to the end post as close to the pin as possible, the aim being to produce the least bending stress from an eccentric connection.

A preliminary assumption of the size of the pin must first be made and the thickness of bearings required for this size determined. The bending moments produced by the stresses in the members connecting on the pin may then be calculated, and if the pin assumed is large enough no further computation is necessary. There are cases where it may, however, be desirable to reduce the size of the pin in order to use smaller heads on the eyebars, but it is assumed here that the desirable and smaller size of pin is assumed originally. The maximum diameter of the head for a *steel* eyebar should not be over $2\frac{5}{8}$ times the width of the bar on account of the difficulty in upsetting the material. The desirable diameter of head is about $2\frac{1}{4}$ times the width of bar, and as an excess of net area in the head over the bar of from 30 to 40 per cent is required, the diameter of the desirable size of pin is about nine tenths the width of bar. By referring to the table of standard steel eyebars, on page 246, it will be noted that two sizes of heads are generally used for each width of bar. The smaller head is the more desirable one to use because of its cheapness in manufacture and as it requires less material. The thickness of the eye should be the same as the body of the bar in *steel* eyebars. The smallest pin should have a diameter equal to three quarters of the width of the bar in order that the bearing pressure of the bar on the pin should not be too great. It is customary to assume the size of the pin in the preliminary calculations from the above considerations, as the cost of manufacture is affected more by a change in the size of pin than in the amount of material required.

The allowed extreme fibre stress on steel pins is, by the specifications under which this bridge is being designed, 20 per cent more than that specified for iron pins, or 18,000 lbs. per square inch. The pin at joint a will be assumed to be $4\frac{15}{16}$ inches in diameter. The

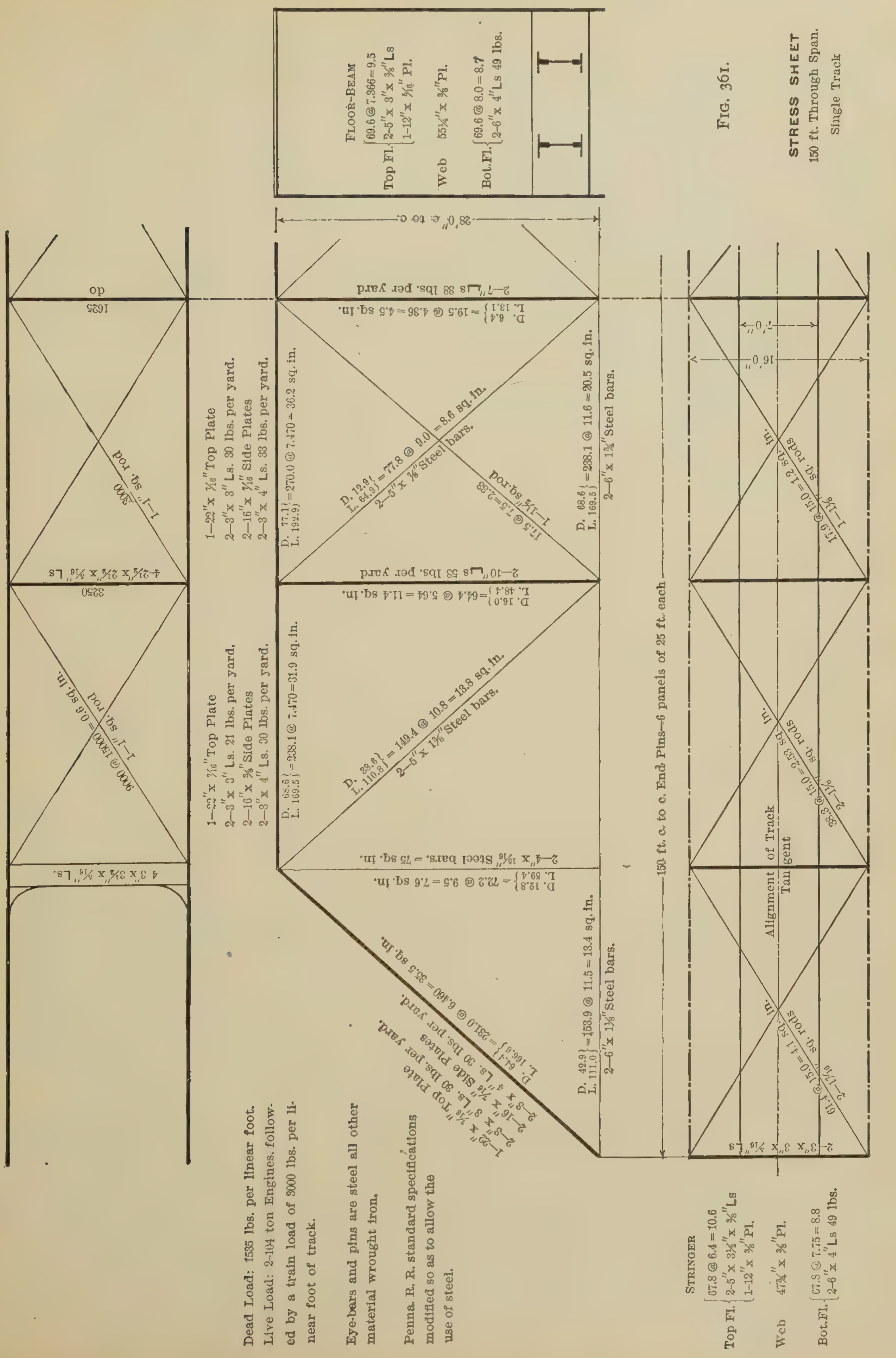

STRESS SHEET
150 ft. Through Span.
Single Track
FLOOR-BEAM
Top Fl. 69.6 @ 7.366 = 9.5
2-5" x 3" x 3/8" Ls
1-12" x 5/16" Pl.
Web 55 1/4" x 3/8" Pl.
Bot. Fl. 69.6 @ 8.0 = 8.7
2-6" x 4" Ls 49 lbs.
STRINGER
Top Fl. 67.8 @ 6.4 = 10.6
2-5" x 3 1/2" x 3/8" Ls
1-12" x 3/8" Pl.
Web 47 3/4" x 3/8" Pl.
Bot. Fl. 67.8 @ 7.75 = 8.8
2-6" x 4" Ls 49 lbs.
Dead Load: 1535 lbs. per linear foot.
Live Load: 2-104 ton Engines, followed by a train load of 3000 lbs. per linear foot of track.
Eye-bars and pins are steel all other material wrought iron.
Penna. R. R. standard specifications modified so as to allow the use of steel.
150 ft. c. to c. End Pins—6 panels of 25 ft. each
28' 0" c. to c.
16' 0"
7' 0"
Alignment of Track Tangent
1-22" x 7/16" Top Plate
2-3" x 3" Ls. 30 lbs. per yard.
2-16" x 7/16" Side Plates
2-3" x 4" Ls. 33 lbs. per yard.
D. 77.1 L. 192.9 = 270.0 @ 7.470 = 36.2 sq. in.
1-22" x 7/16" Top Plate
2-3" x 3" Ls. 21 lbs. per yard.
2-16" x 3/8" Side Plates
2-3" x 4" Ls. 30 lbs. per yard.
D. 68.6 L. 169.5 = 238.1 @ 7.470 = 31.9 sq. in.
D. 88.6 L. 169.5 = 238.1 @ 11.6 = 20.5 sq. in.
2-6" x 1 3/4" Steel bars.
D. 42.9 L. 111.0 = 153.9 @ 11.5 = 13.4 sq. in.
2-6" x 1 1/8" Steel bars.
D. 64.4 L. 166.6 = 231.0 @ 6.460 = 35.5 sq. in.
2-3" x 4" Ls. 30 lbs. per yard.
2-16" x 7/16" Side Plates
2-3" x 3" Ls. 30 lbs. per yard.
1-22" x 7/16" Top Plate
D. 12.9 L. 64.9 = 77.8 @ 9.0 = 8.6 sq. in.
2-5" x 7/8" Steel bars.
D. 38.6 L. 110.8 = 149.4 @ 10.8 = 13.8 sq. in.
2-5" x 1 3/8" Steel bars.
11.5 @ 7.5 = 2.33 sq. rod
1-1 3/4"
D. 6.4 L. 13.1 = 19.5 @ 4.36 = 4.5 sq. in.
2-7" [s 88 lbs. per yard
D. 16.0 L. 48.4 = 64.4 @ 5.64 = 11.4 sq. in.
2-10" [s 55 lbs. per yard.
D. 12.8 L. 59.4 = 72.2 @ 9.5 = 7.6 sq. in.
2-4" x 15/16" Steel bars. = 75 sq. in.
1625
do
3250
4-2 1/2" x 2 1/2" x 5/16" Ls
4-3" x 3 1/2" x 5/16" Ls.
1-1" sq. rod
3000
9000 @ 15000 = 0.6 sq. in.
1-1" sq. rod
17.9 @ 15.0 = 1.2 sq. in.
1-1 1/4" sq. rods
38.3 @ 15.0 = 2.55 sq. in.
2-1 1/4" sq. rods
61.3 @ 15.0 = 4.1 sq. in.
2-1 5/8" sq. rods
2-3" x 3" x 5/16" Ls

Fig. 361.

allowed bearing pressure per square inch of aB on the pin is $1\frac{1}{2} \times 8300$* $= 12{,}450$ lbs. The stress in the end post is 231,000 lbs., requiring $18\frac{9}{16}$ square inches of bearing on the pin. The thickness of the bearing required $= 18\frac{9}{16} \div 4\frac{15}{16} = 3\frac{3}{4}$ inches. As the web plates bear on the pin, the amount of additional thickness to be provided is $3\frac{3}{4} - \frac{7}{8} = 2\frac{7}{8}$ inches. Plates of the following thickness will be used; two $\frac{5}{16}$ inch and two $\frac{9}{16}$ inch thick on the outside of the web plates, and two $\frac{9}{16}$ inch thick on the inside of the webs. They will be arranged as shown in Fig. 362. Plates b are made the same thickness as the top angles. Plates a are as thick as they could be made and leave room enough to drive the rivets through the top plate and angles. Plates c have to make up the required thickness of bearing, or what is needed beyond that used in the web plates, a and b. Plates c will be the hinge or lap plates (i.e., the ones which extend beyond the pin and have a full pin-hole in them).

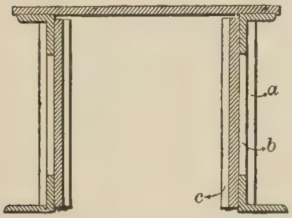


FIG. 362.

327. The Pin Bearing on the Shoe.—The pressure from the dead and live loads on the shoe is vertical and is equal to the vertical component of the stress in the end post, aB, or $231{,}000 \times \frac{28}{37.5} = 172{,}500$ lbs. The bearing area required on the pin is $172{,}500 \div 12{,}450 = 13.85$ square inches. For a pin $4\frac{15}{16}$ inches in diameter a thickness of $13.85 \div 4.94 = 2.80$ inches is necessary. A hinge plate $\frac{5}{8}$ inch thick will be used as the outside plate on each rib of the shoe. The clearance between the outside of the pin plates on aB and the inside of these hinge plates on the shoe must be $\frac{1}{4}$ inch. The distance between the inside faces of the hinge plates must be therefore $17\frac{3}{4}$ inches. The distance centre to centre of the ribs of the shoe is then $17\frac{3}{4} + \frac{3}{4} - 1\frac{7}{16} = 17\frac{1}{16}$ inches. It will be noted that each rib of the shoe is made $1\frac{7}{16}$ inches thick in order to make them both alike, and to use no plate the thickness of which is measured in thirty-seconds of an inch.

328. The Calculation of the Maximum Bending Moment on the Pin a.—The bearings of all the riveted members which connect on the pin 'a' have now been determined for the assumed pin, and the bending moment on the pin must now be calculated to see whether this pin is large enough. The centres of bearings of the various members will be located as shown in Fig. 363. For the bending moment in the horizontal plane the pin is considered as a beam between the centres of the bearings of aB and loaded with two loads at the centres of bearing of the eyebars ab.†

The horizontal component of the stress in aB is 154,000 lbs., and the stress in each bar ab is 77,000 lbs. The maximum bending moment from the horizontal forces on the pin is therefore $77{,}000 \times \frac{15\frac{3}{8} - 11\frac{5}{8}}{2} = 144{,}400$ in.-lbs.

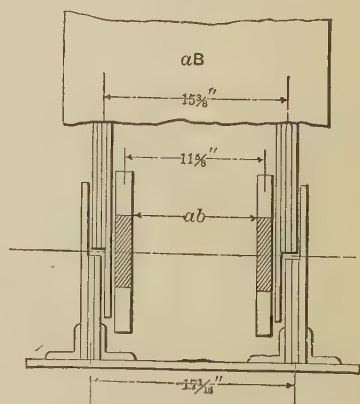


FIG. 363.

For the bending moment in the vertical plane the pin is considered as a beam between the centres of the bearings of the shoe and loaded at the centres of the bearings of aB. The vertical component of the stress in aB is 172,500 lbs., and the vertical pressure on each bearing is 86,250 lbs. The maximum bending moment from the vertical forces is therefore $86{,}250 \times \frac{17\frac{1}{16} - 15\frac{3}{8}}{2} = 72{,}800$ in.-lbs. The maximum moment on the pin is the resultant of these two moments, or $\sqrt{144{,}400^2 + 72{,}800^2}$

* The value of u for aB is $6500\left(1 + \frac{64}{231}\right) = 8300$, the member being wrought-iron. See Specifications on p. 322.

† Throughout this discussion the members of the truss will be designated by the letters at their extreme ends, as shown in Fig. 355.

$= 162,000$ in.-lbs. This requires a steel pin $4\frac{9}{16}$ inches in diameter, allowing 18,000 lbs. per square inch extreme fibre stress. The pin $4\frac{15}{16}$ inches in diameter will be used.

329. **Joint *B*.**—Assume the pin to be $4\frac{15}{16}$ inches in diameter, the same as used at *a*. The thickness of bearing required for *aB* is $3\frac{3}{4}$ inches and for *BC* is $238,100 \div 12,600 = 18.89 \div 4.94 = 3.82$ inches. The hinge plates on *aB* will be put inside and those on *BC* outside, and in each case will be $\frac{3}{8}$ inch thick. A clearance of one quarter of an inch will be left between any hinge plate and the nearest member. The packing of this pin and the distances centre to centre of the bearings of the several members which attach thereto are shown in Fig. 364. The maximum moment on this pin may occur at either of two times: 1st, when the stress in the diagonal *Bc* is a maximum, or, 2d, when the stress in *aB* is a maximum.

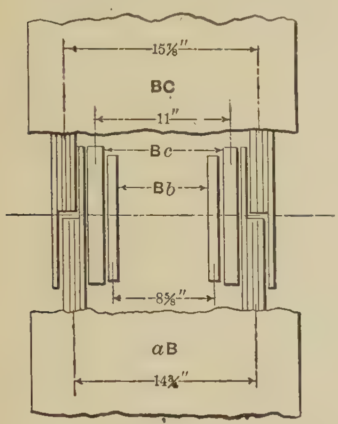


FIG. 364.

1st. *When the stress in Bc is a maximum.*—The stress on *Bc* is then 149,400 and the stress on *Bb* is 20,800 lbs. From the polygon of forces at this point, Fig. 365, we get the corresponding stresses in *aB*, *BC*. The vertical forces are $Bb = 20,800$, $Bc = 111,600$, and $aB = 132,400$ lbs. The maximum vertical moment is therefore $10,400 \times 3\frac{1}{16} + 55,800 \times 1\frac{7}{8} = 136,500$ in.-lbs.* The horizontal forces are $Bc = 99,600$, $aB = 118,200$, and $BC = 217,800$ lbs. The maximum horizontal moment is $49,800 \times 2\frac{7}{16} + 59,100 \times \frac{17}{32} = 152,800$ in.-lbs. The resultant moment is $\sqrt{\overline{136,500}^2 + \overline{152,800}^2} = 205,000$ in.-lbs., requiring a *steel* pin $4\frac{15}{16}$ inches in diameter.

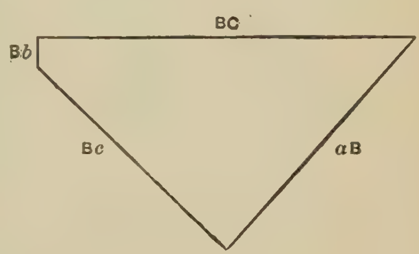


FIG. 365.

2d. *When the stress in aB is a maximum.* The stress on *Bb* is 72,200 and on $aB = 222,400$ lbs.* From the polygon of forces acting at this joint we get $Bc = 125,800$ and $BC = 232,100$ lbs. The vertical forces are $aB = 166,100$, $Bb = 72,200$, and $Bc = 93,900$ lbs. The vertical moment is $36,100 \times 3\frac{1}{16} + 46,950 \times 1\frac{7}{8} = 198,600$ in.-lbs. The horizontal forces are $aB = 148,300$, $Bc = 83,800$, and $BC = 232,100$ lbs. The horizontal moment is $41,900 \times 2\frac{7}{16} + 74,150 \times \frac{17}{32} = 141,600$ in.-lbs. The resultant moment is $\sqrt{\overline{198,600}^2 + \overline{141,600}^2} = 244,000$ in.-lbs., requiring a steel pin $5\frac{3}{16}$ inches in diameter which will be used.

A smaller bending moment could have been obtained for this pin by putting the bars *Bc* outside of *aB* and *BC*, but it is always preferable to pack the bars inside in order to avoid cutting off the horizontal legs of the bottom angles of *BC*, which would have to be done to get the bars close to the bearings of *aB* and *BC*.

The thickness of the bearings required for *aB* and *BC* will be a little less than what was needed for the $4\frac{15}{16}$-inch pin assumed, but owing to the requirements for clearances in the fit of *aB* and *BC* together on the pin the thickness used will be maintained.

330. **Joint *C*.**—A pin $4\frac{7}{16}$ inches in diameter will be assumed in order to use small heads on the 5-inch eyebars. The bearing for this pin on *CD* must be thick enough to stand the pressure caused by the horizontal component of the maximum stress in *Cd*. The stress in *BC* is transferred to *CD* directly by a butt joint. The allowed bearing pressure per square inch is $8400 \times 1\frac{1}{2} = 12,600$ lbs. The bearing area is $52,000 \div 12,600 = 4\frac{1}{8}$ square inches, or for a $4\frac{7}{16}$-inch pin the thickness required is $4\frac{1}{8} \div 4\frac{7}{16} = \frac{15}{16}$ inches. The web plates of *CD* are each $\frac{7}{16}$ inch thick, and the splice plates which rivet to *BC* and *CD* to hold them in position at the butt joint are each $\frac{3}{8}$ inch thick. There is necessarily more bearing at this point than

* The dead load concentration at this joint is neglected in these computations.

is required. The bearing required for Cc is $64,400 \div 12,100 = 5\frac{3}{8}$ square inches. The thickness required is $5\frac{3}{8} \div 4\frac{7}{16} = 1\frac{1}{4}$ inches. One plate $\frac{5}{8}$ inch thick will be used on each side of the post, making the centre to centre of the post bearings $10\frac{5}{8}$ inches. The usual packing of a top chord pin is as shown in Fig. 366, which is a sketch of the joint C. The maximum moment on the pin occurs when the diagonal Cd has its maximum stress. The forces acting on the pin then are $Cd = 77,800$, $Cc = 64,400$, $CD = 51,900$ horizontal and $CD = 6400$ vertical (assuming the dead load panel concentration to be applied at the bearings of CD). The vertical forces are $Cd = 58,000$, $Cc = 64,400$, $CD = 6400$, and the vertical bending moment is $3200 \times 2\frac{13}{32} + 29,000 \times \frac{15}{16} = 34,900$ in.-lbs. The horizontal forces are $Cd = 51,900$ and $CD = 51,900$, and the horizontal bending moment is $25,950 \times 1\frac{15}{32} = 38,100$ in.-lbs. The resultant bending moment is $\sqrt{\overline{34,900}^2 + \overline{38,100}^2} = 51,700$ in.-lbs. This would allow the use of a $3\frac{3}{16}$-inch pin, but a pin $3\frac{15}{16}$ inches in diameter will be used, as this is the smallest pin allowable for a 5-inch bar. The thickness of the bearing plates for Cc will be $5\frac{3}{8} \div 3\frac{15}{16} = 1\frac{3}{8}$ inches. One plate $\frac{11}{16}$ inch thick will be used on each side of the post.

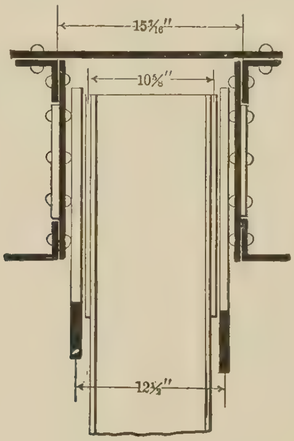


FIG. 366.

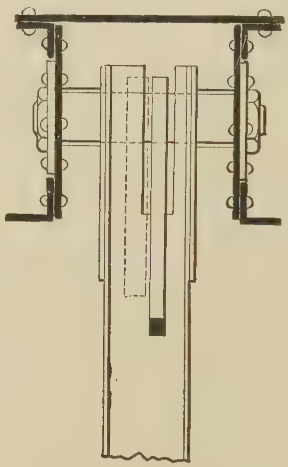


FIG. 367.

331. Joint *D*.—A pin $3\frac{15}{16}$ inches in diameter will be used and the pin packed as shown in Fig. 367. The counter-ties are put on the middle of the pin, the webs of the channels in Dd being cut to let them pass. As there is only one rod each way, the bars are necessarily packed off of the centre, but the stresses in them are so small that the effect is insignificant. The bearing on the pin required for Dd is $19,500 \div 12,900 = 1.51$ square inches or $\frac{7}{16}$ inch thick. One plate $\frac{3}{8}$ inch thick will be used on each side of the post.

332. Joint *d*.—A pin $5\frac{3}{16}$ inches in diameter will be assumed. The only member for which the bearings must be determined is Dd. The maximum stress in the post at the bottom is 78,600 lbs., and occurs when the floor-beam has its maximum concentration from the live load. The dead load panel concentration is also assumed as acting at the centres of bearings of Dd. The allowed bearing stress per square inch on the pin at the foot of the post is $1\frac{1}{2} \times 6500\left(1 + \frac{19,200}{78,600}\right) = 12,100$ lbs. The bearing area required is $78,600 \div 11,100 = 7.08$ square inches, or $7.08 \div 5\frac{3}{16} = 1\frac{3}{8}$ inches thick. Two plates each $\frac{3}{8}$ inch thick will be used on each side of the post. The packing of the several members on the pin will be as shown in Fig. 368.

The maximum bending moment occurs when cd has its maximum stress. The stresses then are $cd = 238,100$, $c'd' = 234,500$, $Cd = 47,200$, $C'd'$* $= 52,700$, and $Dd = 74,500$ lbs.

* Here the primed letters represent corresponding joints on the right of the centre of the bridge.

The vertical stresses are $Cd = 35{,}200$, $C'd' = 39{,}300$, and $Dd = 74{,}500$ lbs. The horizontal stresses are $cd = 238{,}100$, $c'd' = 234{,}500$, $Cd = 31{,}500$, and $C'd' = 35{,}100$ lbs. The vertical moments are as follows:

About $Cd = 19{,}650 \times \frac{15}{16} = 18{,}400$ inch-pounds.
" $Dd = 19{,}650 \times 2\frac{7}{16} + 17{,}600 \times 1\frac{1}{2} = 74{,}300$ inch-pounds.

The horizontal moments are as follows:

About $c'd' = 119{,}050 \times 1\frac{13}{16} = 215{,}800$ inch-pounds;
" $C'd' = 119{,}050 \times 3\frac{3}{16} - 117{,}250 \times 1\frac{3}{8} = 218{,}300$ inch-pounds.
" $Cd = 119{,}050 \times 4\frac{1}{8} - 117{,}250 \times 2\frac{5}{16} - 17{,}550 \times \frac{15}{16} = 203{,}500$ inch-pounds.

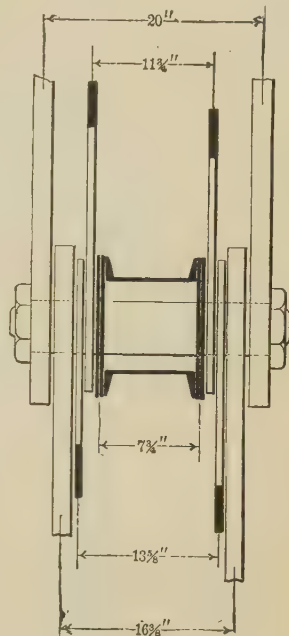


FIG. 368.

The resultant bending moment at Dd is $\sqrt{74{,}300^2 + 203{,}500^2} = 217{,}000$. The maximum bending moment is at $C'd'$ and is due to the chord stresses alone. The diameter of the smallest pin which could be used without exceeding the allowed extreme fibre stress is $5\frac{1}{16}$ inches. $5\frac{3}{16}$ inches will be the diameter of the pin used.

333. Joint *c*.—In order to keep the sizes of the pins as nearly alike as possible a pin $5\frac{3}{16}$ inches in diameter will be assumed. The thickness of bearing for Cc must be determined. The allowed bearing pressure per square inch on the pin for the foot of the post is $1\frac{1}{2} \times 6500\left(1 + \frac{28{,}800}{111{,}500}\right) = 12{,}200$ lbs. The maximum stress in the post *below* the floor-beam is equal to the vertical component of the maximum stress in Bc. The minimum stress is the vertical component of the *dead* load stress in Bc, assuming as is customary that the dead load panel concentration is applied to the pin through the post. The bearing area required at the foot of the post is $111{,}500 \div 12{,}200 = 9.14$ square inches. The thickness of bearing is $9.14 \div 5\frac{3}{16} = 1\frac{13}{16}$ inches. One plate $\frac{3}{8}$ inch thick and one plate $\frac{9}{16}$ inch thick will be used on each side of the post. The packing of the members on the joint will be as shown in Fig. 369.

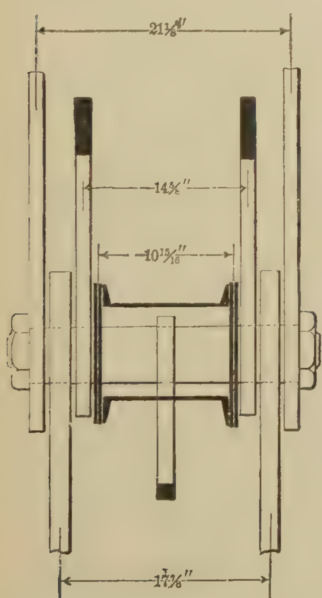


FIG. 369.

There are two cases to be examined for maximum bending moment: 1st, when Bc is a maximum, and 2d, when bc is a maximum.

1st. *When Bc is a maximum.*—The stresses then existing are $Bc = 149{,}400$, $Cc = 111{,}500$, $bc = 123{,}800$, and $cd = 223{,}400$ lbs. The vertical forces are $Bc = 111{,}500$ and $Cc = 111{,}500$. The vertical bending moment is $55{,}750 \times 1\frac{27}{32} = 102{,}800$ in.-lbs. The horizontal forces are $Bc = 99{,}600$, $bc = 123{,}800$, and $cd = 223{,}400$ lbs. The horizontal moments are:

About $cd = 61{,}900 \times 1\frac{5}{8} = 100{,}600$ in.-lbs.,
" $Bc = 61{,}900 \times 3\frac{1}{4} - 111{,}700 \times 1\frac{5}{8} = 19{,}700$ in.-lbs.

The resultant bending moment at any point of the pin between the bearings of Cc is $\sqrt{102{,}800^2 + 19{,}700^2} = 104{,}700$ in.-lbs.

2d. *When bc is a maximum.*—The stresses then existing are $Bc = 120{,}300$, $bc = 153{,}900$,

$cd = 234{,}100$, and $Cc = 89{,}800$ lbs. The vertical forces are $Bc = 89{,}800$ and $Cc = 89{,}800$ lbs. The vertical bending moment is $44{,}900 \times 1\frac{27}{32} = 82{,}800$ in.-lbs. The horizontal forces are: $Bc = 80{,}200$, $bc = 153{,}900$, and $cd = 234{,}100$. The horizontal bending moments are:

$$\text{About } cd = 76{,}950 \times 1\tfrac{5}{8} = 125{,}000 \text{ in.-lbs.}$$
$$\text{“} \quad Bc = 76{,}950 \times 3\tfrac{1}{4} - 117{,}050 \times 1\tfrac{5}{8} = 59{,}900 \text{ inch-lbs.}$$

The resulting bending moment at any section of the pin between the bearings of Cc is $\sqrt{82{,}800^2 + 59{,}900^2} = 102{,}200$ in.-lbs. A pin $4\frac{7}{16}$ inches in diameter, the smallest allowable for a six-inch eyebar, will be used at this joint. The thickness of the pin bearing for Cc will be increased to $9.14 \div 4.44 = 2.06$ inches. One plate $\frac{3}{8}$ inch thick and one plate $\frac{11}{16}$ inch thick will be used on each side of the post.

334. Joint *b*.—In order to understand the packing of this joint and the critical stresses, it is necessary to completely design the details of the hanger Bb to which the floor-beam and lateral rods are attached. Fig. 370 is the detail sketch of this joint. The floor-beam is riveted to a plate, A. This plate A is held up by the bars Bb. The lateral rods are attached to the lower flange of the beam. The permissible tensile stress per square inch on the floor-beam hanger, as plate A is called, is 5000 lbs. per square inch. The stresses in the plate around the pin b_1 are similar to those in an eyebar head. The net area of the plate A at a horizontal section, BC, through pin b_1 must be at least 25 per cent in excess of the net area required at 5000 lbs. per square inch. The net area of the vertical section above the pin b_1 and on the line joining the pin centres of b_1 and B must be three quarters of the net area of the horizontal section BC, as the tensile stress on this area acts perpendicular to the fibres of the iron, and wrought-iron is about two thirds as strong in tension across the grain or fibre as it is with the grain. The bearing of the pin b_1 on the hanger A must not exceed the specified stress of $6500\left(1 + \frac{12{,}800}{72{,}200}\right)1\frac{1}{2} =$ 11,500 lbs. per square inch.

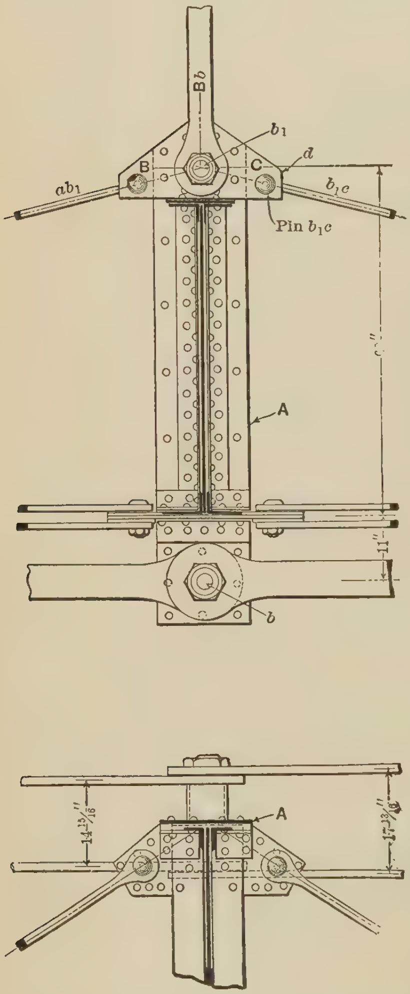


FIG. 370.

The hanger is subjected to a bending stress from the chord component of the lateral rods ab', bc'.* This stress is transferred to the pins b_1 and b by the plate A. The part transferred to the pin b_1 is taken up by the rods ab_1 or b_1c and Bb. The stress from this cause in Bb is small and always neglected in determining the area of Bb. The areas of the rods ab_1, b_1c are determined by this stress alone. The part transferred to the pin b is taken up by the chord bars bc.

The pin b_1 is $3\frac{15}{16}$ inches in diameter. The bearing required on this pin is $72{,}200 \div 11{,}500 = 6.29$ square inches, or 1.6 inches thick. The net area of the hanger required at a horizontal section at the top of the beam is $72{,}200 \div 5000 = 14.44$ square inches, which is satisfied by using a plate $16 \times 1\frac{1}{16}$ inches, making allowance for two $\frac{7}{8}$-inch rivet-holes. The net area at the section BC must be $14.44 \times 1.25 = 18.05$ square inches, requiring a plate $16 \times 1\frac{1}{2}$ inches, making allowance for the reduction of

* Here the primed letters represent corresponding joints in the other truss.

the available area by the pin-hole. The bearing requires 1.6 inches thickness. The plate A will be built of two plates $16 \times \frac{1}{2}$ and $16 \times \frac{9}{16}$ inches each. The required bearing on pin b_1 and the area of the section BC will be satisfied by the addition of two plates, d, $\frac{3}{8}$ inch thick, one on each side of A. The area of the vertical section above the pin b_1 must be $\frac{3}{4}$ of $18.05 = 13.54$ square inches. The total thickness of plates A and d now is $1\frac{1}{16} + \frac{3}{4} = 1\frac{13}{16}$. The distance from the top side of the pin-hole b_1 to the end on the hanger must be, in order to give the required area of 13.54 square inches, $13.54 \div 1.81 = 7.5$ inches.

The chord component of the stress in the lateral rod ab' is $61{,}400 \times \dfrac{25}{29.68} = 51{,}700$ lbs., causing a stress in the rods b_1c of $51{,}700 \times \dfrac{11}{74} \times \dfrac{25.75}{25.0} = 7900$ lbs. One rod one inch square is the smallest rod allowed, and this size will be used for b_1c and ab_1. The bending moment on the pin b_1c will be $\dfrac{7900 \times 1\frac{7}{16}}{4} = 2840$ in.-lbs. A pin $1\frac{3}{16}$ inches in diameter will be used. The plates d are extended to take these pins, making the distance centre to centre of bearings for the pins $1\frac{1}{16} + \frac{3}{8} = 1\frac{7}{16}$, as used in the above computation.

The maximum bending moment on the plate A from the lateral rods is $\dfrac{51{,}700 \times 63 \times 11}{74}$ $= 484{,}200$ in.-lbs. The permissible fibre stress per square inch will be taken as 12,000 lbs., the maximum stress allowed on short columns for the lateral system. The bending moment requires a plate 16 inches wide and $\frac{15}{16}$ inch thick. Plate A will be made $1\frac{1}{16}$ inches thick throughout.

Pin b may now be determined. As there are no members on this pin for which the bearings must be proportioned, the bending moment may be computed and the required pin selected at once. The packing of this pin is shown in Fig. 370.

The bars ab and bc are put so far from the hanger A because it is desirable to have the bars coincide as nearly as possible with straight lines drawn from the centre of each bar on the pin a to the centre of each bar on pin c. This will cause the least deviation of the bars from a line parallel to the centre line of the bottom chord. It is a common requirement of good construction that the bars run as nearly parallel to the centre line of the chord as practicable. If the bars deviate from a line parallel to the centre line of the chord more than one eighth of an inch to the foot, the bars must be bent or the pin-holes in the bars bored as shown in Fig. 371. This involves additional expense and is not desirable construction.

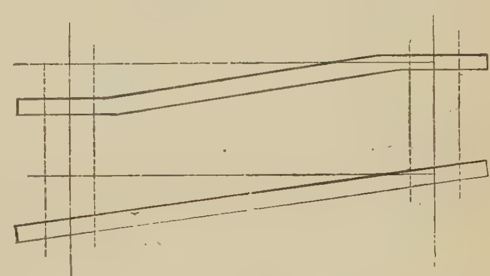


FIG. 371.

The vertical moment on the pin due to the weight of the chord bars alone is small and will be neglected. The maximum horizontal bending moment on the pin from the dead and live load stresses in the chords ab, bc is $76{,}950 \times 1\frac{7}{16} = 110{,}700$ in.-lbs. This would require a steel pin $3\frac{15}{16}$ inches in diameter. The chord component from the wind stress in the lateral rod ab' causes a bending moment of $51{,}700 \times \dfrac{63}{74} \times \dfrac{17.81}{4} = 196{,}000$ in.-lbs., making a total bending moment of $110{,}700 + 196{,}000 = 306{,}700$ in.-lbs. from the dead, live, and wind stresses. For this combination it is usual to increase the permissible extreme fibre stress per square inch one half, or in the present case to 27,000 lbs. This would require a steel pin $4\frac{15}{16}$ inches in diameter which will be used.

The bending moment on the pin b_1 is $\dfrac{72{,}200 \times 3\frac{1}{8}}{4} = 56{,}400$ in.-lbs.

The sizes of all the truss pins have now been determined, and the next problem is to

determine the lengths and number of rivets required for the bearing plates on the various members.

335. **The Lengths of Bearing or Pin Plates** are determined by the following considerations:

1st. Each plate must be long enough to take sufficient rivets to transfer the stress in the plate to the main section.

The stress in a pin plate may be the accumulated stresses in a number of plates outside of the plate in question, but which must pass through this plate to reach the main section. Where there are a number of plates the stress per square inch on the plates nearer the main section increases slightly, hence it is always better to use the thicker plates nearest the main section.

It is not sufficient to merely put the required number of rivets through the plate, but there must also be enough rivets to resist any bending moment due to an eccentric application of the stress with respect to the centre of the group of resisting rivets. In order to fulfil this requirement it is often necessary to use more rivets than the number determined by dividing the bearing stress on the plate by the value of one rivet.

2d. The distance from the edge of the pin-hole to the end of the plate must always be great enough to offer sufficient resistance against splitting on the line AB, Fig. 372, the dangerous section being the one which has the least net area or is the most cut up by rivet-holes.

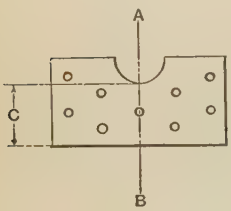


FIG. 372.

The best way, probably, to determine the distance C, Fig. 372, is to consider the plate as a beam balanced over the centre of the pin and acted upon by forces each equal to the value of one rivet applied at the centres of the rivets.

It is a safe rule to make the width of pin plate not less than three fourths of the length. This question is often overlooked in designing, but even as a question of appearance only it deserves attention.

3d. At least one pin plate on each web plate must be used to reinforce the web plate until the stresses are distributed over the entire area of the member. Thus, in aB, the entire stress is first received from the pin on the pin plates and web. The top plate and angles must receive their proportion of the stress through the rivets connecting them to the web plates, and until this stress is transferred the web plate alone would not be sufficient to take it without being subjected to a greater stress than is allowed. The amount of stress taken by the top plates and angles of aB is such a proportion of the total stress on aB as the area of the top plate and angles is of the total area, or 101,300 lbs. To transfer this amount requires *thirty-four rivets* in single shear, the diameter of the rivet being $\frac{3}{4}$ inch. It will readily be seen that it is advantageous and economical to use as many rivets in double shear as possible in order to transfer this stress in the shortest distance. For this reason plate a, Fig. 373, was made wide enough to take the rivets through the top angles. The shortest rivet spaces allowable should always be used at the ends of compression members and continued until the stress has had an apportunity to distribute itself over the entire area of the member. Plates b, Fig. 373, are made long in order to reinforce the web plates.

336. **Pin Plates on aB at Joint a.**—The pin plates are a, $\frac{5}{16}$ inch thick, b, $\frac{9}{16}$ inch thick, and c, $\frac{9}{16}$ inch thick (see Fig. 373). The number of rivets through these plates attaching them to the main section of the end post is as follows:

Plate a.... $\dfrac{231{,}000 \times \frac{5}{16}}{3\frac{3}{4}} = 19{,}250 \div 3000 = 7$ $\frac{3}{4}$-inch rivets in single shear.

" b.... $\dfrac{231{,}000 \times \frac{9}{16}}{3\frac{3}{4}} = 34{,}650 \div 3000 = 12$ " " " " "

" c.... $\dfrac{231{,}000 \times \frac{9}{16}}{3\frac{3}{4}} = 34{,}650 \div 3000 = 12$ " " " " "

This number of rivets will be sufficient if the centre of the group of resisting rivets is on the centre line connecting the pins a and B. In addition to the above it must be observed that there must be $\frac{231{,}000 \times (\frac{5}{16} + \frac{9}{16})}{3\frac{3}{4}} = 53{,}900 \div 3000 = 18$ rivets in single shear attaching a and b to the main section, and finally there must be enough rivets attaching a, b, and c to the main section to transfer the total bearing pressure on the three plates to the main section without exceeding the allowed *bearing pressure* of rivets.

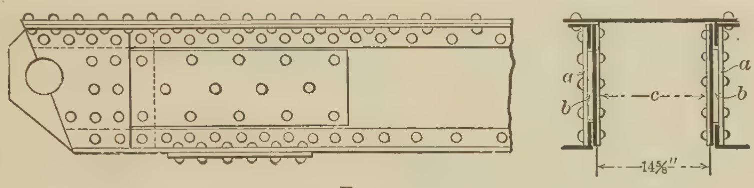


FIG. 373.

In plate a thirteen rivets are used. This is more than would be necessary for this plate, but if the plate were shortened 3 inches both the first and second conditions mentioned as determining the length of pin plates would not have been fulfilled. Plate b is made long in order to stiffen the web in accordance with the requirements of the third condition. Plate c has seventeen rivets, which is a few more than necessary to fulfil the first condition, but if it were made 3 inches shorter and only thirteen rivets used the rivets would be overstressed. Plate c is also the hinge plate. The distance from the edge of the pin-hole to the end of the plate should be not less than 2 inches.

337. Pin Plates on *aB* at Joint *B*.—The plates are a, $\frac{9}{16}$ inch thick, b, $\frac{1}{2}$ inch thick, and c, $\frac{3}{8}$ inch thick. Plate c is the hinge plate. Assuming the centre of the group of resisting rivets to be on the centre line joining the pins a and B, the number of rivets required to attach these plates to the main section singly is as follows:

Plate a.... $\frac{231{,}000 \times \frac{9}{16}}{3\frac{3}{4}} = 34{,}650 \div 3000 = 12$ rivets in single shear.

" b.... $\frac{231{,}000 \times \frac{1}{2}}{3\frac{3}{4}} = 30{,}800 \div 3000 = 11$ " " " "

" c.... $\frac{231{,}000 \times \frac{3}{8}}{3\frac{3}{4}} = 23{,}100 \div 3000 = 8$ " " " "

Fig. 374 is a detail sketch of the bearing of aB on pin B. Plate a is made long enough to fulfil the third condition. Plate b has twenty-two rivets attaching it to the web plate. As

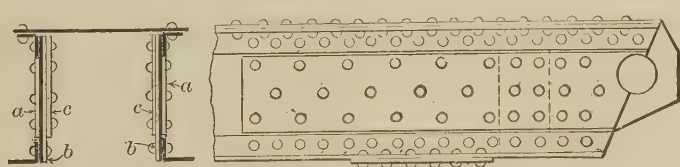


FIG. 374.

b must have enough rivets to transfer the bearing pressure on both b and c, it will be seen that it is as short as it could have been made. Plate c is the hinge plate, and has ten rivets when but eight are necessary.

338. Pin Plates on *BC* at Joint *B*.—The plates are a, $\frac{3}{8}$ inch thick, b, $\frac{3}{8}$ inch thick,

c, $\frac{7}{16}$ inch thick, and d, $\frac{5}{8}$ inch thick. Plate a is the hinge plate. The number of rivets required to transfer the stresses on these plates to the main section is as follows:

Plate a............ $\dfrac{238{,}100 \times \frac{3}{8}}{3\frac{7}{8}} = 23{,}040 \div 3000 =$ 8 $\frac{3}{4}$-inch rivets in single shear.

Plates a and b....... $\dfrac{238{,}100 \times \frac{3}{4}}{3\frac{7}{8}} = 46{,}080 \div 3000 = 16$ " " " " "

Plates a, b, and c.... $\dfrac{238{,}100 \times 1\frac{3}{16}}{3\frac{7}{8}} = 72{,}970 \div 3000 = 25$ " " " " "

Plate d........... $\dfrac{238{,}100 \times \frac{3}{8}}{3\frac{7}{8}} = 23{,}040 \div 3000 =$ 8 " " " " "

Fig. 375 shows how the conditions determining the lengths of pin plates for this joint are met.

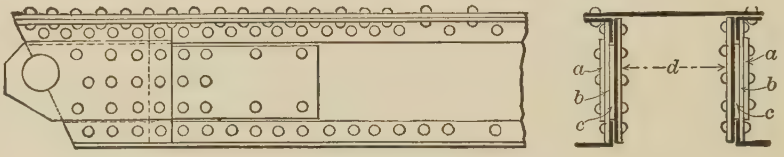


FIG. 375.

339. **Intermediate Top Chord Joint.**—The top chord sections are spliced as shown in Plate I. The splice plates on the web usually take one or two rows of rivets beyond the splice. The splice plate on the top plate usually has but one row of rivets. The duty of the splice is to hold the spliced sections truly in line, and is not relied on to transfer any of the stress. The ends of the joined sections are planed off in order to secure a perfect bearing throughout the joint; the transfer of the stress is supposed to be done by the abutting ends. The position of the splice is determined as follows: The splice must be as near the pin as possible, and also there must be clearance enough to allow the field rivets through the web plates to be driven easily. The splice is usually located by the clearance lines of the post or tie-bars, as they are the only members which interfere with the driving of the field rivets. It is customary now to make the splice on the side of the joint nearer the end of the truss, but this is dependent upon the method of erection.

340. **Post *Cc*.**—At the top pin, C, the bearing of this post is as previously given, $1\frac{1}{4}$ inches thick, and is made up of two plates $\frac{5}{8}$ inch thick, one on each side of the post. The stress on each plate is 32,200 lbs., requiring eleven $\frac{3}{4}$-inch rivets in single shear to transfer this stress to the channels. Twelve rivets are used in each plate, or six through each plate and flange of the channel.

At the bottom pin, c, the required bearing is 2.06 inches thick, and one plate $\frac{3}{8}$ inch and one plate $\frac{11}{16}$ inch thick are used on each side of the post. On the side of the post nearer the track the $\frac{3}{8}$-inch plate is extended in order to attach the floor-beam to it. The number of rivets required to attach the $\frac{11}{16}$-inch plate is $\dfrac{111{,}500 \times \frac{11}{16}}{2\frac{1}{8}} = 36{,}100 \div 3000 = 12.$ The number required to attach the two plates on one side of the post to the channels is $55{,}750 \div 3000 = 19$. Twenty rivets will be used through the pin plates and channel flanges on the outside of the post, and on the inside, or track side, the rivets will be spaced to match the floor-beam rivets. See Plate I.

354. **Post *Dd*.**—At pin D the bearing used is $\frac{3}{4}$ inch thick, or one $\frac{3}{8}$-inch plate on each side of the post. The number of rivets required to attach one of these plates to the flanges

is $9750 \div 3000 = 4$. Six rivets are used through each flange of the channels and the pin plates, as the pitch is usually kept constant at 3 inches centre to centre of rivets and the plates are extended so as to take at least two rivets beyond the point where the web of the channels is cut to allow the counter-rods to pass.

At pin d the bearing is $1\frac{1}{2}$ inches thick, made up of four plates each $\frac{3}{8}$ inch thick, two on each side of the post. One of the plates is extended upwards, to take the floor-beam connection. The number of rivets required to attach one $\frac{3}{8}$-inch plate is $19,650 \div 3000 = 7$, and the number required to attach the two plates on one side of the post to the channel flanges is $39,300 \div 3000 = 14$.

342. **The End Shoes** are shown in Fig. 376. The thickness of pin bearing required has been found to be $2\frac{7}{8}$ inches thick. The bearing pressure on the masonry is limited to 300 lbs.

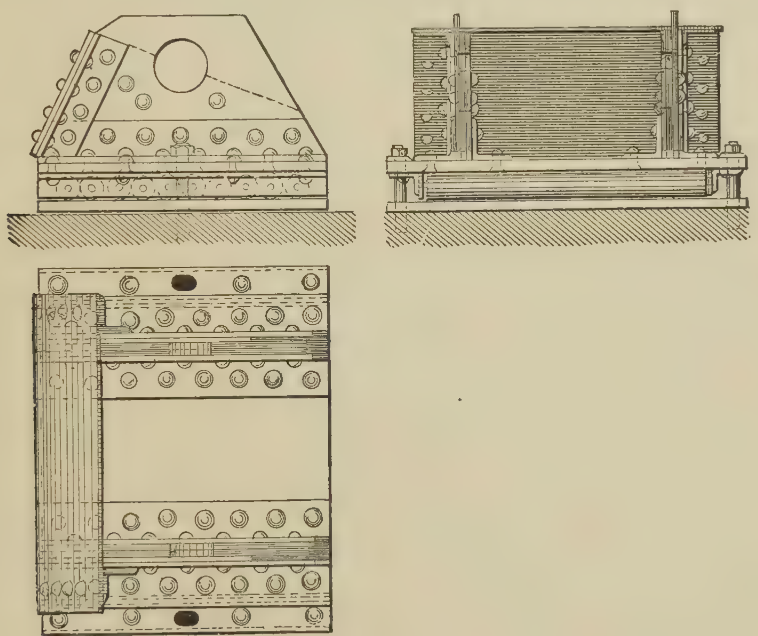


FIG. 376.

per square inch, requiring 575 square inches. The number and length of rollers under the shoe at the expansion end is determined by the formula $p = 250d$, where p is the allowed pressure per linear inch on the roller and d is the diameter of the roller. For $d = 3$, $p = 750$. Therefore $172,500 \div 750 = 230$ inches of 3-inch rollers are required.

The height of the shoe from the sole plate to the pin must be enough to make the ribs of the shoe stiff enough to distribute the pressure uniformly over the masonry or the rollers. The sole and bed plates are usually made $\frac{3}{4}$ inch thick and large enough to give the required bearing area on the masonry. The end reactions of both lateral systems must be transferred to the masonry through the shoe. This requires that they be stiff laterally. This stiffness is usually obtained by the use of a web connecting the two ribs of the shoe, or by a plate over the ends of the ribs corresponding to the top plate of the end post.

343. **Tie Plates and Latticing.**—The specifications require that the tie plate shall be long enough to take one quarter of the total stress on the member through the rivets in one segment. This requires plates on aB or BC long enough to take $\frac{238,100}{4} \div 3000 = 20$ rivets or 5 feet long. The thickness of the tie plates is also limited to one fortieth of the distance

between the rivets in the two segments which the tie plate joins. For the end posts and top chords this requires plates $\frac{1}{2}$ inch thick.

The usual specification for tie plates requires that they shall have a length equal to one and a half times the width of the member. For aB this would be 33 inches. A good rule, and one which requires less material, is to make the tie plates on the inclined end posts, vertical posts, and on the top chords, at the extreme ends only, square, and those at the ends of the intermediate top chord sections 12 inches wide The thickness need never be over $\frac{3}{8}$ inch, and when the distance between the rivets connecting the plates to the two segments is over 20 inches, stiffener angles may be riveted on the plates and a saving in material made.

The pin plates on the vertical posts serve the purpose of tie plates.

The width of lattice bars is fixed by the specifications at $2\frac{1}{2}$ inches for the end post and top chord, $2\frac{1}{4}$ inches for post Cc, and 2 inches for post Dd. The thickness is limited to one fortieth of the distance between rivets for single lattice, and one sixtieth of this distance for double lattice riveted at the intersection of the bars. Bars $2\frac{1}{2}'' \times \frac{3}{8}''$ will be used for aB, BC, and CD, $2\frac{1}{4}'' \times \frac{3}{8}''$ for Cc and $2'' \times \frac{5}{16}''$ for dD. The tie plates for the top struts will be made, as usual, 6 inches wide and $\frac{5}{16}$ inch thick, the lattice bars 2 inches by $\frac{5}{16}$ inch.

344. **Details of the Floor-beams and Stringers** were determined when the calculations were made of the flange sections. The number of rivets required in the connections of the stringers to the beams and of the beams to the posts were previously computed. The detail sketches will sufficiently explain the mode of construction. The end stringers must have bearing enough on the masonry to keep the pressure within the specified limits. They must also be braced together by an X-frame. This detail is shown completely in Plate I.

345. **The Details of the Lateral System.**—The lateral bracing may be truly said to be the *bête noir* of the bridge designer. It is impossible to make any attachment to the trusses which will cause no secondary stresses. The method of attaching the top lateral system used for this truss is one of the best in use, but advantage is taken of the fact that the stresses are small and the members of the truss which act as chords for the wind truss are large and very little affected by an eccentric attachment. The details of the lower lateral system are very common practice and may be said to be as good as any in common use. The attachment was made so far from the lower chord in order to avoid cutting away the lower flange angle of the floor-beam to let the tie bars pass. This will be understood by referring to the detail sketch of joint c. This eccentricity produces a bending moment in aB, Cc, and Dd practically equal to the chord component of the lateral rod into the distance from the plane of the lateral system to the plane of the lower chord pins. A careful analysis of the resulting stresses will show, however, that the stresses per square inch in these members is within the allowed limits for lateral stresses. The details of the pin bearings and the sizes of the pins should be carefully computed by the same methods as those used for the main truss. The attachment to the truss should always have sufficient rivets to take the maximum chord increment, which equals the chord component of the largest rod at the point, and the attachment to the strut or flange of the floor-beam should have sufficient rivets to take the maximum shear at that point. These details will be left to the student to proportion.

Complete detail sketches of this span are shown in Plate I.

CHAPTER XXII.

HIGHWAY BRIDGES.

346. Definition and General Remarks.—Under this general classification of highway bridges is included all bridges used for roadway purposes alone. The factors which control the design of this class of bridges are very different from those affecting the design of a railway structure and make it well-nigh impossible to treat of the highway bridge in general, as each structure usually presents a different problem. The probable maximum live loads in quantity, kind, and frequency of application, the expert attention which the bridge will receive, the ability of the community to pay for a bridge of a capacity beyond their present needs, and, in some cases, the appearance of the bridge when completed are some of the factors which enter into the problem and which the engineer must consider. Usually the lightest structure consistent with absolute safety and one which will require little or no expert attention is required. The economical design of the trusses and of the details of construction result in a larger percentage of saving in a highway than they do in a railway span and are therefore of supreme importance. Where the light live loads and consequently light trusses of the usual highway bridge are taken into consideration it will be seen that for maximum economy of material close, careful, and intelligent designing is necessary. It is claimed to the credit of those engineers who have made this branch of bridge designing a specialty that the highway bridges of this country show more commendable economy of design than do the railway structures. It is a fact that our highway bridges are our only bridges on which much effort has been spent to make the design neat and pleasing in outline.

347. Live Loads.—The live loads which may come upon a highway bridge are a crowd of people, the maximum weight resulting from a closely packed throng being 85 lbs. per square foot, and any concentrated load due to the passage of a heavy wagon or other vehicle over the bridge. In estimating the live load resulting from a crowd of people the probability and also the possibility of a closely packed crowd covering all or part of the span must be taken into account. A good rule is to assume a live load from this cause of 100 lbs. per square foot for spans of 100 feet and under, and 50 lbs. per square foot for all spans over 250 feet, and to reduce the weight per square foot uniformly from 100 to 50 lbs. as the span varies from 100 feet to 250 feet. Consistency would require that the *partial* loads of the longer spans be increased as the length of the load decreases in the above ratio. The above specification would be for bridges subjected to city traffic, and the *upper* limit of 100 lbs. for a span of 100 feet may be reduced for bridges in localities where there is no probability of such dense crowds. The width of the bridge determines to some extent the probability of there being such a weight of people on it.

The concentrated loads are estimated for the special locality and should always include probable heavy loads which the development of the adjacent territory may make necessary. There is, however, a great waste of material in bridges now due to the specification of heavy and improbable, if not impossible, loads.

348. Dead Loads.—The fixed or dead load for a highway span consists of the weight of the iron in the span and of the floor and guards. Owing to the great variety of floors used and the differing specifications as to unit stresses and details, no general rule easy of application has yet been formulated for the weight of iron in highway bridge spans under the various

conditions and specifications. For the ordinary truss span with iron trusses, bracing, and floor-beams and with stringers or floor joists of wood the weight of iron per foot of span with a live load capacity of 1200 lbs. per linear foot is closely approximated by the formula

$$w = 2l + 50,$$

where w = weight of iron per linear foot, and l = the length of the span. This weight does not include the handrails. Each line of handrail weighs usually about 25 lbs. per linear foot. The weight of the joists and flooring can be determined readily from the sizes used. This formula may be used to approximate the weight of iron in spans of a capacity greater than 1200 lbs. per foot by increasing the weight per foot in the same ratio as the capacity is increased, assuming the floor to be as before, plank laid on wooden stringers or joists.

349. **The Various Styles of Floors Used** are as follows:

1st. Plank in one or two layers on wooden joists.

2d. Plank laid on iron joists and a wearing surface of wooden blocks used.

3d. Iron joists covered with corrugated iron or buckle plate on which is laid a bed of concrete to receive the wearing floor of asphalt, granite block, or vitrified brick.

The joists in the first and second cases are spaced from two to three feet apart, depending upon the thickness of the plank flooring and the concentrated loads which may come upon the bridge. For the third style of floor the joists are spaced as far apart as the strength of the corrugated iron or buckle plate flooring will permit. The standard size of buckle plates and the capacity per square foot for different thicknesses may be obtained from the manufacturers on request. The joists for this style are usually spaced three feet or more centre to centre.

350. **Iron Handrails or Fences.**—The ordinary height for iron handrailings is 3 feet 9 inches from the floor level to the top of the handrail. They should be stiff enough to resist any probable force which would tend to bend them out of line or knock them down. Nearly all the standard handrailings used on bridges will fulfil this requirement if braced at distances apart of about 8 feet. The lattice work should be made so that the openings in the fence are not over 6 inches square for the lower half of the fence, and the bottom rail should be within 6 inches of the floor line.

351. **The Allowed Unit Stresses for Highway Bridges** are usually 25 per cent higher than those allowed in railroad structures. The maximum load for which this kind of a bridge is designed is usually rarely applied, and even then its impact is not as destructive as that of a moving train. Wherever the stresses from the assumed loads on a highway bridge are liable to occur frequently and where the impact is appreciable, as in the case of street cars for example, there is no reason why the allowed stresses should differ from those sanctioned by railroad engineers.

352. **The Details of Highway Bridges** should be designed with the same care and with more attention paid to the non-eccentric connection of the several members meeting at a joint than would be given to the design of railway bridge details. The members of the truss are usually smaller and the margin of safety, due to the use of higher unit stresses, is less, so that the secondary stresses caused by eccentric connections have a more destructive effect than they do for a railway bridge. As the details and "non-effective" material such as tie plates and lattice bars are quite a large percentage of the material in a span, it is commendable economy to design very close to the required limits. On page 291 the customary sizes of lattice bars and the spacing of the same are given.

353. **The General Dimensions** to be given to a highway bridge are usually determined by local conditions. The width of roadway is usually made a multiple of 8 feet for each carriageway. The width of the roadway and sidewalks to be provided depends upon the traffic,

as it is desirable to make them wide enough to prevent continual crowding during those hours of the day when the traffic is the greatest. The sidewalk is not often made less than 4 feet wide, or the roadway less than 12 feet. Wheel guards must be placed on the floor on each side of the roadway and so located that when the wheel of a vehicle is running close against it the hub or any part of the vehicle will not strike the iron work of the trusses. This is generally accomplished when the guard is made 6 inches wide.

The overhead clearance or the distance from the top of the floor to the under side of the top lateral bracing for through bridges should be 14 feet or over to allow high loads to pass through. A farmer's load of hay will pass through the ordinary barn door, which is 12 feet high in the clear.

354. **The Panel Length** of a highway bridge in which wooden joists are used is limited to 20 feet or less, depending on the maximum concentrated load which the joist has to carry and the size of joist obtainable. Joists 14 inches high are common sizes and are cheaper than those of greater depth, so that ordinarily the length of panel is limited to that which is the maximum span over which this size will carry the load. For bridges with iron joists the panel length can be varied to obtain the maximum economy in iron alone. Where the joists are supported on the top flange of the floor-beam, as is usual, the size of joist to use may be determined by the permissible fibre stress alone. If the joists are iron and placed between the beams, like the stringers of the span designed in Chapter XXI, they should not be less than one fifteenth of the span, owing to the excessive deflection of shallower joists loosening the rivets in the connection between the joist and the floor-beam.

355. **The Kind of Construction** usually employed in highway bridge construction is I beams or plate girders for spans under 30 feet and riveted or pin-connected trusses for longer spans. The "low" truss or half through bridge is employed for spans of 70 feet and under. Where, however, a deck bridge can be used it is always preferred to the through, as it leaves the deck or floor unobstructed.

356. **The Design of a Highway Span.**—The length centre to centre of end pins will be 105 feet, the panel length 15 feet, and the depth 20 feet. The roadway will be 16 feet wide and the two sidewalks 5 feet wide in the clear. The live load will be assumed at 2400 lbs. per linear foot of bridge. A concentrated load of 10,000 lbs. on two axles 6 feet apart will also be provided for. The allowed unit stresses will be as follows:

Wrought-iron in tension, 12,000 lbs. per square inch.

Steel in tension, 15,000 " " " "

Wrought-iron in compression, $\dfrac{10{,}000}{1 + \dfrac{l^2}{ag^2}}$ lbs. per square inch,

where l = length of member in inches, g = least radius of gyration of the member in inches, and a = 36,000 for two flat ends, 24,000 for one pin and one flat end, and 18,000 for two pin ends.

The extreme fibre stress on steel pins, 22,500 lbs. per square inch.

The bearing stress on pins and rivets, 15,000 " " " "

The shearing stress on pins and rivets, 9,000 " " " "

The extreme fibre stress on yellow pine or white oak, 1200 lbs. per square inch.

For stresses in the lateral system increase the above 50 per cent.

357. **The Design of the Floor.**—The roadway floor will be assumed to be three-inch white oak plank laid on yellow pine joists. The three-inch plank will be laid transversely, and one plank 10 inches wide will carry one wheel of the concentrated load 30 inches without exceeding the allowed fibre stress on white oak. Eight joists will be used, spaced $16 \div 7 =$

2.29 feet or 2 feet 3 inches apart centre to centre. The joists will be figured to carry a live load of either 100 lbs. per square foot or the concentrated load specified. Each joist will have to carry one quarter of the total weight of the concentrated load, as the floor plank are stiff enough to distribute the total load over four joists. The dead load on each joist is 12 lbs. per square foot for the floor plank, and one half of this or 6 lbs. per square foot will be assumed for the joist. The dead load centre moment on the joist is then $\frac{2\frac{1}{4} \times 18 \times 15 \times 180}{8}$ $= 13{,}670$ in.-lbs. The centre moment from the 100 lbs. per square foot live load is $\frac{2\frac{1}{4} \times 100 \times 15 \times 180}{8} = 75{,}940$ in.-lbs. The maximum moment from the concentrated live load is $1000 \times 72^* \doteq 72{,}000$ in.-lbs. The 100 lbs. per square foot live load produces the greater bending moment. The total maximum bending moment, dead and live, is 89,610 in.-lbs., requiring a joist $3'' \times 13''$. The joists are usually sawed to dimensions in even inches, and the width for the roadway joist should never be less than three inches. The weight of the timber in the roadway floor, assuming the weight of the timber to be 4 lbs. per foot, board measure, is as follows:

Floor plank.............	$3'' \times 12''$—16 ft. long	= 48 ft. B.M.	= 192 lbs.
Joists..............eight	$3'' \times 13''$— 1 ft. long	= 26 ft. B.M.	= 104 "
Wheel guards........two	$6'' \times 6''$— 1 ft. long	= 6 ft. B.M.	= 24 "
Hub guards......... two	$3'' \times 8''$— 1 ft. long	= 4 ft. B.M.	= 16 "

Or a total of 336 lbs. per linear foot of bridge.

The sidewalk floor will be two-inch plank laid on three joists for each walk. The live load will be taken at 80 lbs. per square foot. The dead load on the joists is 8 lbs. per square foot for the floor plank and 4 lbs. per square foot assumed for the joist. The maximum bending moment, dead and live, on the joist is $\frac{2\frac{1}{2} \times 92 \times 15 \times 180}{8} = 77{,}600$ in.-lbs., requiring a joist $2\frac{1}{2} \times 12$ inches. The weight of the timber in the sidewalk floors is 164 lbs. per linear foot of bridge. The sidewalk floor plank are usually extended to close the openings between the roadway and sidewalks made by the trusses.

358. The Design of the Floor-beam.—The dead and live loads on the beam are as

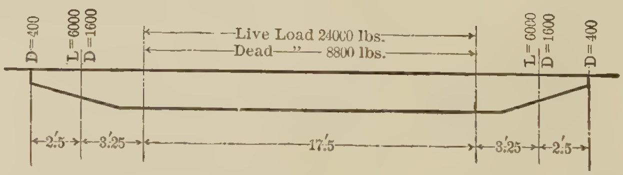


FIG. 377.

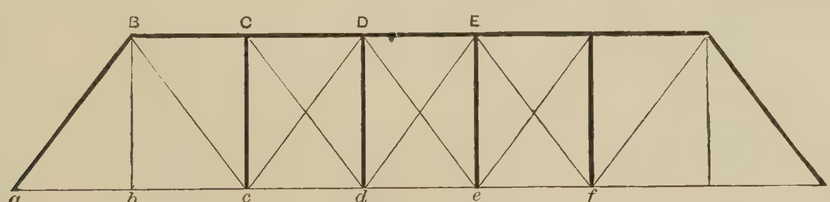
shown in Fig. 377. The maximum bending moment occurs at the middle of the beam where the sidewalks are unloaded and is 64,300 ft.-lbs. The depth of the economical floor-beam, using a web plate $\frac{1}{4}$ inch thick and using one eighth of the gross area of the web as available equivalent flange area, is 20 inches deep. Assuming the depth centre to centre of gravity of the flanges as $18\frac{1}{2}$ inches, the flange stress is 41,700 lbs., requiring 3.48 square inches net area of lower flange or 2.86 square inches net area in the flange angles after deducting the equivalent flange area of the web. Two $3'' \times 3''$ angles weighing 17 lbs. per yard will be used. The top flange will be made the same gross area as the bottom flange. The maximum bending moment from the overhanging load of the sidewalks occurs at the hanger point and is 27,000 ft.-lbs. The flange angles and web plates will be made in single lengths from end to

* See Art. 91. For two equal loads, placed a fixed distance d apart, the maximum moment is one fourth d from the centre and under one of the loads, the other load being three-fourths d on the other side of the centre.

end of the beam. The load on the hanger supporting the floor-beam from the truss pin is 24,400 lbs., and if a unit stress three quarters of that used for long tension members or flanges is used it would require one loop hanger $1\frac{3}{16}$ inches square.

Referring to Plate II, it will be noticed that the lateral rods are in the plane of the top flange of the beam, and that this flange is utilized as the strut for the lateral system. This flange is supported laterally every two feet by the joists, so that the allowed stress from the combination of live, dead, and wind stresses may be as much as 15,000 lbs. per square inch There is, then, no increase of the section of the flange necessary. The floor-beams are riveted to the post to transfer the chord component of the lateral rods. The top flange angles are extended to take the brace from the handrail. Stiffener or distributing angles should be riveted to the web of the beam on each side of the hanger from the truss pin to resist the reaction of the floor-beam. This end reaction is 24,400 lbs., requiring $24,400 \div 2800 = 9$ $\frac{3}{4}$-inch rivets in bearing against the quarter-inch web. Four angles $3'' \times 2'' \times \frac{1}{4}''$ with two vertical lines of five rivets each will be used at each end of the beam. The minimum rivet spacing through the flanges allowable is 4 inches, and is computed as follows: The maximum shear is 16,400 lbs., the ratio of the moment of resistance of the flanges to that of the web is as four to one; hence the amount of flange stress transferred to the flange is four fifths of what it would be if the web were neglected, and therefore $\frac{4}{5}pS = rh$, or $p = \dfrac{5rh}{4S} = 4.0$. In this formula p = rivet pitch, S = the shear, r = value of one rivet in bearing against the web plate, and h = distance between the rivet lines in the top and bottom flanges. The rivets will be spaced 3 inches apart until the bearing points of the first two joists are passed, when the spacing will be changed to 6 inches, using, however, two 3-inch spaces directly under each joist bearing to avoid the use of distributing angles.

359. The Design of the Trusses.—The dead load will be assumed as $2(2 \times 105 + 50) = 520$ lbs. per linear foot for the iron, 50 lbs. per linear foot for the handrailing, and 500 lbs.



Member.	Stresses.				l	g	Area required.	Make-up of Section.	Area	Material.
	Dead.	Live.	Total.	Unit Stress.						
ab–c	− 18.0	− 40.5	− 58.5	15.0			3.9	Two $2\frac{1}{2}'' \times \frac{18}{16}''$ bars	4.0	Steel
cd	− 30.0	− 67.5	− 97.5	"			6.5	Two $4'' \times \frac{18}{16}''$ bars	6.5	"
de	− 36.0	− 81.0	− 117.0	"			7.8	Two $4'' \times 1''$ bars	8.0	"
Bc	− 20.0	− 46.9	− 66.9	"			4.5	Two $3'' \times \frac{3}{4}''$ bars	4.5	"
Cd	− 10.0	− 30.0	− 40.0	12.0			3.3	Two $2\frac{1}{2}'' \times \frac{11}{16}''$ bars	3.4	Iron
De	0.0	− 16.9	− 16.9	"			1.4	One $1\frac{3}{16}''$ square rod	1.4	"
Ef	+ 10.0	− 7.5	0.0	"			0.0	One $\frac{7}{8}''$ tie rod	0.8	"
Bb	− 6.0	− 18.0	− 24.0	"			2.0	Two $1''$ square rod	2.0	"
Cc	+ 10.0	+ 24.0	+ 36.0	7.0	240″	2.73″	5.1	Two $7''$ ⊔s, $25\frac{1}{2}$ lbs. per yd.	5.1	"
Dd	+ 2.0	+ 13.5	+ 15 5	5.5	240	2.0	2.8	Two $5''$ ⊔s, 18 lbs. per yd.	3.6	"
aB	+ 30.0	+ 67.5	+ 97.5	7.6	300	4.0	12.8	One $12'' \times \frac{1}{4}$ top plate Two $9''$ ⊔s, 43 lbs. per yd. Two $2\frac{1}{2}'' \times \frac{7}{16}''$ flats	13.8	"
BC	+ 30.0	+ 67.5	+ 97.5	9.0	180	3.4	10.8	Two $9''$ ⊔s, 54 lbs. per yd.	10.8	"
CD	+ 36.0	+ 81.0	+ 117.0	9.2	180	3.3	12.7	Two $9''$ ⊔s, 64 lbs. per yd.	12.8	"
DE	+ 36.0	+ 81.0	+ 117.0	9.2	180	3.3	12.7	Two $9''$ ⊔s, 64 lbs. per yd.	12.8	"

The stresses in the table are given in thousand-pound units. + denotes compression. — denotes tension.

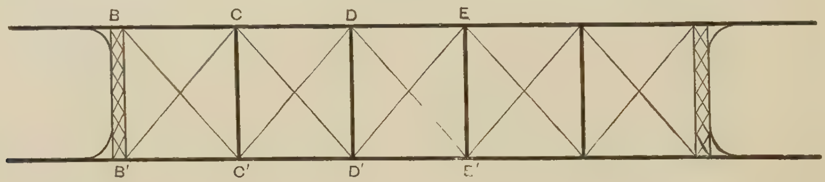
per linear foot for the floor and joists, making a total of 1070 lbs. per linear foot. The panel concentration from this for one truss is $\frac{1}{2}(1070 \times 15) = 8000$ lbs. nearly. Of this, 2000 lbs. will be assumed as concentrated at the top chord joints and 6000 lbs. at the bottom chord joints. The live load is 2400 lbs. per linear foot or 18,000 lbs. per truss panel.

The table on page 347 gives the stresses, required areas, and make-up of the sections.

FIG. 378.

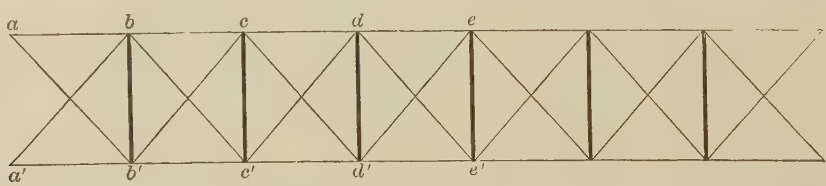
The member *aB* was made as shown in Fig. 378 in order to use a top plate to transfer the wind stress down the post and at the same time keep the pin centres in the middle of the channel webs. The sections *BC*, *CD*, and *DE* have latticing on both their top and bottom flanges, and as they are perfectly symmetrical the neutral axis is in the centre of the webs of the channels.

360. The Wind Bracing.—The top lateral bracing will be proportioned to resist a wind presssure of 150 lbs. per linear foot of the bridge.



Member.	Stresses.		l	g	Area required.	Make-up of Section.	Area.	Material.
	Wind.	Unit Stress.						
BB′						Four 3″ × 2″ × ¼″ Ls	4.8	Iron
CC′	+4.500	7.0	200	1.15	.64	Two 3″ ⊔s, 15 lbs. per yd.	3.0	"
DD′	+2.250	7.0	200	1.15	.32	Two 3″ ⊔s, 15 lbs. per yd.	3.0	"
BC′	−5.9	18.00			.33	One ⅞″ tie rod	.60	"
CD′	−3.0	18.0			.17	One ⅞″ tie rod	.60	"
DE′	0.0	18.0			.00	One ⅞″ tie rod	.60	"

The bottom lateral bracing will be proportioned to resist a static wind force of 150 lbs. per foot of bridge, and in addition a moving wind force of 100 lbs. per foot of bridge.



Member.	Stresses.		Area required.	Make-up of Section.	Area.	Material.
	Wind.	Unit Stress.				
ab′	15.0	18.0	.83	One $1\frac{1}{16}$″ tie rod	.89	Iron
bc′	9.2	"	.51	One ⅞″ tie rod	.60	"
cd′	5.7	"	.32	One ⅞″ tie rod	.60	"
de′	1.5	"	.08	One ⅞″ tie rod	.60	"

The details for this span may be worked out by the student. Complete detail sketches are given in Plate II.

CHAPTER XXIII.

THE DETAIL DESIGN OF A HOWE TRUSS BRIDGE.

361. The Howe Truss has proved the most useful style of bridge ever devised for use in a new and timbered country. It is still very largely used in America for both highway and railway purposes. Railroad bridges of this kind are usually built on the ground by squads of "bridge carpenters," and are very cheap. They are also rigid and perfectly safe if carefully inspected for evidences of decay. They are built wholly of timber except the vertical and lateral tie rods and certain joint castings, splicing members, bolts, etc. They can be put together in such a way that any single stick may be removed and replaced while in service without endangering the structure. One of the standard Howe truss drawings of the Chicago, Milwaukee, and St. Paul Railway is given in Plate III, and the bill of materials at the end of this chapter. This plate shows a span 147′ $2\frac{1}{4}$″ long, composed of thirteen panels of 10′ $11\frac{1}{4}$″ at the bottom chord and 10′ $11\frac{1}{2}$″ at top chord, the height, inside to inside of chords, being 25′. Similar standard plans of the same height and panel length are used for lengths, diminishing by single panels, down to seven, or for a length of 81′ $6\frac{3}{4}$″ centre to centre of bottom chord joints. All these standard plans and the corresponding bills of materials are lithographed and placed in the hands of the bridge carpenters, to work by. All ends are square-sawed, and there is not a mortise or tenon in the whole structure.

Sometimes the timber bottom chord is replaced by eyebars, when it is called a Combination Bridge. But since timber is much stronger in tension than in any other way, there is no good reason for doing this.

At 30 dollars per M, the cost of a cubic foot of timber, in a bridge, would be 36 cents, while the cost of a cubic foot of iron would be about 20 dollars. The working stress on timber may be as much as one tenth of that on iron, so that the relative first cost of timber and iron structures is about as 1 to 4 or 5.

When bridges are to be erected far from existing lines of railroad, iron bridges become impracticable, especially when timber is convenient.

The designing of a Howe truss bridge is mostly a matter of joints and splicing. The timber sections are not computed with that care that is used in iron structures, but material is used liberally and usually far in excess of what the formulas would give. For this reason wooden or combination bridges should not be let out by contract under general specifications, such as are used for iron bridges, unless all the sizes are specified on standard typical drawings like that of Plate III.

By referring to the Table of Strength of Materials at the end of the volume, it will be seen that timber is relatively very weak in lateral compression and in shearing along the grain. For this reason a timber post or strut should never bear on the side of another timber, but always on a wrought- or cast-iron plate, which in turn presents a greatly increased surface to the lateral face of the wood.

362. Chord Splices.—The chords are made up of as long timbers as possible, and but one stick spliced at any section. By referring to Plate III and to the bill of materials accompanying it, it will be seen that the standard length of stick in the upper chord is 43′ $1\frac{1}{4}$″ (cut from 44′ sticks) and in lower chord it is 65′ $7\frac{1}{2}$″ (cut from 66′ sticks), the only variation being for the supplementary end sections. The chords are packed twice in each panel, be-

tween the angle-blocks, and the splice is always made at one of these points. The timber is always sawed for these bridges on special orders, and sticks of these lengths can be obtained in the South, West, and Northwest.

The upper chord splice consists simply of square abutting ends, at the middle of pine packing blocks $2\frac{1}{4}$ inches thick and 10 inches long, dressed, and set into the chord sticks $\frac{3}{4}$ inch on each side, thus leaving a $\frac{3}{4}$-inch air-space between the sticks. When the outside stick is spliced three bolts are used in place of two, as shown on the plate.

The lower chord splice offers more of a problem. It was formerly the custom to make this splice by means of two oak fish-plates notched into the sides of the stick as shown in Fig. 379. There were five ways in which this joint might fail.

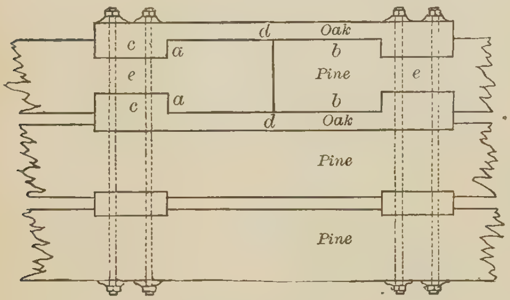


FIG. 379.

1st. At *aa*, by crushing down the fibres.

2d. At *bb*, by shearing off the shoulders of the stick, which would usually be white pine.

3d. At *cc*, by shearing off the shoulders of the splice plates.

4th. At *dd*, by rupturing the splice plates in tension, or partly in shear if they were somewhat cross-grained.

5th. At *ee*, by rupturing the main stick in tension.

These methods are given in the order of their most common occurrence.

Since strength against rupturing at *aa* is only obtained by sacrificing that at *ee*, it is evident that the greatest strength of the joint, so far as these two methods of failure are concerned, is obtained when we have notched in so far that we have an equal strength in these two ways. The endwise crushing strength of white pine is not less than 4000 lbs. per square inch, and the tensile strength is not less than 7000 lbs. per square inch. To realize the greatest strength at this section, therefore, the stick should be notched more than a quarter of its width. To provide for the case of cross-grained wood at this point, it is a good rule to notch one fourth the thickness of the stick on each side.

The shearing strength of white pine is only about 400 lbs., or say one tenth of the strength in crushing endwise. Hence the section in shear at *bb* should be ten times the depth of the notch, or $2\frac{1}{2}$ times the thickness of the stick. If the splicing timbers are also of white pine the above results apply to them too, but if they are of selected, straight-grained, seasoned oak, or better of long-leaf yellow pine (*pinus palustris*), then the thin portions at *dd* need be only about one sixth or one eighth of the thickness of the main timber, and the length of the shoulder, *cc*, only one half that on the white pine timber itself at *bb*. Thus for splicing a white pine stick 8 inches thick with white pine splices, the joint would be 80 inches long and the splices made of 4-inch stuff, notched out 2 inches. If made of long-leaf yellow pine or of straight-grained oak, the joint would be 60 inches long, made of $3\frac{1}{2}$-inch stuff and notched out 2 inches, leaving $1\frac{1}{2}$ inches thickness at *dd*.

No reliance should ever be put upon the bolts. They serve simply to hold the parts together, and would not come into action at all until there had been considerable movement, and then they would act very imperfectly and to an unknown extent. *A good joint will develop its full working strength without any appreciable distortion.*

The great length of these timber splices, and the uncertainty arising from imperfect workmanship and from cross-grained material, which is apt to be more or less wind-shaken or season-checked, so as to offer little resistance to shearing along the grain, has resulted in an entirely new kind of tension splice now adopted on many of our leading railways of the West.

The iron splice is composed of two cast-iron plates on each side of the sticks to be joined.

with short cylindrical spurs fitting into corresponding bored holes in the stick, these being held together by two clamp bars of wrought-iron, having forged hooks at their ends, all as shown in Plate III. The cast-iron plates (and hence the sticks) are drawn tightly together by means of a clamp key which is driven at one end of each clamp bar, the key-seat on the cast-iron plate being inclined to the plane of the key as shown. Each clamp plate here shown has twenty-one spurs, 1 inch long and $1\frac{1}{8}$ inches diameter, making a total area of $23\frac{5}{8}$ square inches bearing area for each plate, or $47\frac{1}{4}$ square inches for each stick. The area of the cross-section of these sticks is 96 inches, so that the compression bearing area is about one half the section of the stick. But since this area is not all cut out at the same section, this bearing area might well be increased, even to the extent of making the spurs $1\frac{3}{4}$ inches long, thus making the bearing area for one stick 74 square inches.

The clamp bar has a minimum section of 3 square inches, but with longer spurs it might be widened to 4 inches and its section made 4 square inches. This would give 8 square inches of section of bar to 74 square inches of bearing area on the wood, or 9 square inches bearing to 1 square inch of iron in tension, which is about the proper ratio.

This arrangement of iron clamp blocks and bars, with tightening key, is a most excellent one and should entirely replace the old wooden fish-plates. By driving out the key, and slightly spreading the chord members, any single stick can be taken out and replaced by a new one.

363. The Angle Blocks and Bearing Plates.—Since timber offers very slight resistance to crushing across the grain, it is necessary to give to all the web struts and vertical tension rods very large bearings on the chords, at both top and bottom, in order to prevent the crushing down of the fibres. The resistance of white pine to crushing across the grain is only about 500 lbs. per square inch, and of yellow pine from 800 to 1000 lbs. The working stress should not be over 300 lbs. for white pine and 500 lbs. for yellow pine.

The angle blocks shown in Plate III are all 18 inches wide and of a length equal to the total width of the chords. The top and bottom blocks are exactly alike. Two ribs project $1\frac{1}{4}$ inches from the lower faces, which are notched into the chords to take the lateral thrust of the struts, and with the patterns here shown the vertical components of the thrust are transmitted through the chords by lateral compression on the timbers. This is well enough if the bearing plates at top and bottom are of sufficient size, which in this design they are.

The bearing plates are here composed of 10- and 12-inch channel bars the full width of the chord sections. Thus in the 147-foot span shown in Plate III the load on the end verticals may be taken at 150,000 lbs. for each truss. The bearing plate here is $12'' \times 43'' = 516$ square inches. At 300 lbs. per square inch this would carry a load of 154,800 lbs., which shows it to be of sufficient size. Being made of channel iron, it is presumed to be stiff

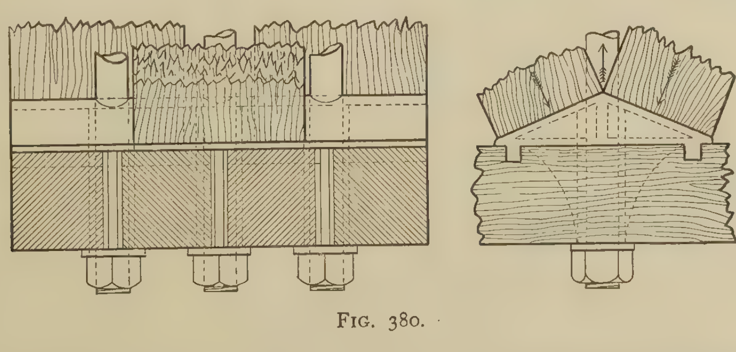


FIG. 380.

enough, if large washers are used under the nuts, to spread the load uniformly over the bearing area.

Another method of transmitting this thrust through the chord section is to extend the

web of the angle blocks entirely through the chords to receive the bearing from the nuts directly. This enables this load to be transmitted wholly through the cast angle block and allows no lateral compression to come upon the timber chords. In this case the shrinkage of the bottom chord timbers would allow the angle block to stand off from its seat somewhat and make a good reservoir for rain-water, which would rapidly rot the timber at a point where it could not be inspected. A close bearing on cast-iron makes a very lasting joint. It would seem, therefore, that the joint shown in Plate III is the best which could be devised.

364. **Miscellaneous Details.**—*Dowels.*—All the web struts are held in place by iron dowel pins $\frac{1}{2}$ inch in diameter by 9 inches long, fitting tight into bored holes in the ends of the struts and entering corresponding holes in the angle blocks. They simply prevent the timbers from slipping out of place. All bridge timbers are apt to be more or less green when put into place, and although most of the shrinkage is in a circumferential direction, there is some shrinkage lengthwise. All Howe truss bridges, therefore, should be tightened up frequently by screwing up the nuts on the vertical rods. If these are not kept tight the counter-struts will become loose and slip off their seats if not held to place by dowel pins or otherwise.

The lateral wind bracing consists of diagonal timber struts and transverse iron tie rods, in the planes of both top and bottom chords. The angle blocks for these lateral systems are shown in Plate III. Instead of dowels there are projecting flanges on the lower sides of the angle blocks which hold the diagonal struts in place.

Washers and Packing.—The upper chord is packed with seasoned pine blocks except at the lateral rods, where $\frac{3}{4}$-inch cast washers (P) are used. The lower chord is packed with the cast washer Y except along the lateral truss rod, where the plane washer P is used the same as in the upper chord. This Y washer is intended to act so as to transfer tensile stress from one stick to another, and in this way cause the chord to act together as one solid stick. For this purpose these washers are rimmed, and these rims are set 1 inch into the timber on each side. The sockets for these circular rims are cut out with a special tool, truly circular, and the projecting rims of the washer are given a "draw" of $\frac{1}{32}$ inch on each side, so that when the packing bolts are tightly drawn, these rims are set snugly into the wood on either side, to a depth of 1 inch, on an outside diameter of 6 inches, there being two of these at each packing section. These sections occur every $5\frac{1}{2}$ feet, so that every $5\frac{1}{2}$ feet a common union of 12 square inches area is made between each pair of adjacent sticks. It is this transfer of stress laterally from one stick to another which causes the total weakening effect of the bottom chord splices to be limited to that due to the splicing of one stick. Thus if stick two has been spliced at a sacrifice of half its strength, and 11 feet farther along stick three is to be spliced, there will be four of these rimmed 6-inch washers introduced on each side of stick three, after passing the splice in two, and these will transfer stress from stick three to sticks two and four, with an equivalent area of 48 square inches of timber, leaving only about a half load in three to be carried past the joint over the two sets of splice bars and blocks. This rimmed washer is a much more efficient packing and transfer block than the solid oak packing blocks notched into the adjacent timbers which were formerly common.

Lip Washers.—To further assist in holding the end angle blocks to place cast-iron lip washers are introduced (pattern I) which have a lip projecting over the free edge of the block, and are bolted through the chord. Each end block on the bottom chord has four of these back of it, the next one three, and the third from the end two. A few are used on the end upper chord blocks also. These washers also hold down this side of the block and so assist in distributing the load, which comes wholly on the opposite slope of the block, evenly over the whole base of the block.

Collision Struts are introduced to stay the end inclined struts in case they should be struck by a derailed car, engine, or projecting timber.

Portal Braces 6″ × 8″ are used as shown in Plate III, being well attached both to the

inclined end posts and to a portal strut at top 10″ × 12″, which in turn is bolted and shouldered upon the ends of the top chord. The absence of any very efficient wind, sway, or portal bracing (and formerly the total absence of any such bracing whatever) has been the cause of many failures of Howe truss bridges, and is still an objection to them. It has been very common to cover in Howe truss bridges on highways, and when this is done and all portal or sway bracing omitted, a comparatively small wind pressure would wreck the structure.

The Floor-beams and Stringers are of timber and usually rest directly upon the bottom chord. Sometimes they are hung below it, as shown in Plate III. In this case a large bearing area must be provided both on top of the chord and at the base of the beam, to prevent the cutting in of the washers. The stringers are also of timber, held well apart by means of packing washers or thimbles, three inches long, thus giving not only a free circulation of air, but allowing live coals from the engine an opportunity to roll off and drop between them. The remaining portion of the floor system is not peculiar to this style of truss.

Corbels are usually introduced under the ends so as to save the bottom chord timbers and keep them away from the sills or wall plates. They are packed with the plain washers and bolted to the bottom chord.

365. **Working Stresses.**—As stated in Art. 361, the timber members of a Howe truss are not usually nicely computed and dimensioned as is always the case with wrought-iron and steel, but certain maximum loads should not be exceeded.

Prof. Lanza, who is now the leading authority on the strength of timber, gives as the results of actual tests on large beams and columns the following:*

Working Fibre Stress in Cross-breaking, using a Factor of Safety of Four, on Actual Tests of Full-sized Beams.

White pine and spruce........................	750 lbs. per square inch.
Georgia (long-leaf) yellow pine.................	1200 " " " "
White oak..	1000 " " " "

Columns.—For the ultimate strength of timber columns use the formulas given on p. 151, which are based directly on Lanza's published results of full-sized column tests. A factor of safety of four or five should be sufficient.

In applying the formulæ to composite columns made up of several sticks bolted together at intervals, give to each stick its proportionate share of the total load to be carried over that member, and then assume that it stands alone and unsupported. This is the only safe rule. Even though they are firmly bolted, with packing blocks or washers notched into the sides, these grow loose in time and do not resist initial lateral bending. They should never be assumed to act as one solid stick.

Shearing.†—For shearing and crushing it is probably safe to use a factor of safety of two or three. Pending the final reports on the U. S. Timber Tests now in progress, use for working values of the shearing strength along the grain, for

White pine..	100 lbs. per square inch.
Long-leaf yellow pine............................	200 " " " "
Short-leaf " "	150 " " " "
White oak..	300 " " " "

* Applied Mechanics, Fourth Ed., pp. 670, 677, 684, and 685.

† These values of working stresses in shearing, crushing, and tension are taken from the results of the U. S. Timber Tests now being conducted at Washington University by Prof. Johnson for the Forestry Division of the Agricultural Department.

Crushing across Grain.

Take for working values, on seasoned timber,* for

White pine....	300 lbs. per square inch.
Long-leaf yellow pine........................	500 " " " "
Short-leaf " "	450 " " " "
White oak.......	1000 " " " "

Crushing Endwise (Short Blocks).

Take for working stress, for dry seasoned timber: *

White pine	2500 lbs. per square inch.
Long-leaf yellow pine........................	3000 " " " "
Short-leaf " "	2800 " " " "
White oak.....................................	2750 " " " "

Tension.—Wood fibre is much stronger in tension than in any other way, and as a result it may be said that wood seldom or never breaks in pure tension in actual service. In fact, it is very difficult to break it in tension in a laboratory test. In structures timber usually fails in shearing, in cross-breaking, or in crushing. It must always be assumed, in long sticks in tension, as in the bottom chord of a Howe truss, that the grain runs more or less across the line of the stick, and a liberal allowance must be made for the reduction of the section by framing it, so that although the tensile strength of the fibre may be ten (or in the case of long-leaf yellow pine nearly twenty) thousand pounds per square inch, yet it is not wise to rely on a working stress in tension of more than about 1000 or 2000 lbs. per square inch.

366. **Weights and Quantities.**—The following table of loads, quantities, and weights is taken from a complete scheme of stress diagrams and sizes for both deck and through Howe truss bridges from 30 feet to 150 feet in length, for the Oregon Pacific Railway (A. A. Schenck, Chief Engineer), published in *Engineering News*, April 26, 1890. The live load assumed was two 88-ton engines followed by a train load of 3000 lbs. per foot. For deck bridges add 20 per cent to the weight of the timber and deduct 20 per cent from the weight of the wrought-iron.

WEIGHTS AND QUANTITIES FOR HOWE TRUSS BRIDGES.

Length of Span.	Style of Truss.	Height of Truss.	No. of Panels.	Assumed Loading per Foot.			Total Load per Foot. Lbs.	Estimated Quantities.			
									Wrought-iron.		
				Trusses. Lbs.	Floor. Lbs.	Train. Lbs.		Timber. Feet B. M.	Rods not Upset. Lbs.	Rods Upset. Lbs.	Cast-iron, Lbs.
30 feet	Pony	9 feet	4	360	500	5,060	5,920	10,160	2,170	......	970
40 "	"	11 "	4	400	500	4,600	5,500	13,360	2,960	......	1,210
50 "	"	11 "	6	450	500	4,200	5,150	19,020	5,610	......	2,880
60 "	"	12 "	6	540	500	3,860	4,900	22,780	6,790	......	3,660
70 "	"	13 "	7	600	500	3,640	4,740	29,930	9,260	8,210	8,260
80 "	"	14 "	8	620	500	3,600	4,720	35,390	11,660	10,260	9,970
90 "	"	15 "	9	720	500	3,560	4,780	42,710	15,170	13,440	12,530
90 "	Through	25 "	8	720	500	3,560	4,780	41,880	17,880	15,150	12,260
100 "	"	25 "	9	800	500	3,500	4,800	48,890	22,580	18,950	14,290
110 "	"	25 "	10	880	500	3,400	4,780	54,770	25,820	22,290	15,930
120 "	"	25 "	11	940	500	3,300	4,740	62,040	30,890	26,010	18,290
130 "	"	25 "	12	1,000	500	3,200	4,700	70,130	37,050	30,180	20,830
140 "	"	25 "	13	1,050	500	3,150	4,700	78,160	40,820	33,020	23,210
150 "	"	25 "	14	1,100	500	3,100	4,700	86,630	48,090	39,140	27,060

* If timber may be green or wet use one half these values. See Johnson's *Materials of Construction*, Chapter XXXII.

STANDARD, 1891.

CHICAGO, MILWAUKEE & ST. PAUL RAILWAY.
BRIDGE AND BUILDING DEPARTMENT.
BILL OF MATERIAL FOR STANDARD HOWE TRUSS BRIDGE. (PLATE III.)

Drawing No. 4006. 147 ft. 2¼ in. SPAN.

TIMBER.

Bill	No. of Pieces.	Description.	Section in inches.	Length in feet.	Cut to Length in feet and inches.
A	16	Top chord	8×12	44	Two 43′ 1¼″ Fourteen 43 10
	2	" "	8×12	38	37′ 7½″
	2	" "	8×12	34	32 1¾
	2	" "	8×12	28	26 7¾
	2	" "	8×12	22	21 2¼
	4	" "	8×12	16	Two 15′ 8¼″ Two 10 2¾ Two 4 8¾
	3	" " fillers	4×12	14	
	10	Bottom chord	8×15	66	65′ 7½″
	4	" "	8×15	52	51 8⅝
	4	" "	8×15	42	40 9⅝
	4	" "	8×15	30	29 10⅛
	2	" "	8×15	16	Four 7′ 11⅝″
	2	" " fillers	4×15	16	
	8	Braces	14×14	28	
	8	"	12×14	28	
	10	"	12×12	28	
	8	"	10×12	28	
	16	"	10×10	28	
	12	"	8×10	28	
	12	"	8× 8	28	
	2	Portal struts	10×12	20	
	4	" braces	6× 8	14	
	4	Collision struts	6×14	14	
	2	Stringers	5×12	18	17′ 7″
	4	"	5×12	16	Two 14′ 11¼″ Two 9 3½ Two 6 7¾
	54	"	5×12	14	Fourteen 13′ 11¼″ Forty 13 7
	4	"	5×12	12	11′ 11¼″
	8	Corbels	8× 8	10	Sixteen 5′ 10″
	4	Wall plates	12×12	26	
	140	Top chord packing blocks	2½×12		0′ 10″
	126	Bridge ties	6× 8	12	
		Guard rail	5× 8	310	
B	4	Lateral braces	8×10	18	
	16	" "	8× 8	18	
	16	" "	6× 8	18	
	12	" "	6× 6	18	
	54	Floor-beams	10×16	20	
C	8	Lateral braces	8× 8	18	
	4	" "	6×12	18	
	8	" "	6×10	18	
	16	" "	6× 8	18	
	12	" "	6× 6	18	
	50	Floor-beams	10×16	20	
	4	Blocking on wall plates	8×8	12	

WROUGHT-IRON.

Bill	No. of Pieces.	Description.	Section in inches.	Length in feet and inches.
A	24	Truss rods	2½ diam.	27′ 11″
	24	" "	2¼ "	27 10½
	28	" "	2 "	27 10
	12	" "	1⅝ "	27 9½
	2	Lateral rods	2 "	20 11
	4	" "	1⅞ "	21 7
	4	" "	1¾ "	21 6½
	4	" "	1½ "	20 9½
	4	" "	1⅜ "	20 9
	4	" "	1¼ "	20 8½
	4	" "	1⅛ "	20 8
	34	Truss packing bolts	¾ "	3 9
	236	" " "	¾ "	3 1
	12	Brace " "	¾ "	3 0½
	8	" " "	¾ "	2 11½
	14	" " "	¾ "	2 7½
	16	Portal brace "	¾ "	1 10½
	16	" " "	¾ "	1 8½
	12	" " "	¾ "	1 4½
	80	Stringer packing "	¾ "	1 3½
	64	Guard rail "	¾ "	0 11¼
	16	Lip washer "	¾ "	2 2
	24	" " "	¾ "	1 6
	8	" " "	¾ "	1 3
	32	Wall plate drift	¾ "	1 10
	192	Dowels	½ "	0 9
	8	Bearing channels, No. 7		
	8	" " " 8		
	8	" " " 9		
	8	" " " 10		
	8	" " " 11		
	8	" " " 14		
	18	Clamps with keys	3 × 1	6 0
	195	Spikes	½ sq.	0 9
		6½ inches of thread on each end of truss and lateral rods		
B	52	Floor-beam bolts	¾ diam.	2 9½
	100	Stringer drift bolts	¾ "	1 6
	8	Floor-beam " "	¾ "	1 10
C	200	Floor-beam bolts	1⅜ diam.	2 10½
	106	Stringer drift bolts	¾ "	1 6
	72	Plates marked BP 1	6 × ½	2 10
	16	" " BP 2	6 × ½	2 10
	12	" " BP 3	6 × ½	2 10

CAST-IRON.

Bill	No. of Pieces.	Description.	Pattern Mark.
A	4	Angle blocks	X 3
	32	" "	X 4
	4	" "	X 5
	12	" "	X 6
	52	Lateral "	Z
	18	Clamp "	R 1
	18	" "	L 1
	180	Packing washers for chords	P
	240	Lower chord packing washers	Y
	44	Packing washers for counters	V
	80	Stringer packing blocks	S
	48	Lip washers	T
	4	Standard washers for 2 in. bolts	
	8	" " " 1⅞ " "	
	8	" " " 1¾ " "	
	8	" " " 1½ " "	
	8	" " " 1⅜ " "	
	8	" " " 1¼ " "	
	8	" " " 1⅛ " "	
B	1088	Slotted washers for ¾-in. bolts.	
C	200	Standard washers for 1⅜-in. bolts	
	984	Slotted " " ¾-in. "	

NOTE.—For spans with supported floor-beams use bills A and B, and for those with suspended floor-beams use bills A and C. Cross ties and guard rails to be of white oak; top chord packing blocks of dry white pine; all other timber of white or Norway pine or Douglas fir, as directed by the engineer. Truss rods to have 6½ inches of thr ad on each end with square nuts, and bolts to have 2½ inches of thread with square heads and nuts. Screw threads cut on original size of rods. Details of wrought- and cast-iron are shown on drawing No. 2854. Quality of material must conform to the standard specifications of the bridge and building department. When spans rest on masonry omit from the bill the item of 32 wall plate drift bolts, ¾ inch diameter by 1 foot 10 inches long.

367. **Long Span Howe Truss Bridges.**—In Plate IIIA are shown the details of a Howe truss bridge of 250 feet span, designed and built for the Oregon division of the Southern Pacific Railway by Mr. W. A. Grondahl, Resident Engineer. These bridges are constructed from "Oregon fir" timber, it being practicable to obtain large sticks of this wood as long as 70 feet. This enables a new departure to be made in wooden bridges, and Mr. Grondahl has solved the problem in an eminently satisfactory manner. On the Pacific coast this timber is cheap, while iron is very dear, and hence it proves to be economy to build of the cheaper but shorter-lived material.

The drawings given in Plate IIIA very clearly indicate the method employed. The live load for which the bridge was designed consisted of Cooper's Class Extra Heavy A (see p. 79), or of two 100-ton engines, followed by a train load of 3000 lbs. per foot.

The peculiar features of this new Howe truss are:

First. Its great height (45 ft. out to out), and its long panels (31 ft. 3 in.), which are, however, subdivided in the lower chord, as in the Baltimore truss, Fig. 97, Art. 78.

Second. The use of eyebars and large steel pins in making the bottom chord splices.

Third. The use of timbers 10 in. × 21 in. × 63 ft. long in making up the chord members, and of 19 in. × 20 in. × 50 ft. long in the web system.

Fourth. The omission of all counter-braces where they are not required by the analysis.

In the working out of this design the following features should be noted:

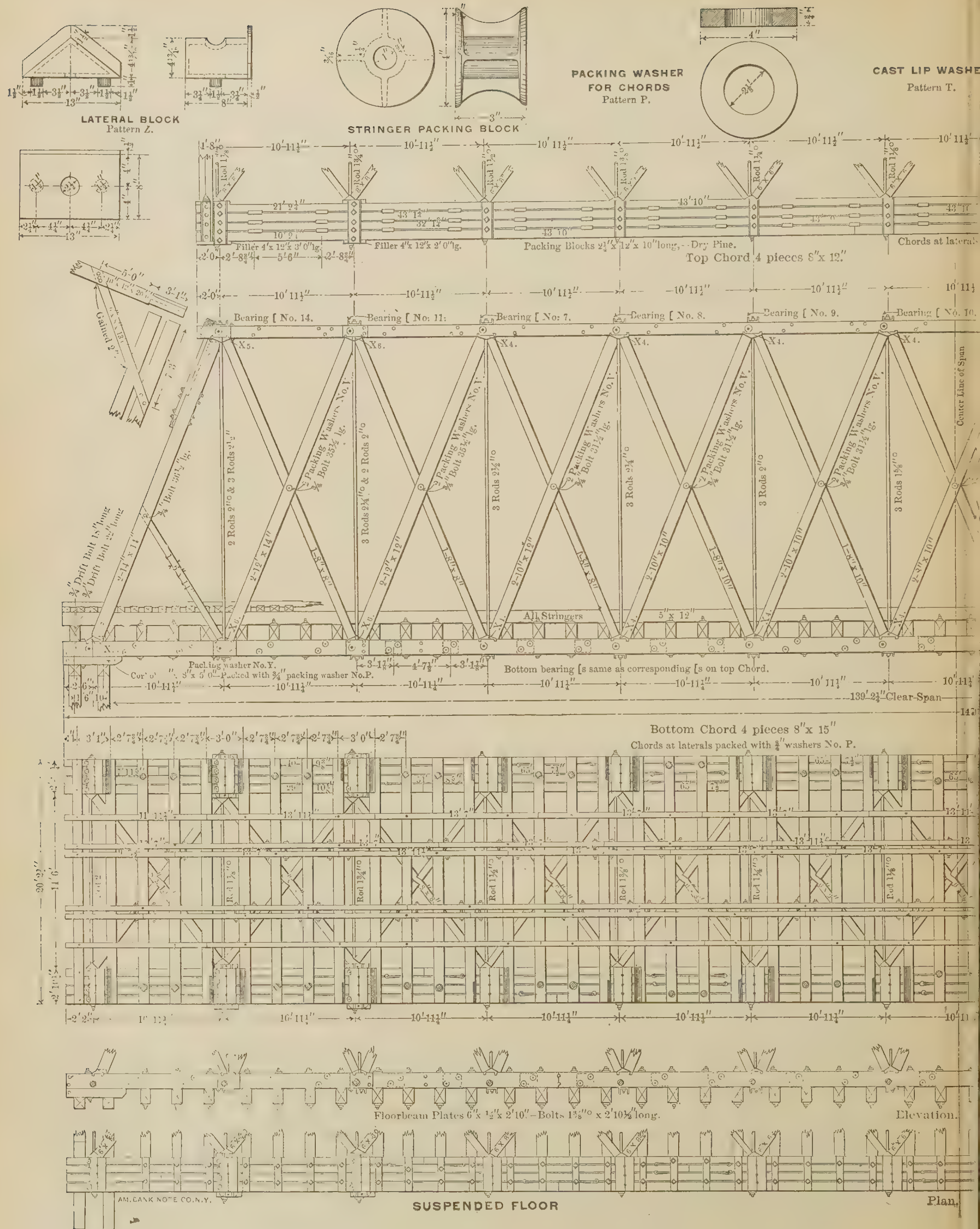
The Suspension Joint in the middle of the web braces, as shown in Figs. 6, 7, and 8. The suspension stirrups hang from a spool 7 in. in diameter, on the top of which rests a small strut for sustaining the weight of the upper chord, which member is held in place by an inverted stirrup passing under the spool. The spool is held in place by the inclined parts, into which it is seated to a depth of 6 inches at each end, as shown in Fig. 6.

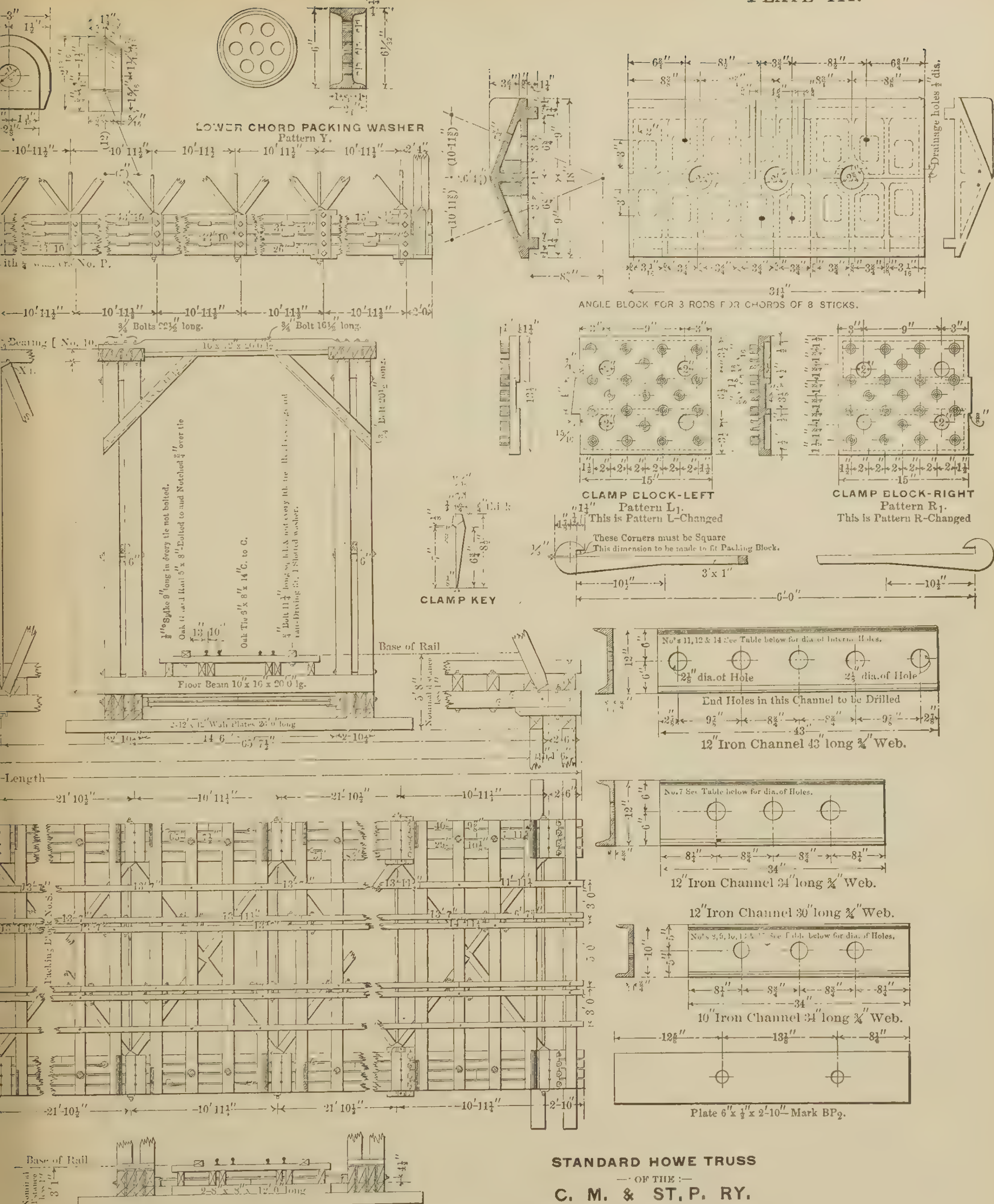
The Bottom Chord Splice as shown in Figs. 9 to 15. Two eyebars, from five to eight feet long, are laid beside the member at the splice, and $3\frac{1}{8}$-inch pins used, which make a snug fit in the timber. To prevent these pins from bending, and to increase the bearing area on the wood, other pins are inserted in front of the main pin, and cast-iron plates, one inch thick (Figs. 14 and 15), set in so as to make a close fit, thus distributing the pull on the eyebars upon three, four, five, or six pins, as the pull increases from the end towards the centre, as shown in Figs. 10, 11, 12, and 13. After these plates and pins are all in place the whole combination is tightened up by turning an eccentric collar on the pin through one end of the eyebars, as shown in Fig. 9. The small circles there shown are the recesses into which are fitted the lugs of the spanner used to turn these collars 180°, thus drawing the end pins about $\frac{1}{8}$ inch nearer together. This takes the place of the "clamp key" used in the C., M. & St. P. designs, Plate III. The ends of the spliced sticks are held in place by a one-inch bolt and washers placed vertically in the joint, as shown in Fig. 10.

The Floor-beams are bunched as near to the bottom chord joints as possible, three 10″ × 18″ sticks making one beam.

The Transmission of Loads through the Bottom Chords is effected by means of cast-iron "false tubes," shown in Figs. 21 and 22. These bear on the cast-iron angle blocks above and on the gib plates which receive the nuts and washers below. They are cast to a bevel, or slope, so that they can be driven, or drawn into a tight fit in the wedge-shaped slots cut in the sides of the sticks (Figs. 17, 18, 19, and 20), these latter being held laterally by two $\frac{3}{4}$-inch bolts.

These and other features of this truss, of less significance, have so far extended the capacities of the Howe truss as to open up a new field of application for it in the Northwest or wherever such timbers as here described can be obtained. Recent tests of this timber made by Prof. Johnson show it to be superior to white pine in strength and stiffness. It is of a straight and even grain free from knots, wind shakes, and season checks, and promises to become the most valuable structural timber in the world.


LATERAL BLOCK
Pattern Z.
STRINGER PACKING BLOCK
PACKING WASHER FOR CHORDS
Pattern P.
CAST LIP WASHE
Pattern T.
Filler 4"x 12"x 3' 0" lg.
Filler 4"x 12"x 2' 0" lg.
Packing Blocks 2¼"x 12"x 10" long,- -Dry Pine.
Top Chord 4 pieces 8"x 12."
Chords at laterals
Bearing [No. 14.
Bearing [No. 11.
Bearing [No. 7.
Bearing [No. 8.
Bearing [No. 9.
Bearing [No. 10.
Center Line of Span
¾" Drift Bolt 18" long
¾" Drift Bolt 22" long
2 Rods 2" & 3 Rods 2½"
3 Rods 2¼" & 2 Rods 2"
3 Rods 2¼"
3 Rods 2"
3 Rods 1⅞"
All Stringers
5"x 12"
Packing washer No. Y.
packing washer No. P.
Bottom bearing [s same as corresponding [s on top Chord.
139' 2¼" Clear-Span
Bottom Chord 4 pieces 8"x 15"
Chords at laterals packed with ⅝" washers No. P.
Floorbeam Plates 6"x ½"x 2' 10".–Bolts 1⅜" x 2' 10½" long.
Elevation.
SUSPENDED FLOOR
Plan.
AM. BANK NOTE CO. N.Y.



NOTE.
All contacts of wood and wood to be thoroughly painted with white lead.
All bolts 3/4" diameter except as noted for suspended floor
Standard slot washers for 3/4" bolt.

STANDARD HOWE TRUSS

— OF THE —

C. M. & ST. P. RY.

BRIDGE & BUILDING DEPARTMENT

147'-2 1/4" HOWE TRUSS SPAN

Chicago, Illinois, April, 1891.

CHAPTER XXIV.

THE DESIGN OF SWING BRIDGES.

368. **Different Types of Swing Bridges and Determining Conditions.**—Swing bridges are known generally as Centre-bearing and Rim-bearing, according to the manner in which the bridge, when swinging, is carried at the centre pier. If the entire dead load, when swinging, is carried on a vertical pin or pivot, it is called Centre-bearing. If the entire dead load, when swinging, is carried on a circular girder, called a drum, which in turning moves upon rollers, it is called Rim-bearing. Again, these two conditions are combined into one, so as to make the bridge partly rim-bearing and partly centre-bearing. The conditions which generally determine the style of bearing to be used in any given case are usually concomitant with the style of the bridge proper to be used. In general, plate girder swing bridges have a centre bearing, and truss bridges have a rim bearing, or a rim bearing and centre bearing combined. However, in practice, the condition which may finally determine the style of bearing to be used is the vertical height available under the bridge, or the depth from the base of rail to the top of pier. This distance is usually limited, and compels the designer to resort to various devices to properly carry the bridge when swinging.

369. **Plate Girder Swing Bridges.**—These should be used for all lengths up to 100 feet.* For lengths from 100 to 160 feet the riveted truss design is preferable.

The joint at the centre for plate girder bridges is usually centre bearing. Fig. 381 shows the cross-section taken near the centre of a plate girder swing bridge. The main girders *A*, *A*, for a through bridge, are usually spaced 14 feet apart. The part marked *C* is the centre casting, which receives the centre pin or pivot *P*, upon which the cross-girders rest. The parts marked *B* and *B* are supports for the main girders *A* and *A* when the bridge is closed. These supports relieve the pivot from carrying any live load, except such as comes directly on the cross-girders. When the bridge is opened, the supports *B* and *B* being fixed to the masonry, the entire weight is carried on the pivot *P*. In designing the pivot, care should be exercised to have the wearing parts accessible at all times, so that they can be cleaned or replaced by new parts if necessary.

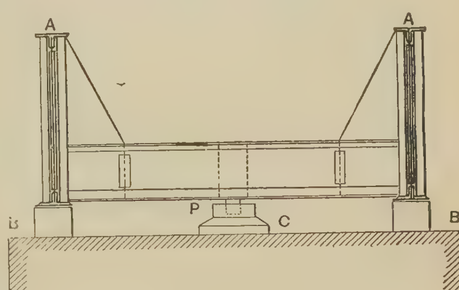


FIG. 381.

Fig. 382 shows the elevation and plan for a centre bearing, or pivot. When the pin turns, the sliding takes place between friction disks, which are usually made of hard steel, phosphor-

* This is about the limiting length for shipping a girder on cars with safety. Plate girders 136 feet long have been shipped on cars for a short distance, but it is dangerous in any case, and especially when shipments are made to a great distance over all kinds of roads.

bronze, or gun-metal. In order to insure that the sliding shall take place between the disks only, the upper and lower disks are provided with short dowels which fit in corresponding sockets in the pin and centre casting, and prevent their sliding. The surfaces of the disks are grooved where they come in contact with each other. This is to insure lubrication; the recess in the centre casting being kept full of oil at all times, the oil finds its way into the grooves.

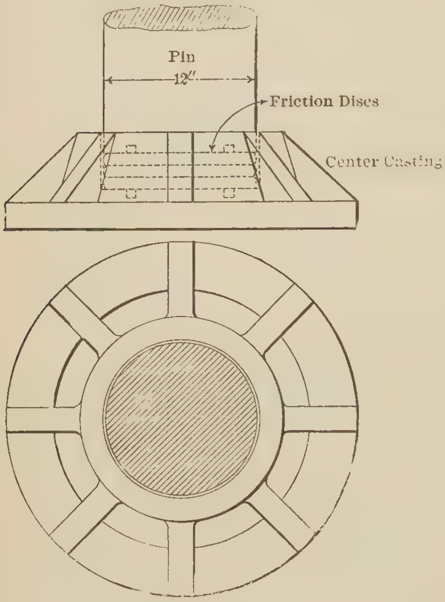


FIG. 382.

The safe load, on the disks, may be taken at 3000 lbs. per square inch, when the bridge is turning. On small disks the pressure may be as high as 6000 lbs. per inch. In general, the safe load per square inch on a disk depends on the frequency of the turning, the angular velocity, and the lubrication of the disk. If the disk be small, so that the oil grooves cut a comparatively large surface from the disk and insure lubrication, the safe load may be higher than for a larger disk. At 10,000 lbs. pressure per square inch there is danger from abrasion. The pin and centre casting may be of cast-iron or cast-steel.

370. Riveted Pony Truss Swing Bridges.—When a riveted truss design is used instead of the plate girder bridge, the centre bearing is generally the same as for a plate girder. The great advantage in using the riveted truss design for lengths from 100 to 160 feet is that the trusses can be made in two halves complete, and then coupled up, over the centre, by means of eyebars with pin connections.

Fig. 383 illustrates a method of coupling up the two halves of a riveted truss swing bridge. The upper chord *AA* being at all times under tension, we can here use eyebars in the place

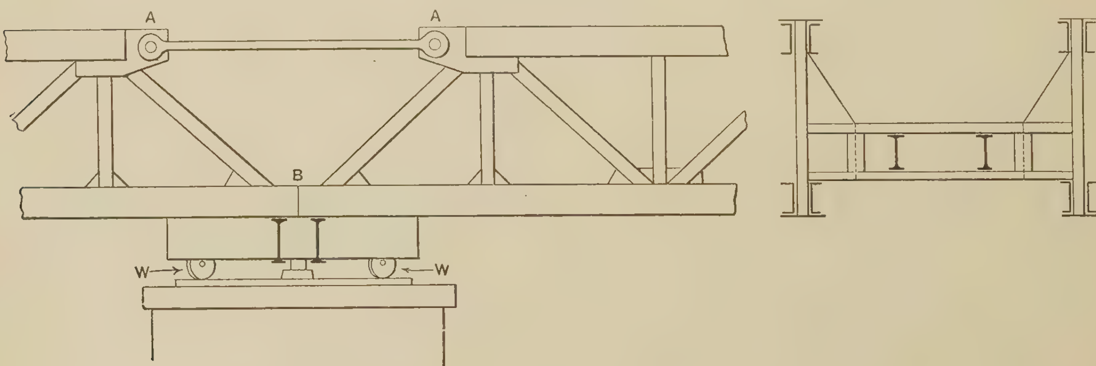


FIG. 383.

of riveted members. The lower chord at *B* being at all times under compression, a butt joint can be used, with such splicing as is ordinarily used for such joints. The great saving in this design is in the facility with which the coupling at *A* can be made in the field. The wheels *W* are balance wheels, usually four in number, to balance the bridge when swinging. These wheels are not supposed to carry any direct load. They are simply intended to carry any unbalanced load when the bridge is swinging, caused by wind, or the unstable equilibrium of the bridge itself. The usual method for designing these wheels is to assume an unbal-

anced load of say 1000 lbs. at one end of the bridge when swinging. As their duty, at all times, is not severe, they are usually made to run on a heavy T rail, which is bent to a circle and laid directly on the masonry.

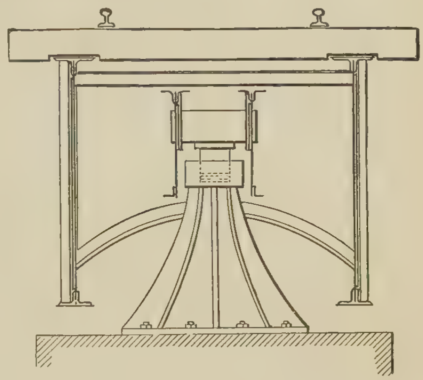


FIG. 384.

These balance wheels are necessary in all forms of centre-bearing bridges, whether girder or truss designs. All of the previous sketches are intended for through bridges. When the distance from the base of rail to the under side of the bridge is sufficient, a deck bridge is used. The method of supporting a deck bridge, at the centre, does not differ materially from that of a through bridge. In a deck bridge, however, it is desirable to have the support for the centre as high up as is possible, that is, as near to the centre of gravity of the load as will permit. This necessitates raising the centre casting up to the required level. This is sometimes accomplished by placing it upon a pedestal. The usual way, however, is to change the form of the centre casting. Fig. 384 shows the cross-section of a deck plate girder swing bridge near the centre. The centre casting has now the shape of a frustum of a cone, and is called the cone.

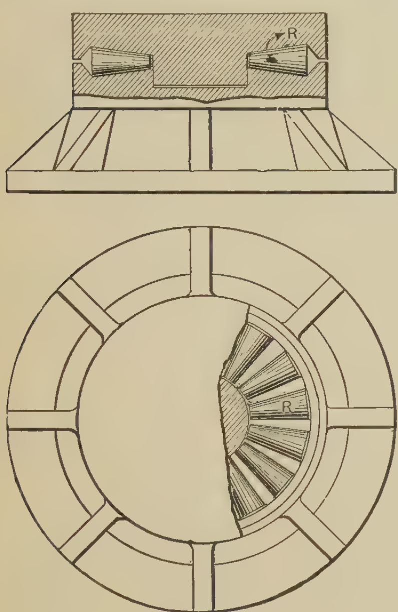


FIG. 385.

371. Centre Bearing on Conical Rollers.—In all of the previous designs we have contemplated a sliding friction at the centre by means of friction disks. In spans where rapid and frequent swinging is required, it is desirable to get rid of the sliding friction at the centre. Here we put the load on a nest of conical rollers. Fig. 385 shows a section in elevation and a plan of a centre bearing on conical rollers.* The conical rollers R are made of hard steel. The box is made of cast-iron or cast-steel. In using this form of centre great care should be taken to have the bearing on the rollers at all times true and level. Any inequality of bearing on moving parts results in unequal wearing. If some of the rollers become worn more than others, owing to unequal bearing, imperfect workmanship, or other causes, they soon lose their proper relative position in the nest, and begin crowding and wearing each other. Then the nest becomes clogged with the grindings, and the efficiency of the bearing is soon destroyed.

In the recent designs for conical rollers this trouble is anticipated, and in order to guard against it the rollers are encircled by a collar, or *live ring* as it is called. Each roller is then held in its proper position by a small rod which passes through its centre. The outer ends of these rods are fastened to the live ring. The inner ends come together on a hub called the spider. This device insures the same movement in all of the rollers at the same time. The safe load on friction rollers is given by the formula $p = 300d$, Art. 255. For hard-steel rollers on cast-iron beds use $p = 250d$. For conical rollers use the average diameter for d.

The bearing on the masonry at the centre may be taken at the same values as for fixed spans, when the bridge is closed and fully loaded with live load. When the bridge is empty and swinging, the above values may be doubled. For example, if the permissible bearing on

* This form of centre bearing was invented by Mr. Wm. Sellers, and is known as the Sellers Centre.

the masonry is 300 lbs. per square inch for fixed spans, then for swing bridges it would apply only when the bridge is closed and fully loaded. When the bridge is empty and swinging, we may use double this value, or 600 lbs. per square inch. The practice simply demands a higher value for the bearing on the masonry when the bridge is swinging than when it is closed, and about double the value seems rational.

372. **Truss Swing Bridges.**—As truss spans can be built either with pin or riveted connections, a discussion of one will apply to both. In this country the common practice is to use pin connections for long spans. This applies to swing bridges for all lengths above those which are too long for using the plate girder or riveted truss design. In what follows, all truss bridges will be understood to be pin connected. All trusses for swing bridges should be simple in design, so as to be free from all ambiguities in their stresses under all possible conditions of loading and temperature, as well as the conditions resulting from a rapid variation in the depth of the trusses, which necessarily involves the moment of inertia of the cross-section. In all of the analysis on swing bridges it has been assumed that the moment of inertia of the continuous girder is constant. Any great departure from this assumption invalidates the correctness of the assumption and vitiates the values of the computed reactions.* No simple classification of different types of truss swing bridges can be made. In general, the most preferable design, everything else being equal, should be a single truss system free from all adjustable members. The advantage of using long panels can be obtained, as in all fixed spans, by secondary trussing. Inclined upper chords for through bridges are introduced where economy of design dictates their use. In general, it may be said that the outline for the trusses of a swing bridge is governed by the same rules as for fixed spans. The most economical depth at the centre is about the same as for a fixed span having a length equal to the total length of the swing bridge.

373. **Unusual Forms of Trusses.**—Perhaps a good way to illustrate the best forms for trusses to be used would be to show some which are objectionable. An unusual form is the

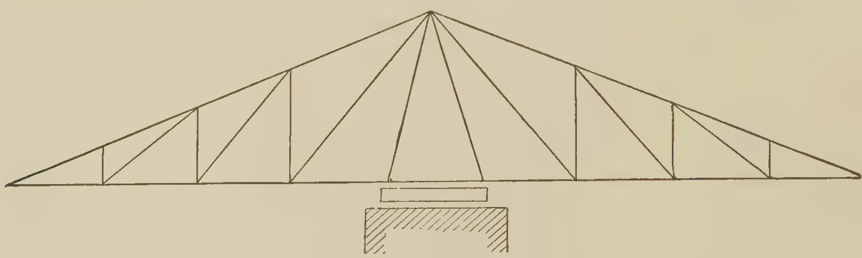


Fig. 386.

triangular pattern. In this, all of the lines of the truss form triangles. Fig. 386 shows the outline for a triangular form of truss.

The principal objection to this form of truss is the variation in the depth of the truss from zero at the ends to the maximum at the centre. The low depth of the truss at the ends not only vitiates the value of the reactions obtained by assuming a uniform moment of inertia, but it also results in excessive deflections when the bridge is closed and the live load is on one arm. These excessive deflections tend to bend the chords. As the chord joints are not articulated, the bending may be sufficient to give the chords a permanent set. This has actually occurred in two instances where this form of truss has been used. However, this form of truss presents some advantages which should not be lost sight of. The low inclined chords at the ends will aid in deflecting a possible derailed car which might strike the bridge. The form of the truss, also, is such as to reduce the areas exposed to wind pressure at the ends of

* For the ordinary American practice the errors introduced by the varying moment of inertia practically compensate for the errors coming from neglecting the web members in computing the continuous girder moments. See Art. 178*a*, p. 196.

the arms, making it somewhat easier to handle during high winds. This form of swing bridge was at one time very commonly used for short spans, but has now given way to more economical forms, more especially the plate girder and the riveted truss design.

Fig. 387, shows another form of truss for swing bridges which is objectionable.

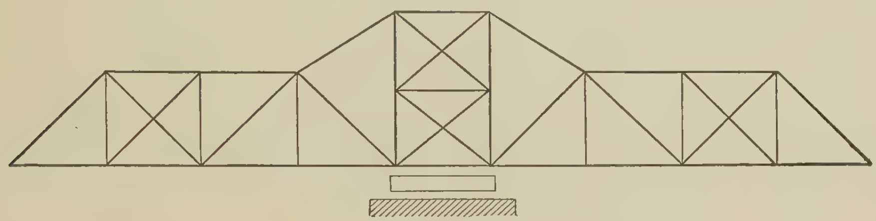


FIG. 387.

The objection to this form of truss design lies principally in the central portion directly over the turntable. Here the character of the design involves an uncertain distribution of loading on the turntable. Again, the counter-stresses in the web system are provided for by the use of adjustable rods, which are objectionable in any truss design, owing to their tendency to get out of adjustment. However, the most important objection to this form of construction is the lack of economy in the design.

374. **Standard Forms of Trusses.**—As previously stated the most desirable form of truss design is that which is free from adjustable members and from all ambiguities in the stresses arising from complex systems of trusses. Fig. 388 shows a standard form of truss for spans

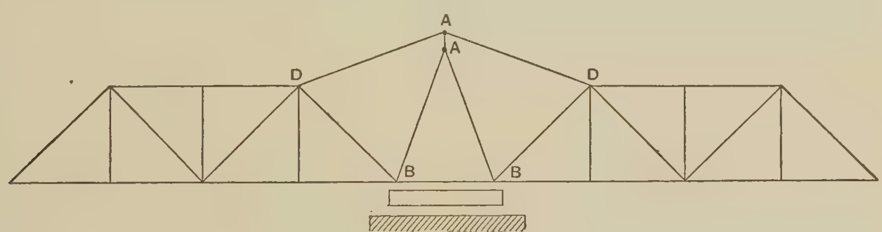


FIG. 388.

of any length. The principal advantages in this form are, economy in design, a minimum number of members, the absence of all adjustable members, a freedom from all ambiguities in the stresses, etc. In this design, the members *AD* should have such an inclination that when the bridge is fully loaded, the members *AB* will carry the greater part of the load to the points *B* on the turntable. This will insure a better distribution of the maximum load on the turntable. Another advantage in this form of truss lies in the use of the links shown in the figure by the member *A–A*. Aside from their primary use in equalizing the loads on the turntable at points *B*, they have proven useful in case the bridge, when swinging, becomes unbalanced. Such a condition may arise in case the bridge is struck by a passing steamer, which may lift one arm, while the other arm deflects downward. If the joints at *B* will yield so as to allow the arms to deflect considerably, the bridge may remain intact on the turntable.*

There are a great many standard forms of trusses for swing bridges; in fact, too many to be described here. The authors have shown in Fig. 388 what they consider the best form of truss for swing bridges. However, conditions may arise which may compel the designer to modify this design. For instance, in single track swing bridges it frequently happens, when it becomes necessary to handle the bridge by steam power, that the engine must be placed over

* Such an accident occurred to the swing bridge of the St. Louis Southwestern Railway over the Red River at Garland City. The bridge was built from C. Shaler Smith's plans.

the track. In that event it becomes convenient to have vertical centre posts to support the shaft leading vertically down from the engine-room to the drum. This shaft (there are sometimes more than one shaft) is always vertical, and the vertical posts at the centre make a convenient support for journal-boxes holding the shaft. This condition may modify the form of

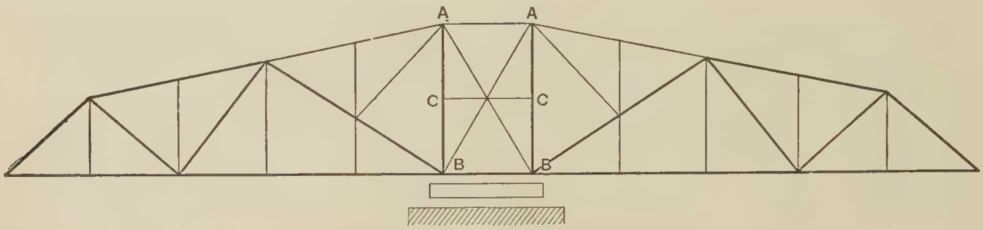


FIG. 389.

truss to that shown in Fig. 389. Here the centre posts AB are vertical, and the engine-house is usually supported on the cross-girder CC.

375. Methods of Supporting Truss Swing Bridges at Centre.—Truss swing bridges are sometimes made centre bearing, similar to plate girder bridges, where the available depth from base of rail to the top of centre pier is limited, so as to preclude the use of any other form of support at the centre. However, such conditions have to be provided for by special designs. The usual method of supporting the trusses of a swing bridge is on a turntable. As previously stated, if the trusses of a swing bridge rest on a circular girder, called a drum, which in turning moves upon rollers, the bridge is said to have a rim-bearing turntable. The word Turntable comprises the entire turning arrangement at the centre. Fig. 390, shows the

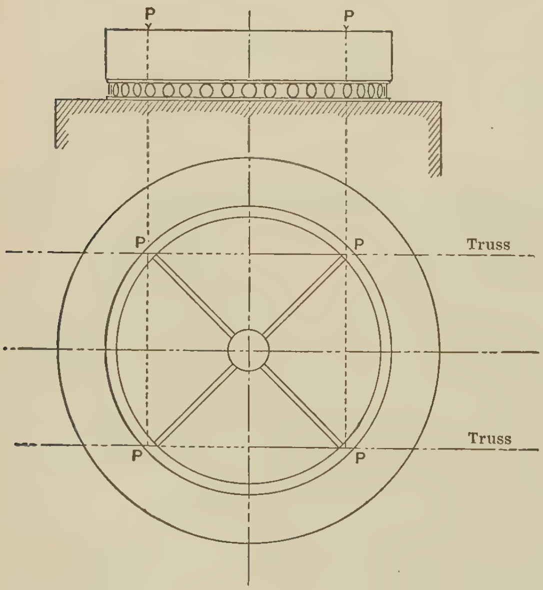


FIG. 390.

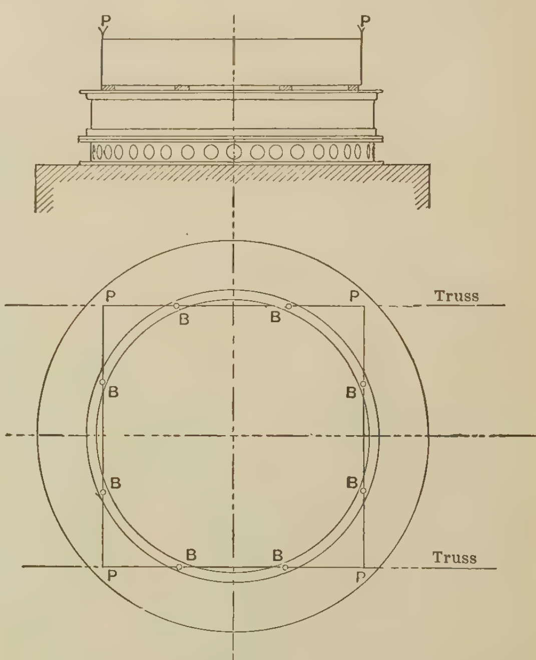


FIG. 391.

outline, in plan and elevation, of a rim-bearing turntable loaded at four points marked P. This form of loading on a turntable is now avoided excepting perhaps for highway bridges.

The objections to loading a bearing rim at four points only is that it is impossible to get a uniform load on the rollers, no matter how stiff we make the circular girder or drum.

The fact is, in this form of turntable, the wheels which are directly under the loaded points carry the entire load. The result is that the rollers and the tread upon which they roll soon show signs of unequal wearing.

The common method of loading a rim-bearing turntable is to distribute the load from the trusses equally over eight symmetrical points. Fig. 391, shows the outline, in plan and elevation, for a rim-bearing turntable loaded at eight points. The loads from the trusses come on the four corners of a square box *PPPP*. The four sides of this box are the girders which rest directly on the drum at the eight points marked *B*. If the loads at *P* are equal, the loads at *B* must be each equal to $\frac{1}{2}P$. The points *B* in the drum can be spaced at equal distances, by assuming the proper diameter for the drum. If *a* equals the side of the square *PPPP*, then the diameter of the circle, necessary to space the point *B* at equal distances on the drum, $= 1.0824a$.

Plate IV shows the general design for a rim-bearing turntable.

All swing bridges, whether centre bearing or rim bearing, must rotate about some fixed point or centre pin. In a purely rim-bearing turntable this centre pin carries no load. Its duty is simply to hold the turntable in position with reference to its centre of rotation. The fact that the centre pin carries no load is one of the objections to the rim-bearing turntable. The centre pin usually turns in a centre casting which rests directly on the masonry (see Plate IVA. If the rollers all travelled in the proper circle, the centre pin would not tend to move laterally. As a matter of fact, the rollers are constantly getting out of line or out of their proper circle, so that the centre pin is frequently subjected to a series of displacements laterally. If the loads are comparatively light, the pin and centre casting can be made to withstand this tendency to displacement by anchoring the centre casting to the masonry.

The most approved design for a turntable has the weight distributed, so that a portion of the loads goes to the centre pin. This form of turntable is called the centre-bearing and rim-bearing turntable combined.

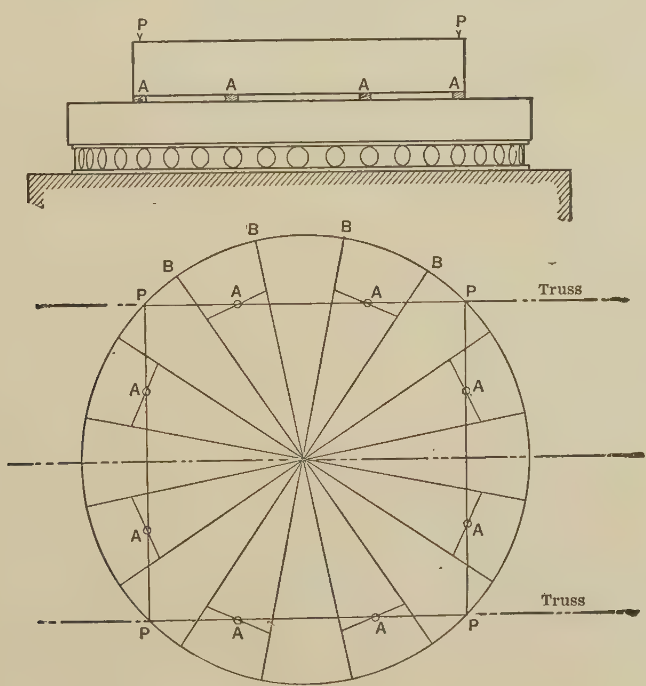


FIG. 392.

In this kind of a turntable, the load on the centre pin not only relieves the load on the rim, whereby the bridge turns easier, but it also aids in holding the centre pin in its proper

position, and in this way helps to keep the rollers in the proper circle. Fig. 392 shows the outline, in plan, and elevation for a rim- and a centre-bearing turntable.*

The loads from the trusses come on the four corners of the square box *PPPP*. The four sides of this box are the girders, which rest on eight points marked *A*. The four corners of the box are perfectly free; so that, although the corners *P* come directly over the circle of the drum, there is no load transmitted to the drum directly under the point *P*. The eight points *A* now carry the entire load from the four points *P*. The load at *A* is then transmitted to the two adjacent radial girders *BC*, and these girders in turn carry a portion of the load to the centre, and the remainder to the rim. In this manner, the load which comes on the rim is uniformly distributed over sixteen points. This form of turntable gives the best distribution of loads, and consequently turns easier than any other form of turntable in use in this country; that is, excepting, of course, the purely centre-bearing swing bridges, which, properly speaking, have no turntable.

The safe load on static rollers may be taken the same as given for small rollers, Chap. XVIII; that is, $p = 1200\sqrt{d}$. The safe load on moving rollers may be taken at $p = 600\sqrt{d}$. This value of p has ample margin for the variation in the hardness of the different materials used for rollers and treads, unequal bearing of rollers, etc.

376. End Lifting Arrangements.—In the early designs for swing bridges very little attention was paid to the proper elevation of the ends of the arms when the bridge is closed; and even at this late day a great many important swing bridges are built by contractors with an utter disregard of the condition of the end supports. In fact, the majority of swing bridges have no provisions for lifting the ends whatever; while others have all kinds of make-shifts, which generally shirk their duty entirely. It is safe to say that in this country it is the exception to find a swing bridge where proper provision is made for raising the ends when the bridge is closed. It has been shown, in Chap. XII, that if the ends are not raised we cannot obtain the conditions necessary for a beam continuous over three rigid supports. In other words, if the ends are not raised, we must, in our analysis for finding the stresses, make an assumption which will satisfy this condition of the ends under the extreme variations of temperature.† Now, it is difficult to say just what this condition should be. Furthermore, if the ends are left free to hammer, under extreme variations of temperature, the ends may be thrown out of line so far as to cause derailment of a train coming on the bridge. In fact a swing bridge, wherein no proper provision is made for raising the ends, is a dangerous structure at all times.

In the previous analysis on swing bridges, Chap. XII, two conditions of the ends were assumed, in computing the stresses due to dead load only. First, Case I, when the ends were just touching their supports without producing any positive reactions. Then, in Case II, the ends were assumed to be raised, so that the reactions were equivalent to those of a beam continuous over three level supports. Now, inasmuch as we combine the live load stresses with either Case I or Case II, it follows that the ends should at all times be raised so as to satisfy a mean between those conditions assumed for Cases I and II. The ends could probably be raised high enough to satisfy Case II for the time being, but under extreme variations of the relative temperature of the two chords this condition would be changed; so that, in order to satisfy all conditions of temperature as well as loading, the ends must be raised enough to take out at least one half the deflection of the ends of the arms due to dead load under Case

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* This form of turntable is now used exclusively by the Detroit Bridge and Iron Works.

† By "variations of temperature" in this chapter is meant simultaneous differences of temperature of the two chords (upper and lower) of a drawbridge. The lower chord may be entirely in the shade while the upper chord is entirely in the sun, and this may cause a difference of temperature in the two chords of 40° or 50° F. At night the variation may be in the opposite direction.

I, span swinging. This would still leave the ends of the arms below the true level of the lower chord; that is to say, one half the deflection would still remain in the truss.

The proper way to eliminate this remaining deflection in the arms is to shorten the upper chord, usually in the centre panel or those adjacent to the centre, just enough to raise the ends to the true level of the lower chord.

For example, assume a swing bridge 400 feet long and 50 feet deep over the centre The computed deflection of the ends of the arms under Case I is, say, 4 inches. Now if the ends were raised $2\frac{1}{4}$ inches by means of the end lifts, there remains $1\frac{3}{4}$ inches of the end deflection to be eliminated by shortening the upper chord. The amount that the upper chord must be shortened on each side of the centre is $1\frac{3}{4} \times \frac{50}{200} = 0.44$ inch $= \frac{7}{16}$ of an inch. This subject will be taken up in detail further on.

The simplest form of an end lift for a swing bridge is that in which the ends of the arms are provided with wheels which rest on the crowns of inclined roller beds. Fig. 393 is an elevation of the end of a truss swing bridge, showing the wheels R resting on the beds B, which are bolted to the masonry.

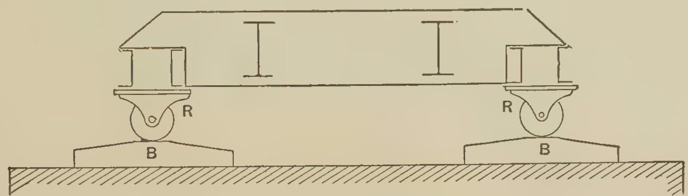

FIG. 393.

The roller beds B are set at a proper elevation to give the end reactions, as previously shown. In this form of an end lift it becomes necessary to use some form of automatic latch, to stop the wheels from rolling over the top of the beds and down on the other side. When the span is long, so that it can acquire considerable momentum in turning, the automatic latch can no longer be relied on to bring the bridge to a sudden stop. The fact that the ends must be lifted by the very energy which the bridge acquires in turning implies that the bridge must have a good angular velocity when near closing, so as to force the wheels up the inclined beds. This energy, or shock, which is taken up by the latch is transmitted to the masonry. For short spans, where the ends are to be lifted about $1\frac{1}{4}$ inches or less, this method of lifting is effective and inexpensive. It is especially adapted for use in short spans, which have to be opened quickly; the rollers being once started down their inclined beds, the same energy is imparted to the moving mass of the bridge which was necessary to raise the ends.

When it becomes necessary to lift the ends of a plate-girder swing bridge, it is customary to fix the end wheels on the masonry, and provide the ends of the arms with beveled plates, to correspond with the inclined beds previously shown. Fig. 394 gives in elevation the end

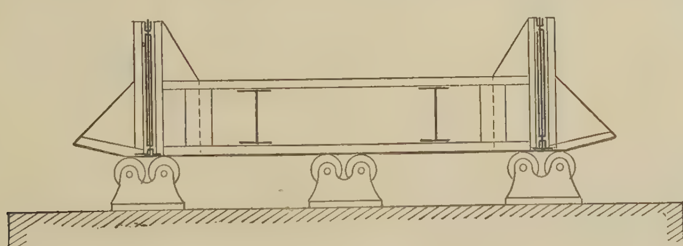

FIG. 394.

of a plate-girder swing bridge, showing three stands of end wheels, with two wheels in each stand. The inside stand of wheels carries no load of any kind when the bridge is closed.

Its duty is merely to assist in steadying the bridge when coming on or rolling off the end wheels. It should be stated here that for all short-span swing bridges the ends should be raised relatively higher than for long spans. For example, if the theoretical deflection of the ends due to dead load, Case I, were such that we should have to give the ends only a ½-inch lift, for practical reasons we would make it probably 50 per cent more, or ¾ inch, to allow for extreme variations in temperature, which cannot be provided for by any assumptions that we might make.

There are other simple forms of end lift, such as the wedge, for instance, which, however, do not lift the ends, and therefore do not properly belong here. What they do is simply to give the ends of the bridge a firm bearing for certain conditions of temperature, and prevent the ends from hammering.* Under certain conditions of temperature changes, the ends may not touch their supports at all, which necessitates a new adjustment for the wedges. Again, under other conditions of temperature, the wedges may be held so firmly in place as to make it difficult to draw them out from under the ends of the arms, so that any form of end lift which has to be adjusted for different conditions of temperature is usually left out of adjustment, because it is impracticable to handle the bridge otherwise.

There is another kind of end lift, suitable for short spans, which should be explained here. In the end lift just described the shock transmitted to the masonry, when closing the bridge, may be so objectionable as to preclude its use. The form shown in Fig. 395 should then be used.

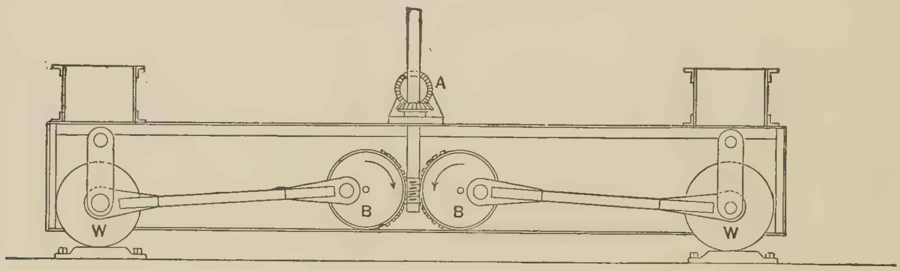

FIG. 395.

In this form of end lift the bridge is first swung into line and latched; then, by turning the vertical shaft A, the wheels B revolve about their centres, and at the same time force the suspended wheels W under the ends of the arms.

Let Fig. 396 represent the motion of the parts of the end lift shown in Fig. 395. Let h represent the required lift of the arms in feet. Let R represent the load on wheel W when

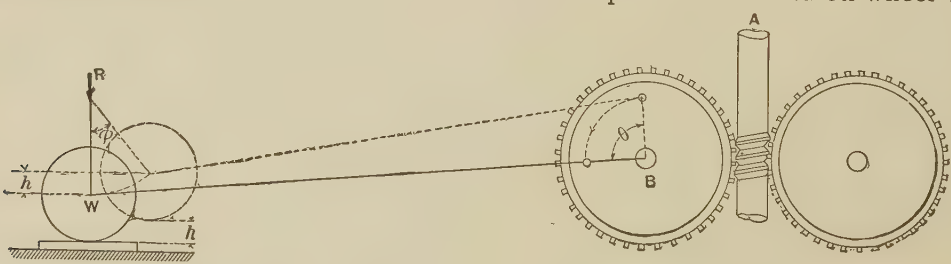

FIG. 396.

the maximum lift has been attained. Then the total work done in raising the end $= \frac{R}{2} \times h$.

As the time for opening or closing a swing bridge is usually limited, let t equal the number of seconds required to raise the ends; then the mean power required to raise the ends

$$= \frac{R \times h}{2t}.$$

* See Plate IV.

This form of end lift is well adapted for use in spans under 250 feet long, where the distance at the abutment from the base of the rail to the masonry is limited.

The Screw-jack or Direct Lift.—In this form of end lift the load at the ends is lifted directly by means of screw-jacks placed at the four corners of the bridge. These jacks are usually four in number. They are all connected by means of shafting and gearing, and are worked by a motor from the centre of the bridge. The screw-jack is well adapted for use in long spans where the value of h, or the distance through which the ends must be lifted, is a large factor. The mean power required to raise the ends is, as before, $\frac{R \times h}{2t}$. Fig. 397 shows a form of screw-jack under one end of a bridge.

No attempt is here made to show the actual form of screw-jack as generally used, but the illustration is adapted to show simply how a screw-jack could be made to exemplify the principles used. In Fig. 397, if the wheel W is turned we turn the screw in or out of the cylinder C, which correspondingly lowers and raises the end of the span. The lower end of the screw-shaft turns in a socket in the pedestal P.

In any form of lift wherein the multiple of force of the moving parts of the mechanism remains a constant, the power required varies directly with the load. If the load is uniformly varying, the power increases uniformly to the end of the lift.

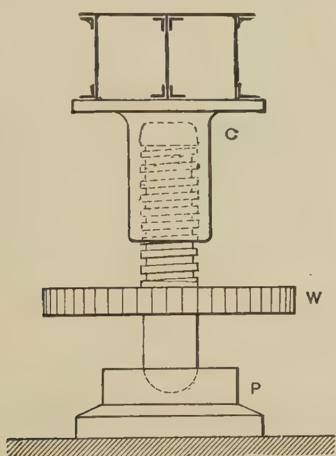

FIG. 397.

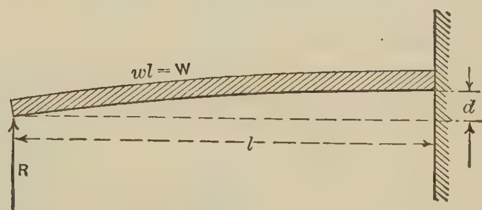

FIG. 398.

In Fig. 398, let l be the half length of a swing bridge and $wl = W$ the weight of one arm. Let R represent the final force applied to the end lift, which we can assume to be a screw-jack. Then the deflection

$$d = \frac{Wl^3}{8EI} - \frac{Rl^3}{3EI}, \quad \ldots \ldots \ldots \quad (1)$$

in which d is positive downward.

From eq. (1) we have

$$R = \left\{ \frac{3}{8}W - d \times \frac{3EI}{l^3} \right\}. \quad \ldots \ldots \ldots \quad (2)$$

In eq. (2), when $d = 0$, $R = \frac{3}{8}W$; which is the reaction for a beam continuous over three supports and uniformly loaded.

It has been shown that the full amount of the end deflection should not be taken out by the end lifts, but rather only one half this amount, or perhaps a little more than one half, to allow for variations of temperature of the two chords; so that R should not be $\frac{3}{8}W$, but

rather nearer $\frac{3}{16}W$. The value of R then varies from zero to about $\frac{3}{16}W$; and in any form of end lift, such as the screw, the force applied would vary in the same ratio.

Now, in order to equalize the horse-power expended in lifting the ends, the speed of the engine should vary inversely as the force required. This means that the engine would sometimes be racing, while near the end of the lift, when R becomes nearly $\frac{3}{16}W$, the piston speed would be very much reduced; while under unfavorable conditions the engine might become stalled. To preclude the possibility of such an occurrence, the engine should be able to develop the full power required at the average piston speed: that is, $\frac{R \times h}{t} =$ mean power required; wherein $R = \frac{3}{16}W$, $t =$ time for lifting in seconds, and h is the value of d* in equation (2) when $R = \frac{3}{16}W$. Now, it frequently so happens that the maximum horse-power required is for lifting the ends, and not for turning the bridge; so that probably an engine of less capacity could be used if some means were employed by which the multiple of force used in the end-lifting arrangement were made to vary with the load. This is practically accomplished in the end lift known as the ram, which is described further on.

The objections to the screw-jack apply in general to all similar forms of end lifts—that is, where no provision is made for varying the multiple of force with the load. Notable among the similar forms of end lifts is the hydraulic jack. Here the load is lifted by a hydraulic ram instead of a screw; and the medium for transmitting power is glycerine or alcohol, instead of gearing and shafting. Although this is a most useful means of doing work in all forms of mechanism, it is objectionable for lifts for swing bridges on account of its tendency to leak. Swing bridges have been built which can be raised and lowered bodily by means of hydraulic jacks at the centre. The work done in raising a bridge bodily is considerably more than the work done in raising the ends. Their ratio is about as five to one. Among such examples may be mentioned the Harlem River Bridge of the Manhattan Elevated Railway in New York.† This bridge is lifted bodily from the centre by means of a hydraulic jack.

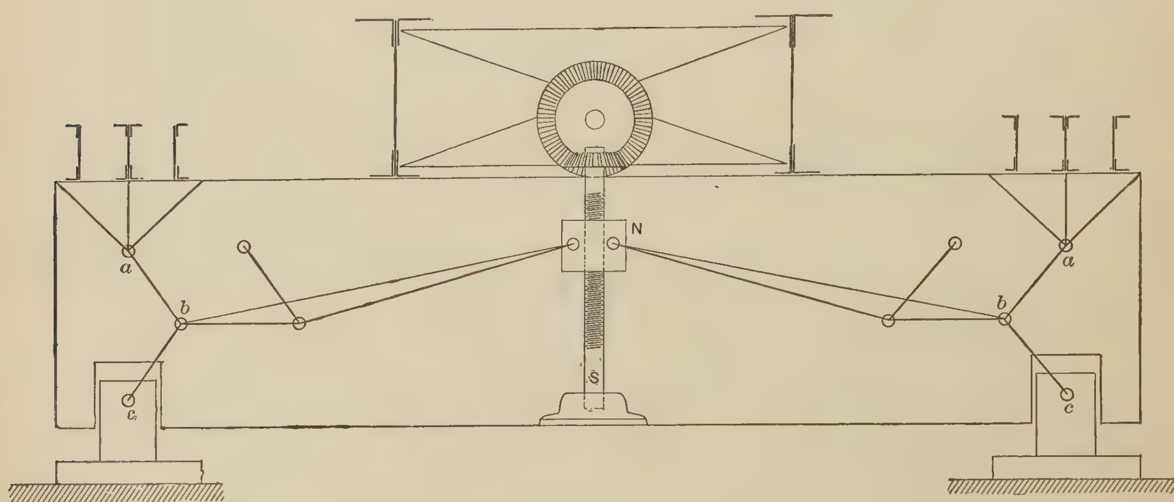

FIG. 399.

377. The Ram with Toggle-joints.—In this form of end lift the multiple of force is varied, so that it is a minimum at the beginning of the lift and a maximum at the end.

Fig. 399 shows in outline the form of mechanism employed in the ram. No attempt is

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* This value of d, however, is obtained in another way more accurately by computing the actual deflection of the ends of the arms, and taking $h \overset{=}{>} \frac{d}{2}$.

† This bridge was designed by Mr. Theodore Cooper, C.E.

here made to show the actual form of the parts used. The screws S, one at each end of the bridge, are turned by means of shafting and gearing from the centre of the bridge. As the screws turn, the nuts N, called the cam nuts, travel in guides, up or down, on the screws, which action shortens or lengthens the toggle-joints, and at the same time lowers or raises the ends of the bridge.

In Fig. 400, let R represent the end reaction, which equals the load to be lifted. As previously shown, R may vary from zero to $\frac{3}{16}W$. Let a, b, c represent the toggle-joint fixed at a. The toggle is worked in and out by means of a horizontal bar H, which is coupled at e to

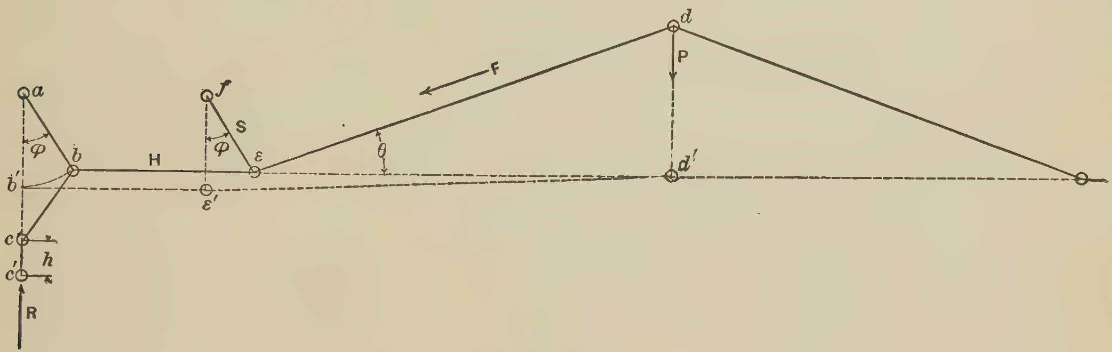


FIG. 400.

the arms fe and ed. The distance dd' represents the travel of the cam nut, which receives a thrust $= P$ from the screw.

Let F represent the thrust in the arm ed from the cam nut at d; then we have

$$F = \frac{P}{2} \operatorname{cosec} \theta,$$

$$S = \frac{P}{2} \sec \phi,*$$

$$H = F \cos \theta + S \sin \phi;$$

$$\therefore H = \frac{P}{2}(\cot \theta + \tan \phi).$$

But $$H = 2R \tan \phi;$$

$$\therefore P = 4R\left(\frac{\tan \phi}{\cot \theta + \tan \phi}\right). \quad \ldots \ldots \ldots \quad (3)$$

From eq. (3) we see that when $R = 0$, $P = 0$; also, when $\phi = 0$ we again have $P = 0$. That is to say, P, which is the thrust on the cam nut, and consequently may represent the effort of the motor in lifting the ends, is zero for minimum and maximum values of R.

In Fig. 401, the curve Oad represents the effort of the engine during a piston-travel Od, when the lifting arrangement is such as previously described. The straight line Oe represents the effort of the engine during the same piston-travel when lifting the ends with a direct lift, such as the screw-jack.

* Since the vertical component in fe = vert. comp. in $ed = \frac{P}{2}$, as be is horizontal.

From the figure we see that the maximum effort of the engine in the first case is much less than in the second—which proves that the ram is a better form of lift than the screw.

The dotted curve $Oa'd'$ represents the curve of effort for an end lift, wherein the horizontal bar H described in Fig. 400 is omitted, and where deb, which is now one straight member, comes into a horizontal position when abc becomes vertical. This curve shows that without

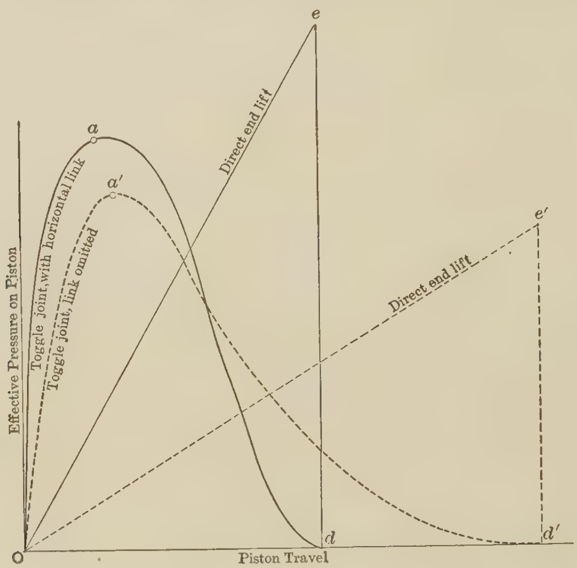


FIG. 401.

the horizontal bar H the ram is not as efficient as the screw, for which the curve of effort is represented by the straight line Oe'.

These curves were all plotted from actual examples, and clearly show the advantage in using the ram as compared with any other known form of end lift. For any given case the areas of the figures Oad and $Oa'd'$, and the triangles Oed and $Oe'd'$, are all equal to each other, as they represent the total effective work done, $\frac{R \times h}{2}$, regardless of the manner in which the ends are lifted.

378. **Machinery for Operating Swing Bridges.**—The machinery for operating swing bridges comprises all the machinery necessary for raising and lowering the ends, as well as for turning the bridge. Swing bridges are usually operated either by hand power or steam power. Electricity has been applied for operating them; but here, as in all similar applications of electricity, it is only a means of transmitting power from the steam-engine or the turbine. Gas-engines have also been employed for driving the machinery for swing bridges; but in any case, whatever may be the source of the power employed, the motor which drives the moving parts of the machinery must be attached to the motor shaft, so that, in designing the machinery necessary for operating a swing bridge, the kind of power to be employed need not enter into consideration at all until, perhaps, we come to provide the necessary space for the motor, its attachments to the motor shaft, etc.

The simplest means for turning a swing bridge would be to pull the ends of the bridge into position, for opening or closing, by means of a rope attached to one end; and this is a good method to employ for turning any swing bridge whenever the machinery becomes disabled and no other means are at hand for applying the available steam or hand power.

The force necessary for turning is usually applied to the circular girder or drum in rim-bearing bridges. In centre-bearing bridges the force is applied to some member of the bridge

proper, usually a cross-girder or floor-beam. It frequently happens that one arm of a swing bridge moves over dry land. In that case it may be convenient to apply the force necessary for turning at one end of the bridge. This will enable the designer to place all of the machinery for operating the bridge on dry land. However, for many reasons it is desirable to have the motor and all the necessary machinery on the bridge self-contained, so that the foregoing plan cannot be recommended.

In this article, only such machinery will be described as is ordinarily used in operating swing bridges; that is, where the force necessary for turning is applied, near the centre of the bridge, through one or more pinions which mesh with the teeth on a circular rack. The pinions are keyed to vertical shafts, and when more than one pinion is used they are usually placed in pairs diametrically opposite to each other. The vertical shafts are connected through bevel gears with a horizontal shaft, which in turn is connected by gearing to the motor shaft, usually called the engine shaft.

379. **Resistances.**—The work to be done in operating a swing bridge is: first, the lifting of the ends, which of course includes the work done in overcoming the resistances of all the moving parts, as well as the internal friction of the engine; second, the work done in turning the bridge, which includes the work done in overcoming the resistances due to inertia, wind pressure, and the resistances of all the moving parts. The frictional resistances are: rolling friction between the rollers and the top and bottom treads; sliding friction between the disks under the centre pin, and between the collars on the spider-rods and the rollers; and the resistances of the machinery, which may include both rolling and sliding friction. The resistance of inertia is the force required to accelerate the motion of the bridge. The resistance due to wind pressure is indeterminate and can only be arrived at approximately. If the wind blew perfectly steady, so that the pressure is uniformly distributed over the exposed area, it would offer no resistance to the turning of the bridge, as the pressure on one side of the centre would equalize that on the other whatever the direction of the wind. The wind, however, never blows steadily, but in gusts, and the pressure is never uniformly distributed, so that for a long-span swing bridge the velocity at one end seldom equals that at the other. When it becomes necessary to open and close a swing bridge in all kinds of weather, the motor used should be capable of developing sufficient power to turn the bridge when the maximum and minimum velocities of the highest wind under which it is considered safe to operate a swing bridge are acting on the opposite ends at the same time.

Although it will be shown that the unbalanced wind pressure may offer the greatest resistance to turning and frequently determines the horse-power to be used in operating the bridge, the various resistances to be overcome will be here considered.

In what follows it will be convenient to reduce all the various resistances to lifting and turning to work done in foot-pounds per second, including the losses due to the transmission of power through the gearing, etc.

Lifting the ends.

Let $R = \frac{3}{16}W$ represent the total load to be lifted;

$h =$ the total lift in feet;

$t =$ time in seconds for lifting.

Then, since the maximum rate at which the work is done is that at the end of the lift where the reaction is R, we have for the power required,

$$\text{Power} = \frac{R \times h}{t} = \frac{3Wh}{16t}.$$

This represents the maximum rate at which the work is done, in foot-pounds per second, at the ends which are to be lifted, and does not include the power lost in the resistances of the moving parts of the machinery.

Turning the Bridge.—In determining the power necessary for turning, it will be most convenient to first find the resistances to turning, and then determine their equivalents at the pitch circle of the rack, since it is at this point that the force to overcome them is applied.

Rolling Friction.

Let R_1 = radius of the drum;
R = radius of rack circle;
W_r = weight on rollers under drum;
ϕ_1 = coefficient of rolling friction;
F_r = force at rack required to overcome rolling friction.

Then

$$F_r = \phi_1 W_r \frac{R_1}{R}. \qquad (4)$$

Sliding Friction between Disks.

Let R = radius of the rack circle;
W_s = weight on the centre pin;
d = diameter of disks;
ϕ_2 = coefficient of sliding friction between disks;
F_s = force at rack required to overcome sliding friction.

Then

$$F_s = \phi_2 W_s \frac{d}{3R}. \qquad (5)$$

Collar Friction.—For the friction of the washers and collars at the ends of the spider rods,

Let r_1 = interior radius of collar;
r_2 = exterior radius of collar;
r = radius of the rollers;
ϕ_3 = coefficient of collar friction.
W_r = weight on the rollers;
R and R_1 the same as before;
F_c = force at rack to overcome collar friction.

Then the force with which the rollers are pressed against the collars $= W_r \times \frac{2r}{R_1}$.

From mechanics, the lever arm of the friction or the radius of the circle at which the total friction may be considered to act $= \frac{2}{3} \times \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$.

The ratio of the force at the rack to the force at the centre of the track required to overcome the collar friction $= \frac{R_1}{R}$. Then

$$F_c = \frac{2}{3} \times \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \times \frac{1}{2r} \times \frac{R_1}{R} \times \phi_3 W_r \times 2\frac{r}{R_1} = \frac{4}{3} \times \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \times \frac{\phi_3 W_r}{R}. \qquad (6)$$

Wind.—To find the effect of the wind, let p = the unbalanced wind pressure acting on one arm, and let f = the distance of the centre of pressure on any member from the vertical plane through the centre of rotation and parallel to the direction of the wind, the direction of the wind being assumed to be normal to the plane of the truss; let a = area of that member, and let F_w = a force which placed at the rack would balance the total unbalanced wind pressure. Then, since R = the radius of the rack circle, we have

$$F_w = \frac{\Sigma(paf)}{R}. \qquad (7)$$

However, it is sufficiently accurate to consider the total wind pressure as acting at a distance g from the centre = one quarter the length of the bridge; then letting P = the total wind pressure, we have

$$F_w = \frac{Pg}{R}. \quad \ldots \ldots \ldots \ldots \quad (8)$$

Inertia.—The work done in overcoming the inertia of the bridge is equivalent to the kinetic energy acquired by the total rotating mass when the maximum angular velocity has been attained, and this same amount of work will have to be done in bringing the bridge to rest; although, in the latter case, the resistances due to friction, and perhaps wind, retard the motion of the bridge and assist in bringing it to rest.

In order to determine the kinetic energy acquired, it is necessary to find the moment of inertia of the entire bridge with reference to its axis of rotation. However, it is sufficiently accurate to consider the bridge to be a rectangular parallelopipedon of uniform density and of the same weight, whose length and width are the same as those of the bridge, and whose depth, or dimension parallel to the axis of rotation, is any convenient quantity.

To find the mass which, placed at the pitch circle of the rack, would produce the same moment of inertia:

Let R = radius of rack circle;
M = mass at rack circle;
I = moment of inertia of entire bridge.

Then

$$M = \frac{I}{R^2}.$$

Let v = maximum linear velocity at rack circle;
t = time for uniform acceleration;
a = rate of acceleration $= \frac{v}{t}$.

Then the kinetic energy attained for the velocity v at the end of t seconds $= \frac{1}{2}Mv^2$, and the constant accelerating force applied at the rack circle is

$$F_a = Ma = \frac{Iv}{R^2 t}. \quad \ldots \ldots \ldots \ldots \quad (9)$$

380. **Summary of Resistances in Turning.**—Summing the resistances at the rack circle, we have

$$F = F_r + F_s + F_c + F_w + F_a, \quad \ldots \ldots \ldots \ldots \quad (10)$$

which is the force to be applied at the rack circle to overcome all the external resistances.

Resistances of the Machinery.—The losses due to the resistances of the moving parts of the machinery are usually expressed in percentages of the net resistance at the rack circle or at the ends of the bridge. These losses can only be determined by experiment, but are nearly a constant for any given arrangement of machinery. For swing bridges the loss due to resistance of the machinery is usually taken at 100 per cent of the work done at the pitch line of the rack circle in turning the bridge, or at the ends of the bridge in lifting the ends; then

$$\left.\begin{array}{l} \text{Total power required in lifting } = 2\left(\frac{3}{16}\frac{Wh}{t}\right) \text{ ft.-lbs. per second;} \\ \text{“ \quad “ \quad “ \quad “ turning} = 2Fv \text{ ft.-lbs. per second.} \end{array}\right\} \quad \ldots \quad (11)$$

The time t for lifting, and the time t for acceleration in turning, having separate values.

381. Design for Engine.—*For Lifting the Ends.*—The total horse-power required, including all resistances of the machinery, is

$$H.P. = \frac{2\left(\frac{3Wh}{16t}\right)}{550}; \quad \ldots \ldots \ldots \ldots \ldots (12)$$

in which t = time in seconds for lifting;
h = total lift in feet;
W = total load in pounds to be lifted.

For Turning the Bridge.—If it is desired to uniformly accelerate the motion of the bridge during the first half of the quarter turn, and uniformly retard the motion during the last half, then t, the time for acceleration, becomes one half of the whole time consumed in opening the bridge.

If it is desired to open the bridge by accelerating it until it attains a given velocity, which is to be maintained until the motor is reversed or the brakes put on, when the bridge is brought to rest, then,

Let T = time required for opening the bridge;
$L = \frac{1}{4}$ circumference of rack circle;
v = maximum linear velocity at rack circle;
l = space on rack circle in which bridge is to be accelerated;
l_1 = space on rack circle in which bridge is to be retarded;
$L - (l + l_1)$ = space on rack circle in which bridge is to have a uniform velocity of v;
t = time for acceleration;
t_1 = time for uniform velocity v;
t_2 = time for retardation.

Then

$$T = t + t_1 + t_2;$$

$$t = \frac{2l}{v}, \qquad t_1 = \frac{L - (l + l_1)}{v}, \qquad t_2 = \frac{2l_1}{v}.$$

$$\therefore v = \frac{L + l + l_1}{T}. \quad \ldots \ldots \ldots \ldots \ldots (13)$$

When $l = l_1 = \frac{L}{3}$, which is usual, we have

$$v = \frac{5L}{3T}. \quad \ldots \ldots \ldots \ldots \ldots (14)$$

The value of v is then substituted in the equation for the power required in turning (eq. 11). Then the total horse-power required in turning is

$$H.P. = \frac{2Fv}{550}, \quad \ldots \ldots \ldots \ldots \ldots (15)$$

where F is the total force applied at the rack circle by eq. (10), and v is the maximum linear velocity at the rack circle.

Having determined upon the horse-power to be used, which determines the elements of the motor, we can then work back from the motor through the various parts of the machinery to the rack circle, or to the ends of the bridge, and make every part capable of resisting the greatest force which can come upon it from the motor. In that case it is customary to

assume all the parts to move without any resistance, which is an error on the side of safety. Then the total work done by the motor in foot-pounds per second = H.P. × 550.

Assume the motor to be a steam-engine.

Let n = greatest number of revolutions of engine, or maximum engine speed, per minute;

$\frac{l}{2}$ = length of engine crank;

d = diameter of cylinder in feet;

m = ratio of diameter of cylinder to stroke, or $m = \frac{d}{l}$;

p = steam pressure in pounds per square foot.

Then for an engine taking steam during the full stroke,

$$\text{H.P.} \times 550 = 2pl\frac{\pi m^2 l^2 n}{4 \times 60} = W; \quad \ldots \ldots \ldots \quad (16)$$

$$m = \sqrt{\frac{W \times 120}{p\pi l^3 n}}. \quad \ldots \ldots \ldots \quad (17)$$

But $d = m \times l$, from which knowing m, and assuming a value for l, we can find d.

All motors used for swing bridges should be reversible and capable of exerting their maximum force during any part of a revolution. If the motor be a steam-engine, the engine should be of the double-cylinder type * with a link motion for reversal. The size of the cylinders should be such that the engine would be capable of exerting the full force required during any part of the stroke. With such a motor, a swing bridge can be turned in either direction, and by reversing the motor it can be used as a brake. This is very important in bridges which have to be opened or closed rapidly, or handled during high winds.

382. Constants.—The coefficient of rolling friction, ϕ_1, as determined in the experiments on the Thames River Bridge,† was 0.003. The coefficient ϕ_2, for sliding friction, may be taken at 0.1. The coefficient ϕ_3, for collar friction, may be taken the same as for sliding friction. In the formula for the effect of unbalanced wind pressure, p may be taken as the pressure due to a wind velocity of thirty miles per hour, or $p = v^2 \times .004 = 3.6$. However, when the unbalanced wind pressure is due to a wind velocity of thirty miles per hour it is the predominating resistance. Such an unbalanced wind pressure may never occur when the bridge is swinging; and it is customary, when it is considered, to neglect all other resistances. In other words, the motor would then be able to hold the bridge only against the unbalanced wind, as a brake, without attempting to turn the bridge against the wind.

Let P = total unbalanced wind pressure;

g = distance of centre of pressure from the centre of bridge;

R = radius of rack circle;

F_w = force at rack circle to balance P.

Then $F_w = \frac{Pg}{R}$; and the total horse-power

$$\text{H.P.} = \frac{F_w \times v}{550}; \quad \ldots \ldots \ldots \quad (18)$$

in which v is the velocity at the rack circle when the unbalanced wind pressure occurs. The value of v is taken about one half the maximum value, or when no unbalanced wind pressure is considered.

* This form of engine is commonly known as the marine engine.

† See a paper on the Experimental Determination of Rolling Friction in operating the Thames River Bridge, in Vol. XXV of the Transactions of the American Society of Civil Engineers, by Alfred P. Boller, Jr., and H. J. Schumacher.

DESIGN OF A 216-FOOT SWING BRIDGE.

383. Data.—The following data will be taken:

A single-track swing bridge 216 feet long, to be operated by hand power exclusively.
Two trusses placed 16 feet apart (14 feet in the clear).
Four panels in each arm.................... 25 ft. 0 in. = 100 ft. × 2 = 200 feet.
One centre panel........... .... = 16 "

Total length centre to centre of end pins.. .... = 216 "

Depth of trusses 25 feet at ends (21 feet clear).
" " " 40 feet " centre (" " ").
Distance from base of rail to under side of bridge not limited.
Distance from base of rail to top of centre pier not limited.

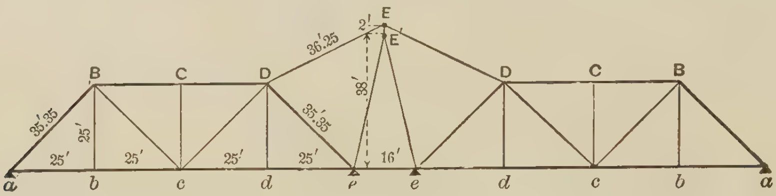


FIG. 402.

The links at EE', Fig. 402, being at all times practically vertical, the stresses in the two members $E'e$ are at all times equal to each other.

384. Floor System and Laterals.—Having fixed upon the outline of the trusses, the design of the floor system, that is, the stringers and floor-beams, follows as described in Chapter XXI.

The next step will be to design the wind trusses, or lateral systems.

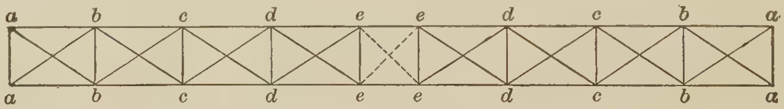


FIG. 403.

Bottom Laterals.—Make all the diagonals of angles with riveted connections, so as to resist both tension and compression.

When it is possible to do so, the laterals should be riveted to the flanges of the stringers at their intersection.

The laterals in the middle panel *ee*, Fig. 403, may or may not be omitted, according to the arrangement of the turntable.

Top Laterals (Fig. 404).—Make all the diagonals of angles. Make all the bracing for portals at B, D, and E of angles. Make all the struts at C and E of angles.

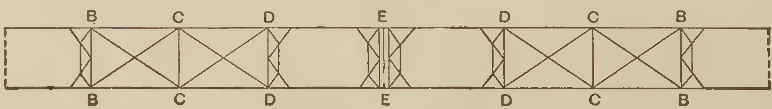


FIG. 404.

When the bridge is swinging, the entire wind pressure on the top lateral system goes to the turntable. The wind forces accumulating at D are transmitted to the turntable by means of direct bending in the posts De (see Fig. 402).

The laterals in the panel DE are omitted. The lateral wind force at E is transmitted to the turntable by means of direct bending in the posts $E'e$.

When the bridge is closed and the ends are raised, the portion of the wind forces which goes to the abutment is transmitted by means of direct bending in the end posts aB.

The stresses in the lateral systems are analyzed as shown in Chap. XII. The bending moments in the posts are computed on the assumption that the ends are fixed as shown in Chap. X, Art. 151. These bending moments are used in proportioning the posts, and combined with the direct stresses from the vertical loads.

385. Dead Load.—Having dimensioned all of the parts which are not influenced by the stresses in the main trusses, their weights may be computed. We need therefore only assume the weight of the main trusses. However, we know that the weight of the main trusses for any swing bridge is about the same as for a fixed span of the same length minus the weight of the turntable to be used.

We can then assume for computations :

Total weight of two main trusses........	=	135,000 lbs.
" " " floor system (computed)	=	70,000 "
" " " lateral systems (")	=	17,500 "
Total weight of iron-work above turntable	=	222,500

Check : $5l^2 - 50l =$ total weight of iron-work.

Assume dead load per foot $= w$ (iron) + 400 lbs. (track).

The stresses are then computed for the various cases as shown in Chap. XII, and the members are proportioned as shown in Chap. XVII.

Having proportioned the main trusses, their weight is computed, so that we finally get the computed weights of all the iron-work above the turntable.

386. The Turntable, including the turning and end-lifting arrangements.

Let us assume that the total weight of the iron-work above the turntable came out as assumed, $5l^2 - 50l = 222,500$ lbs.

Weight of iron-work above turntable	=	222,500 lbs.
" " track = 216 × 400	=	86,400 "
Total dead load on turntable	=	308,900 "
Maximum live load on turntable (Case IV)	=	(say) 405,000 "
Total live load + dead load on turntable	=	" 713,900 "

The turntable to be used is to be rim-bearing and centre-bearing combined. (See Fig. 405 for general outline of turntable.) The total load on the turntable comes on the four corners P of the square formed by the longitudinal and transverse girders, as explained in Art. 375 of this chapter.

If P is the load on each corner, then one half goes each way to p, so that the load at p is $\frac{P}{2}$. The load at p is carried equally to the two adjacent radial girders by means of short cross-girders aa, called diaphragms. The load at a is then again divided, a portion going to the centre C and the remainder to the sixteen points d on the rim. For this case

$$\text{the load on the rim} = \frac{4P \times 8.8}{11.3} = 3.115P,$$

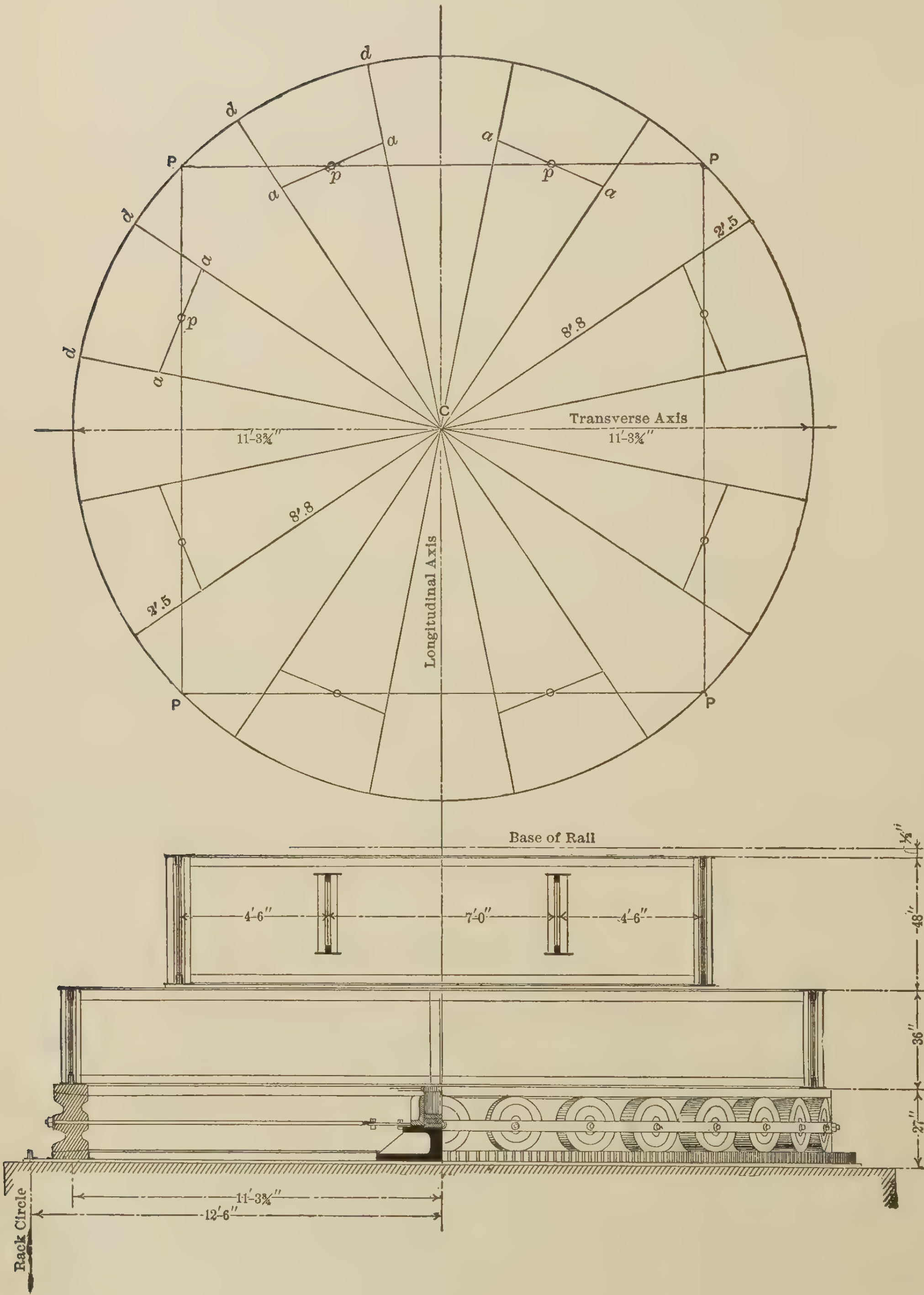
$$\text{and the load on the centre} = \frac{4P \times 2.5}{11.3} = 0.885P.$$

The longitudinal and transverse cross-girders are designed as simple beams supported at p, and loaded at each end by a load of $\frac{1}{2}P$. The diaphragms aa are designed as beams supported at each end, and loaded at the middle by a load of $\frac{1}{2}P$.

The radial girders Cd are designed as beams supported at each end, and loaded by a load of $\frac{P}{4}$ at 2.5 feet from one end.

In designing the circular girder or drum, the analysis for a straight beam will not apply. However, it is assumed, in order to provide ample strength and stiffness, that the circle is composed of a series of straight beams of a length equal to two segments dd, or one eighth of the circumference of the circle. These beams are assumed, furthermore, to be disconnected, that is, free at their ends, and loaded in the middle by a load $= \frac{P}{4} \times \frac{8.8}{11.3}$, for this case, with end supports.

Total dead load on turntable..	=	308,900 lbs.;
Maximum live load on turntable....................................	=	405,000 "
Turntable weighs (say)..	=	86,300 "



d
d
d
d
d
a
a
a
a
a
a
p
p
p
P
P
P
P
2'.5
8'.8
8'.8
2'.5
C
Transverse Axis
11'-3¾"
11'-3¾"
Longitudinal Axis
Base of Rail
½"
4'-6"
7'-0"
4'-6"
48"
36"
27"
11'-3¾"
12'-6"
Rack Circle

FIG. 405.

Total dead load on rollers .... $= \frac{395,200}{4} \times 3.115 =$ say 307,800 lbs.;

" live load " " .... $= \frac{405,000}{4} \times 3.115 =$ " 315,400 "

" dead load " centre .... $= \frac{395,200}{4} \times 0.885 =$ " 87,400 "

" live load " " .... $= \frac{405,000}{4} \times 0.885 =$ " 89,600 "

Assumed diameter of rollers .... = 18 inches;

Number of rollers .... $= \frac{22.62 \times \pi}{1.5} =$ say 40 rollers 18 in. diameter;

Permissible bearing on rollers (static).... $= 300 \times 18 = 5400$ lbs. per linear inch;

" " " " (moving).. $= 150 \times 18 = 2700$ " " " "

Total number linear inches required (static) $= \frac{623,200}{5000} = 124$ inches;

" " " " " (moving) $= \frac{307,800}{2500} = 123$ inches.

Use forty rollers, 18 in. diameter, 4 in. face = 160 linear inches, to be of cast-iron with chilled faces. The treads for rollers should be made of wrought-iron or steel. The latter is preferable and may be of the same grade as that used for track rails for railways.

Permissible bearing on centre (static) .... = 6000 lbs. per square inch;

" " " " (turning) .... = 3000 " " " "

Area required in disks (static) .... $= \frac{177,000}{6000} = 29.5$ square inches;

" " " " (turning) .... $= \frac{87,400}{3000} = 29.2$ " "

Use a centre pin $6\frac{1}{2}$ inches diameter..................... = 33.2 " "

" 3 disks $6\frac{1}{4}$ " " .... = 30.7 " "

The upper and lower disks to be hard steel and the middle disk to be phosphor-bronze. The centre casting requires for bearing on the masonry $\frac{177,000}{300} = 590$ square inches. Use casting with base 27.5 inches diameter = 594 square inches.

The next step in order should be the turning and end-lift arrangements; but before we can proceed with the latter it is necessary to know the end deflection.

387. Framing of the Trusses for Proper Camber and End Deflection.—The lengths of the members should be such that with a proper elevation of the ends the bottom chord is a horizontal line under the maximum load.

(I.) Compute the normal lengths of the members, assuming the elevation of the ends to be normal.

(II.) Increase or diminish the lengths of the members according as they are under compression or tension, respectively, when the full live load is on the bridge. This change of length is rigidly $\frac{pl}{E}$ for each member, where p is the total live and dead load stress per square inch in that member. Since such measurements of length are made only to the nearest $\frac{1}{32}$ inch, the change of length used is taken so as to give the final length in even thirty-seconds of an inch.

(III.) Find the change produced in the elevation of the ends due to the change in the lengths of the members by the formula given in Art. 200, Chap. XV, $D' = \Sigma\frac{pul}{E}$; wherein $\frac{pl}{E}$ is to be taken as the "change of length used" in (II) for each member. The value of D' in this case is called the camber deflection.

(IV.) Find the end deflection, span swinging, dead load only acting, by the formula $D = \Sigma\frac{pul}{E}$. The value of D in this case is called the true deflection.

(V.) Assume that one half of the deflection, from (IV), is eliminated by lifting the ends; then shorten the upper chord, near the centre of the bridge, so as to raise the arms the remaining one half of the end deflection from (IV) plus the whole deflection caused by the change in the length of the members, or condition (III). Let $D =$ deflection from (IV), or the true deflection; let $D' =$ deflection from (III), or the camber deflection.

Assume that $\frac{1}{2}D$ is eliminated by lifting the ends; then the remaining $\frac{1}{2}D + D'$ is to be eliminated by shortening or lengthening the upper chord near the centre of the bridge, according as the value of $\frac{1}{2}D + D$ is plus or minus. However, although the value of D' is usually minus or an upward deflection, it is never greater than $\frac{1}{2}D$; so that $\frac{1}{2}D + (-D')$ is always plus, which necessitates shortening the upper chord, and never lengthening it.

388. The Turning Arrangements.—Let us assume the time for opening or closing the bridge by hand power to be 240 seconds; then if the time for uniform acceleration, t, = 120 seconds, the maximum velocity at the rack circle, which is twice the mean velocity, v, $= \dfrac{25 \times 3.14}{4 \times 120} = 0.16$ foot per second. If the bridge is to be operated exclusively by hand power, no provision is made for swinging the bridge against unbalanced wind pressures; so that the force F_w at the rack circle necessary to overcome wind is here neglected. From eq. (10) we have

$$F = F_r + F_s + F_c + F_a = \text{force at rack circle};$$

$$F_r = \phi_1 W_r \frac{R_1}{R} = 0.003 \times 307{,}800 \times \frac{11.3}{12.5} \ldots\ldots\ldots\ldots = 831 \text{ lbs.};$$

$$F_s = \phi_2 W_s \frac{d}{3R} = 0.1 \times 87{,}400 \times \frac{0.5}{3 \times 12.5} \ldots\ldots\ldots\ldots = 116 \text{ “}$$

$$F_c = \frac{4}{3} \times \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \times \frac{\phi_3 W_r}{R} = \frac{4}{3} \times \frac{26}{8} \times \frac{0.1 \times 307{,}800}{12.5 \times 12} = 860 \text{ “}$$

$$F_a = \frac{Iv}{R^2 t} = \frac{395{,}200 \times \overline{62.5}^2 \times 0.16}{32.2 \times \overline{12.5}^2 \times 120} \ldots\ldots\ldots\ldots\ldots\ldots = \underline{416} \text{ “}$$

$$F = 2223 \text{ “}$$

The total force required to overcome all resistances equals twice the net resistance = 4446 lbs. With two men on the turning lever at 50 lbs. each, the multiple of force by gearing $= \dfrac{4446}{2 \times 50} = 44.46$. The time to open or close, with men walking at the uniform rate of 4 ft. per second $= 44.46 \times \dfrac{2\pi R}{4} \times \dfrac{1}{4} = 222$ seconds = 3 minutes 42 seconds.

The maximum rate of walking would then be 8 feet per second. If the velocity v is generated in less than 120 seconds, the maximum rate of walking would be less, to open in the same time.

389. The End-lifting Arrangements.—The total power required in lifting the ends, eq (11), $= 2\left(\dfrac{3Wh}{16t}\right)$ in foot-pounds per second, in which

$W =$ total dead load above turntable = 308,900 lbs.;
$h =$ lift in feet.................... = 0.1 ft.;
$t =$ time required for lift in seconds.

With two men on turning lever at 50 lbs. each, the multiple of force by gearing $= \dfrac{2 \times 3 \times 308{,}900}{16 \times 2 \times 50} = 1158$.

The time required to raise the ends, with men walking 4 ft. per second, $= 1158 \times 0.1 \times \frac{1}{4} = 29$ seconds.

In case it should become necessary to operate the bridge more rapidly, it would be advisable to introduce a steam-engine or some other form of motor.

Assume the time is to be limited, so that the total time for opening or closing shall not exceed 75 sec-

onds, including the time necessary for raising or lowering the ends. Let the time of opening be 60 seconds and the time for acceleration = 30 seconds; then the maximum velocity at rack circle,

$$v_r = \frac{25 \times 3.14}{4 \times 30} \dots\dots\dots\dots = 0.65 \text{ ft. per second};$$

$$F_a = \frac{Iv}{R^2 t} = \frac{395,200 \times \overline{62.5}^2 \times 0.65}{32.2 \times \overline{12.5}^2 \times 30} \dots\dots = 6661 \text{ lbs.};$$

$$F_r + F_s + F_c \text{ as before} \dots\dots\dots\dots = \underline{1807} \text{ ``}$$

$$F = 8468 \text{ ``}$$

The force required at the rack circle for such a rapid acceleration will be found to be ample to hold the bridge against any unbalanced wind pressure, so that the wind is here again neglected.

$$\text{Horse-power required} \dots\dots = \frac{2Fv}{550} = \frac{2 \times 8468 \times 0.65}{550} = 20 \text{ H. P.};$$

Let t, the time for lifting, = 5 seconds.

$$\text{Horse-power required} = \frac{2\left(\frac{3Wh}{16t}\right)}{550} = \frac{2 \times 3 \times 308,900 \times 0.1}{16 \times 550 \times 5} = 4.2 \text{ H. P.};$$

Entire time required for opening or closing = 65 seconds.

Use 20 horse-power engine with double cylinder and link-motion for reversal.

Then, from eq. (16), H. P. × 550 = 11,000 foot-pounds per second.

From eq. (17) we have

$$m = \sqrt{\frac{11,000 \times 120}{p\pi l^3 n}};$$

in which p = pressure in pounds per square foot . . . . = say 5760 lbs.;
l = length of stroke . . . . = say 0.7 ft.;
n = maximum engine speed or revolutions per minute = 250.

Then

$$m = \sqrt{\frac{11,000 \times 120}{5760 \times 3.14 \times 0.343 \times 250}} = 0.91.$$

Diameter of cylinders = $m \times l$ = say 7 inches.

Two cylinders 7 in. diameter, 8 in. stroke, with a maximum piston speed of $\frac{250 \times 8 \times 2}{12}$ = say 333 feet per minute, which is below the maximum piston speed generally allowed.

After determining the elements of the motor to be used, the next step would be to design the machinery, making every part strong enough to resist the maximum force which can come upon it from the motor. This branch of the design of swing bridges will not be treated here, as it properly comes under the head of machine design, and is fully treated in standard books on this subject.

CHAPTER XXV.

TIMBER AND IRON TRESTLES AND ELEVATED RAILROADS.

I. Timber Trestles.

390. **Use of Timber Trestles.**—Timber trestles are used in America under the following conditions:

(*a*) When the first cost is less than that of an earth-fill.

(*b*) When water-way must be left and it is not practicable to build either bridges or culverts on the first construction of the road.

(*c*) When the financial condition of the corporation will not admit of a more permanent kind of structure.

It has been estimated that there are at present in the United States at least 2400 miles of railway wooden trestle, of which at least 800 miles are likely to be permanently maintained as such. It is very common to build wooden trestles on a new line of road, and when it needs renewal to make an earth-fill. This can be done very cheaply after the line is constructed; and if the earthwork comes high on the first construction of the line, it may be economical to do this. Wooden trestles are built of all heights up to 150 feet, and although a multitude of patterns have been successfully employed, there are certain guiding principles which lead to a nearly uniform best practice. No discussion will be here given of simple pile bents, as not properly being framed structures. Neither will the subject of pile driving or the bearing resistance of piles be here entered upon.* The foundation of a framed timber bent is preferably composed of masonry or solid rock, but next to this piles are to be preferred.

391. **The Framed Bent.**—Some illustrations of properly framed timber bents are shown in Plate VI, those in the upper portion showing the practice of the Pennsylvania Railroad Company, and in the lower portion that of the Norfolk & Western Railway. There are always two vertical struts and two or more inclined ones, called "batter posts." The batter is 2½ in. to 3 in. to one foot. These, together with the sill and cap and the diagonal braces make up the frame, or "bent" as it is called, which is the elementary form of all trestle-work. When the timbers are all in pairs it is called a "double bent" see (the pattern used by the Toledo, St. Louis & Kansas City Railroad, Plate IX). As in the case of a Howe truss bridge, the timbers here are not nicely proportioned for their loads, but the sizes are made large to allow for decay. Purely empirical rules of practice are followed in this matter, and the various types shown in Plates V to X have been selected as offering good examples to follow.

392. **Bearing Joints.**—In Plate VI two kinds of bearing joints are shown. The mortise and tenon (Pennsylvania Railroad Standard) is giving place to the split cap, shown in the Norfolk & Western plans, Plate VI. Here the posts are notched to receive the two cap or sill pieces, a smooth surface being made on the outside at top to receive the stringer, and at bottom to rest on the bearing timbers. The advantages of this latter joint are that it facilitates repairs, and secures better timber for the caps and sills, since they are only half the thickness.

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* See Baker's "Masonry," John Wiley & Sons, for the best treatise on Foundations, including piles.

Another form of this joint is that shown in Plate IX, where all the bearings are made on cast-iron plates. This adds materially to the life of the joint and insures a more uniform distribution of the pressure.

Another method of accomplishing the same thing is by means of wrought-iron plates, as shown in Fig. 406. This joint has been used by the New York, Lake Erie & Western Railway, and is more fully described in *Engineering News* for Nov. 5, 1887. The plates may be stamped out and spiked on. They would probably more than save their cost in the time saved on the ground in the framing which is here rendered unnecessary.

In all cases where timber is used in compression across the fibres, attention must be given to the area presented to the imposed load, using the working stresses given on p. 354.

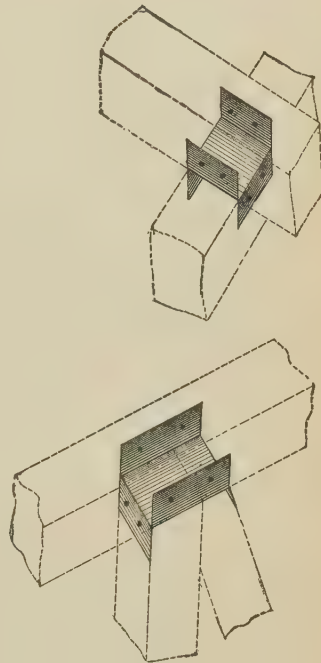


Fig. 406.

393. **Floor Systems.**—*Stringers.*—The stringers should consist of two or three sticks each, well separated from each other by cast-iron washers, and bolted together. They should extend over two spans and terminate at alternate bents. *They should never be notched down upon the caps.* This greatly weakens the stick as a beam, causing it to fail by shearing, or splitting back from the base of the notch. They should be spliced by means of large packing blocks, which are notched down over the cap and bolted to the stringers, as shown in Plate VI. On the Minneapolis & St. Louis Railway (Plate V) these packing pieces are simply cullings from the stringer pieces, sawed to 5½-foot lengths, and inserted with plain cast-iron washers for air circulation. The stringers are held laterally upon the caps either by bolting through or, better, by cast-iron brackets, as shown in Plate V.

Corbels should not be used. They add to the cost, and furnish large joints for the storage of water, and hasten decay. They are shown on Plates V and VIII, but they should be omitted from all wooden-trestle construction. The bearing area and beam strength should be sufficient without them. Thus in case of a span of 16 feet, with an equivalent live and dead load of 6000 lbs. per foot, or of 48,000 lbs. coming to the cap from one side, there should be at least three 8-inch stringers, and if the cap is 12 inches wide the bearing area is $6 \times 24 = 144$ square inches. This gives a pressure of 330 lbs. per square inch, which is about the limit of allowable bearing stress on white pine across the grain,* and might be permitted. If yellow pine were used for this work, three 6-inch sticks would be sufficient.

For very high, and therefore expensive, bents they should be placed farther apart, as in Plate VII, and the stringers braced out from the bent as shown. The shortening of the span by means of corbels, as in Plate V, is of doubtful economy or expediency. Neither is it advisable to truss the stringers with iron rods and king-posts, as is sometimes done. The additional strength from this source, with shallow depths to the trussing, is very small, and too great reliance is apt to be put in such a combination.

Packing.—The stringers should never be more than 8 inches thick, and are usually from 14 to 18 inches deep. They should be separated two inches or more apart, and packed with the splicing timbers at the bents, as shown in Plates V and VI. These splice or packing pieces are notched over the cap timber, but the stringers are not notched or framed in any way. When corbels are used the packing timbers may be dispensed with, and cast-iron spools or separators used for packing, as shown in Plate V.

* See p. 354, for working stresses. If smaller caps are used, a sufficient bearing area demands the use of corbels which should be made of white oak or some other durable hard wood. See Plate X*a*.

The ties, guard-rails, safety or rerailing devices, etc., are not peculiar to this form of structure.

394. Sway Bracing.—These are designed to stay the bents against lateral forces of all kinds and to add to the lateral rigidity of the structure. The batter posts assist greatly to this end, but when the bents are more than 18 or 20 feet high some kind of sway bracing should be employed. This bracing usually consists only of 2-inch to 4-inch plank, from 8 to 12 inches wide, spiked to the sides of the bents, as shown in the plates. This is a very unscientific and inefficient kind of brace. This method of fastening cannot possibly develop the full strength of the brace. If the brace is thick, so as to have strength as a strut, it makes the leverage on the spikes so great that their strength is greatly reduced. If the brace is thin it has little strength as a strut, and besides the spikes have little bearing area. It is not practicable to insert simple struts in the plane of the bent, as "bridging," since this would require too many joints and too much cutting and fitting. The ordinary method of cover-planks, attached on the two faces of the bent, seems to be the only feasible scheme. It now remains to provide the most efficient connection. The greatest objection to the use of spikes is the small bearing area on the wood. Large treenails (wooden pins) would be much better. If 3×6-inch plank be used, and treenails $1\frac{1}{2}$ inches in diameter, then the bearing area is $4\frac{1}{2}$ square inches and the shearing area of the pin is $1\frac{3}{4}$ square inches. If the plank be of white pine and the treenail of long-leaf yellow pine or of white oak, the strength of the joint would be very much greater than if it were made of one 3×12-inch plank fastened with three $\frac{1}{2}$-inch square ship-spikes. The spikes will also rust rapidly just at the joint where the strength is most required. The treenails should be turned to a uniform diameter about $\frac{1}{32}$ of an inch greater than that of the hole. The holes in the braces could be bored before erection, and then the braces lightly spiked to place, when the holes in the posts and caps could be bored in exact line with the outer holes. This would make a reasonably rigid and permanent joint, which would develop its full resistance without appreciable distortion.

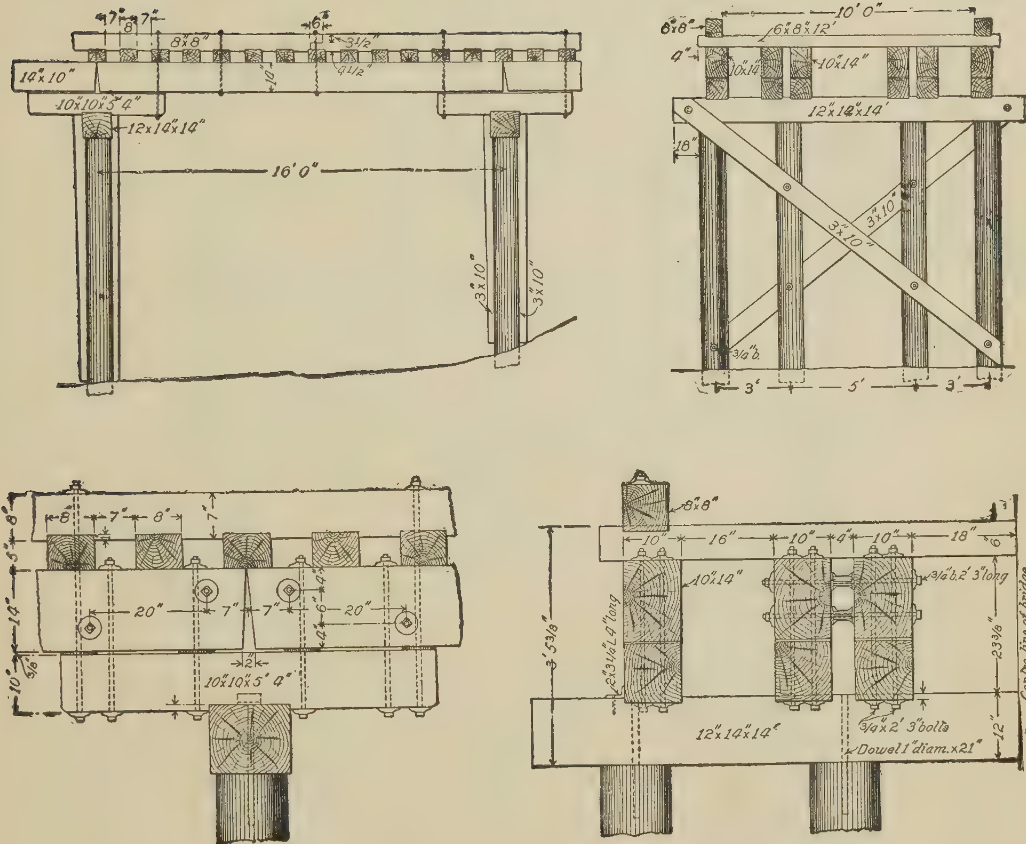
When the bents are more than about 20 feet high they should be divided into panels or stories, and sway bracing introduced into each panel. It is not possible to compute the stress on the sway bracing, except for assumed wind pressures. In fact, it never is computed except for very high structures. On low trestles the sway bracing is designed to resist the lateral throw of the engine and train rather than the wind pressure.

395. Longitudinal Braces.—It is not uncommon to merely join the bents together by outside horizontal girts, or waling-strips, as shown on Plate IX, the diagonal braces being entirely omitted. This is a grave omission. For short reaches of low trestling it may be allowable to omit these diagonal braces between the bents, where there are provided substantial abutment resistances to take up the pull or thrust of the train; but it is not wise to rely upon such abutment action.

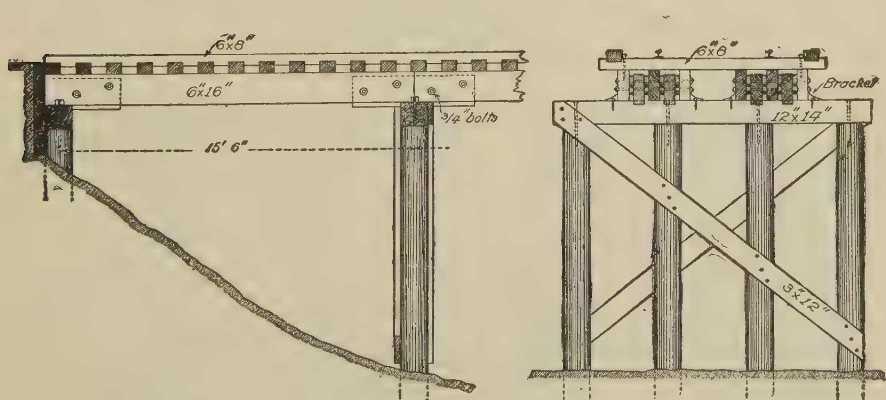
The pull of two locomotives at the head of a train may be as much as forty or fifty thousand pounds. When these are upon a stretch of trestle-work, something must resist this longitudinal external force. When braking a train also, the horizontal thrust may be much greater than this, and these great longitudinal pulls or thrusts should be amply provided for.

In this case a series of direct end-bearing struts may be introduced, as is done on the Pennsylvania Railroad design in Plate VI. Here a single row of $8'' \times 8''$ braces are introduced, joining alternate top cap-sills with the intermediate bottom sills in single-story trestle-work, or with the intermediate cross-girts in multiple-story trestles, as in Plates VIII and IX. With this joint there is a series of direct lines of struts from top to bottom, all end-bearing, as any strut should be. There is no hindrance to inserting these braces in this manner. It is very bad practice, therefore, to insert them in any other way, as is done in Plates VII and IX, where they are simply spiked or bolted on the outer sides; and to omit them altogether, as is so often done, even in high trestle-work, is simply criminal.

PLATE V.

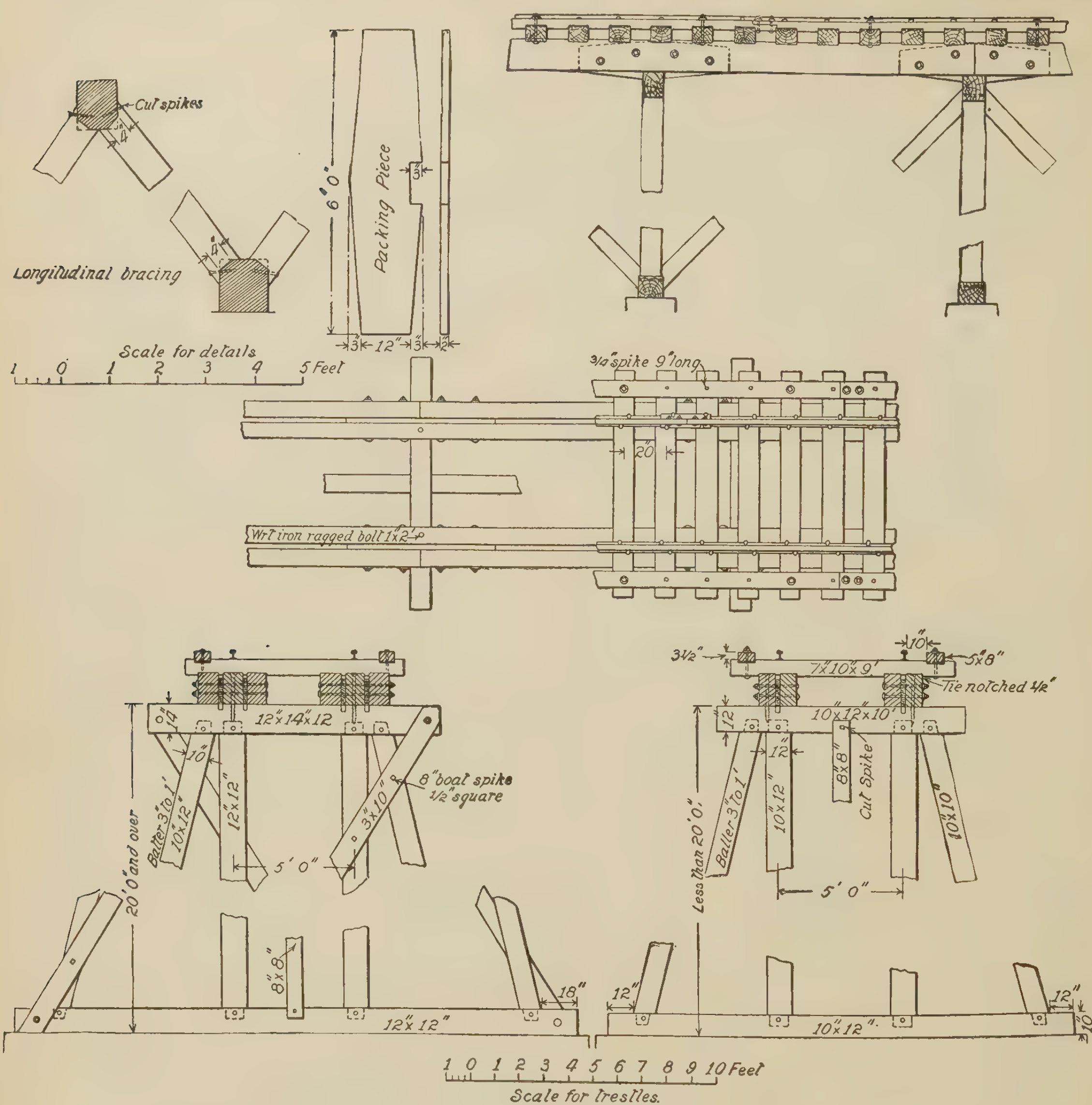


STANDARD PILE-TRESTLE, CHICAGO & NORTHWESTERN RAILWAY.

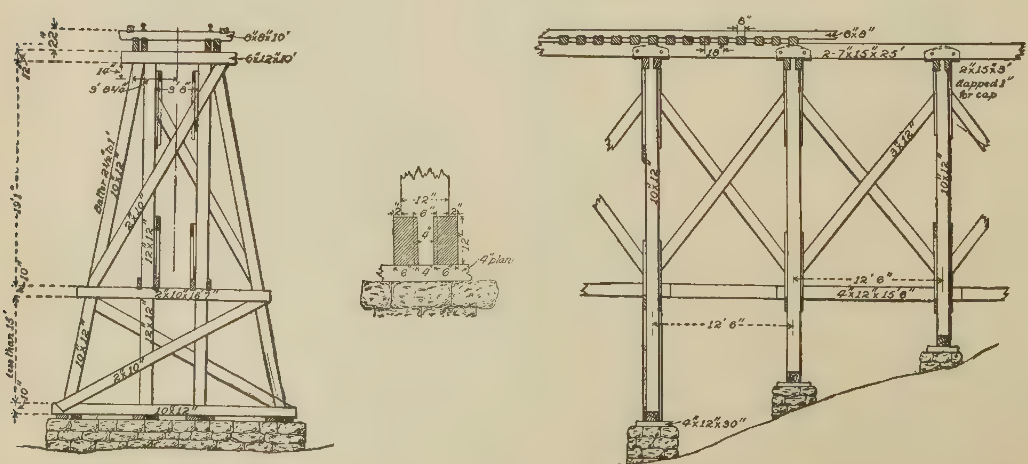


STANDARD PILE-TRESTLE, MINNEAPOLIS & ST. LOUIS RAILWAY.

PLATE VI.

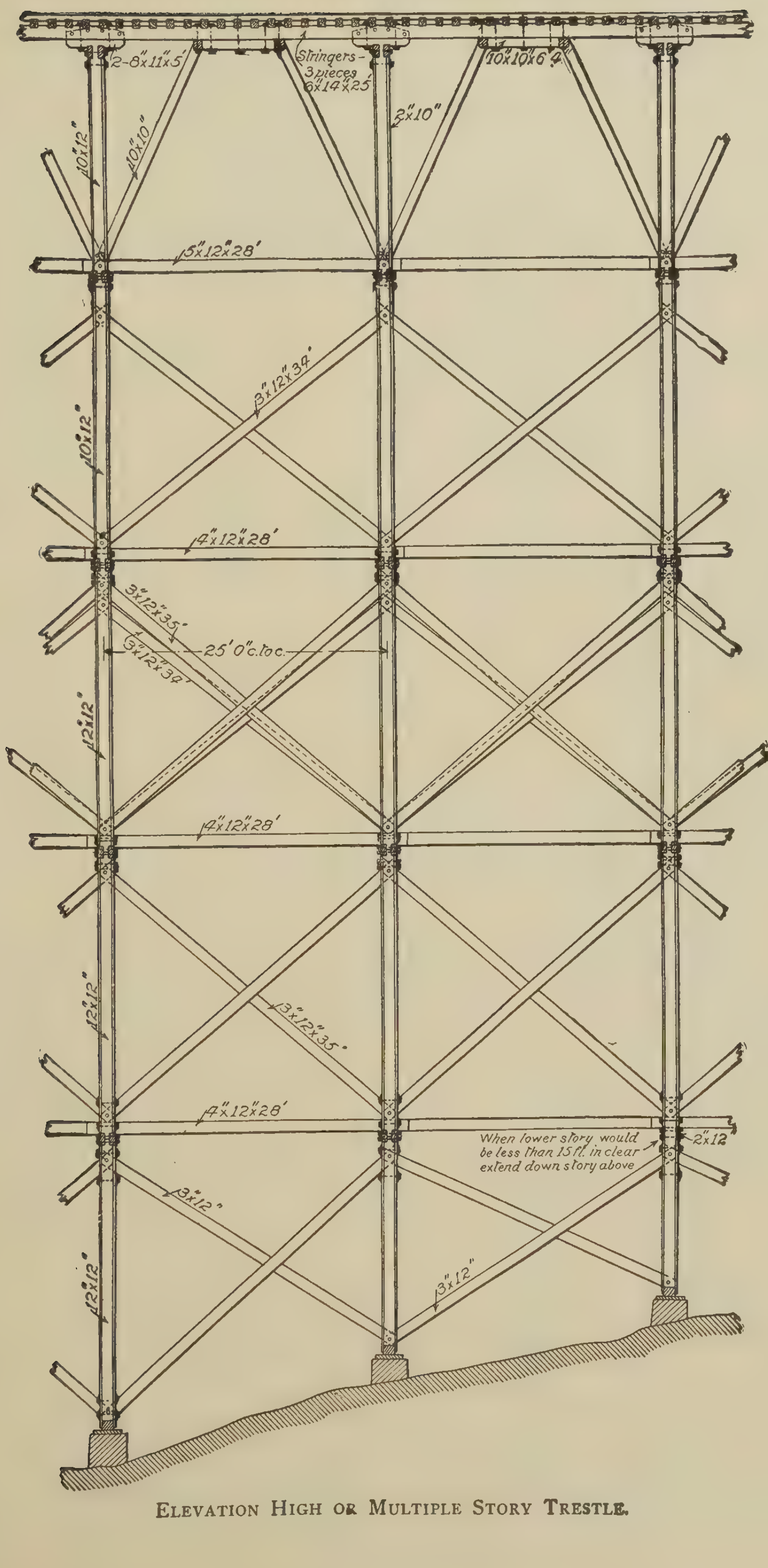


STANDARD FRAMED TRESTLES, PENNSYLVANIA RAILROAD.



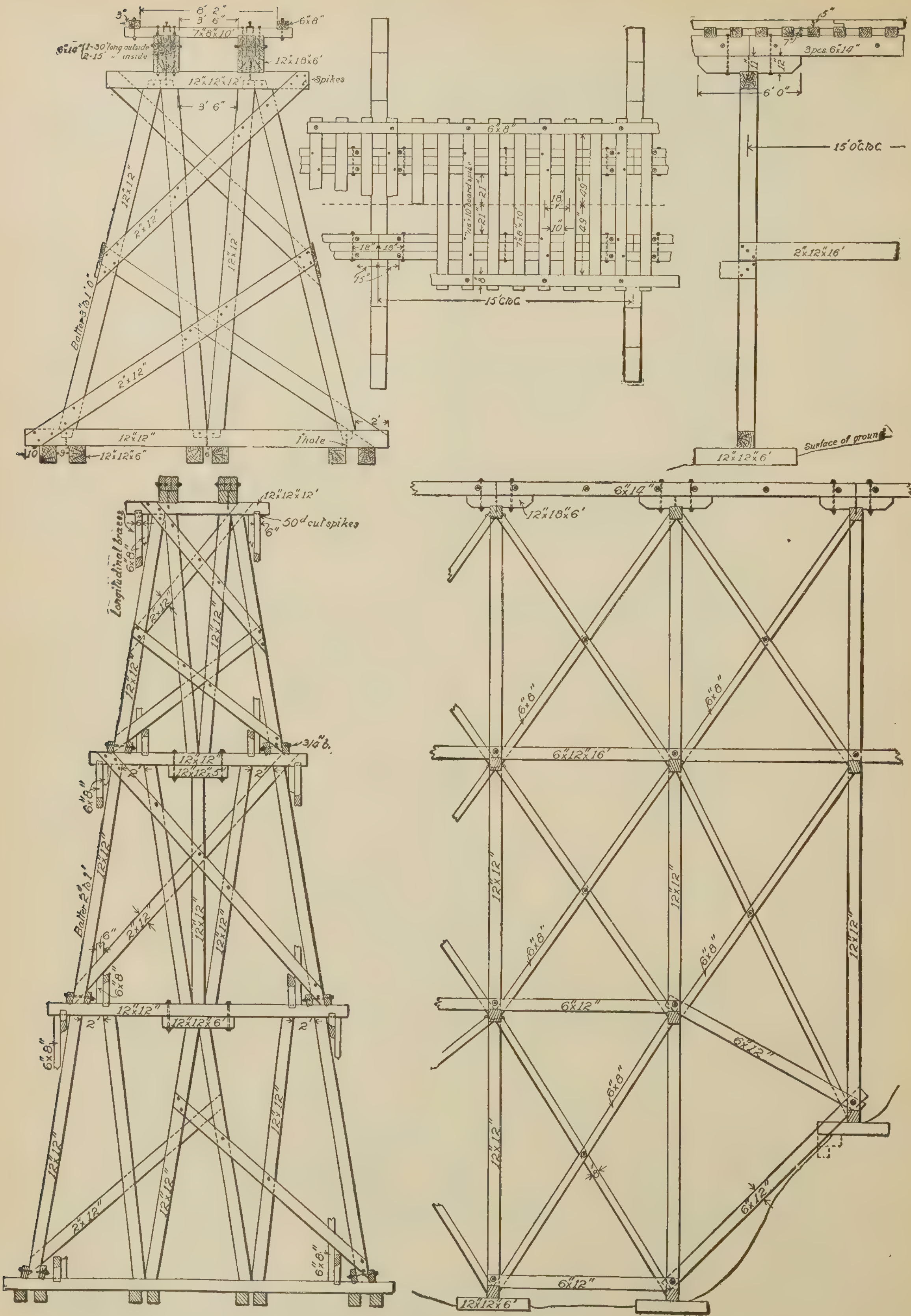
STANDARD TRESTLES, NORFOLK & WESTERN RAILROAD.

PLATE VII.



ELEVATION HIGH OR MULTIPLE STORY TRESTLE.

PLATE VIII.

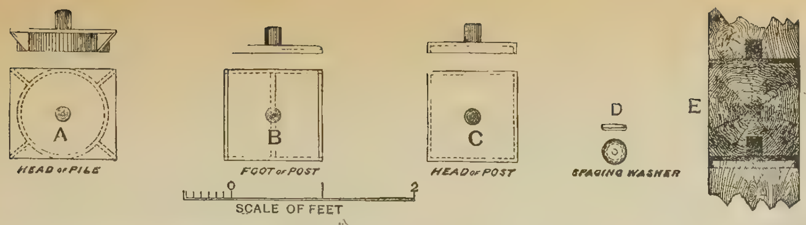


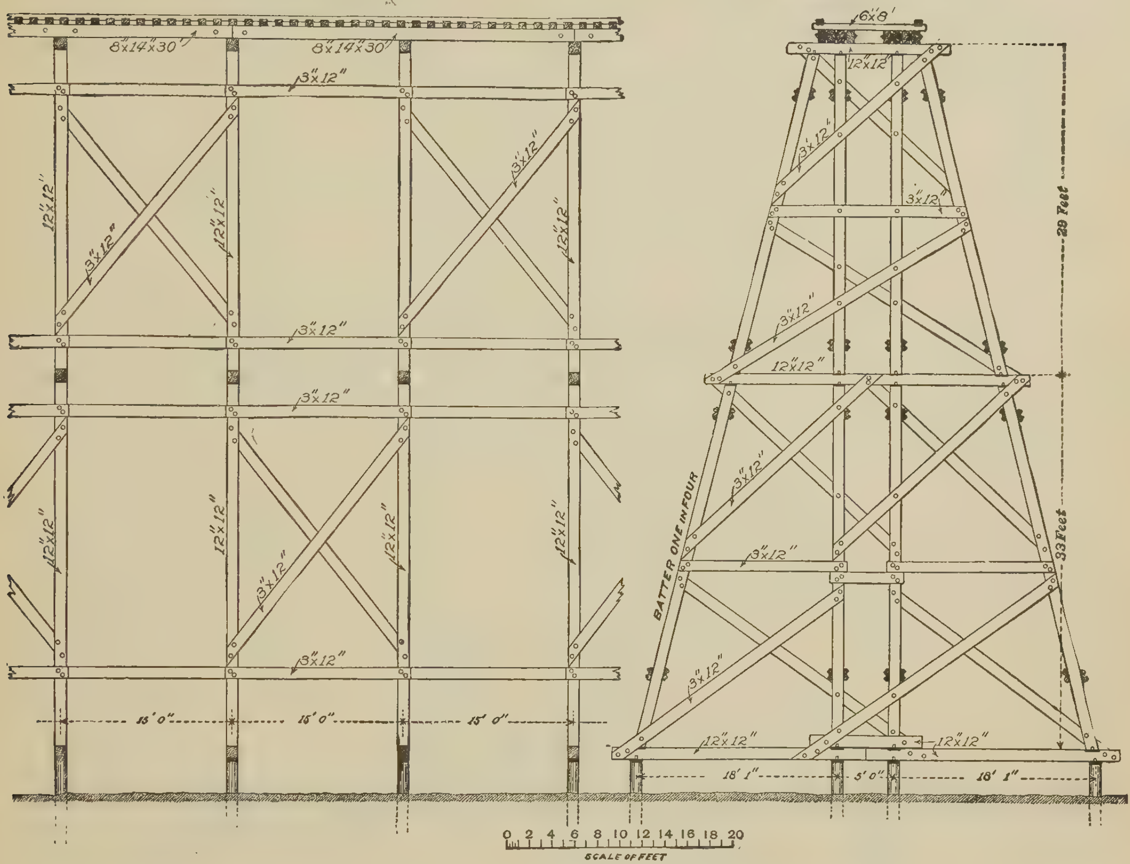
SCALE OF FEET

0 1 2 3 4 5 10 15 20 25

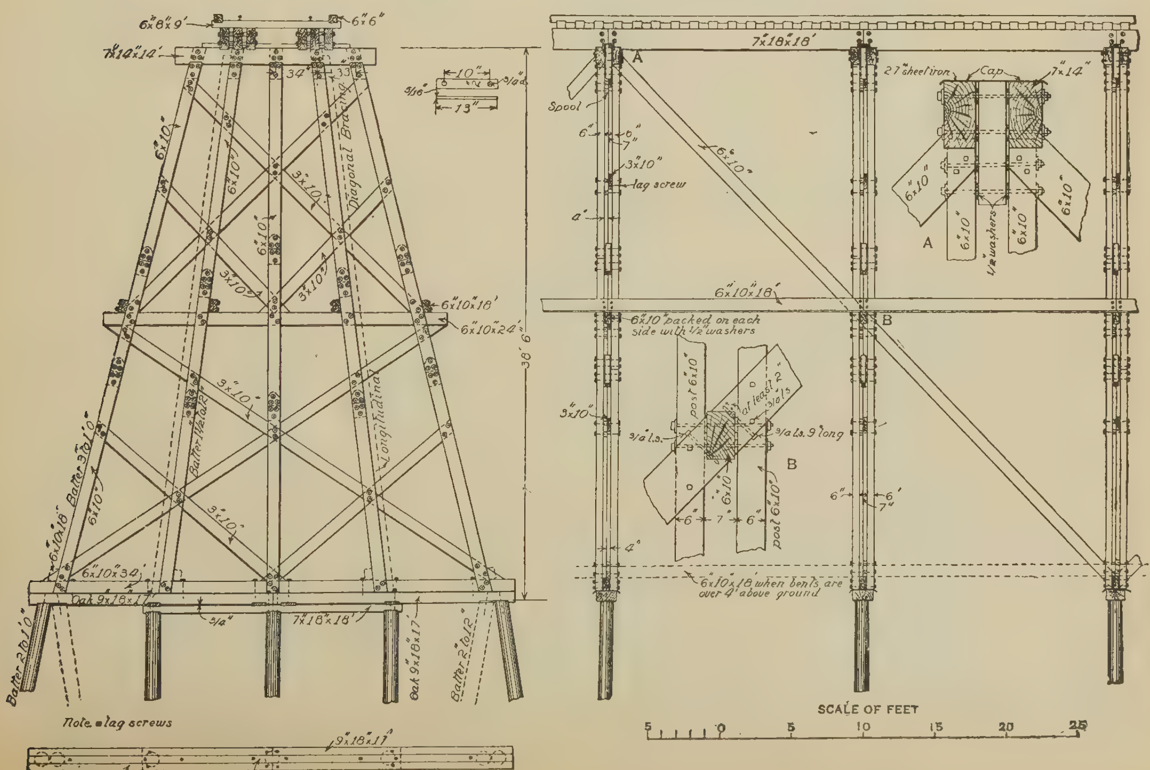
FORMER STANDARD TRESTLE, CHARLESTON, CINCINNATI & CHICAGO RAILROAD.

PLATE IX



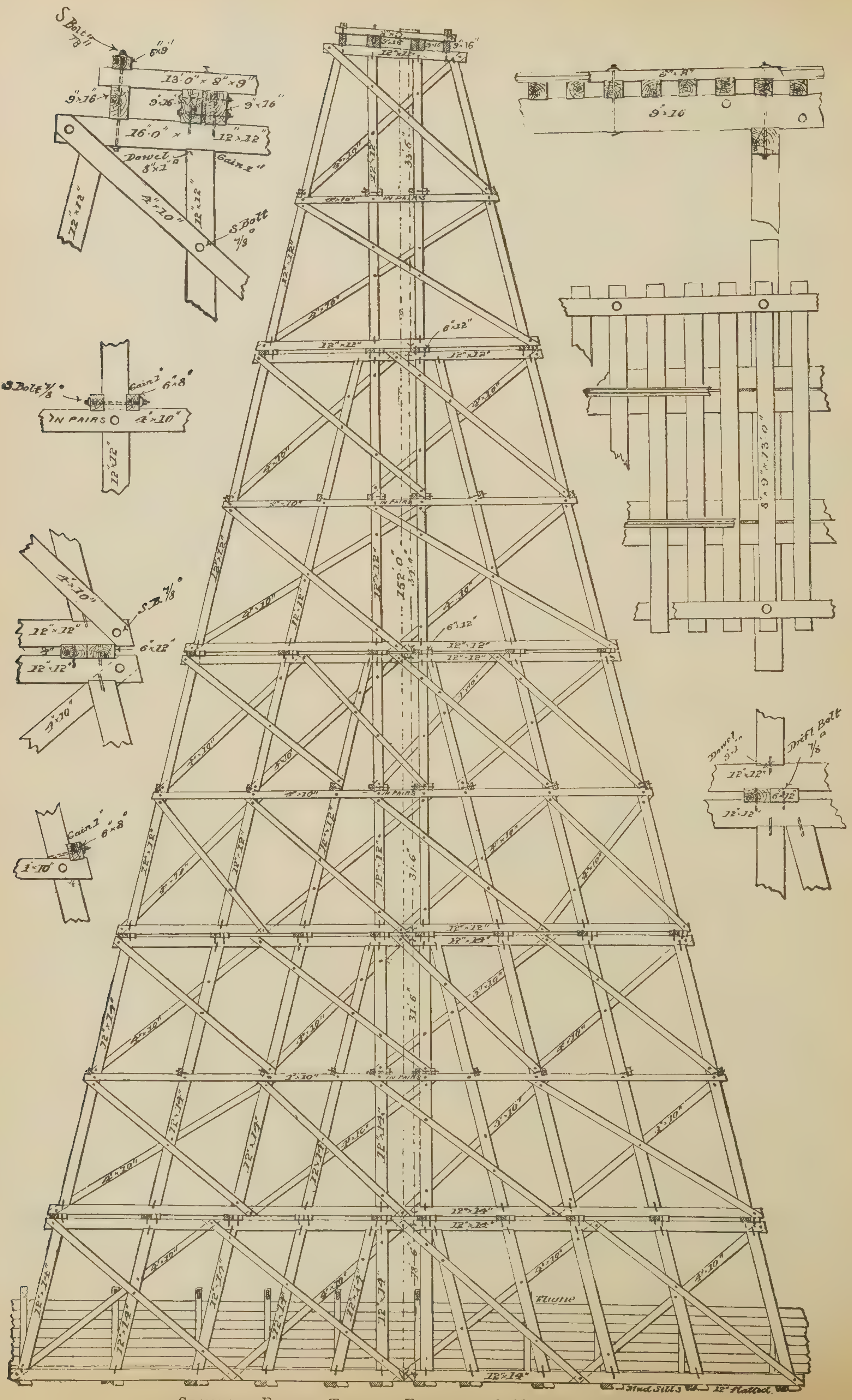


HIGH FRAMED TRESTLES, MINNEAPOLIS & ST. LOUIS RAILWAY.

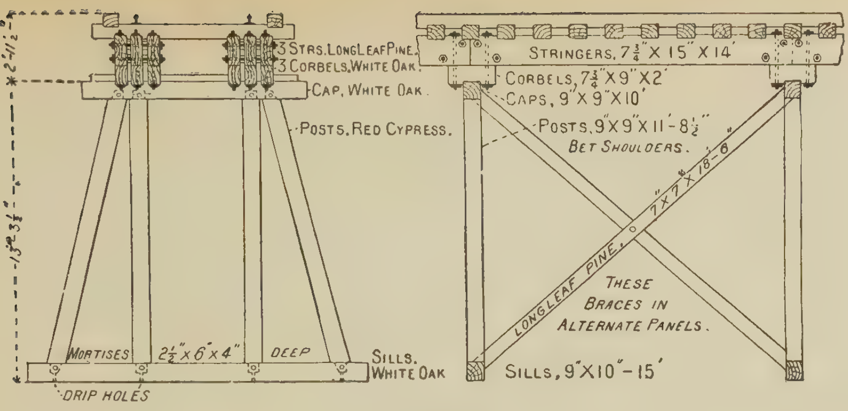


STANDARD FRAMED TRESTLE, TOLEDO, ST LOUIS & KANSAS CITY RAILROAD

PLATE X.

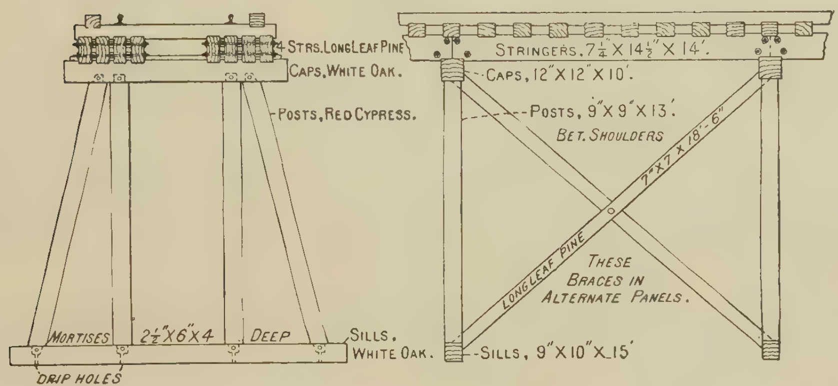


STANDARD FRAMED TRESTLE, ESQUIMALT & NANAIMO RAILWAY.



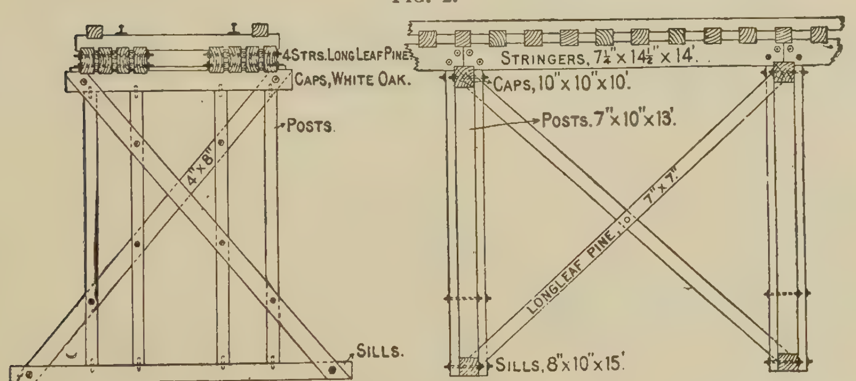
SCALE: $\frac{1}{8}$ INCH = 1 FOOT.

FIG. 1.



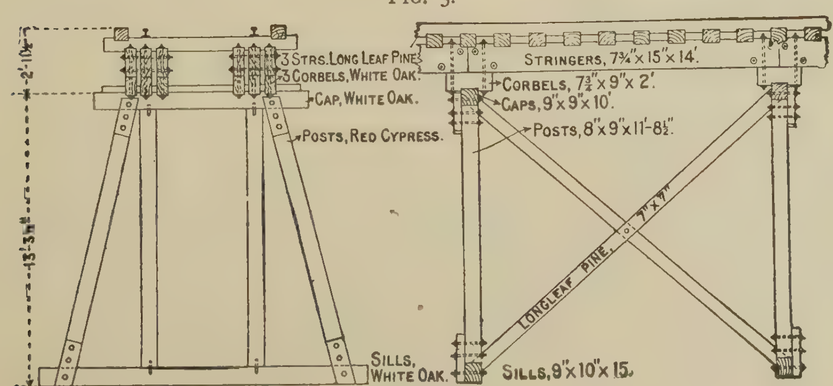
SCALE: $\frac{1}{8}$ INCH = 1 FOOT.

FIG. 2.



SCALE: $\frac{1}{8}$ INCH = 1 FOOT.

FIG. 3.



SCALE: $\frac{1}{8}$ INCH = 1 FOOT.

FIG. 4.

The ordinary practice in the design of timber trestles has been investigated by Mr. A. L. Johnson,* chief computer on the U. S. Timber Test work, and he has found that the current factors of safety in such structures vary, for different members and for different kinds of stress, from less than unity to 25, the former being for crushing or bearing stress across the grain. In other words, these structures are now very irrationally designed. He recommends the two designs in Figs. 1 and 2, Plate X*a*. Here are given both the species and the sizes best suited to the several parts, the design in Fig. 1 being with corbels and that in Fig. 2 without them. In these designs the following factors of safety were used:

Stringers in cross-breaking........................	5 +
Stringers in deflection, $\frac{1}{200}$ span....................	2 +
Stringers in end-bearings........................	4 +
Cap in bearing value..............................	3 +
Posts as columns	7.4 +

Mr. G. Lindenthall, in commenting on the designs, proposes* the designs shown in Figs. 3 and 4, Pl. X*a*, these being slight modifications of Mr. Johnson's designs. Here all mortises are omitted, wooden dowel-pins and splice-planks being used in their place. Mr. Lindenthall strongly recommends, also, the covering over of the stringers with galvanized sheet-iron to protect them from the weather and from the lodgment of live cinders, which causes the destruction of so many wooden trestles by fire. Mr. Bouscaren has tried planking over the structure and covering it with gravel, with good results.

The following table of working unit stresses is based on factors of safety of 10 in tension, 5 in longitudinal compression, 4 in lateral compression, 6 in cross-breaking, 2 in modulus of elasticity, and 4 in shearing, and on the most reliable information obtainable to date:

AVERAGE SAFE ALLOWABLE UNIT STRESSES IN POUNDS PER SQUARE INCH RECOMMENDED BY THE COMMITTEE ON "STRENGTH OF BRIDGE AND TRESTLE TIMBERS," AMERICAN ASSOCIATION OF RAILWAY SUPERINTENDENTS OF BRIDGES AND BUILDINGS, FIFTH ANNUAL CONVENTION, NEW ORLEANS, OCTOBER, 1895.

Kind of Timber.	Tension.		Compression.			Transverse Rupture.		Shearing.	
			With Grain.						
	With Grain.	Across Grain.	End Bearing.	Columns under 15 Diameters.	Across Grain.	Extreme Fibre Stress.	Modulus of Elasticity.	With Grain.	Across Grain.
Factor of safety................	10	10	5	5	4	6	2	4	4
White oak..........................	1,000	200	1,400	900	500	1,000	550,000	200	1,000
White pine.........................	700	50	1,100	700	200	700	500,000	100	500
Southern, long-leaf, or Georgia yellow pine..........	1,200	60	1,600	1,000	350	1,200	850,000	150	1,250
Douglas, Oregon, and Washington fir or pine:									
Yellow fir.........................	1,200	....	1,600	1,200	300	1,100	700,000	150	....
Red fir............................	1,000	....	....	....	....	800	....	....	....
Northern or short-leaf yellow pine.	900	50	1,200	800	250	1,000	600,000	100	1,000
Red pine...........................	900	50	1,200	800	200	800	600,000	....	....
Norway pine........................	800	....	1,200	800	200	700	600,000	....	....
Canadian (Ottawa) white pine.....	1,000	....	....	1,000	....	....	....	100	....
Canadian (Ontario) red pine......	1,000	....	....	1,000	....	800	700,000	100	....
Spruce and Eastern fir...........	800	50	1,200	800	200	700	600,000	100	750
Hemlock............................	600	....	....	800	150	600	450,000	100	600
Cypress............................	600	....	1,200	800	200	800	450,000	....	....
Cedar..............................	800	....	1,200	800	200	800	350,000	....	400
Chestnut...........................	900	....	....	1,000	250	800	500,000	150	400
California redwood...............	700	....	....	800	200	750	350,000	100	....
California spruce.................	....	....	....	800	....	800	600,000	....	....

* See *Bulletin No. 12, Forestry Division U. S. Agricultural Department*, 1896.

396. Bent and Post Splices.—When the trestle is higher than about forty or fifty feet it becomes necessary to splice the posts. There are several ways of doing this. Entirely separate bents can be constructed in sections or stories, and set one on top of another, as shown in Plate X, with an air space between the sills, except under the posts, where seasoned oak bearing-planks are carefully framed in.

Or the posts may abut end to end, with side notches cut at the joints for cross-girts, as shown in Plate VII.

Or cast-iron bearing caps can be employed as shown in Plate IX, these resting on one common sill, or cross-girt, at each splicing section. All these are good joints when well executed, but the last named is probably the most lasting, and the one which settles or crushes down the least.

397. Working Stresses.—The safe working stress per square inch for different timbers for different kinds of resistance is given pretty fully in Chapter XXIII, on Howe Trusses. As there shown, it is important to load timber in compression longitudinally only. So in a timber trestle it would be best to avoid crushing the timber across the grain at all if possible. or at any rate introduce as small an amount of timber under this kind of stress as possible, Thus in the Norfolk & Western Railroad plans in Plate VII, the posts are given end bearings continuously from top to bottom, being simply notched at the joints. On the other hand, Plate VIII shows 24 inches of timber under a lateral crushing load each 25 feet in height.

II. Iron Trestles.

398. General Design.—Iron railway trestles for single track usually consist of a series of plate-girder spans supported on iron columns, as shown in Fig. 407. The columns are

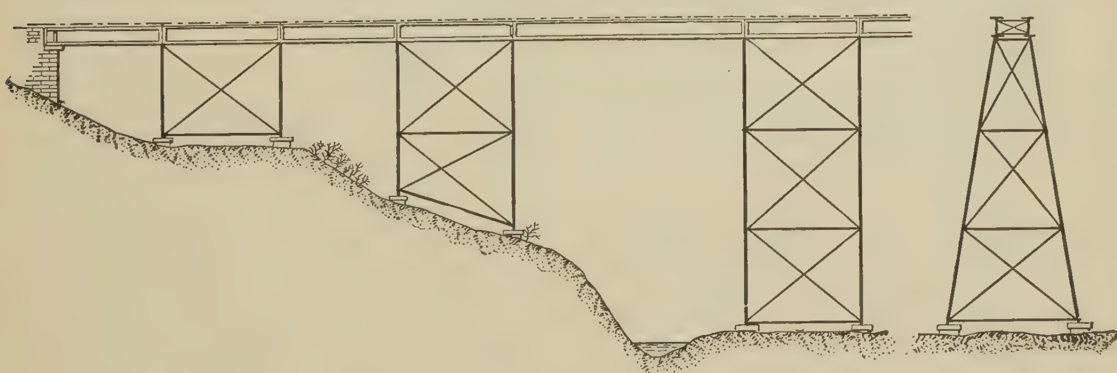


Fig. 407.

braced together transversely in pairs to form bents, as shown in the cross-section, Fig. 407. The bents are then braced together in pairs to form towers. The towers are designed so as to have sufficient stability, both longitudinally and transversely, to withstand any force which may tend to overturn them. The only force that would tend to overturn them in a transverse direction on a straight track is the pressure of the wind on the structure and on the train. The friction of the wheels on the rails caused by suddenly applying the brakes to a moving train is the force which tends to overturn the towers in a longitudinal direction or parallel to the track. If the structure is on a curve it must be made stable against a lateral force, in addition to the wind force, equal to the centrifugal force of the train load. Transverse stability is secured by giving to the posts of the bent a batir, in order to secure a base of sufficient width to prevent overturning. Longitudinal stability is obtained by making the span which connects the bents that are braced together to form the tower of sufficient length. The plane of each bent should always be vertical. The tower span is now generally made 30 feet

long, and the spans between the towers are varied with the height of the towers; the higher the trestle the longer the intermediate or variable span may be made with economy. The approximate length for the intermediate span for maximum economy for the height of the trestle under consideration is usually determined upon first by a rough calculation, or by means of an established formula, if the subject has been previously investigated. The arrangement of spans and towers is then made. It is preferable and cheaper to make the intermediate spans of the same length instead of varying them slightly for variations in the height of the trestle, as it makes the work simpler and gains a great advantage in manufacture by having a greater number of duplicate parts. It is, however, advisable in long viaducts, where much saving in material can be secured, to vary the intermediate span length for a change in height, if a number of duplicate spans of each length can be used.

The lengths of the end spans of a viaduct are sometimes dependent upon the kind of abutments that are built at the ends. If T abutments are used and the earth filled in around it, the first bent from the abutments must be placed so that the fill does not cover its base. If wing walls are used to confine the embankment, the length of the end span may be anything.

The outlines of the bracing are usually determined upon by its appearance when shown on a scale-drawing; the main object is to avoid having the diagonal rods make too acute an angle with the vertical. The foot of each post in a tower should be braced by struts, both longitudinally and transversely, in order that the tower may expand and contract as a whole. If the posts are not so braced, the friction between the base-plate of the column and the bed-plate will produce a transverse stress and bending in the column. From this condition it can be readily seen that the struts must be made stiff enough to overcome the friction. Another benefit derived from the use of these struts is that the transverse force is then resisted by both the masonry pedestals under a bent to the greatest advantage. If the struts were omitted the windward pedestal would have to resist more than half of the force, while at the same time its vertical load would be diminished, and it would therefore be less stable than the leeward pedestal, which would have less than half the force to resist and would have its vertical load increased.

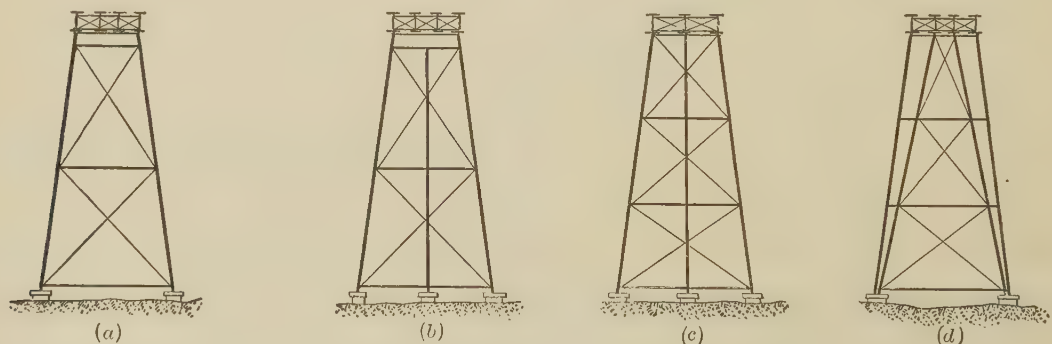


FIG. 408.

In some cases of low trestles it is sufficient to omit longitudinal diagonal bracing and substitute knee braces or brackets between the longitudinal girders and the columns. The longitudinal stiffness of such a structure is measured by the resistance of the columns to flexure in a longitudinal direction. A height of from 15 to 20 feet should be the limit for such designs.

Rocker bents, or those hinged at both top and bottom, may often be used when it is impracticable to use a full-braced tower. They serve to shorten the length of the spans and brace the structure transversely, and on this account are sometimes employed for economical reasons.

For double-track trestles the same general arrangement of spans and towers is adhered to, but the cross-section is varied, as shown in Fig. 408. Each bent is composed of two, three, or four columns braced together. In designs (*a*) and (*b*), Fig. 408, the girders are supported on a cross-girder at the top of the bent. In (*c*) and (*d*) the girders rest directly on the caps of the columns. The plan (*d*) is a favorite way of constructing a double-track trestle from an old single-track structure, or it would be used where it was desirable to so construct a single-track trestle that it may readily be changed to double track. In the latter case only the two inner girders, columns, and bracing would be built to accommodate a single track, and the outer girders and columns added when provision is to be made for two tracks.

A very important difference in the stresses in the bracing for single and double track bents is that in the latter the bracing, if the columns are battered, is stressed by the live load, and should, in order to be consistent with the rest of the design, be proportioned for lower unit stresses.

399. Lateral Stability and the Batter of the Columns.—It is now generally specified that the bents must not require anchorages in order to secure them against overturning. This avoids the necessity of building large masonry pedestals for the columns, which would be required if an anchorage were needed; and further does away with long anchor bolts and the accurate measurements required to locate them while building the pedestals. In order to comply with the condition that no anchorage must be required, and also to provide for the excessive wind pressures given in some of the standard specifications, there is no doubt that in many cases an excess of material is used, and that the total cost is above that which is actually necessary. All that is actually necessary is that the structure be made stable for the highest wind pressure, which is generally assumed to be 50 lbs. per square foot. The question of anchorage should be made one to be settled by a comparison of the total cost, including pedestals, with and without anchorage.

The batter of the columns varies from $1\frac{1}{2}$ to 3 inches to the foot. If it is desirable or required that no anchorage be provided, it is necessary that the wind and centrifugal forces combined with the vertical load produce no resultant tension at the foot of the windward column, or at the foot of the inner column if on a curve. The wind surface of a train is assumed to be 10 feet high, and the centre of pressure $7\frac{1}{2}$ feet above the rails. The line of action for the centrifugal force of a train is taken as 5 feet above the rails. The amount of the centrifugal force depends on the weight of the train, the degree of the curve, and the velocity with which the train is moving. For a speed of thirty miles per hour the centrifugal force is approximately one per cent of the weight of the train for each degree of curvature. For speeds of forty, fifty, and sixty miles per hour it is approximately two per cent, three per cent, and four per cent respectively of the weight of the train for each degree of curvature. Thus for a train weighing 4000 lbs. per linear foot of track moving at a rate of forty miles per hour on a six-degree curve the centrifugal force would be $4000 \times .02 \times 6 = 480$ lbs. per linear foot of track.

The exposed wind surface of the structure is taken as the surface of the girders and floor, and twice the surface of the bent or tower, as seen in longitudinal elevation.

By taking moments about the foot of the leeward column the moment of the vertical loads must be equal to or greater than that of the wind forces if no tension is allowed at the foot of the windward column. By making the base of the bent wider, *i.e.*, by increasing the batter of the columns, this can always be accomplished. By dividing the moment of the wind force by one half of the total vertical load on the bent the distance which the columns must be spread apart at the base is found at once. The lightest train that will not blow over for the assumed wind pressure is taken for the live load on the trestle when computing the width of base. For a 30-pound wind pressure acting on a train surface 10 feet high, beginning $2\frac{1}{2}$ feet above the rails, the lightest train load is $300 \times 7\frac{1}{2} \div 2\frac{1}{2} = 900$ lbs. per linear foot of track,

This assumption reduces the vertical load on the column to the minimum amount, and hence requires a wider base for stability. The columns are given the same batter in all the bents of one structure, as it improves the appearance to have all the columns on one side in the same plane, and it also simplifies the manufacture to have all bevels alike.

It is often advantageous, when a viaduct is on a curve, to give the outer columns of the bents more batter than the inner columns. This increases the stability of the bent as against the combination of the wind and centrifugal forces for the same width at the base. The bracing of the bents for this case is stressed by the vertical load because of the different inclinations of the columns.

The width centre to centre of the girders or the columns at the top is variable, and is independently determined for the trestle under consideration. The closer the girders are spaced the smaller will be the size of the necessary cross-ties and the resulting weight of the floor, and the bracing between the columns will be shorter and will be subjected to a smaller stress, requiring smaller members, all of which reduce the cost of the structure. On the other hand, the supported floor will be very narrow and the danger from a derailed train increased. In case the girders are spaced close centre to centre, it is imperative that ample and efficient rerailing and safety appliances be used; it is, however, advisable to use them on all trestles. Another view to take of this question is that generally taken by the passenger departments of railroads; namely, that the travelling public always feel much safer when they can see what is supporting the train from the car windows. This fancied security may be obtained by making the floor wider, and is appreciated by the public. A good mean is to space the girders 8 or 9 feet centres, as this does not require unusually large cross-ties and gives a safe floor.

When an old wooden trestle is to be replaced by an iron structure, and traffic over the trestle is to be maintained, it is always economical to so design the iron-work that it may be erected without disturbing the old bents. This is done by spacing the girders far enough apart to clear the legs of the old bents and locating the columns so that they will not be opposite them. The iron when erected will then envelop the old work, and after changing the cross-ties the latter can be taken down.

400. Length of the Tower Span and Longitudinal Stability.—It is the common practice to make the tower span of a viaduct 30 feet long. While this length is generally used, it is not always true that such towers do not require anchorage against overturning longitudinally. If it is assumed, as is usually specified, that the friction on the rails produced by suddenly applying the brakes to a moving train is one fifth of the vertical load, and further that this force is resisted by *all* the towers of the structure acting together, the limiting height of tower is about three and one half times its width longitudinally if no anchorage is allowable. The assumption that all the towers act together to prevent overturning is a fair one, as any longitudinal deflection of the towers would result in closing up expansion joints and thus distribute the force. The coefficient of friction used is probably very excessive. In order that a series of high towers may act together to resist overturning with absolute certainty, no expansion joint need be provided if the deflections of the towers from temperature produce no prohibited stresses.

401. Economic Length of the Intermediate or Variable Span.—No general formula for the length of the variable span of a viaduct can be given which will, with the many different practices prevalent among designing engineers, give reliable results. The length of this span for maximum economy depends upon the laws which govern the variation of the iron weights required in the spans and in the towers or bents, and also on the cost of the masonry pedestals. If the live load and the specifications as to allowed stresses and details of construction are fixed, and also if the designs are made consistent with maximum economy for the various span lengths and heights of bents, it may be possible to derive an empirical formula which will

give results reliable enough to serve as a guide in determining the economical length of this span. Such a formula will be given, and its origin and limitations explained, for the purpose of enabling those who have need of it to derive one that will suit their purposes.

A series of trestles varying in height from 25 to 150 feet were completely designed for a constant tower span of 30 feet, with intermediate spans of 30, 40, 50, 60, and 70 feet, all plate girders. The weights for each combination of height of trestle and length of intermediate span were calculated and the results tabulated.

A live load of 104-ton engines (Cooper's Class Extra Heavy A) was used in determining the stresses in the girders and bents. The track was straight. The wind pressure was assumed to be 30 lbs. per square foot on a train surface 10 feet high, on one surface of the floor and girders as seen in elevation, and on twice the surface of the bent as seen in elevation. The centre of pressure on the train surface was $7\frac{1}{2}$ feet above the rails. The track (rails, cross-ties, etc.) was assumed to weigh 400 lbs. per linear foot of track. The permissible unit stresses were 8000 lbs. per square inch on the net area of the tension flanges of girders, 8000 lbs. per square inch reduced for length by Rankine's formula for dead and live load stresses in the columns of the bents, 12,000 lbs. per square inch reduced for length by Rankine's formula for lateral struts and for *dead*, *live*, and *wind* stresses in the columns of the bents, and 15,000 lbs. per square inch in tension on all bracing *rods* for the towers. The top flange of all girders was made of the same sectional area as the bottom flange, and the moment of resistance of the web was neglected. The longitudinal bracing of the towers was proportioned to resist the stresses resulting from a longitudinal force of 800 lbs. per linear foot, but the columns were not increased in area on account of this force. It is usually assumed that the sudden braking of a train on a trestle is of rare occurrence, and the chances of it being done for the maximum train load and during a heavy wind are very remote. The thickness of the metal used was limited to three eighths of an inch as a minimum everywhere except for bracing, where five sixteenths of an inch metal was allowed.

The plate-girder spans were found to vary in weight according to the formula

$$w = 9l + 110, \qquad (1)$$

and the weight of the towers was very closely approximated by the formula

$$w_1 = 10.3h - \frac{hl}{25}; \qquad (2)$$

where w = weight per foot of the spans, l = length of span, w_1 = weight of the towers per foot of track, and h = height of the towers from the rail to the masonry. With these results as a basis the total weight per foot of the trestle, W, is

$$W = \frac{16{,}200}{l_1} + 9l_1 - 430 + 10.3h - \frac{hl_1}{25}; \qquad (3)$$

where l_1 = length of the intermediate span plus 30 feet, or the distance between *centres* of towers.

Assuming that the four pedestals* under a tower cost as much as five thousand pounds of iron, the equivalent iron weight per foot of trestle which would represent the cost of the pedestals is $5000 \div l_1$. Hence the total *equivalent* weight of iron per linear foot of trestle, including pedestals is

$$W = \frac{16{,}200}{l_1} + 9l_1 - 430 + 10.3h - \frac{hl_1}{25} + \frac{5000}{l_1}, \qquad (4)$$

* The pedestals are assumed to cost the same for each pedestal, regardless of the length of span, which is generally true.

from which W is a minimum when

$$l_1 = \sqrt{\frac{21{,}200}{9 - \frac{h}{25}}},$$

or

W is a minimum for an intermediate span of 30 feet when $h = 77.5$
W " " " " " " " " 40 " " $h = 117.0$
W " " " " " " " " 50 " " $h = 142.0$
W " " " " " " " " 60 " " $h = 159.0$
W " " " " " " " " 70 " " $h = 172.0$

One very important factor has been omitted in the above investigation, and that is that the cost of manufacture per pound of the iron-work in the girder spans and the bents differs, and as the spans are the cheaper this would tend to make the economical span longer for a given height than that found by the formula.

It will be noticed that the expression for the weight of the towers, $10.3h - \frac{l_1}{25}h$, gives a *constant* weight per square foot of the area included between the rail and the tops of the masonry pedestals for a constant length of span, l_1. Thus for $l_1 = 60$ the weight per square foot of the above area is 7.9 lbs., and this decreases one twenty-fifth of a pound for each foot of increase in the length of l_1. This gives an easily remembered expression for the weight of the towers and is one very widely used. Rapid approximate estimates of the amount of iron required in the construction of trestles may be made this way.

402. Comparison of the Cost of Embankment and of Iron Trestle.—It is often desirable to know at what height it becomes cheaper to use an iron trestle in preference to an embankment. The length of the intermediate span for this case is manifestly the shortest allowable, or 30 feet, as has become the established practice. The weight per foot of the girders or spans would then be the weight per foot of 30-foot spans, and the weight per square foot of the area of the profile is that for a length of span, l_1, equal to 60 feet. If we know the constants to use to get the above weights and the cost per pound of iron-work erected in place, the cost of the iron-work per linear foot is readily obtained. The quantity of the fill and the cost per yard are easily obtained. Having the cost of iron and of fill for various heights, the height at which iron is the cheaper is readily found. This may be reduced to a formula when the constants are determined. Thus for girders and towers, as in eq. (3), the weight of the trestle would be $380 + 7.9h$ per linear foot. The number of cubic yards per linear foot in a fill would be $(b + sh)\frac{h}{27}$. The cost of these two would be equal when

$$p(380 + 7.9h + 83) = (b + sh)\frac{h}{27} \cdot p_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where p = cost in cents of the iron per pound erected, h = height of the trestle or fill in feet, b = width of the fill on top in feet, s = slope of the sides of the fill, and p_1 = costs in cents per cubic yard of the fill in place. In the above equation 83 is the number of pounds per linear foot of trestle which is equivalent to the masonry pedestals. If $p = 4$, $p_1 = 25$, $b = 14$, and $s = \frac{3}{2}$, we obtain

$$1852 + 31.6h = \left(14 + \frac{3h}{2}\right)\frac{25h}{27} = 12.96h + 1.39h^2, \quad \text{or} \quad h = 43.8 \text{ approximately.*}$$

* The cost of the abutments is not included in the above formula, but an absolutely correct expression would take them into account. It would be extremely difficult to introduce this factor, as their cost *per foot* of trestle, neglecting the question of the kind of abutment used (i.e., T, U, or Wing abutments), depends on the length and height of the trestle, both being variable quantities. If it were a question of a fill throughout, or part fill and part trestle, such a formula as the one given would not be reliable enough for practical use. But if the query were when to stop the fill and begin the trestle, the error would be slight.

403. Design of the Columns.—The factors which should govern in the selection of the form to use for a trestle column are the adaptability of the section for the correct attachment of the bracing, and its stiffness and value as a strut. The four-Z-iron posts fulfil the first condition admirably, but owing to the small sizes of Z iron rolled it requires more material in the effective section than is required for the two-channel or two-channel and cover-plate form. However, as the Z column requires no latticing and has but two rows of rivets, it is often the cheaper section, and if so is a very desirable one to use.

While it is not usually done, it is no doubt advisable to make the attachments of the bracing struts to the columns very rigid in order to add to the stiffness of the column. For high trestles it would be advisable to reduce the allowed stress per square inch on the columns in order to reduce the amount which they will shorten under load and thus reduce the deflection of the top of the trestle.

404. Lateral and Longitudinal Bracing.—The bracing for the towers is usually laid out on a scale drawing so as to appear to the best advantage as bracing for the columns. The heights of the stories for the longitudinal bracing is commonly made as nearly equal to the length of the tower span as practicable, and in order to brace the column in both directions at the point of attachment of the longitudinal bracing the stories of the lateral or transverse bracing are made the same height. The attachments of all bracing should be so made that the neutral axes of the members intersect on the neutral axis of the column. Trestle columns are comparatively long struts and should be as free from eccentric stress as it is possible to design them.

The tension members of the bracing are usually adjustable rods, but of late years angle iron with riveted connections has been extensively used. The latter are stiffer but are more expensive, and it is extremely doubtful if it is possible to secure accurate adjustment and well-fitting riveted attachments when they are used.

The bottom flanges of the girders of the tower span are generally utilized as the top longitudinal strut.

405. Stresses in the Towers.—The external forces which may act on a trestle tower are the wind pressure on the exposed surface of the structure and the train, the centrifugal force of the train if the trestle is on a curve, the friction of the wheels on the rail, the weight of the structure and the train, and the reactions of the pedestals. The wind, the centrifugal force, and the vertical load if not applied centrally, or if the bent is not symmetrical, stress the transverse bracing. The friction of the wheels alone puts a stress on the longitudinal bracing. The bottom struts of both the longitudinal and transverse bracing must be made strong enough to resist a stress equal to the friction of the column base on the bed-plate in order to provide for the effects of temperature. The amount of this friction may be assumed for safety as twenty-five per cent of the *dead* load on the bed-plate.

The stresses in the towers may be found by the methods given in Part I. The stresses for *all* the external forces should be computed, as it is a very common mistake to overlook the effect of the vertical load on the bracing.

The stresses in the towers are usually found analytically as follows: Referring to Fig. 409, which is the general case of a single track bent with columns of different batters, the stresses for the loading shown would be:

Maximum stress in $AB = \{ W_1(a+k) + W_2(b+k) + W_3(c+k) + W_v j \} \div (c+k)$;

" " " $CB = \{ W_1 a + W_2 b + W_3 c + W_v x \} \div y$;

" " " $CD = \{ W_1 a + W_2 b + W_3 c + W_4(c+d) + W_v x \} \div (c+d)$;

" " " $ED = \{ W_1 a + W_2 b + W_3 c + W_3(c+d) + W_v x \} \div z$;

" " " $AD = \{ W_1' a_1 + W_2' b + W_3' c - W_v x \} \div y_1$, or $\{ W_2' b + W_3' c \} \div y_1$;

Maximum stress in $CF = \left\{ W_1'a_1 + W_2'b + W_3'c + W_4'(c+d) - W_v x \right\} \div z_1$, or $\left\{ W_2'b + W_3'c + W_4'(c+d) \right\} \div z_1$;

" " " $BD = \left\{ W_1(c+d-a) + W_2(c+d-b) + W_3 d + W_v f \right\} \div w$;

" " " $DF = \left\{ W_1(c+d+e-a) + W_2(c+d+e-b) + W_3(d+e) + W_4 c + W_v g \right\} \div u$;

" " " $AC = \left\{ W_1'(c+d-a_1) + W_2'(c+d-b) + W_3'd + W_v h \right\} \div w_1$;

" " " $CE = \left\{ W_1'(c+d+e-a_1) + W_2'(c+d+e-b) + W_3'(d+e) + W_4'e + W_v i \right\} \div u_1$.

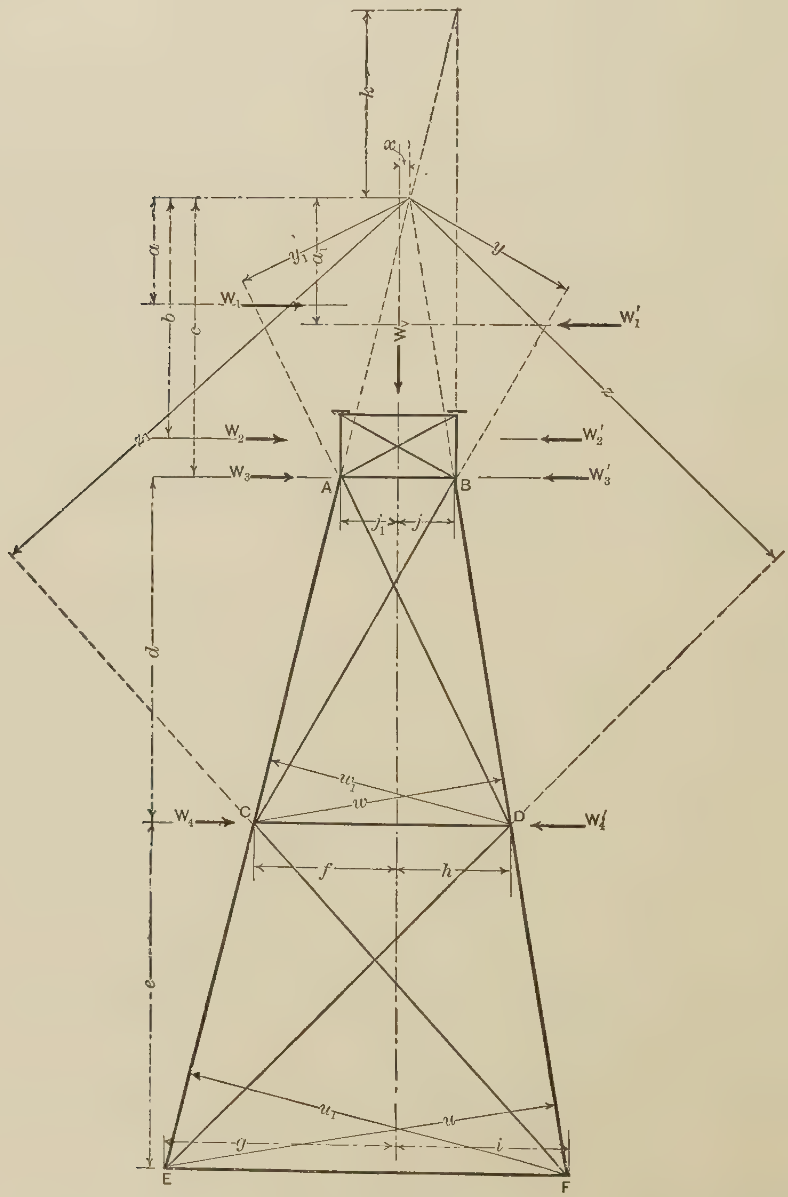


FIG. 409.

The maximum stresses given above for BD, DF, AC, and CE are the maximum compressive stresses only. The two expressions for AD and CF mean that these members may possibly receive their maximum stresses when the train is not on the trestle. This would be the case if $W_v'x$ were greater than $W_1'a_1$, as these two expressions include all the forces due to the train.

In the above expressions W_v is that part of the weight of the train, track, and girders* which is supported by the bent, and the force is assumed to act vertically through the centre of the track if the rails are of the same elevation, or as many inches from the centre of the track towards the low rail as the outer rail on a curve is elevated above the inner rail, if the rails have different elevations. W_1 and W_1' are the combined effects of the wind pressure on the train and the centrifugal force. The wind pressure on a train is usually taken as 300 lbs. per linear foot of track, acting 7½ feet above the rails. The centrifugal force is assumed to act 5 feet above the rails. W_2 and W_2' represent the wind pressure on the girders and floor, generally taken as 30 lbs. per square foot if the train is on the structure, and 50 lbs. per square foot if the structure is not loaded. W_3, W_3', and W_4' represent the wind pressure on the tower concentrated at the joints. This pressure on the bents is generally assumed as 225 lbs. per foot of height of bent for the loaded structure, and 375 lbs. for the structure not loaded. Thus W_3 and W_3^1 would be taken as $225 \cdot \frac{d}{2}$ and $375 \cdot \frac{d}{2}$, for the loaded and unloaded structure respectively, and W_4, W_4' would be $225 \cdot \frac{d+e}{2}$ and $375 \cdot \frac{d+e}{2}$ similarly.

Bents with the columns inclined differently, such as the one illustrated, are seldom used except for trestles on a curve, and the example is given mainly to show the effect of the load W_v on the bracing. For double track bents with inclined columns a train on one track only will always stress the bracing.

The bracing is proportioned to resist the stresses due to all of the above external forces except that of the vertical train load at the limiting allowed unit stresses used for wind bracing generally. For stresses resulting from the vertical train load the limiting stresses are those used in the counter-ties and posts of trusses. For the stresses in the columns from wind, centrifugal force, and the friction of the wheels on the rails, the practice varies. Some specifications require that the area of the column be increased for these stresses beyond that required by the vertical load by an amount in square inches equal to the stress divided by the value of the column per square inch as a lateral or wind strut. Other specifications require no increase in section until the stresses from the above forces increase the stress per square inch fifty per cent above that allowed for the vertical load on the column and then add section until the total stress per square inch from dead load, live load, wind and centrifugal force does not exceed that amount. It is generally assumed that the wind force and that of the friction of the wheels on the rails are not coexistent forces. The rational way to proportion the columns would seem to be to use low permissible stresses for the working or duty stresses, which are those due to the dead and live loads, including centrifugal force, and to increase the permissible stresses as much as is consistent with safety for wind stresses in combination with the dead and live load stresses. In the case of a trestle near a switch or a station where trains are continually using air-brakes, it would be advisable to include friction stresses among the duty stresses.

406. The Masonry Pedestals supporting a tower are usually made as small as it is possible to make them and not exceed the permissible pressure on the foundation. They should be made as low as possible, a good rule being to have them extend one foot above the surface of the ground, in order to make them stable against the lateral pressure of the wind. In all cases their stability should be investigated and ample provision made for the extreme case.

407. Details of the Towers.—Figs. 410, 411, and 412 show the ordinary details for a tower with Z-columns. The struts are all riveted to the columns, and the rods have pin connections. The intersections are as near the neutral axis of the column as it is practicable to make them. If the rods are very large, or if they are stressed by the live load, it is imperative that the intersections of the neutral axis of the struts and rods at a joint be on the neutral axis of the column; but for the usual case of small rods a slight eccentricity is usually

* An absolutely correct expression would separate W_v into two parts, that due to the train load and that of the track and girders. It is not done here, as it would complicate the illustration, and as it is only necessary in the unusual case of a bent having columns of different batters.

permitted. If posts composed of two channels are used, the transverse bracing may be connected by pins through the neutral axis of the column. The details shown will serve to show one method of connecting the bracing, but any other which accomplishes the same results would be satisfactory.

The Caps of the Columns.—At this point provision must be made for supporting the girders and for attaching the transverse and longitudinal bracing. The girder rests on a cap plate riveted to the top of the column by means of auxiliary angles as shown. The top of the column is planed to an even surface, on which the cap plate bears and transfers the load of the girder directly through the bearing. The attachment of the transverse bracing is usually simple, the strut and the column resisting the two components of the stress in the rod. In the case shown, Fig. 410, a bracket is riveted to the column, to which is riveted the strut,

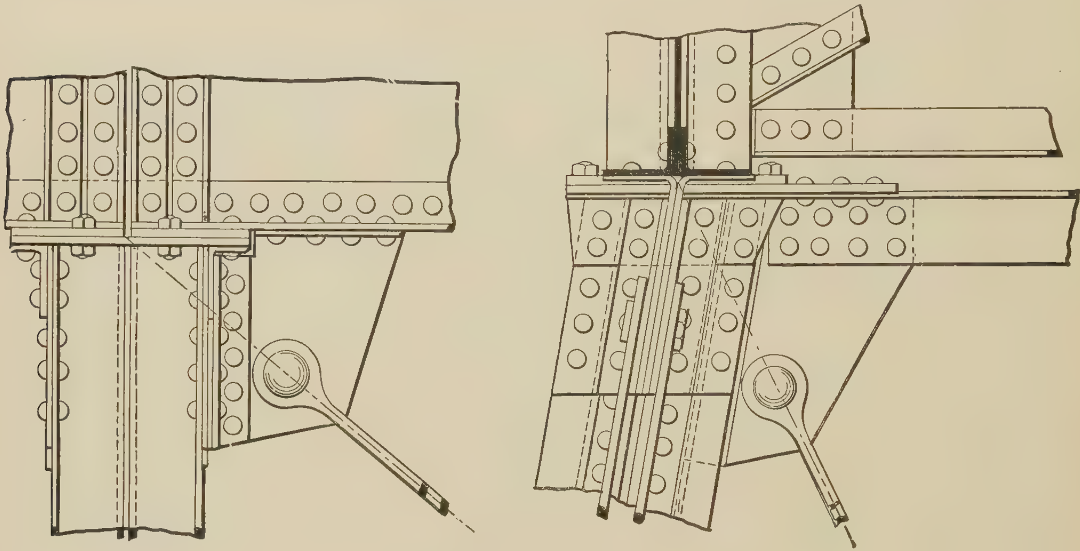


FIG. 410.

and the rods are connected to it with a pin. The attachment of the top transverse strut must be made sufficient to take the horizontal component of the stress in the column. This component is best transferred to the strut by rivets through the cap plate extended as in the sketch. The top strut is usually short, and can be made cheaply of two angles.

Intermediate Connections to the Column.—At the intermediate joints the attachments of the bracing are very simple, as the only requirement is that the rivets through the column and through the struts be sufficient to transfer the respective components of the stresses. If the rods are small, a slight eccentricity in these attachments is allowable.

A splice in the column should be made a short distance *above* the joint by planing the ends of the two sections for an even bearing and riveting splice plates on all four sides to hold the sections in line, but relying on the butt-joint for the transfer of the stress.

408. Column Bases.—At the foot of the column the bracing is attached as before, but here provision must be made for the distribution of the pressure over the masonry, so as not to exceed the permissible bearing pressure. The column is planed to an even surface and transfers the load directly to the sole-plate, which is stiffened by gusset plates and angles riveted to it and to the column. The sole-plate at an expansion point should rest on a bed plate, in order that the friction or resistance to the movement of the column at this point may be as little as possible. The bed-plate must be made large enough and sufficiently stiff to distribute the pressure without exceeding the limiting bearing pressure per square inch.

The holes for the anchor bolts should be made circular in the bed-plate and oblong or slotted in the sole-plate, to allow the latter to slide on the former.

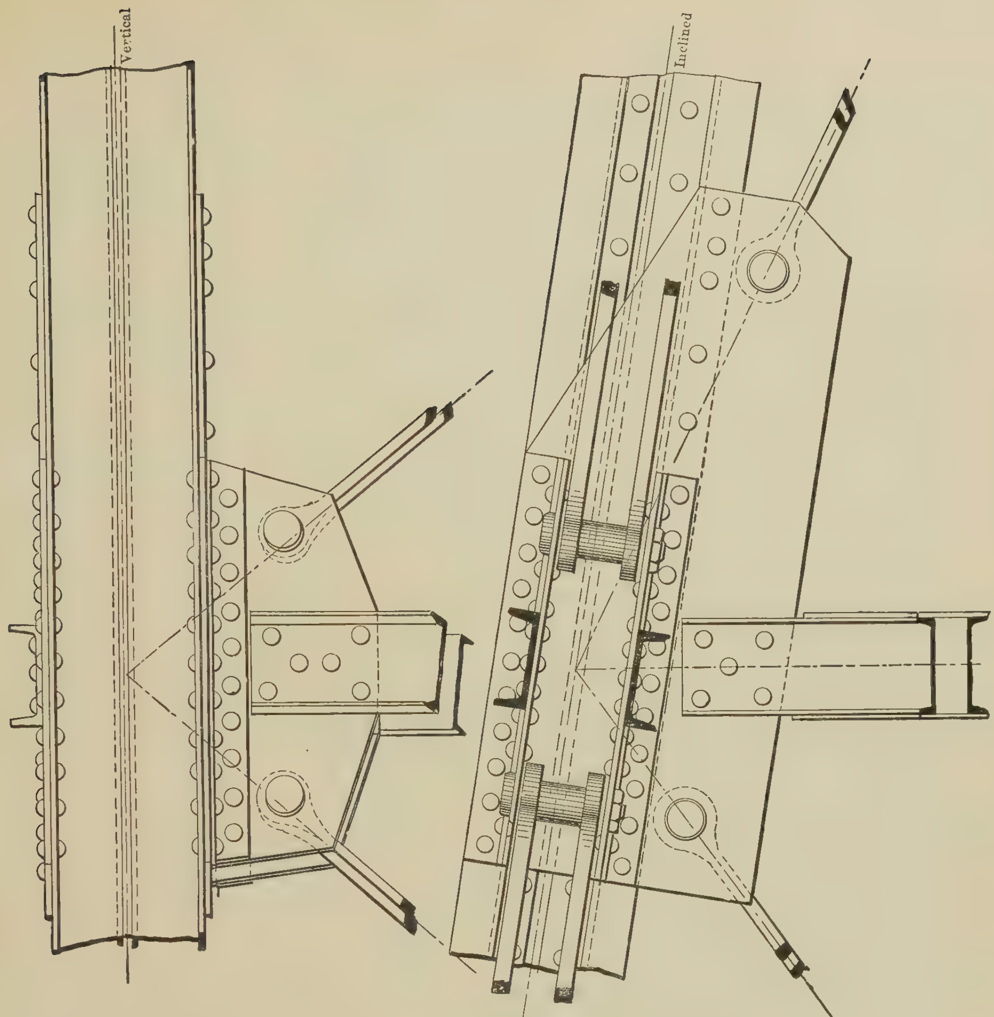


FIG. 411.

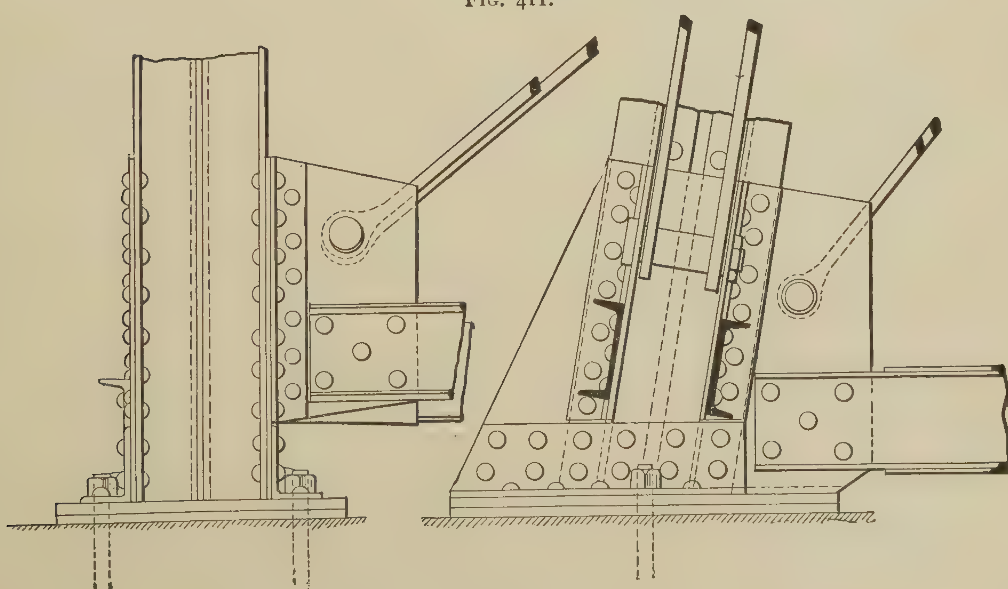


FIG. 412.

III. ELEVATED RAILROADS.

409. Characteristic Features.—An elevated railroad differs from a trestle in being lower, and in the fact that it usually occupies the streets of a city. The height is the least allowable for wagon traffic beneath it, and the provision for this traffic usually prohibits the use of vertical diagonal bracing, either longitudinal or transverse. It is therefore a railway, of two or more tracks, supported upon a series of short iron or steel columns, providing from fourteen to sixteen feet clearance, without complete systems of vertical diagonal bracing. Such railroads generally have a very heavy traffic also, especially in the number of the trains; and this demands a very rigid structure, not liable to be racked to pieces. Because the diagonal vertical bracing cannot extend to the foundations, there may be very large bending moments produced in the columns, from lateral wind and centrifugal forces, and from the rapid slowing up of trains when stopping, which produce great longitudinal thrusts. If the columns are free to turn at either end, as they would be if pin-connected or insecurely attached at these points, the bending moment is about twice as great in them as it is when they are rigidly held by adequate anchorage and top connections. It is very important, therefore, to provide such anchorage and connections as will effectually fix the posts in direction at their extremities, this being one of the most important features of this class of structures.

As to styles of superstructure, it may be said that both the girders and floor-beams are now made up with solid webs, or as plate girders. If a lighter structure is desired than can be obtained in this way, then a riveted lattice girder may be used; but great care should be given to its design and construction, a sufficient number of rivets used in making the joints, and all gravity lines made to intersect in a point at every joint if possible. Both legs of all web angles, also, should be joined to the chord sections, so that these members can receive their loads over their entire cross-sections and not through one leg only, as is so often done. As for pin-connected trusses for such structures, they should not even be considered.

In the following articles on this subject only such matters will be discussed as are peculiar to this class of structures, and a few of the best examples illustrated.

It goes without saying that an all-masonry or embankment support for an elevated railway (except on street-crossings) is far superior to one of steel girders and columns; but when the road occupies the public streets or alleys this is impracticable, and in any case it is more expensive. The elevated railway system of Berlin is built in this way, largely for the purpose of preventing the noise which always accompanies a rapidly moving train upon a metallic structure. The tracks of the Pennsylvania Railroad Company in Philadelphia are also carried upon a masonry structure. On the Berlin system the tracks on the bridges which cross the streets are embedded in gravel, also for the purpose of preventing the noise of passing trains.

The details of an elevated structure should be worked out with great care, with a view to permanent stability as well as to economy. The duplication of parts causes small savings in design to accumulate to considerable sums in many miles of the structure.

410. The Live Loads.—Elevated roads built for standard traffic are of course designed to carry the same engine and train loads as the bridges of the same line. Roads built for city or suburban passenger rapid-transit service are designed for the particular styles of motors and cars which are expected to run upon them. It has been common, however, in this class of structures to greatly underrate the future requirements, so that within a few years of the inauguration of the system the actual loads are far in excess of those for which the structure was built.

The following wheel-load diagram, Fig. 413, which gives the loads on one rail, or one half the train loads, was used in designing the Chicago and South Side Elevated in 1890.

This provides for forty-four-ton engines and thirty-two-ton cars, which are both considerably greater than are actually used.

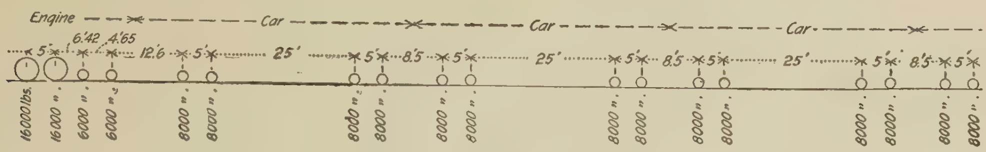


FIG. 413.

It has been customary to provide for thirty-ton * engines and twenty-eight-ton cars, having a wheel base as shown in Fig. 414, where the loads on one rail are again shown.

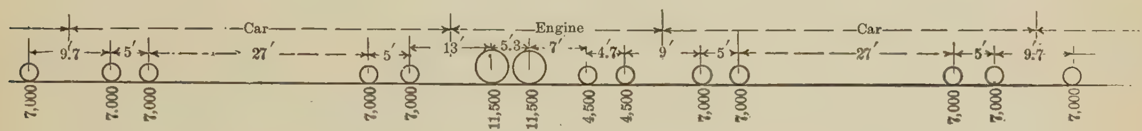


FIG. 414.

The computations of stresses, and the dimensioning of all the parts, have been fully explained elsewhere in this work.

411. The Lateral and Longitudinal Stability and stiffness of these structures is dependent upon the stiffness of the columns. Three different methods have been used, in which the object sought was to secure the greatest stiffness with the least cost. The old method was to anchor the columns firmly to adequate foundations, and connect the cross and longitudinal girders to them loosely at the top. In this plan the column was assumed to be anchored at the bottom and "free-ended" at the top. The bending moment which the column was proportioned to resist was equal to the horizontal force at the top multiplied by the height of the column. Another method is the exact reverse of the above, as in this case the connection of the cross and longitudinal girders with the column is made rigid, and the connection with the foundation is made only sufficient to resist lateral displacement. The columns in this case are assumed to be fixed or anchored at the top and "free-ended" at the bottom. The bending moment for which the column is to be proportioned is the same as in the former method. The latest method is a combination of these two, as in this plan the columns are anchored rigidly to adequate foundations at the bottom, and to the cross and longitudinal girders at the top. Changes of temperature produce a small stress in the structure when this plan is used. It is probably insignificant, but should always be computed. The columns are therefore assumed to be fixed in direction at both ends, and the bending moment which they are proportioned to resist is about one half of the product of the horizontal force into the height of the column. The first and third methods require expensive and reliable foundations, while the second method requires the least amount of foundation possible in any case. The first and second plans require about the same amount of iron in their construction, and more than is required by the third plan.

If the design of the structure is such that each track is supported by a single line of columns that have no connection with the columns under the other track, it is absolutely necessary to "fix" or anchor the columns to the foundations, as this is the only means of securing lateral stability.

Double-track structures have been supported on a single line of columns, and in such cases the anchorages must be able to resist the bending moment due to the eccentric loading of the tracks in addition to that of the lateral forces.

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* Thirty-five-ton engines have been used on some of the elevated passenger roads in New York City.

The wind pressure on elevated railway structures is taken at 30 lbs. per square foot of exposed surface, and the protection afforded by adjacent buildings is usually neglected. The longitudinal force is assumed to be that due to the sudden application of air-brakes to a moving train, checking the wheels and causing them to slide on the rails. The coefficient of friction is taken as $\frac{1}{5}$.

Expansion Joints are provided in elevated railway structures at intervals of about 200 feet. For riveted trusses the longitudinal girders slide on the top flange of the cross-girder, as shown in Fig. 420. The usual expansion joint for plate-girder construction is shown in Plate XI. The longitudinal girder rests freely on a shelf or bracket built out from the cross-girder. It is customary to have the longitudinal girders on one side of the cross-girder rigidly connected to the cross-girder at an expansion joint.

When the expansion joint is a suspended one, as when the girder and cross-beam have about the same depth, the construction shown in Figs. 415 and 416 may be followed.*

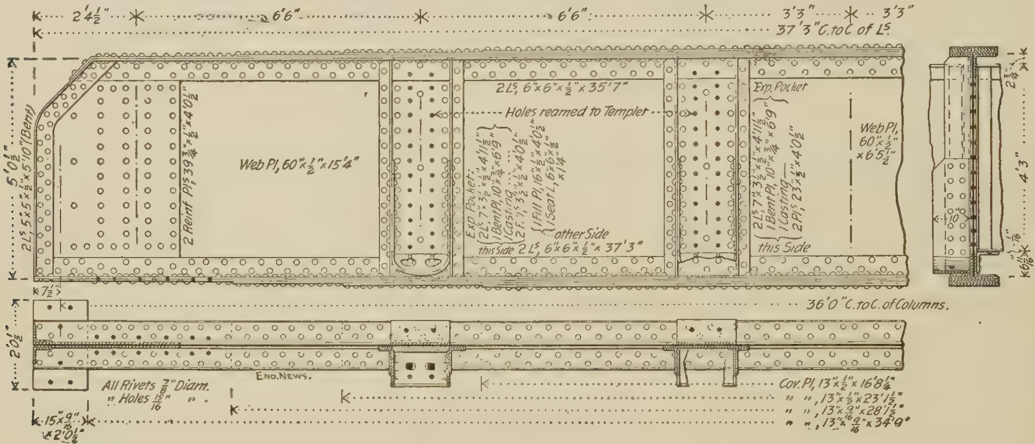

FIG. 415.

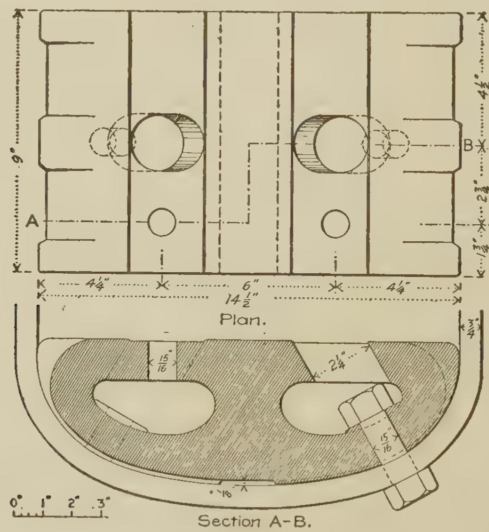

FIG. 416.

The merit of this joint lies in the cast-iron filler piece having a curved bottom which fits the curve given to the pocket-plate. By the aid of this cast-iron support the upward pull on

* These are from plans prepared for the Quaker City and Northeastern Elevated Railways of Philadelphia. See *Engineering News*, May 25, 1893.

the sides of the pocket-plate is transmitted uniformly over the bearing area of the longitudinal girder.

The casting is bolted to the pocket-plate, and the girder slides upon it.

Foundation Joints.—An accurate and even bearing of the columns upon the cap-stones of the foundations is of the utmost consequence. It is impossible to make these parts fit exactly after the structure is riveted up, however much care may have been taken in the shop and foundation work. If the bearing is not even, the weight of the structure with its load will come upon one side of the column section, thus giving to it an excessive bending moment which it was not designed to carry, and greatly overstressing a portion of the cross-section.

To insure an even bearing it is absolutely necessary to wedge up the post, after riveting up, until it bears evenly on all sides, and until it is raised about three eighths of an inch from the stone. Then fill this space with Portland cement or iron-rust cement, made of iron filings or trimmings mixed with sal-ammoniac. This should be rammed to place with a thin steel or hard wooden blade, and allowed to harden before the wedges are removed, and the anchor-bolt nuts screwed down hard. If the posts are socketed in cast-iron pedestals and sealed in place by long iron-rust joints, as described in Art. 413, the ideal conditions are probably realized.

412. General Case of Horizontal Forces Acting on an Elevated Railway with Columns fixed at the Ground.—In Fig. 417 let the horizontal force P act upon the train and structure, so that its centre of pressure is at the height h from A and h' from A', the supports being on ground not level transversely. Let the lateral system $CDD'C'$ be supposed to be rigid as compared to deflections of the columns, so that the conditions assumed in Art. 151, Chap. X, are fulfilled. Then we have, from eq. (20), Art. 151,

$$x_0 = \frac{z}{2}\left(\frac{z + 2c}{2z + c}\right);^* \quad x'_0 = \frac{z'}{2}\left(\frac{z' + 2c'}{2z' + c'}\right). \quad . \quad . \quad . \quad (6)$$

To find the relative values of H and H', the sum being equal to P, we may resort to the use of Proposition III, Art. 200, Chap. XV, according to which the load P divides itself between the columns in direct proportion to their relative rigidities; or inversely as their deflections at the points D and D', for the same horizontal forces, or shears, acting between A and D in the one case and between A' and D' in the other. Hence we may write

$$\frac{H}{H'} = \frac{\Delta'}{\Delta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where Δ and Δ' are unequal horizontal deflections of D and D' respectively for *the same horizontal reaction* acting throughout their lengths from A and A' to D and D'.

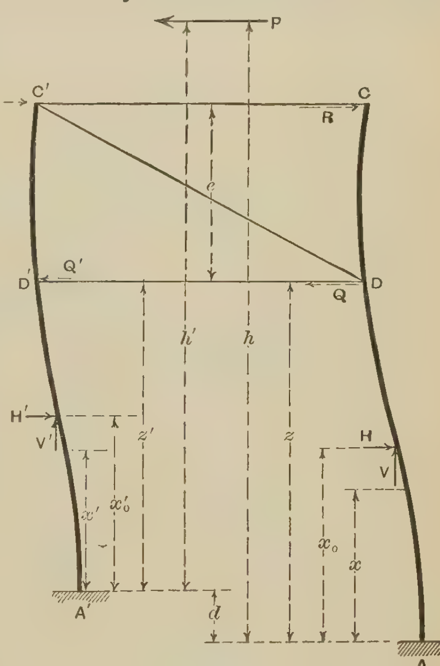

FIG. 417.

For this condition it was shown in Art. 151, eq. (20), that the point of inflection is situated at a distance above the base equal to (Fig. 417)

$$x_0 = \frac{z}{2}\left(\frac{z + 2c}{2z + c}\right) = \frac{z}{2}\left(\frac{3z + 2e}{3z + e}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Since the column may be supposed to be cut at this point of contraflexure the given horizontal reaction, K, may be supposed to act here towards the left on the lower portion (Fig. 417) and towards the right on the upper portion of the column, or the moment at any part of the column below D may be expressed by the equation

$$M_{x<z} = K(x_0 - x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

To find the deflection of the point D with reference to A, we have from the equation of the elastic line

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{K}{EI}(x_0 - x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

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* In this equation $c = z + e$ of Fig. 417.

Integrating,

$$\frac{dy}{dx} = \frac{K}{EI}\left[x_0x - \frac{x^2}{2}\left(+ C = 0\right)\right], \quad \ldots \ldots \ldots \quad (11)$$

since for $x = 0$, $\frac{dy}{dx} = 0$.

$$y = \frac{K}{EI}\left[\frac{x_0x^2}{2} - \frac{x^3}{6}\left(+ C = 0\right)\right], \quad \ldots \ldots \ldots \quad (12)$$

since for $x = 0$, $y = 0$.

For $x = z$, we have $y = \Delta$, the deflection at D, or

$$\Delta = \frac{K}{EI}\left[\frac{x_0z^2}{2} - \frac{z^3}{6}\right], \quad \ldots \ldots \ldots \quad (13)$$

and similarly for the deflection at D' for the same force K, in the other column,

$$\Delta' = \frac{K}{EI'}\left[\frac{x'_0z'^2}{2} - \frac{z'^3}{6}\right]. \quad \ldots \ldots \ldots \quad (14)$$

From eq. (8) we have

$$x_0 = \frac{z}{2}\left(\frac{z + 2c}{2z + c}\right) = \frac{z}{2}\left(\frac{3z + 2e}{3z + e}\right), \quad \ldots \ldots \ldots \quad (15)$$

and similarly,

$$x_0' = \frac{z'}{2}\left(\frac{3z' + 2e}{3z' + e}\right), \quad \ldots \ldots \ldots \quad (16)$$

where $e = c - z$, as shown in Fig. 208*a*.

Substituting these values of x_0 and x_0' in eqs. (13) and (14), we obtain

$$\Delta = \frac{z^3K}{12EI}\left(3\left[\frac{3z + 2e}{3z + e}\right] - 2\right) = \frac{z^3K}{12EI}\left(\frac{3 + 4\frac{e}{z}}{3 + \frac{e}{z}}\right), \quad \ldots \ldots \ldots \quad (17)$$

and similarly,

$$\Delta' = \frac{z'^3K}{12EI'}\left(\frac{3 + 4\frac{e}{z'}}{3 + \frac{e}{z'}}\right). \quad \ldots \ldots \ldots \quad (18)$$

Whence, from eq. (7), we obtain

$$\frac{H}{H'} = \frac{\Delta'}{\Delta} = \frac{z'^3I}{z^3I'}\left(\frac{\left[3 + 4\frac{e}{z'}\right]\left[3 + \frac{e}{z}\right]}{\left[3 + 4\frac{e}{z}\right]\left[3 + \frac{e}{z'}\right]}\right). \quad \ldots \ldots \ldots \quad (19)$$

But $H + H' = P$, therefore,

$$H' = \frac{P}{\frac{z'^3I}{z^3I'}\left(\frac{\left[3 + 4\frac{e}{z'}\right]\left[3 + \frac{e}{z}\right]}{\left[3 + 4\frac{e}{z}\right]\left[3 + \frac{e}{z'}\right]}\right) + 1}, \quad \ldots \ldots \ldots \quad (20)$$

and

$$H = P - H'. \quad \ldots \ldots \ldots \quad (21)$$

If $I = I'$, both of these disappear from the formulæ.

If $z = z'$, or if the supports are on a level, then eq. (20) becomes

$$H' = \frac{PI'}{I + I'};$$

$$= \frac{P}{2} \quad \text{when } I = I'. \quad \ldots \ldots \ldots \quad (22)$$

If $e = 0$, which corresponds to the case of *a column fixed in direction at D*, as when the columns are joined by a plate girder extending from C to D, we have, from eq. (20),

$$H' = \frac{Pz^3 I'}{z'^3 I + z^3 I'}. \qquad (23)$$

To find the point of application of H, or the point of inflection, we have from eqs. (15) and (16):

For $z = z'$, or for supports on a level,

$$x_0 = x_0' = \frac{z}{2}\left(\frac{3z + 2e}{3z + e}\right). \qquad (24)$$

For $e = 0$, or for columns fixed in direction at D, as by a plate girder from D to C, we have

$$x_0 = \frac{z}{2}; \qquad (25)$$

or the point of inflection is midway between A and D.

For $e = z$, or for open bracing in the upper half of the bent, we find

$$x_0 = \tfrac{5}{8}z. \qquad (26)$$

or the point of inflection is $\frac{5}{8}$ of the distance AD from A.

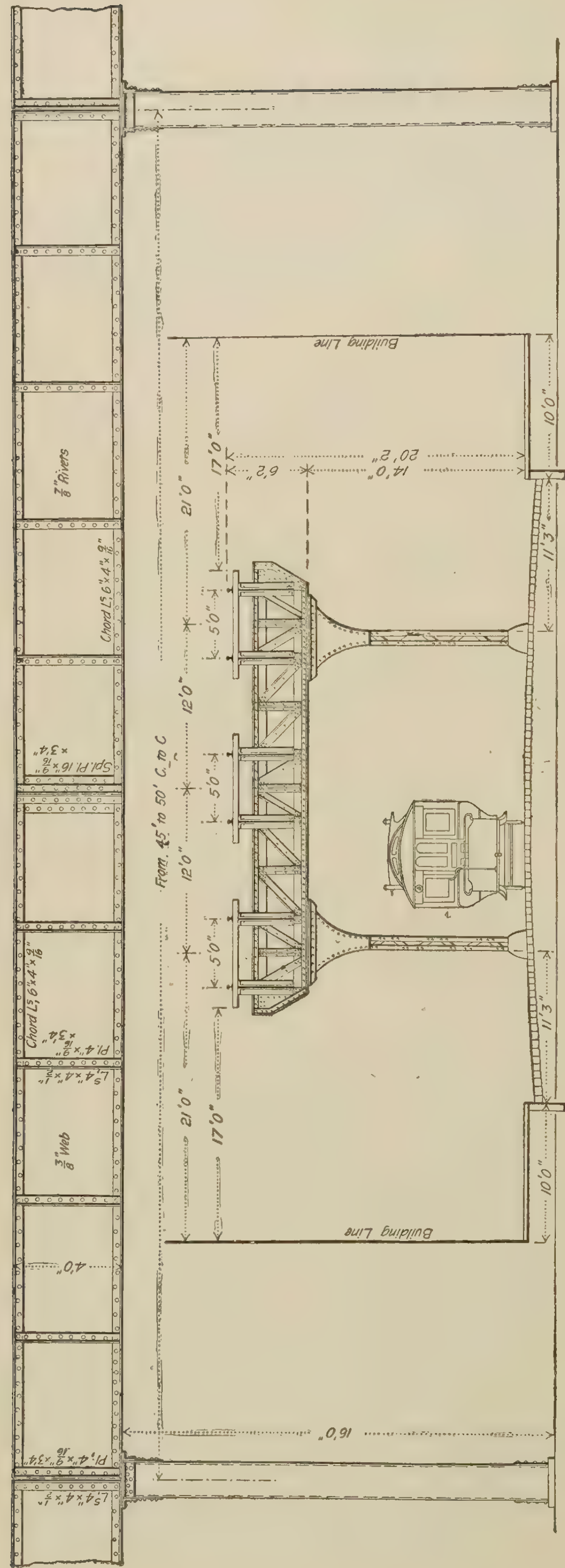
After having found H and H' and their points of application for any case, the resulting bending moments and direct stresses are readily computed by the ordinary methods of Art. 115.

413. Selected Examples.—In Plate XI are shown the general and detail drawings of the elevated portion of the *Merchants' Terminal Railway of St. Louis.* It is of the plate-girder style of construction throughout, and carries a double track for standard railway traffic. It is provided with an expansion joint in every second panel, and the posts are effectually anchored to the ground. The bearings on the foundation-stones were made by packing iron filings and sal-ammoniac between foot-plate and cap-stone after the structure was riveted up. The columns are assumed to be fixed in direction at both top and bottom. Wherever it was possible both the lateral and the longitudinal bracing reached to the bases of the columns. Where this was not possible an auxiliary lattice longitudinal bracing was used where the plate stringer was higher than 14 feet from the pavement, as shown in the plate.

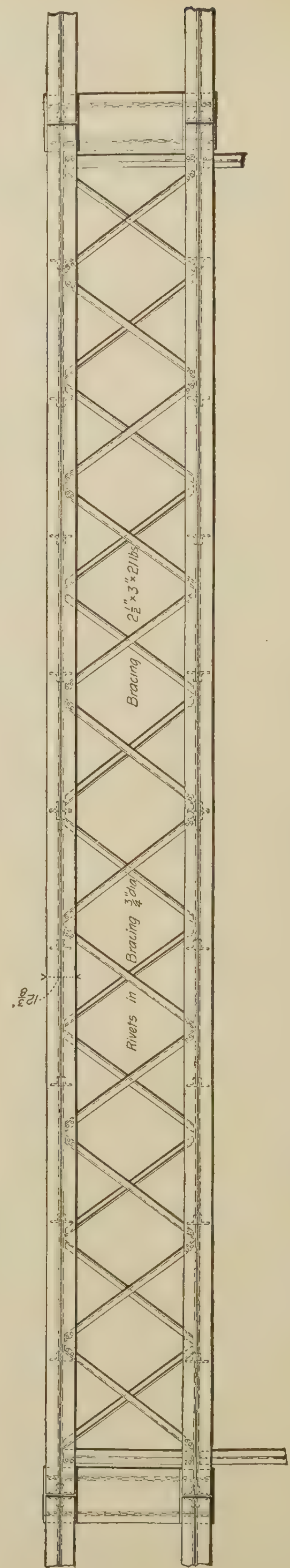
This structure was built by the Phœnix Bridge Company, under the direction of Robert Moore, M. Am. Soc. C. E., as Chief Engineer, and is thought to be one of the best examples of standard elevated railway construction.

The Chicago and South Side Elevated Railway is about eight miles long, and extends from the heart of Chicago to Jackson Park. It is built mainly on a purchased right of way 30 feet wide, alongside an alley. It is designed for passenger traffic only. The general plans are shown in Figs. 418 and 419. The stringers are plate-girders forming spans of from 35 to 60 feet. The standard structure has no cross-beam, properly speaking, the girders being carried directly by the columns, the tops of which spread sufficiently to do this. In place of the cross-beam is a system of diagonal bracing, as shown in Fig. 419.* The columns are firmly anchored to a foundation of masonry 7 feet square at the base and reaching to a depth of 10 feet. The upper portion of this is a cast-iron cap 24 inches high, provided with sockets $21\frac{1}{2}$ in. deep to receive the channel bars forming the columns. This casting is bolted to the masonry by four $1\frac{1}{4}$-in. anchor rods, and the channel bars are absolutely joined to the cast-iron base by means of an iron-rust cement composed of iron-filings and sal-ammoniac, which was tamped into the surrounding spaces after the structure was riveted up. Mr. Robert S. Sloan, M. Am. Soc. C. E., was Chief Engineer, and it was erected by the Keystone Bridge Company of Pittsburg.

* For a detailed and illustrated description of this structure, with its passenger stations, see *Eng. News* of Jan. 16, 1892.



CROSS-SECTION OF STRUCTURE ALONG SIXTY-THIRD STREET.



STANDARD STRUCTURE AT SIDE OF ALLEY.

Fig. 418.—The Chicago and South Side Elevated Railway (for Passenger Traffic). Built 1890–92.

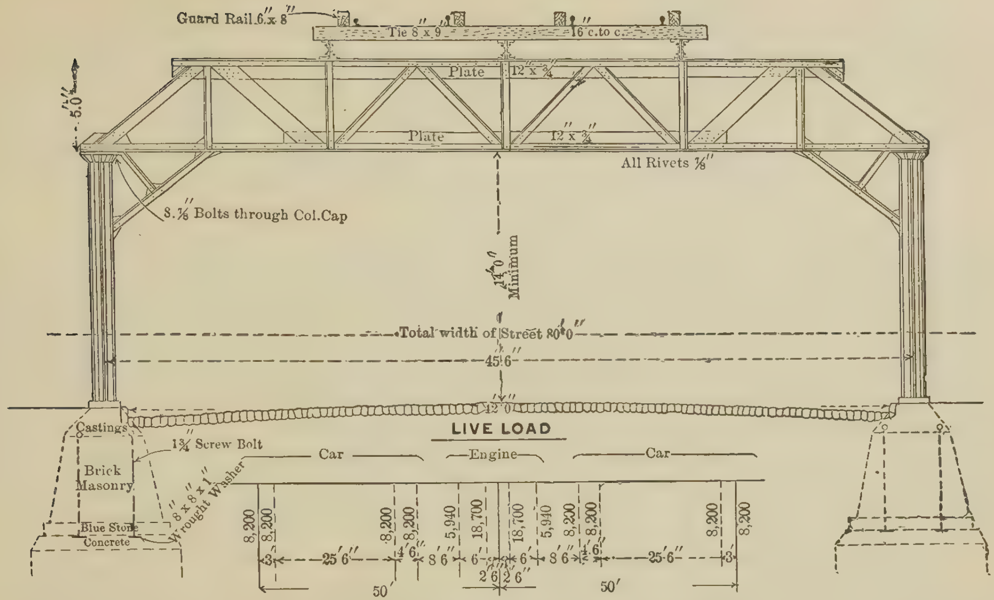


FIG. 420.—KINGS COUNTY ELEVATED PASSENGER RAILWAY OF BROOKLYN, N. Y., BUILT IN 1887.

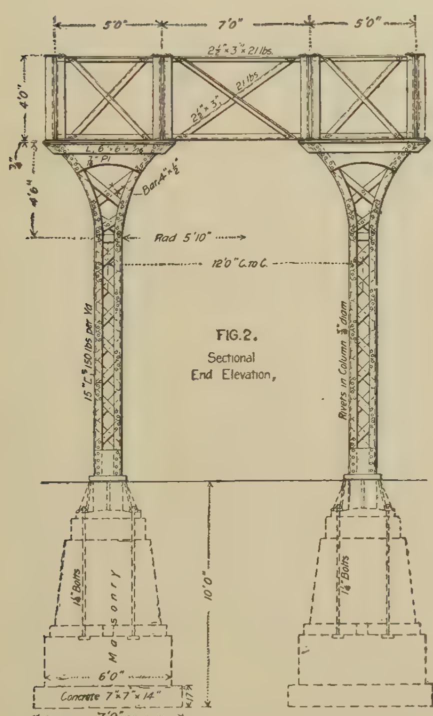


FIG. 419.—CROSS-SECTION OF THE CHICAGO AND SOUTH SIDE ELEVATED RAILWAY.

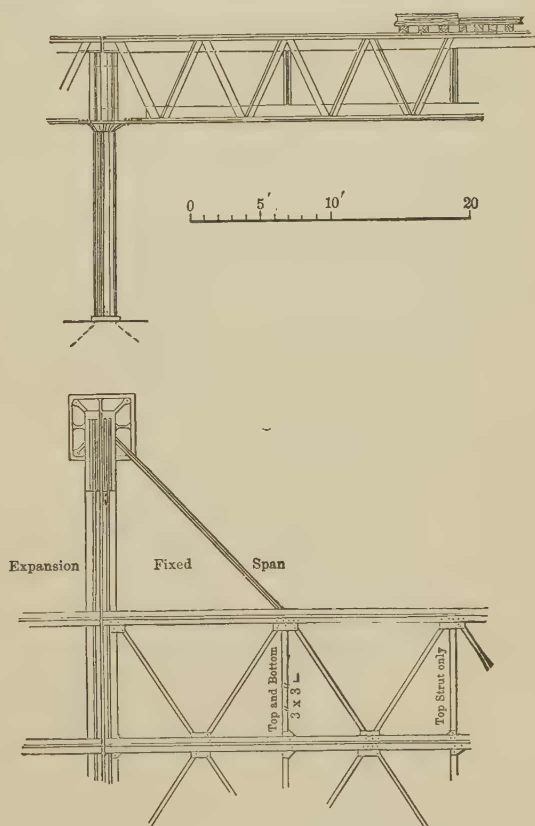


FIG. 420a.—KINGS COUNTY ELEVATED.

The Kings County Elevated Passenger Railway of Brooklyn, New York, is illustrated in Fig. 420. This structure was designed in 1885 and built in 1887. The charter required it to be of the lattice-girder type, and to be placed over the centre of the street. It was designed and built by the Phœnix Bridge Company. The cross-girders are in pairs, resting on Phœnix columns, being joined only through the top plate of the column. The structure is cheap in first cost, but not as rigid as it should be for heavy traffic. The columns are not very rigidly attached at either top or bottom, and as for longitudinal stability, it has very little. It is by no means an ideal structure, but a fair example of the earlier forms of rapid-transit elevated railways.

Plate XII contains the general drawings of the four-track steel viaduct of the New York Central and Hudson River Railroad erected in Park Ave., New York City, 1893.* It rests on three rows of columns, carrying three steel plate girders of about 65 feet average span, and of a depth of 7 feet. The columns are bolted to broad cast-iron bases, which in turn rest on masonry piers, so that they may be considered as fixed at the ends. At top they are braced in all ways by means of curved plate brackets.

There are neither cross-girders nor cross-ties properly speaking. The structure is braced laterally by means of a lattice construction between the main girders on the deck portion and by the column brackets and gusset knee-braces on the through portion. The floor is of solid box-construction, made up of $\frac{3}{8}$-inch steel plates, the rectangular elements being 17 inches high and 15 inches wide. These extend entirely across the whole floor and rest on the three girders in the deck portion and are riveted between the girders on the through portion. This forms a solid steel floor, on which the rails rest directly as shown in the plate. The floor acts as so many eye-beams, or plate cross-girders, having a height of 17 inches, and $\frac{3}{8}$-inch webs placed every 15 inches throughout the entire length of the structure.

This road will carry some five hundred trains a day upon its four tracks, and is supposed to embody the latest and best practice in elevated railways for standard traffic. The space between the column bases and the cast-iron wheel-guards is filled with Portland-cement mortar. The drawings are so complete that is not necessary to describe them further.

414. Economical Span Lengths and Approximate Cost.—The cost of different kinds of construction varies but little, if each kind is designed for maximum economy. The economical span lengths vary. For plate-girder construction a span length of from 40 to 50 feet will result in the greatest economy of total cost (including foundations). For riveted trusses the span length should be from 50 to 60 feet. The economical and proper depths for riveted trusses necessitate raising the level of the track, as the clearance or height from the street surface to the under side of the iron-work must remain the same: and this constitutes a very important and valid objection to this style of construction.

The weight of iron in the superstructure of an ordinary well-designed double-track elevated railroad structure for passenger traffic varies from 2000 to 2500 tons per mile, and the cost per mile of such a structure, including superstructure, track, station buildings, and foundations, varies from $180,000 to $225,000. The cost per mile of a standard double-track elevated railway suitable for heavy freight traffic, including foundations, but not including station buildings, will be from $325,000 to $360,000.

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* Described in *Engineering News*, May 25, 1893. Walter Katté, M. Am. Soc. C.E., Chief Engineer; Geo. H Thomson, M. Am. Soc. C.E., Engineer of Bridges.

CHAPTER XXVI.

THE ÆSTHETIC DESIGN OF BRIDGES.*

INTRODUCTION.

415. **Development of Æsthetics.**—The character of a people, a nation, or an age is manifested in its artistic productions. The forms which are bred into the mind from infancy determine its peculiar taste for beauty. Out of the multitude of nature's phenomena the human mind strives to conceive the laws governing their relationship. It is possible to formulate a clear and harmonious conception of the seemingly optional purpose of things, only by comprehending the deep necessities of nature's laws. Originally man had only natural products after which to pattern, and the primitive ideas of art were improved through the progress of generations until the different races of the earth gave rise to the various architectural forms. These forms are peculiar to the surroundings, the necessities, and the degree of civilization of a people.

When we utilize the laws governing gravitation and the strength of material for the use and convenience of man, we create artificially that which nature has exemplified. A pleasing effect will be produced by accustomed forms, which shows that we are strongly influenced by surroundings and governed by our environment. When human creations contradict nature, then they cease to comply with our æsthetic feeling. In the course of architectural development we depart from the rudimentary ideas. As long as the mind keeps pace with this departure it retains the feeling of æsthetic satisfaction. When the simple form and purpose of a structure become so disguised that the mind cannot grasp them, then the effect will be to create dissatisfaction, which is contrary to all ideas of beauty.

Since the civilized world lays a strong claim to good taste, and since bridges are monumental to the advances of civilization, their design may well be subjected to the laws governing æsthetics. The theme of this chapter, therefore, will be to consider the purpose of a bridge from a higher standpoint than that of absolute necessity, and to characterize its form with the attribute of beauty.

416. **Hindrances to Artistic Design.**—There are two considerations which singly or combined would oppose artistic design. One of these is given by local conditions, such as legal requirements, inadequate building material, or unsuitable location. These, when unavoidable, will often excuse the engineer if his design is not artistic, but in many cases good judgment combined with sound æsthetic principles will aid in producing a better result than would ordinarily be obtained. The other and perhaps more common reason is found in financial considerations. These difficulties will, however, not excuse the majority of gross violations of æsthetic design which we find everywhere. It might be in place here to attempt an explanation of some of the foremost causes of such violations.

With the aim of obtaining cheap work competition is invited, resulting in favoring the lowest bidder. Unless the artistic appearance of a structure is imposed as a necessary feature, it is rarely, if ever, considered by contractors. Their sole object is to satisfy only the absolute requirements of strength and dimensions. Another cause is the general lack of good taste.

* Through an oversight, which is sincerely regretted by the authors, no credit was given in the first editions of this work to Prof. R. Baumeister for valuable suggestions for this chapter found in his "Handbuch der Ingenieur-Wissenschaften;" also for the cuts in Plates XIV, XV, and XVI, and for designs 1 to 21 and 30 to 32 of Plate XX.

It is hoped by bringing this subject before American engineers to realize an advance in the æsthetic design of bridges.

Out of the consideration that the general public lays stress on the appearance of its bridges in cities, parks, etc., and often appropriates money to supply this want, we realize an advance in artistic taste. At the same time the financial consideration is regarded as the most important feature in public works; therefore the question of æsthetics is barred out on general principles. But rightly considered, these two factors are not necessarily contradictory. As above stated, there are instances where the engineer is bound by so many unavoidable restrictions that a pleasing or satisfying result cannot be attained. But where such limitations do not exist and the competent designer is left to his own discretion, the primary form may as well be artistic as otherwise. It costs nothing to display good taste.

FUNDAMENTAL PRINCIPLES.

417. **Artistic Analysis.**—While the object of structural analysis demands scientific perfection of every material consideration, the artistic analysis consists in supplying æsthetic satisfaction or beauty of appearance. The superiority of artistic over simple natural beauty is due to the clearer display of the laws governing taste. We will classify the primary ideas underlying æsthetic design in the order of their importance:

418. **Symmetry** is the fundamental idea of æsthetics. The symmetrical order of a series of spans with respect to a centre line gives the impression of clearness of principle. The mind finds no cause to question the general disposition (see Pl. XXIII). Spans of different lengths placed without regard to symmetry would call forth criticism (Pl. XXII). When a channel span does not fall in the middle of a river, or the foundations compel unfortunate location of piers, in each case the cause for unsymmetrical disposition remains hidden. Where the form clearly shows its adaptation to the natural profile the laws of symmetry may be violated without disturbing the good general effect (Fig. 2, Pl. XXIV). If, however, the possibility for symmetrical balance is visible, the result will be unfavorable (see Pl. XXII).

419. **The Style of a Structure** should be in conformity with the surrounding landscape. A bridge in a wild surrounding of rocks and forests over a swift mountain stream should be bold in appearance. The masonry should be coarse, unfinished, only showing clearly the traces of artistic order in contradistinction to nature's wilds (Fig. 2, Pl. XXVII). If the rocky cliffs be very massive, the structure should accordingly convey that impression. In such a landscape nothing can be more appropriate than a heavy masonry arch.

There should be no doubt as to the relative importance of landscape or bridge. If the latter be large in proportion, it should stand out in relief, robbing the landscape of its supremacy (Figs. 1 and 2, Pl. XXI, and Fig. 1, Pl. XXXI). On the other hand, if the bridge be comparatively insignificant, then it is proper rather to underestimate its value. The result will be decidedly in favor of producing a pleasing effect.

In a thinly-wooded surrounding, as in a public park, a structure should also have a graceful and more finished appearance (Fig. 1, Pl. XXVII). If it is a city bridge in the vicinity of immense buildings, perhaps of beautiful designs, then we must look to a majestic harmony with the prevalent style of the neighboring architecture (see Pls. XXV to XXXI, and details on Pls. XIV to XVII). The masonry will consist mostly of cut stone to give the neat and clean outline characteristic of the city.

In a general way, we must be careful not to carry any one feature to the extreme, as this would destroy the harmony and thus detract from the general appearance.

420. **The General Form** should never disguise the purpose of a structure, but should aid in impressing the mind with visible strength and proper adaptation to purpose.

Similar means must be employed to accomplish similar ends. This maxim establishes order and purpose of form. A bridge consisting of several masonry arches and several metal spans of equal lengths will not appear harmonious in design unless these are properly grouped (see Figs. 7 and 9, Pl. XXII). Even then the question arises, why would not the same material have answered for all the spans? Certainly in such a case it would be more in accordance with good taste to place longer metal spans in the centre or most important part of the bridge, and to mark distinctly the lesser importance of the adjoining masonry arches, as on Pl. XXIII. This divides up the structure into main bridge and approaches.

The artistic form strives to make clear the relation between the different members; not only to emphasize certain ones, but to show the dependence of one upon another. This explains the preference of artistic form to sober design. When the static principles underlying a structure are inconceivable to the public mind, the latter will not be impressed with the idea of safety or beauty. In such instances the safety is actually demonstrated by tests which are often intended to win over public prejudice. It is therefore easy to understand why the recent developments in engineering are not generally considered beautiful. No matter what technical advantages they may possess, we must accustom ourselves to the new form and develop a liking.

A stream spanned by a bridge of less length than the breadth of stream would make the structure appear inadequate (Fig. 5, Pl. XIV; Figs. 1 and 4, Pl. XVII). The main spans may be carried past the banks of a river, thus giving an unnecessary importance to the bridge. We readily fall upon these difficulties of design where the water-level is subject to great changes. It is plain then that any omission or any unnecessary addition would tend to belie the form of its purpose.

Let it be generally understood that in no case should the artistic ideas governing form contradict any static consideration. Both have their origin in the same natural laws, and when they conflict the static correctness should always be preferred to beauty.

421. **The Dimensions** of the various parts of a structure should bear a harmonious relation to the whole. This constitutes harmony of proportions.

For a given length of a bridge, all things considered, there will be certain lengths for the various spans which will give the best general effect. Again, the subdivision of a span into panels should bear a certain relation to both its height and length. So also a railing should consist of panels equal to a definite fraction of the truss panel.

All these questions are usually determined from the statical considerations of the problem, but frequent opportunity is given to vary such proportions in favor of good appearance without interfering materially with the technical or practical conditions. We are often compelled to dimension a secondary member unnecessarily large just for the sake of looks, which can easily be done by choosing a flat or angle instead of a rod.

When cast-iron or wood is used for a column we commonly find abnormal forms. The rest of the structure being wrought will require much less material and the result will be a lack of harmony in dimensions.

422. **Ornamentation** should be regarded as distinct from simple or rudimentary form and should be utilized only to reinforce the same.

It is not the object of ornamentation to change the general character of a structure to such an extent as to hide the underlying principles. Its proper application is to aid the mind in the conception of the purpose, by contrasting different members of a structure with each other. This law is so frequently violated that we are often at a loss whether or not to believe what we see. Covering a framework with boards and paint to imitate a stone arch; a horizontal girder blended with arch construction; a foot-bridge modelled after a Roman aqueduct; or decorating a girder to represent a temple, which latter hangs in the air instead of resting

on the ground—all these are adulterations and belie the underlying principles by exhibiting false forms.

When we ornament the ring of an arch, emphasize the line of a roadway, or surmount a pier by a statue or other decoration to mark the importance of such parts, we add to the clearness and hence to the æsthetic appearance.

This subject will be taken up in detail in the articles on Ornamentation.

INFLUENCE OF BUILDING MATERIAL, COLOR, AND SHADES AND SHADOWS ON THE ÆSTHETIC APPEARANCE OF A BRIDGE.

423. **Material.**—One of the foremost considerations upon which the popularity of a structure depends is the building material. Stone occupies the highest rank, as it possesses properties requisite to æsthetics which are peculiar to no other material. As a consequence of its low unit strength and the necessary manner of bonding, we obtain massive forms compared to wood or metal. To the average mind, stone lends the appearance of solidity and strength, and emphasizes the monumental character of architecture. The general public is most familiar with the relation existing between the weight and strength of this material, so that even the uneducated eye can discern whether a structure seems unusually bold or heavy. Wood is not sufficiently durable for permanent structures and is not considered here.

Iron and steel are in strong contrast to stone. The relation of strength to weight in iron gives rise to such light forms that the mind is often impressed with wonder. Metal designs display such light masses compared to masonry that it is difficult to accustom ourselves to this material. Another fact which speaks in favor of stone is that it occurs nearly everywhere in nature, whereas metallic iron is a manufactured product. These differences in the choice of material are inherent in the customs and surroundings of a people.

When the application of iron and steel to bridge-building has become characteristic of the architecture of a nation, then this material will be popular from an æsthetic standpoint.

Let it not be supposed from the preceding that the choice of artistic form is limited by the material. It is possible to symbolize the same natural laws in almost any material. A column, whether of stone, wood, or iron, will not change its character. When, however, artistic form is applied to construction, the material is intimately related thereto. For example, the capital and base of a stone column should receive different dimensions and shapes from those of a cast-iron column.

We cannot realize absolute stability, or an ideal balance between loads and forces, for these are relative properties varying with the kind of material. In viewing a structure we involuntarily infer its material from its form even if paint has been utilized to disguise natural color. Artistically considered, it is improper to imitate a stone ornament in tin or wood, as the strength and weight of the latter, as also the manner of workmanship, contradict such forms. Even if the progress of the arts should make it possible to produce any or all forms out of any material, we would regard such progress wrongly applied when the attempt is made to destroy characteristics rather than to emphasize them. Such disregard of the natural properties of material would eventually lead to coarse, miniature forms, lacework on an abnormal scale, or even to an æsthetic lie. Technical miracles have no claim to beauty, since they lack harmony of form.

The general effect is most likely to be pleasing when but one kind of material is used (see Figs. 1, 2, 3, and 5, Pl. XXI; Figs. 1, 2, and 3, Pl. XXIV; and Fig. 1, Pl. XXXI). Great diversity in the static balance or in the technical execution of members, especially when these are of different materials, compel the spectator to change the scale with which he measures the relative magnitude of forces (see Figs. 2 and 9, Pl. XXII). He may find it difficult or even impossible to obtain a general harmonious impression. Such examples are numerous; as when

masonry arches and iron superstructure of same length of span are combined in one bridge; or a brick arch between metal spans; or ornaments made of different material from the main structure. On the other hand, it is often possible to obtain rather a fortunate result by varying the material, as in a light metal span between masonry approaches (see Pl. XXIII).

Both the laws of harmony and of contrast are entitled to their places in art. It is not, however, admissible to hide or ignore a member with the view of producing uniformity of effect. For example, a rod used to take up the horizontal thrust of an arch, to cover the lack of stability of supporting columns. Instead of giving this member its architectural significance, thin, thread-like rods are frequently employed. When we fail to comply with such æsthetic wants, we allow an unfortunate sense of wonder to fill the lack of technical truthfulness. We dispense with the solution of one of the most important problems in architecture.

424. Color.—It is a well-known fact that colors produce an impression on the mind which may be pleasing or otherwise, quite similar to the effect of a musical chord or discord upon the ear. An object may appear in perfect harmony with its surroundings, but when placed elsewhere would lose its entire beauty. It would seem then that the question of color should be regarded as one of considerable weight in the artistic design of an engineering structure. Hitherto this matter has been left to the developed taste. Since our vision does not afford us the same aid in choosing color that we obtain from statics and dynamics in the choice of form, it would seem proper to add this feature to practical æsthetics.

All colors may be divided into two classes, primitive and collective. The former class comprises the rainbow colors, while the latter are primary colors diluted with white or gray. As may be supposed, this difference is not physically marked, but is of purely æsthetic character. In the scale we ascend from the primitive colors, blue, yellow, and red, to their primitive combinations, violet, green, and orange, and finally to the collective color white with its gray variations. Both classes have admirers. The masses are more inclined to favor the intense pigments, while the educated minority show a preference for the softer shades. This is not the result of a lack of appreciation for the bright colors, but of the development of taste.

The choice of color should be in accordance with the material to be covered. We avoid the use of paint when it is not necessary to the preservation of the material. Stone occurs in such variety of tints and requires no preservative, so that we can usually satisfy our wants without resorting to artificial means. Iron and wooden parts, therefore, would involve the question of paint. When a single color is used, let this be a soft tint in harmony with the stone and other surroundings. Different shades may be employed only to reinforce the static principles of a structure. It would, however, be decidedly against good taste to imitate material by the use of paint. An iron structure disguised to represent wood cannot produce a pleasing effect, since the form would appear too light. The same with the reverse, when the form would seem abnormally heavy.

In suiting the color to the surrounding landscape the question of relative magnitude or importance is greatly involved. If the bridge is small, it may be well to effect a contrast or relief by the choice of complementary colors. As in a wooded locality, red or white would be well chosen. If, on the other hand, the structure is so large that the landscape appears comparatively insignificant, a wide range is offered without incurring the loss of contrast.

Coloration should emphasize the static differences of the main parts. In some instances it may be desirable to employ various colors to aid in the decoration of a structure. Then the paint becomes part of the ornamentation. Heavy masses or the principal members should in this case receive a heavy color, while members carrying little or no load should be made to appear slender. Also a lack of relative proportion may be partly overcome by this means. We designate a color as being heavy when it seemingly increases the size of an object. Livelier shades may be employed for members which characterize the features or outlines of a bridge. More subdued tints would be applicable to the less conspicuous surfaces. Mem-

bers used in the same capacity should, however, be treated alike. There is only one admissible exception to this rule, as when we vary two colors in regular succession. The stones of an arch ring may alternate red and gray without destroying the clearness of purpose. The beauty of a structure is increased by ornamentation, but let it be remembered that this beauty may be easily destroyed by extremes and by overloading.

The general color effect should agree with the character of a structure. Bridges, as a rule, demand a more sedate appearance, for which reason it is an easy matter to exceed the bounds of good taste. Grace and neatness may be displayed by resorting to different shades for railings, smaller ornaments, or even for slender rods, as the suspenders or hangers of a suspension bridge. But, after all, we are compelled to restrict the diversity of color to contrasting only the main parts, as superstructure to masonry, etc. The tendency should be always to complete the general effect by supplying the missing features, thus bringing about harmony between color and the character of a structure.

We attribute certain properties to colors, as warm, cold, light, heavy, bright or lively, and subdued, all of which are derived from impressions made upon the mind. These terms are explanatory in themselves, but it is well to remember that red and yellow always produce warmth, while blue, purple, and neutral tint do the opposite. A heavy color seemingly increases the size of an object by contrast, a light one tends to diminish apparent volume by destroying contrast. Bright and lively are applied to the primitives, and collective colors are usually subdued shades of the former. We utilize these properties in bringing about æsthetic unison between a structure and its surroundings, since these should possess the same character.

425. Shades and Shadows.—What has been said of color also applies to shades and shadows to a very great extent. All shapes and forms would be destitute of any modulation and would appear as planes were it not due to the shading produced by one part projecting over another. Upon this fact depend all principles of ornamentation. In designing copings, etc., we must consider the shadows, as the whole form is characterized by them. Smooth-face stones when laid would appear as one mass were the joints not of another color, whereas in rough-face stone the joints are indicated by shadows. Great art may be displayed in this feature of a design, but the good result is too often left to chance.

ORNAMENTATION.

426. In Bridge Building little opportunity is afforded to give members their ideal architectural significance, because their forms are determined by the technical requirements of strength, durability, and cost. We may well be satisfied when the general outline of a bridge exhibits a graceful and pleasing appearance, as in Pls. XXI and XXII. Only in particular cases, as for arch rings, portals, piers, cornices, and railings, we are enabled to develop artistic forms. (See Pls. XIV to XVII.) Therefore, to complete the æsthetic value of a structure, it is proper to elaborate these decorative elements by supplying ornaments. In so doing we must consider the quantity, distribution, scale, choice, and style both of the ornaments and of the structure itself.

427. Quantity of Decoration.—It is contrary to the principle of æsthetic economy to overload a structure or any part thereof with ornaments, even if these details are pleasing and properly chosen. The result would be to suppress or disguise the purpose of the main members and to exhibit an unbecoming wastefulness. The plain or elaborate character of an entire structure must not be contradicted by any of its parts. In this branch of engineering there is little danger of extravagance, but rather a lamentable insufficiency. The relation between a structure and its decorations should impress the observer with æsthetic satisfaction.

428. The Distribution of Ornaments may be utilized to emphasize certain parts without opposing the artistic perfectness of a bridge. The more important river spans would deserve being ornamented rather than their approaches; likewise an arch rather than its wing- and

retaining-walls. (See Pls. XIV to XVII.) Buttresses may be more elaborate than the wall to which they belong.

As a rule, members carrying heavy loads should receive less decoration than those taking little stress. Hence the base of a pier would be plain compared to its coping or capital (Figs. 3, 4, 6, and 9, Pl. XVI); so also main trusses relatively to the railings and cornices (Figs. 6 and 7, Pl. XIV and Pl. XXIII). The parts situated closer to the observer are more valuable æsthetically than distant ones, for which reason the end portals, railings, etc., should be ornamented. (Examples in Pls. XIV to XVII.)

On the other hand, such contrasts must not be too strong. A statue would seem out of place on a rough block, or fine ornamental details on an otherwise plain plate girder. The attempt to attain a proper equilibrium between the architecture of a portal and the adjoining bare superstructure has often been unsuccessful. The tendency is to give a preference to the former, especially when cheap cast forms are at hand, while the comparative treatment of the latter remains neglected. Many designers have given way to the temptation of elaborating in cast-iron, while leaving wrought metal in its plain and unfinished condition. It shows poor taste in design when we find neat columns or piers supporting homely roof trusses, plate girders, etc.

429. **Scale of Ornamentation.**—The dimensions of a bridge determine the relative sizes of its ornaments. Small decorations, even in large quantities, are not suitable to huge structures, and *vice versa*. It is difficult to attain a happy medium in this important feature, because a bridge is subject to examination from two main points of observation: the one away from the structure, from which the general appearance would be judged; the other on the roadway, from which portals, trusses, railings, etc., would be viewed. We depend upon artistic tact for the preservation of clearness.

When comparison is possible between two objects of similar form, though differing widely in dimensions, the seeming adaptation to purpose will be disturbed. Small forms should never be a mere reduction of large ones when both are applied to the same structure. Even when the same purpose is accomplished by each, it is well to modify the former, usually by omitting some of the details of the latter. We see such examples where the capital or head of a pier is repeated in the small columns of the railing, or when spans of same design, differing greatly in length, succeed each other in a bridge (Figs. 7 and 10, Pl. XXII). In a few instances the repetition of a form in various sizes is admissible when the material is changed; as a long metal arch with adjoining masonry arches. (See Pl. XXIII.)

We must not forget that all structures, even the most gigantic, are erected by and for man; hence statuary, columns, and all other ornaments shaped to represent objects belonging to the animal or vegetable kingdom should approach the natural dimensions. The impression made by exaggerated artistic forms is not one of intellectual greatness, but simply a material hugeness, wherein grace and delicate details have been lost.

430. **Choice of Artistic Form.**—The perfect artistic form of an object should recall the structural purpose of its parts without necessitating close observation or analysis. The strength of a member, its direction, application, and manner of resisting forces, should be readily comprehensible. It is easiest to illustrate this idea by citing some of the many gross violations where attempts are made to decorate but without success. On some of the old covered bridges the wooden suspenders have been disguised to represent compression columns, by supplying bases and capitals. On the Callowhill Street Bridge in Philadelphia, the main trusses take the shape of an arched passage, the vertical struts acting as piers, with the top chord representing a series of arches. Adding a frame-like decoration to wall spaces between buttresses is out of place, because the forces to be resisted have no bearing on such forms. The plain surface of a beam or girder is not appropriate for a frieze-like ornament extending from end to end. The centre of the beam should be marked by some special feat-

ure, from which the decoration may extend towards the ends. It is also advisable to choose ornamental details involving no particular direction, as isolated figures or panels (Fig. 6, Pl. XIV, also Pls. XVIII and XIX).

431. **Style.**—The chief end of art is not attained by copying exactly the forms of nature or those of textile fabrics, but by reproducing the analogy between the static functions of natural products to satisfy artificial wants. Exact imitations are limited in architecture by difficulties relating to their execution in stone, wood, or metal. The objects to be accomplished by style are; to prepare the natural forms for artistic purposes; to maintain the structural elements, but supply the necessary coarseness to correspond with the building material; and, lastly, to neglect extremely fine details by changing the scale which might slightly exceed the natural. In other words, we create style by a liberal translation of nature's forms.

The extent to which style may be applied depends upon the magnitude and location of the member, as on the material and character of the whole structure. Every age adopts a style according to its peculiar conception of nature, which explains the great divergence in architectural forms while the constructive principles have remained the same.

All that has been said on the subject of ornamentation is really defining "artistic beauty;" and after all there is nothing that will be of more service to the designer than a well-developed taste for that which is beautiful. Artistic taste may be acquired by close observation and study; but even then not everybody is susceptible to a broad understanding of "æsthetics." We are easily influenced by new impressions, and especially by the arts from other countries.

Within the limits of our environment our ideas of beauty naturally assume a domestic character. Every other country has its own style, and when we become familiar with each we begin to realize that our domestic ideas are only primitive. Each style has claims to beauty peculiar to itself, and we must learn to value them, since they are as justifiable as the claims of our own style.

ÆSTHETIC DESIGN.

432. **Division of Subject.**—What has been said in the previous articles is more of a general character. With the aim of increasing the value of the present chapter we will consider briefly the design of the principal elements of a bridge with respect to general æsthetic form and ornamentation. In doing so the most natural subdivision of the subject would be into *Substructure*, *Superstructure*, and *Roadway*.

433. **Substructure: Piers and Abutments.**—We divide up a site into a number of spans, the points of division being marked by piers, and the extreme points by shore piers or abutments. The piers serve the purpose of supporting the superstructure, of resisting the horizontal thrust resulting from wind and moving loads, and (when located in water) of withstanding wave and current action. From static considerations these forces will be resisted by a pier of certain form; but when the question of beauty is involved, we are usually compelled to modify this form beyond the requirements of absolute necessity.

The relation between height and thickness should be in æsthetic harmony with the length of span and apparent load to be carried. For long spans and small clearance the supports should convey the idea of massiveness (Fig. 5, Pl. XXI). In the other extreme of short spans and high piers a slender design, as an iron bent or tower, would seem proper (Fig. 4, Pl. XXI). A very high viaduct may even be constructed with single bents resting on pins, top and bottom, the horizontal thrust being taken up by fixed towers placed at regular intervals. Such a disposition would add grace and variety to the structure.

434. **The Scale for Details** must be deduced from the general dimensions. A pier is composed of three parts—base, body, and capital or coping. Æsthetic stability is added by

widening out the base. In rivers where the stage of water is variable it is advisable to start the base a little above high water and spread with the increasing depth, thus retaining a visible base for all conditions (Fig. 5, Pl. XXI). The base, being most subject to the action of the external forces, should not be decorated. The only admissible addition might consist of a coping to separate the base from the body. This is desirable, for it idealizes the architectural significance of the supporting power of a foundation (Fig. 3a, Pl. XIV).

435. **The Body of a Pier** is subject to many variations. The general style should agree with that of its base and coping. A batter is very desirable, as it emphasizes the stability against horizontal forces. Offsets in the vertical lines, to replace batter, are to be avoided, since expense is the only motive to prompt such design. Ornamentation should tend in a vertical direction, but it is well to maintain clearness of form and purpose by choosing artistic shapes, using decorations only to a very limited extent (Figs. 3, 4, and 5, Pl. XIV and Pl. XVII). When the body is carried above the supports, as for an arch or deck bridge, this point should be characterized by some form of cornice or coping. The portion above the supports is of minor importance; it should therefore receive smaller dimensions and may be more elaborately decorated (Figs. 1 and 2, Pl. XV; Figs. 3, 4, and 6, Pl. XVI; Fig. 5, Pl. XXI).

436. **The Coping.**—In treating the coping, which really takes the place of the capital of a column, a wide range both of form and purpose is offered. The common and most frequent style of coping consists of a heavy course of cut stone projecting over the body of the pier. Besides distributing the load over the area, it emphasizes the point where the weight is applied and adds a finish or covering.

The coping is subject to more ornamentation than any other portion of a pier, and may, according to circumstances, develop into the form of a complete capital. In some cases the body is carried to the roadway, where it forms a prominent point of the railing. The cornice and balustrade may then unite in offsetting the coping (Figs. 1 and 6, Pl. XIV; Figs. 1 and 2, Pl. XV; Figs. 3, 4, and 6, Pl. XVI; and Fig. 1, Pl. XVII).

For an even number of spans a pier will fall in the bridge centre. This pier should be more conspicuous than the others, both as regards size and ornamentation. The object is to mark the centre of the structure, an important factor to symmetry and artistic effect (Fig. 5, Pl. XXI, right-hand pier is centre of bridge).

437. **The Abutment** performs the function of a pier, but usually embodies other features which necessitate a more massive form. Generally an abutment constitutes the connecting link between main structure and approach or embankment, in which case the massiveness is required to serve the purpose of a retaining wall. Batter of the front face adds greatly to visible stability. For any wall resisting geostatic pressure the theoretical line of the face is a curve, which answers the æsthetic requirements best, as it conforms to the popular conception of the way in which walls usually fail, viz., by bulging out or overturning. The simpler form approaching this is obtained by a straight batter, while the one with a vertical front displays the least stability, and hence is wanting in artistic effect.

Abutments may also be used as office buildings, and then admit of very elaborate designs. In case of a suspension bridge the foot of a tower is usually made to serve this purpose. When an abutment covers all these requirements it may be developed into the form of an end portal or archway. The design of this portion of a bridge is most susceptible to architectural beauty (Figs. 4 and 7, Pl. XV, and Figs. 1, 2, 5, and 9, Pl. XVI).

It is to be remembered that the artistic details of piers and abutments should be developed upward and never down; that the coping takes precedence over the body, and this over the base, in the order of æsthetic value. It is barely possible to formulate any rules for the details regarding the horizontal subdivision of a pier or other body of masonry into courses, etc. These subdivisions depend upon the position of the observer and the height, thickness,

surroundings, loading, material, shades and shadows, etc., of the object. (For examples see Plates XIV to XVII.)

438. **Superstructure.**—The question of artistic design is more neglected in the general forms of trusses than in any other portion of a bridge. The reasons previously mentioned for neglecting æsthetics in bridge building bear more strongly on iron and steel work than on masonry. Competition is far greater among manufacturers of the former class, and especially when plans are prepared by them. The question of extreme economy of both labor and material forms the basis for such designs. Therefore, when stress is laid on artistic effect, plans should be prepared by the engineer, or else let it be understood in advance that design as well as cheapness will be considered. This will tend toward elevating the standard of designing without neglecting economy.

With the possible exception of city bridges, ornaments may be entirely omitted without materially injuring the æsthetic value of a structure. The essential considerations are form, symmetry, and adaptation to surroundings and purpose. When these are complied with, so that a structure as a whole will convey a pleasing and harmonious effect, then we have accomplished the end of an artistic design.

To insure a successful result æsthetically considered, certain relations between the various parts of a bridge must be maintained. These have received mention in the foregoing, viz., the proportion of height of substructure to superstructure; of height to thickness of piers; of base and coping to body of pier; even the thickness of masonry courses (with due regard for material) to the general scale. A gradual curve over the length of a bridge, with the highest point in the centre, adds grace and prevents the optical illusion of a long horizontal line appearing sagged. Technical conditions frequently necessitate such grades, which are either regular curves or broken lines with intersections in pier centres. Much may be done to improve appearance by grouping spans according to importance and location, but in such cases the change should be abrupt rather than gradual; as, for instance, a heavy pier may be inserted, allowing the portion to one side to act as main structure, the other as approach (Fig. 5, Pl. XXI). It would also seem inappropriate to repeat the form of a large span in a small one, thus producing the effect of a miniature or model from which the larger was designed. The small span should have a character of its own, to imply its difference of purpose.

Should the number of spans in a bridge be even, then the centre pier should have some extraordinary feature to mark its importance. When the number is odd, the centre span should be longest, and if possible contain the highest point of the grade line (Figs. 3 and 5, Pl. XXI).

In the case of a long viaduct the appearance may be greatly improved by a systematic grouping of piers or spans; as by inserting a masonry pier between a number of metallic towers or bents at regular intervals, or by making spans alternate in length. This will relieve monotony which would otherwise be a serious objection.

The most pleasing effect is produced by an arch, whether of stone or metal, but is often avoided on account of expense (Pls. XXVI and XXXI). The next form is probably the inverted arch, or suspension bridge (Fig. 1, Pl. XXII), then follow trusses with curved chords (Fig. 1, Pl. XXVIII, and Fig. 2, Pl. XXX). Still less pleasing in appearance are parallel chords (Figs. 2 and 3, Pl. XXVII); and last in order of beauty, irregularly broken lines, as in a cantilever (Fig. 6, Pl. XXII). The relative amount of material for an arch or suspension bridge is much less than for a truss, yet the cost of the former is usually higher, for reasons depending on manufacture and building.

If we wish to regard clearness of purpose as the prime factor in determining what is beautiful, then it is plain that the suspension system should be most popular, its principles being plain to nearly every one. The arch would rank next, and trusses last. It might be

added that a beam or plate girder is simplest of conception, which is true within the limits to which they are applicable. The truss, however, in virtue of its internal complications loses its resemblance to a beam, and is therefore less popular.

It is also easy to understand why a deck bridge should be more æsthetic than a through bridge. When the supports are at the top chord (where they should be for a truss) the structure is in stable equilibrium, and the sway system can be properly developed, which favors architectural perfection. It is to be regretted that deck bridges are so limited in their adaption.

Ornamentation if desirable should be applied in accordance with the rules given in the articles on that subject.

439. **Roadway.**—The purpose of all bridges is to carry loads. For railroad and highway accommodations this is accomplished over a roadway, for aqueducts through tubes, etc. Hence the symbol of utility is exhibited by the line carrying these loads.

In accordance with the foregoing principles, it would seem proper to develop this line to its full æsthetic value. We accomplish this end by adding weight in the form of cornices, ornaments, railings, etc., to the line of the roadway. The simplest addition of this kind, for a metal bridge, consists of the floor system itself, and of a plain coping-stone for a masonry structure. Both represent in themselves continuous, heavy lines, but these may be farther developed into artistic forms. As long as we avoid extremes, either by supplying too much, or by destroying the harmony with the style of other parts of the structure, a decided improvement will be realized by developing this member.

The parts carrying the roadway receive artistic form by the addition of a cornice, the character of which depends upon the material and style of the structure. Masonry is best adapted to such ornamentation, since arches are always deck bridges and the natural finish is produced by the cornice and balustrade.

It would seem proper to omit cornices from through bridges, since there is always a strong tendency to cover the bottom chord to such an extent as to destroy its æsthetic significance. This severe criticism may be justly applied ot the Callowhill Street Bridge in Philadelphia, where both chords are completely disguised.

Plate XVIII shows a number of cornices and string courses to be used in buildings and which serve as suggestions for cornices for both stone and metal bridges; some alterations, however, would be necessary to suit them to the latter material. They all require heavy structures and would be out of place on slender designs. Plates XIII to XVII give good examples for the various styles of bridges.

440. **Railings.**—For trusses, the only available decoration of the roadway would seem to be the railing. The top chord of a deck span may be surmounted by a cornice, but great care must be exercised not to disturb the proper balance. A railing serves as a protection and reminds the observer of the intended purpose of a structure. Hence the lack of supplying this member to a public passage-way would invariably recall an insufficiency of design.

In classifying the forms of railings or balustrades the technical considerations would depend principally upon the materials, stone, wood, or metal, while the æsthetic properties are determined by the static requirements to which the material is made to conform. A plain parapet wall is the simplest balustrade and is frequently applied to stone bridges. The first motive to decoration is the addition of a cover or coping, which serves to protect the wall against water. The complete form, however, would necessitate a base. The artistic idea of a railing, therefore, embodies three parts: coping, wall or web, and base. These, though derived originally from stone architecture, retain their æsthetic value when applied to wood or metal designs.

The relation between the heights of these parts to the whole is important. There is a tendency to elaborate the base and coping beyond their proper proportion. This should be

carefully avoided, as it makes a bridge appear too light for its purpose. Naturally, the web of a railing is most subject to ornamentation, and is usually divided into panels, which may be solid or otherwise to harmonize with the entire structure.

Plate XX gives a number of designs for railings which may be used in the following manner: Figs. 1 and 6 are intended for execution in brick; Figs. 2 to 5 in stone or terra cotta; Figs. 7 to 10 in cast-iron. Fig. 11 may be either cast- or wrought-iron and is the corresponding metal form to the stone railing shown in Plate XIV, Fig. 1. Figs. 12 to 32 are intended for wrought-iron, some of which have cast trimmings and posts. Figs. 29 and 31 represent cast frames with wrought fillings. Other handsome designs may be found in plates XIV to XVII.

Before closing the subject of "Æsthetic Design" there is one other feature, not bearing on design but on execution, which ought to receive mention here. Nothing detracts more from the general appearance of a structure than an untidy, slovenly surrounding. Not infrequently quantities of refuse building material are left on the site of an otherwise handsome bridge. A little attention in this direction would add considerable credit to the parties concerned.

COMMENTS ON THE PLATES.

441. **Plate XIV.**—Fig. 1 may be considered artistic when we allow for the character of its surroundings, among massive buildings in the city of Hamburg. The arch is plain and heavy, with just enough detail to separate the ring from the other masonry. The railing and cornice are in good proportion, and are well combined with the coping of the pier. The æsthetic appearance might be somewhat improved by diminishing the cross-section of the portion of the pier above the springing of the arches in some such manner as in Fig. 1, Pl. XV.

Fig. 2 is at fault, owing to the omission of a coping at the springing of the arches, as a consequence of which the latter have no visible support. There is no necessity for extending the full section of pier to the height indicated.

Fig. 3. The base of the tall pier might be modified as shown in *a*, otherwise the design is good.

Fig. 4. The tower above the roadway seems abnormally heavy in proportion to the pier. This is caused by the change of batter, which might have been avoided.

Fig. 5 is a very neat design, though a slight batter of piers and abutments would add to its beauty.

Fig. 6 is open to the same criticism as Fig. 2. The coping at the springing line should have been carried around the pier.

Figs. 7 and 8 are very skilfully treated.

442. **Plate XV.**—All the figures of this plate, though differing widely in character and purpose, are good examples of artistic design. The pier in Fig. 3 combines usefulness with a pleasing appearance, and was actually intended for a means of defence. Such forms are out of date, yet retain their beauty even to the present day. The towers in Fig. 7 are somewhat lacking in æsthetic stability. The defect might easily be remedied by slightly inclining them toward each other.

443. **Plate XVI.**—Fig. 1 is of a character similar to Fig. 3, Pl. XV, and would hardly be applicable to this country. The buttress adjoining the arch in Fig. 7 might properly be improved by a slight batter to the left, with a corresponding widening of its base. The other examples on this plate are hardly subject to any criticism.

444. **Plate XVII.**—Fig. 1 exhibits a very tasteful design for a city bridge. The shore span seems rather short as the abutment is placed in the water, but the conditions justified this disposition owing to the previously existing quai-walls. The centre of the arch is properly

offset by the keystone, and is further emphasized by a similar ornament in the railing. The recesses in the roadway at the piers and abutments are very appropriate and add considerable to the decoration. The cornice and railing over the pier, though different from that of the continuous roadway, agree well with the general style of the structure. The arch ring is well developed and stands out in relief from the plain surfaces of the external segments.

Fig. 2. Here the line of the supports is carried through in the form of a secondary cornice, actually dividing the bridge into two stories, which idea is artistically embodied in the centre pier. The entire design is plain and complies well with the requirements of a railroad bridge.

Fig. 3 is a more elaborate structure for a railroad, and is well suited to its location.

Fig. 4 illustrates the utility of brick in erecting an inexpensive bridge. All the decorations are of this material produced by color-effect and otherwise. The pier would be more graceful if the portion above the lower coping were not quite so wide.

445. **Plates XVIII, XIX,** and **XX** are considered in articles pertaining to roadway.

446. **Plate XXI.**—Fig. 1 owes its pleasing apppearance solely to its form. The structure is very plain otherwise, but suits its purpose admirably.

Fig. 2 is of similar character, but adapted to railway traffic. The dimensions of the arch increase toward the springing, which illustrates, architecturally, the technical requirements.

Fig. 3. This immense structure justly demands supremacy over its surroundings. The graceful lines of the cables stand out in bold relief against the sky, thus bringing about perfect harmony with the landscape, which is characterized by the masts and rigging of thousands of vessels in the harbor. The bridge is strictly plain and owes its beauty to its form. The towers have the appearance of being unfinished, and should be capped with an appropriate design.

Fig. 4 is a plain though very effective design for a viaduct. It might not be as economical as a plate girder, but the appearance is worth the difference. It is a fine example of artistic construction in wrought-iron.

Fig. 5 represents one of the few bridges that may be called a model. Every detail discloses artistic tact.

447. **Plate XXII.**—As a general criticism, all these designs lack symmetry except Nos. 1, 2, and 8, which defect is not excusable, as may be seen from Pl. XXIII. No. 1 is a correct solution of the problem, and presents a very graceful appearance.

No. 2 is wanting in æsthetic stability, as the thin cables of the approaches do not counterbalance the mass of the main span.

No. 3 conveys the idea of extreme economy. Such a structure would not be very creditable to a wealthy city.

No. 4 is a succession of arches which bear no artistic relation to each other. The heavy pier in the bridge centre is not called for, since the long span is really more important. The change of grade was unnecessary, as shown by the other designs.

Nos. 5 and 9. If the left half had been repeated on the right-hand side of the centre pier, both designs would have been acceptable. Of course this would convert the probable cantilever into two braced arches and two cantilever-arms.

No. 6 would seem like a monstrosity in a city like New York. Such a bridge would be more suitable to the wilds of the far West.

No. 7 contains such a variety of arches that the common mind might wonder whether there could be any circumstance where arch construction is not possible. Of course the designer probably had some reason for choosing this disposition, yet the manner of solving the problem is justly open to criticism.

No. 8 is too heavy and would not harmonize with the surroundings.

No. 10 would represent a fair design, though the centre span might have been increased about 50 feet. Even then the river spans seem rather insufficient.

448. Plate XXIII.—The four designs for Harlem River or Washington Bridge illustrate very clearly how the unsymmetrical forms on Pl. XXII might have been avoided. The drawing by Mr. C. C. Schneider is a model in every respect. The metal arches are highly artistic, yet plain, and appeal to the eye as the important members of the bridge. The more ornamental roadway is massive, but in very good proportion to the arches. The masonry approaches exhibit a distinct character, peculiar in themselves. The style of the shore piers is exactly repeated in the river pier. The coping which marks the springing of the masonry arches is carried through the entire stone work. The railing and cornice are uniform over the bridge, but are emphasized in the main structure by the secondary arch system below. The harmony between the various parts is admirably preserved.

The design by Mr. W. Hildenbrand is less pleasing in effect. The arches of the approaches resemble too much the secondary system of the roadway, which latter appears rather heavy compared with the main arches which carry them. The manner of bracing the vertical columns destroys to a great extent the effective character of the braced arches which stand out so clearly in Mr. Schneider's design.

The defects in the contract drawing are nearly all remedied in the bridge as built, and are so plainly visible that nothing more need be added.

449. Plate XXIV.—Figs. 1, 2, and 3 are very neat designs, each suited admirably to its purpose. The beauty is due entirely to the general form, and decorations would be quite unnecessary. The lack of symmetry is clearly justified by the profile.

The St. Louis and Washington bridges are considered elsewhere.

450. Plate XXV.—Fig. 1. This most beautiful structure well deserves mention here, and illustrates the extent to which æsthetic design may be applied to bridges. It is hoped that we may soon be able to include similar artistic feats among American productions.

Fig. 2. The Salzburg Bridge, together with its landscape, exhibits a very picturesque appearance. The only defect is in the peculiar truss ornamentation, which partially disguises the technical outline of the superstructure.

451. Plate XXVI.—Fig. 1 is a railroad- and highway-bridge combined, which accounts for the heavy form, though the design is in good keeping with the surroundings.

Fig. 2. The iron highway bridge in Basel is a very pretty structure. The masonry is rather old-fashioned, but this is required by the character of the locality. The river piers are still in want of the necessary statuary.

Fig. 3 shows what results may be obtained without resorting to ornamentation. The bridge represents a most effectual piece of work, comparatively cheap and to the purpose.

452. Plate XXVII.—Fig. 1 illustrates a type of bridge which should be more frequently seen in the parks of our large cities. It is a very handsome design, with the exception of the railing. One of the patterns on Plate XX, as Figs. 21, 25, or 30, would have been more in place.

Fig. 2 is in itself far from being a beautiful structure, though it is in artistic balance with the rugged landscape. Nothing elaborate is called for, hence the design answers its purpose.

Fig. 3 represents the earliest form of metal bridges, with piers adapted to defensive purposes. Even at the present time these might prove useful. The general disposition is nevertheless in conformity with æsthetics. The centre pier is developed to the proper degree, and the shore piers, or abutments, are offset by the fortified towers, which include the end portals and toll offices. The lattice trusses rank among the least artistic, but in this case, with its many systems, it approaches closely the beam or tubular form.

453. Plate XXVIII.—Fig. 1 is a modern example of the fortified type of bridges. The abutment is a perfect fort, to which the heavy trusses are well suited.

Fig. 2 is a very graceful and highly ornamental bridge, of which the city of Mayence may well be proud.

Fig. 3. This is one of the earliest forms of metal arches, and was used largely as a pattern for the Eads Bridge in St. Louis. It is a plain yet effective design and highly creditable to its engineer.

454. **Plate XXIX.**—Fig. 1. The Rialto Bridge, over the Grand Canal, in Venice, is historical in bridge architecture. It is the peculiar character of the adjoining dwellings that lends beauty to this monument to early engineering. The same design repeated elsewhere would undoubtedly prove an artistic failure.

Fig. 2 is a beautiful structure and very creditable to the city of St. Louis. It is beyond question one of the neatest suspension bridges in this country, and would bear repetition elsewhere.

Fig. 3 represents the model bridge of Philadelphia. Similar structures erected at Market Street, Girard Avenue, and Callowhill Street, would have been more in place than the present crossings of the Schuylkill at these points. The Girard Avenue Bridge is very handsome, but does not compare with the one at Chestnut Street. Instead of the abrupt change of grade in the centre of this bridge, a gradual curve might have been chosen.

455. **Plate XXX.**—Fig. 1. The portal of the New Hamburg Bridge is one of the finest pieces of engineering architecture in existence. The superstructure, which is an exact copy of that of the old bridge shown in Fig. 2, would hardly be duplicated elsewhere, though it looks well. The fact that two structures of this design were placed side by side is to be deplored.

456. **Plate XXXI.**—Fig. 1 of this plate has received mention in speaking of Plate XXIV.

The St. Louis Bridge with its world-wide reputation scarcely needs any comment here; it is added as a model of æsthetic design.

456*a*. **Plate XXXI*a*.**—Fig. 1 * is a fine view of the Tower Bridge recently constructed across the Thames at London, near the Tower of London, from which it takes its name. This design is the result of some twenty years' continuous study and discussion, and although it spans a total opening of only about 800 feet, it has been constructed at a cost of over $4,000,000. A large part of this cost has been incurred purely for the sake of æsthetic effect. The four elegant stone towers simply serve the purpose of enclosing and covering from view four pairs of steel towers which carry the loads imposed by the suspension cables. The two side openings are 270 feet each, and the centre opening 200 feet. The roadway of the central span is arranged to open on the bascule principle, and while it is open passengers are transferred to the tops of the towers by elevators and cross over on two independent footways, each twelve feet wide, at a height of 141 feet above high tide. This bridge was opened for service in June, 1894, and has continued in very successful and satisfactory operation to the present writing (May, 1895). The design of this bridge is due to J. W. Barry, Engineer, and Horace Jones, Architect, both of whom were employed by the city of London to prepare and execute the design. It forms the latest and best illustration of the proper course a corporation should pursue in securing the best results where an engineering design is expected to have a pleasing architectural effect. Neither the engineer nor the architect alone, however competent in his own field, is equal to the successful execution of such a project. The result has more than justified the action of the corporation of the city of London in this matter.

Fig. 2 † illustrates the Black Friars Road Bridge across the Thames at London, which was finished in 1869. Mr. Joseph Cubitt was the engineer engaged by the city corporation. The piers are ornate, but in excellent taste, the cylindrical column being composed of polished red granite, and the pedaments and capitals of Portland stone. The external wrought-iron ribs are covered with ornate cast-iron facia, and the slightly curved roadway is guarded on either side with a parapet of Gothic design, three feet nine inches high. This bridge is an excellent example of simplicity and good taste, obtained at a moderate cost.

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* See *Engineering*, Jan. 4, 1895. † See *Engineering*, Feb. 8, 1895.

456*b*. Plate XXXI*b*. — Fig. 1 * of this plate shows the Charing Cross railway bridge, completed in 1869 and widened in 1884. But little attempt has been made in this bridge to produce a pleasing architectural effect, and the pile-like character of the piers gives to the bridge the air of a temporary structure. This bridge was built by the railroad companies who use it, and it well illustrates the difference between a good and a poor architectural design. Even such attempts as have been made at ornamentation on the brick piers appear strained and out of harmony with the rest of the design.

In Fig. 2 † we have a view of the Victoria Road Bridge across the Thames, which was completed in 1858, after a design by Mr. Thomas Page. The total length of this bridge is 700 feet, the central opening being 333 feet. Both the abutments and the piers are enclosed in stonework, giving a very pleasing architectural effect. The loads are really carried inside of these stone shells on iron columns and concrete masonry. As the anchorage abutments are on the river banks, and the two piers are placed in the river, the entire structure rests over the water, and nothing has been lost by having the approaches extend over the land. It offers less obstruction to river traffic than an arch bridge would, and it has a more pleasing appearance, especially by way of contrast.

456*c*. Plate XXXI*c*.—This plate contains an additional view of the Tower Bridge London, shown in Fig. 1, Plate XXXI*a*.

456*d*. Plate XXXI*d*.—This plate gives a view of the famous suspension bridge at Buda-Pesth, which is frequently called the handsomest bridge in the world.

NOTE.—The author is greatly indebted to the following gentlemen, who have so kindly assisted in procuring desirable illustrations:

Messrs. Wm. R. Hutton and Leo von Rosenberg, for the use of Plates XIII, XIX, XXII, XXIII, and XXIV from the monograph on the Washington Bridge;

Mr. John C. Trautwine, Jr., for view of Chestnut Street Bridge, Philadelphia;

Mr. Carl Gayler, M. Am. Soc. C. E., Designer, and the King Bridge Co. Contractor, for view of Grand Avenue Bridge, St. Louis;

The *Cosmopolitan Magazine*, for the plates of Fig. 3, Plate XXI; Fig. 1, Plate XXIX; and Fig. 1, Plate XXXI;

Prof. C. L. Crandall, of Cornell University, for the use of Plate XVIII, and a number of photographs from the magnificent collection belonging to that institution;

Mr. James Dredge, Editor of *Engineering*, for the four views of London bridges shown in Plates XXXI*a* and XXXI*b*.

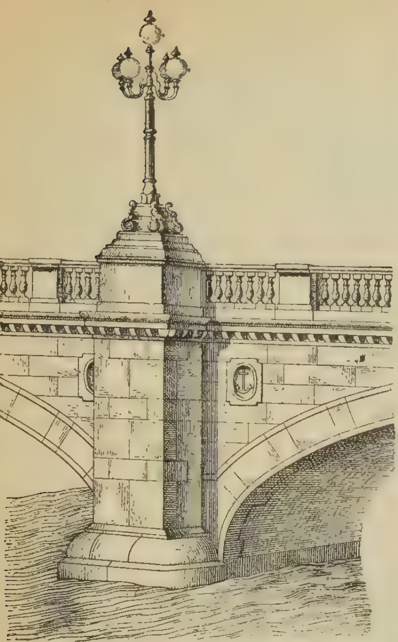
* See *Engineering*, Feb. 22, 1895.

† See *Engineering*, April 5, 1895.

Central Pier.

WASHINGTON BRIDGE.

Fig. 1.



Lombards Bridge in Hamburg.

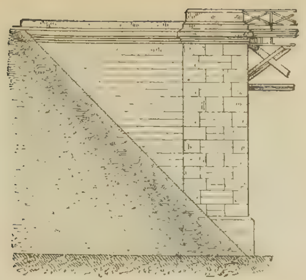
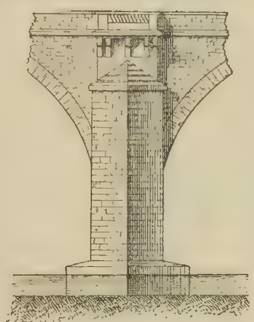


Fig. 2.
Old Heidelberg Bridge.



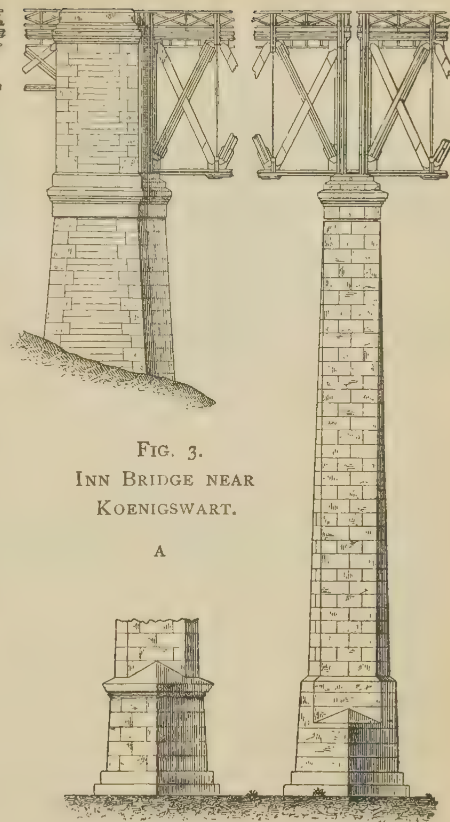
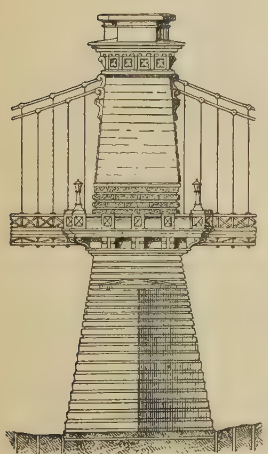


Fig. 3.
Inn Bridge near Koenigswart.

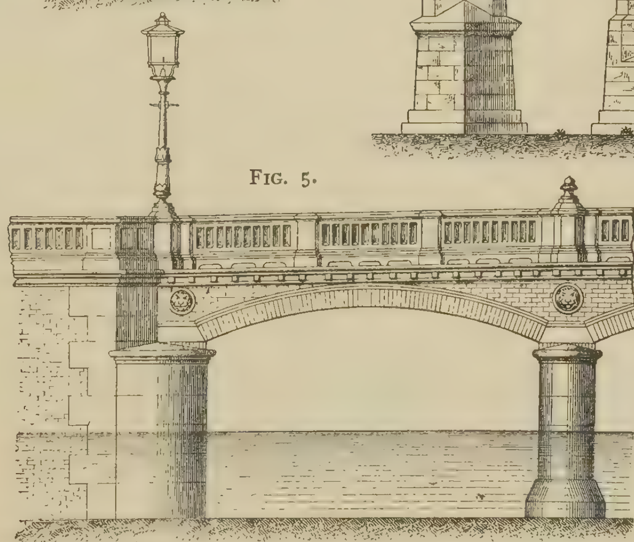
A

Fig. 4.



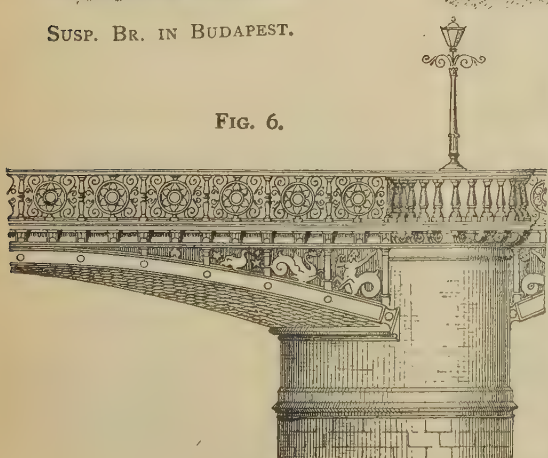
Susp. Br. in Budapest.

Fig. 5.



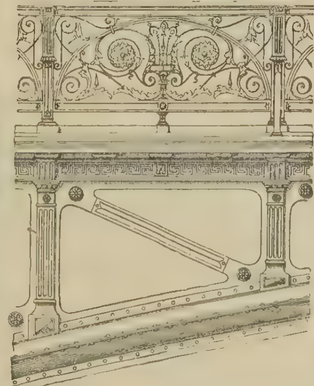
Roesendamms Bridge in Hamburg.

Fig. 6.



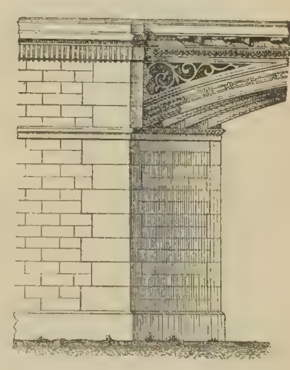
Unterspree Bridge in Berlin.

Fig. 7.



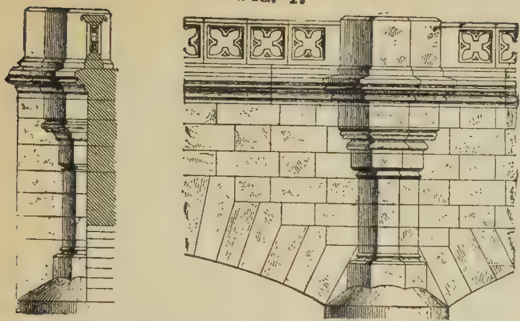
Tegetthoff Bridge, Vienna.

Fig. 8.



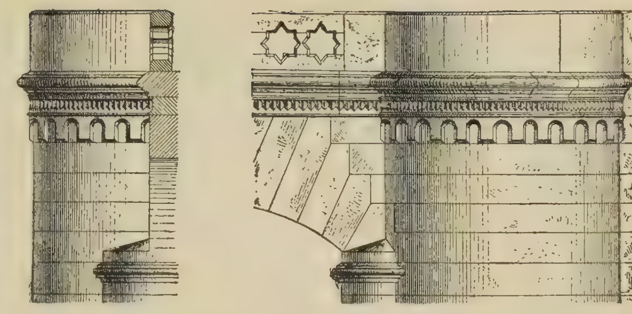
Road-Crossing, Freiburg.

FIG. 1.



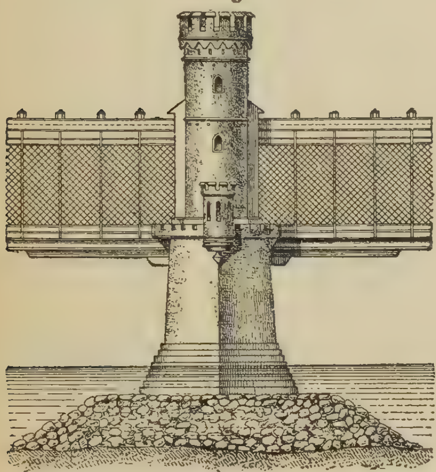
LAHN BRIDGE NEAR FRIEDELHAUSEN, GERMANY.

FIG. 2.



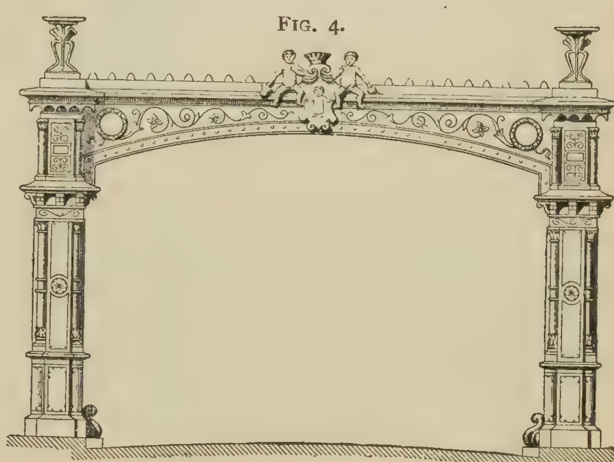
NIDDA BRIDGE NEAR VILBEL, GERMANY.

FIG. 3.



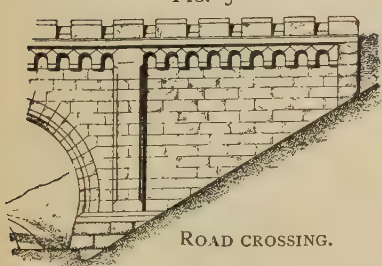
WEICHSEL BR. NEAR DIRSCHAU, GER.

FIG. 4.



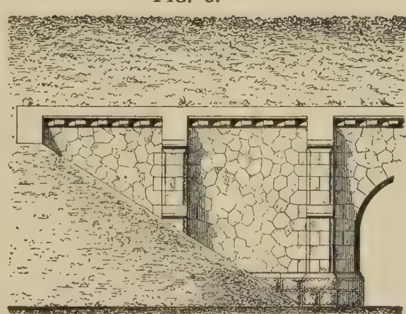
WETTELSBACH BRIDGE IN MUNICH, GERMANY.

FIG. 5



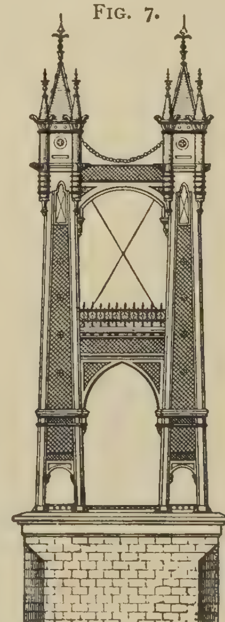
ROAD CROSSING.

FIG. 6.



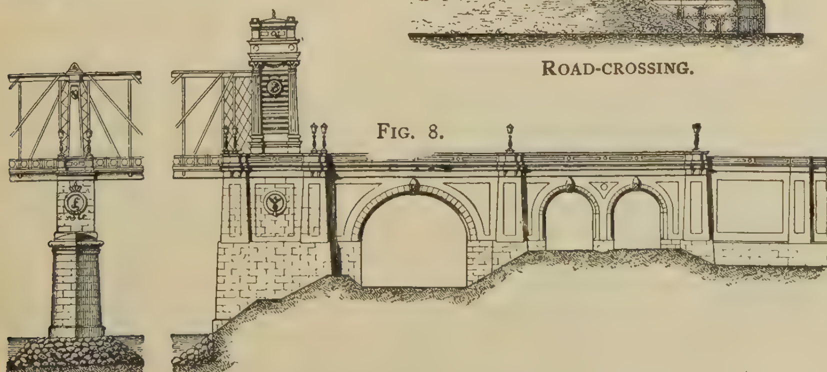
ROAD-CROSSING.

FIG. 7.



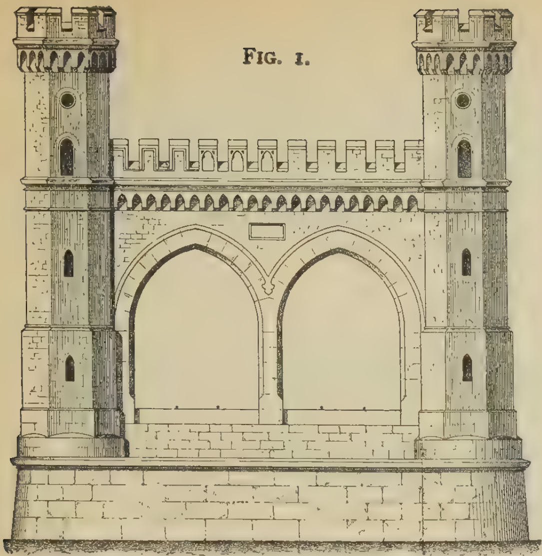
MONONGAHELA BR., PITTSBURG.

FIG. 8.



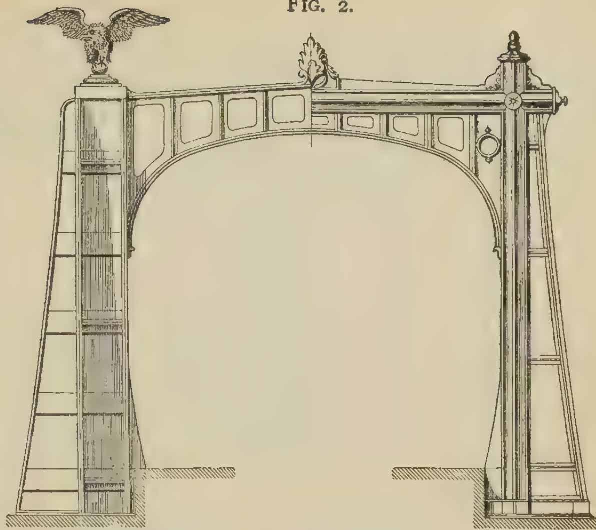
RHINE BRIDGE IN MANNHEIM.

FIG. 1.



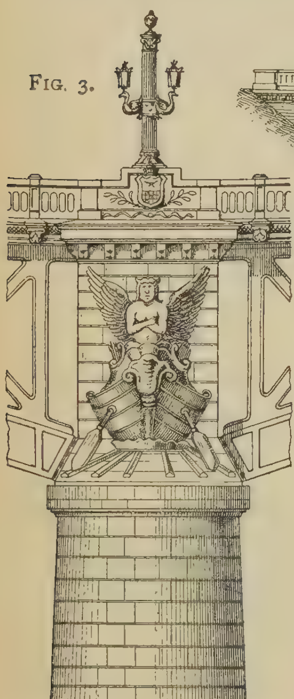
PORTAL OF LAHN BRIDGE, NASSAU.

FIG. 2.



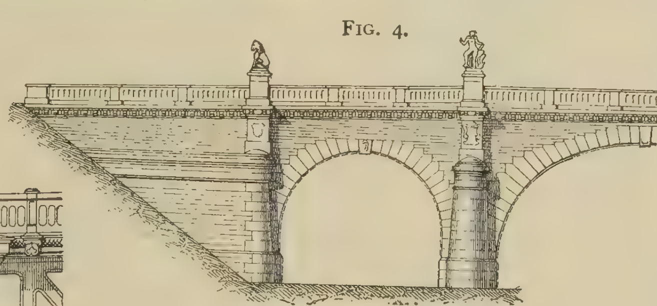
PORTALS OF DANUBE BRIDGES, VIENNA.

FIG. 3.



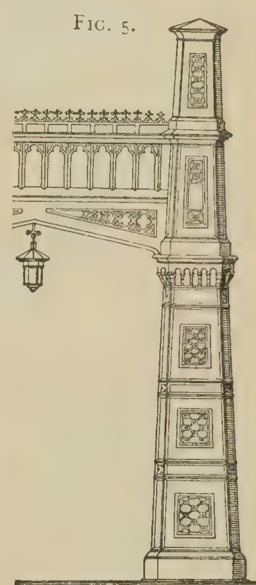
MARGARET BRIDGE, BUDAPEST.

FIG. 4.



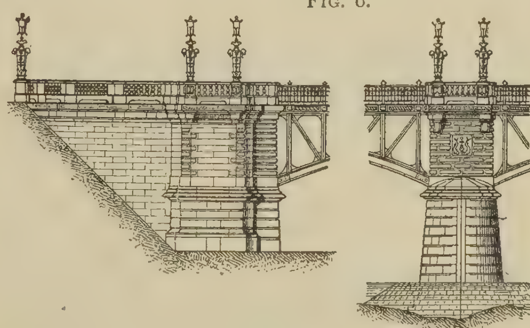
ZOLL-ELBE BRIDGE IN MAGDEBURG.

FIG. 5.



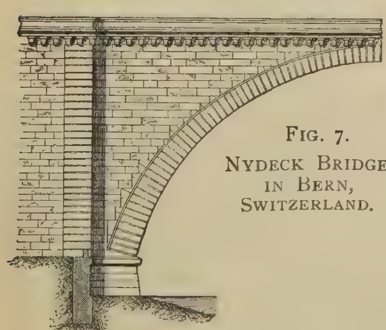
PORTAL OF SUSPENSION BRIDGE IN MUELLHEIM.

FIG. 6.



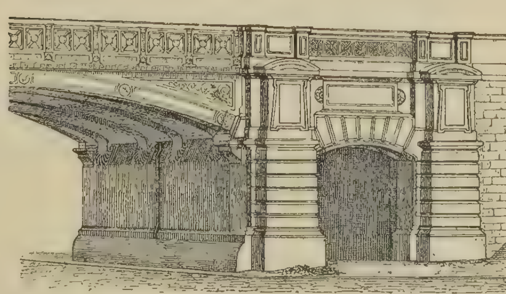
NEUENHEIM BRIDGE, HEIDELBERG

FIG. 7.



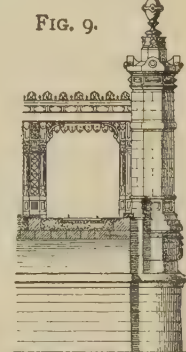
NYDECK BRIDGE IN BERN, SWITZERLAND.

FIG. 8.



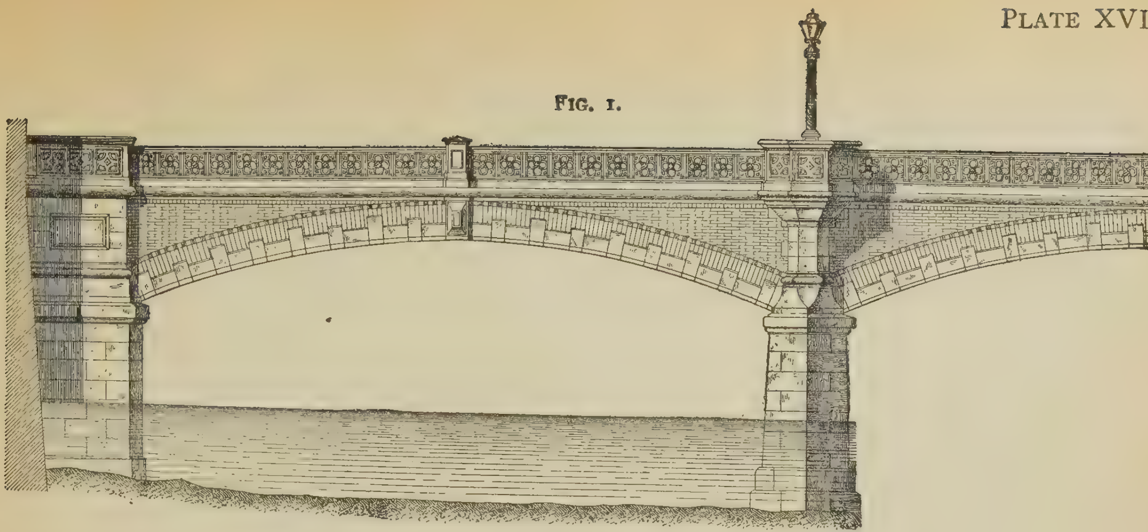
STREET-CROSSING IN STRASSBURG.

FIG. 9.



CAROLA BRIDGE NEAR SCHANDAU.

FIG. 1.



WANDRAHMS BRIDGE IN HAMBURG

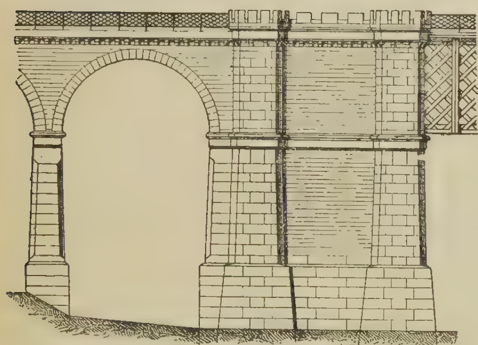
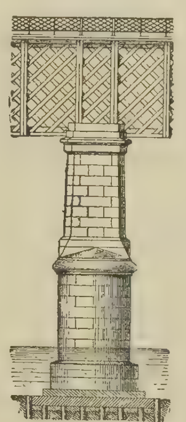
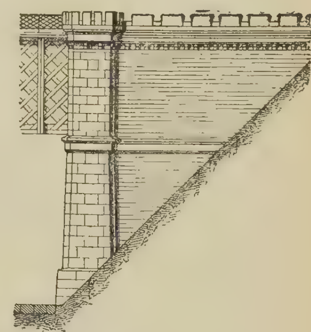


FIG. 2.





RHINE BRIDGE NEAR WALDSHUT, GERMANY

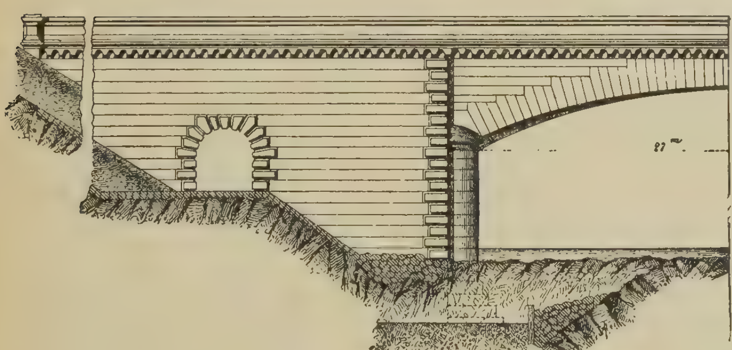
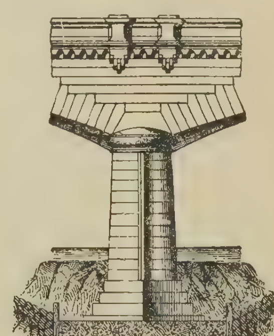
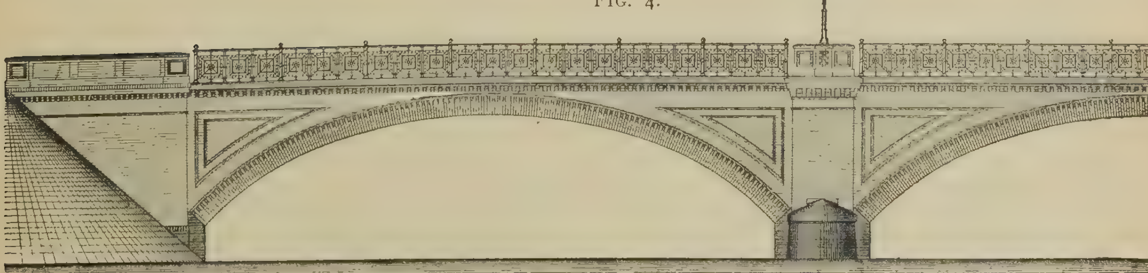


FIG. 3.

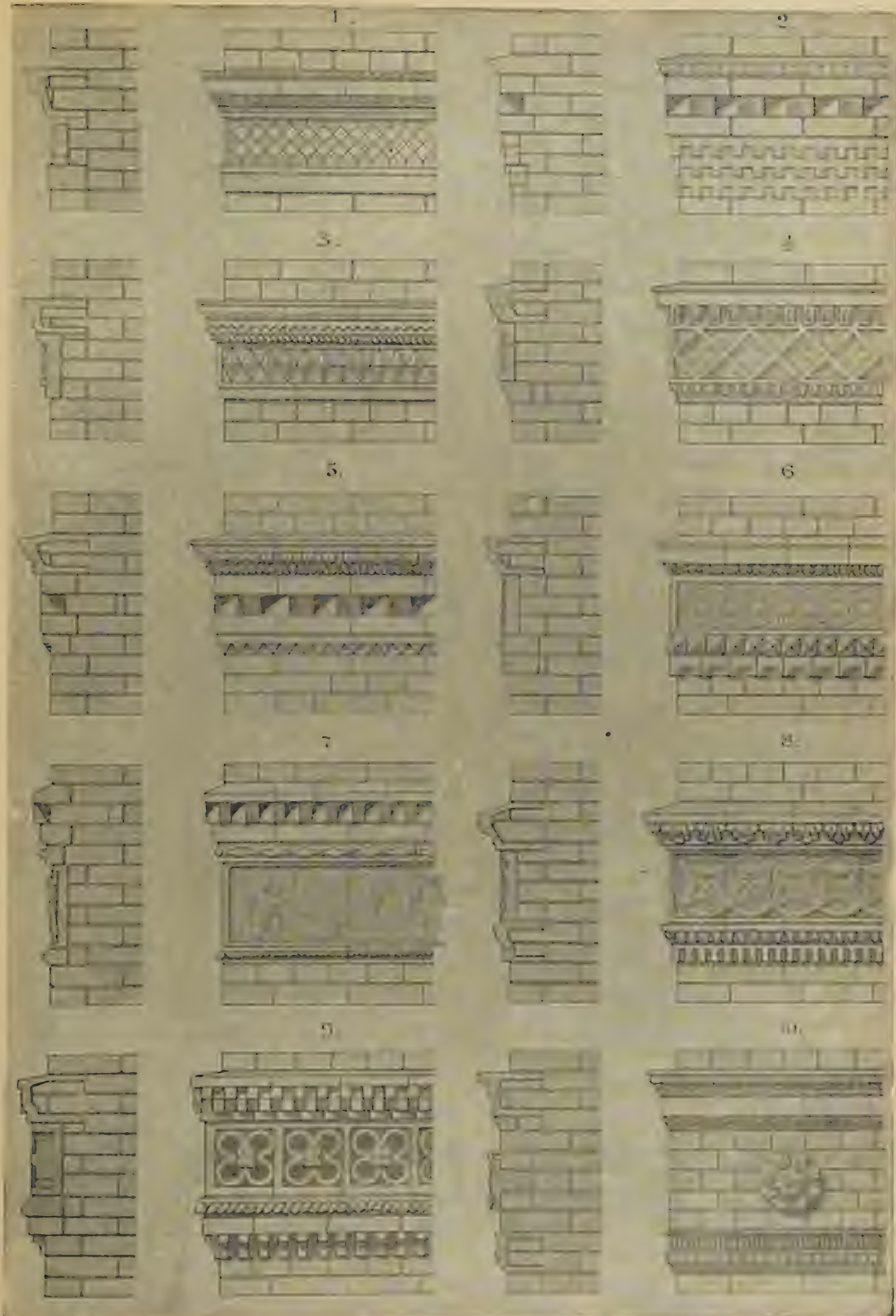


NECKAR BRIDGE NEAR LADENBURG, GERMANY.

FIG. 4.



NEW BRIDGE IN COEPENICK, GERMANY.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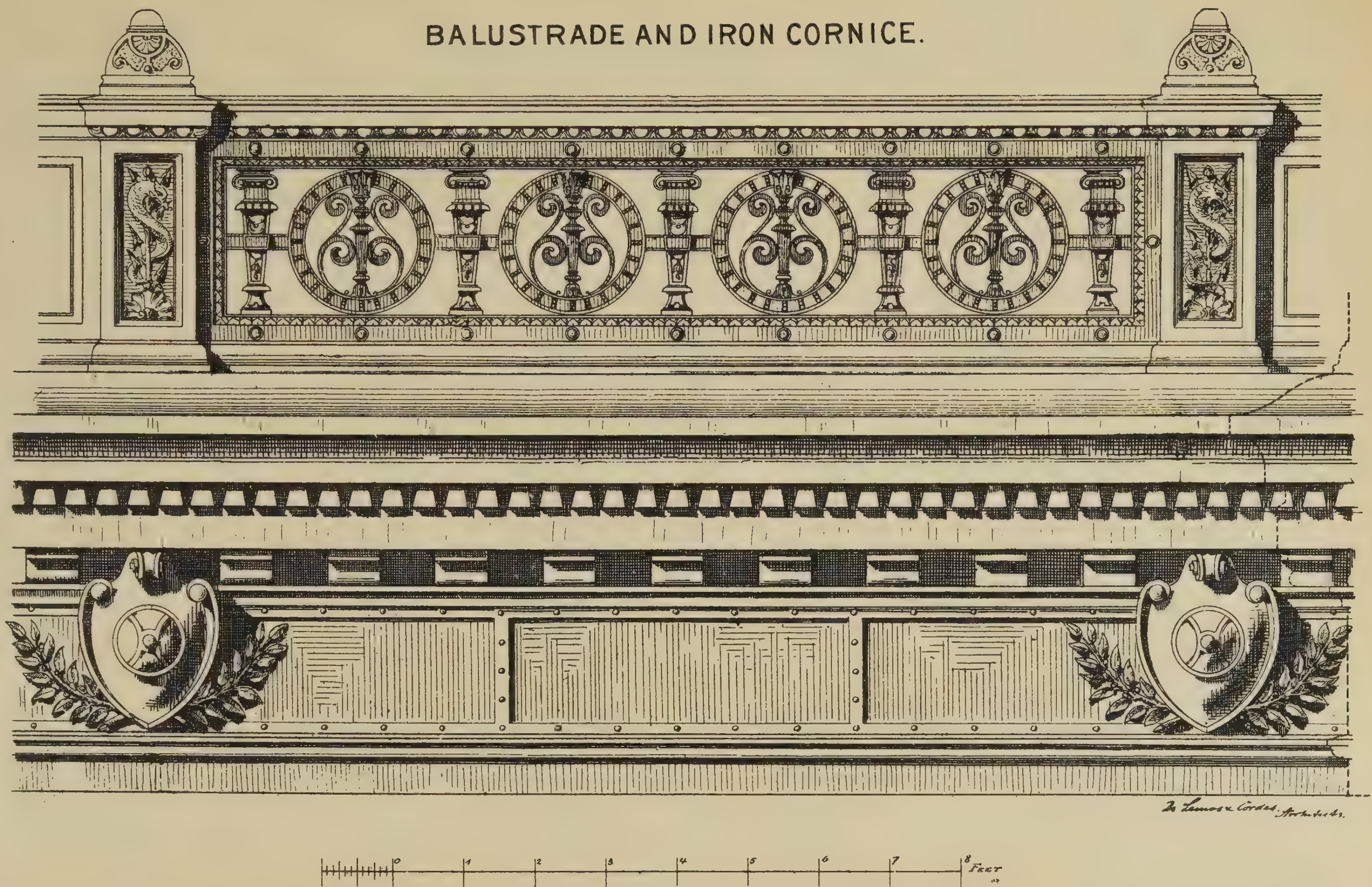
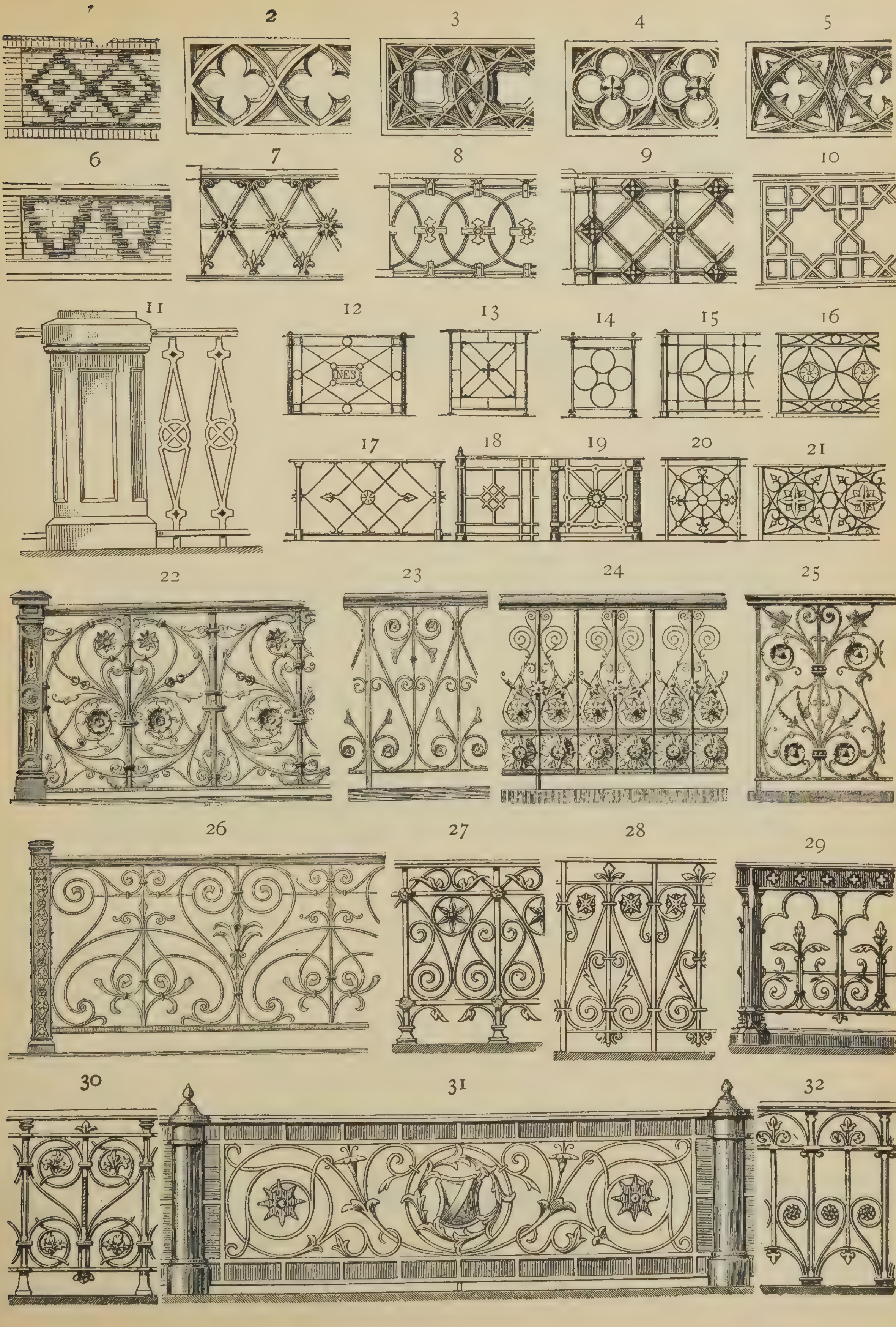
BALUSTRADE AND IRON CORNICE.



RAILINGS. PLATE XX



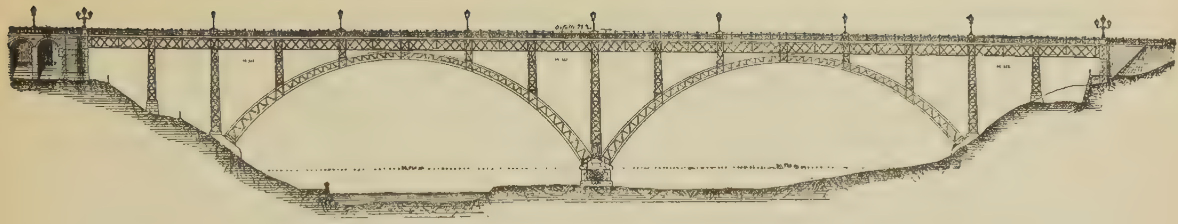


FIG. 1.—KIRCHENFELD BRIDGE, IN BERN, SWITZERLAND.

FIG. 2.—SCHWARZWASSER VIADUCT, SWITZERLAND.

FIG. 3.—NEW YORK AND BROOKLYN SUSPENSION BRIDGE.

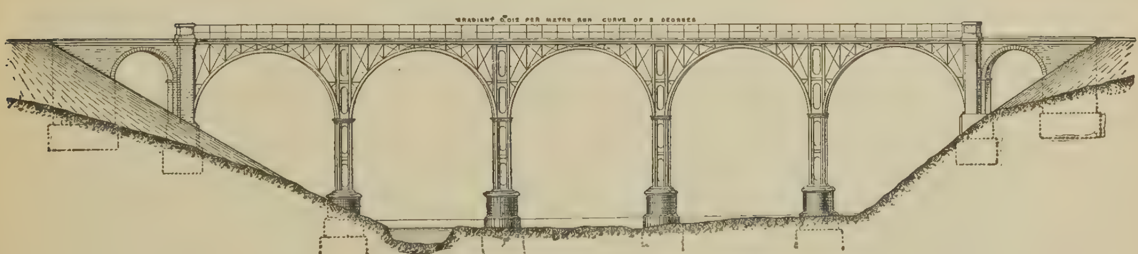


FIG. 4.—VIADUCT OVER THE RIVER RETIRO, BRAZIL.

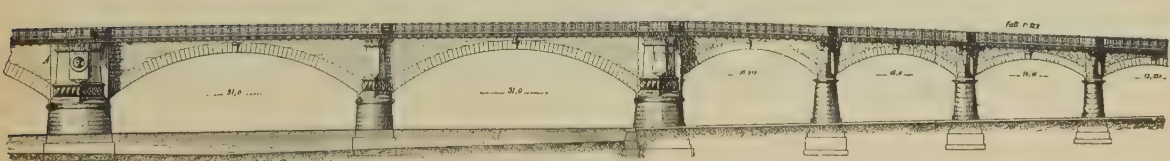


FIG. 5.—ALBERT BRIDGE, IN DRESDEN.

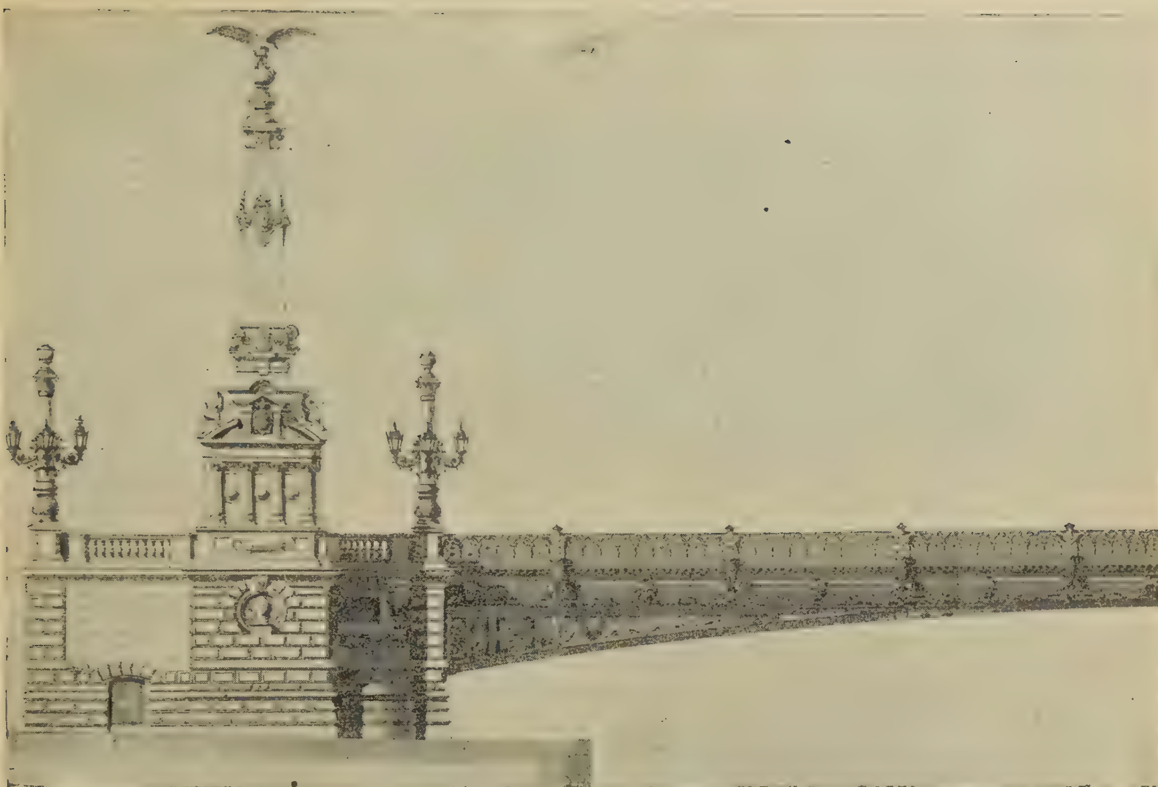


FIG. 1.—STEPHANIE BRIDGE OVER DANUBE CANAL IN VIENNA.

FIG. 2.—METAL BRIDGE OVER INN RIVER IN SALZBURG, TYROL.

FIG. 1.—BRIDGE OVER RHINE RIVER IN CONSTANCE, BADEN.

FIG. 2.—BRIDGE OVER RHINE RIVER IN BASEL, SWITZERLAND.

FIG. 3.—RAILROAD BRIDGE NEAR LOERRACH, BADEN.

FIG. 1.—BRIDGE OVER CANAL, BELLE ISLE PARK, DETROIT.

FIG. 2. HIGHWAY BRIDGE OVER WEHRA RIVER, WEHR, BADEN.

FIG. 2.—BRIDGE OVER THE RHINE, COLOGNE, GERMANY.

FIG. 1.—RAILROAD BRIDGE OVER THE RHINE AT MAYENCE, GERMANY.

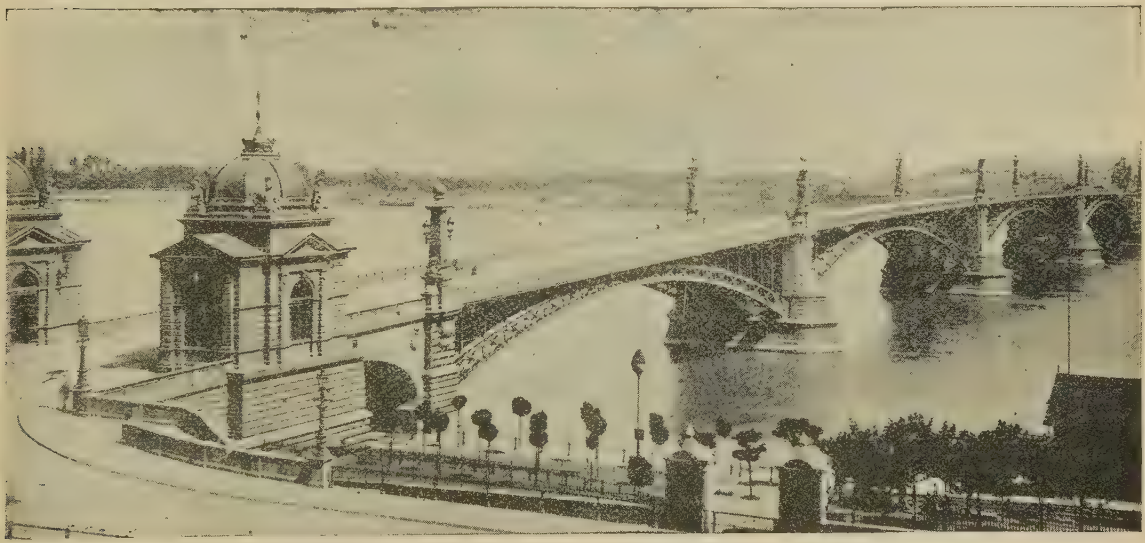


FIG. 2.—HIGHWAY BRIDGE OVER THE RHINE AT MAYENCE.

FIG. 3.—BRIDGE OVER THE RHINE AT COBLENZ.

FIG. 1.—THE RIALTO BRIDGE OVER GRAND CANAL, VENICE.

FIG. 2.—GRAND AVENUE SUSPENSION BRIDGE IN ST. LOUIS, MO.

FIG. 3.—CHESTNUT STREET BRIDGE OVER SCHUYLKILL RIVER, PHILADELPHIA.

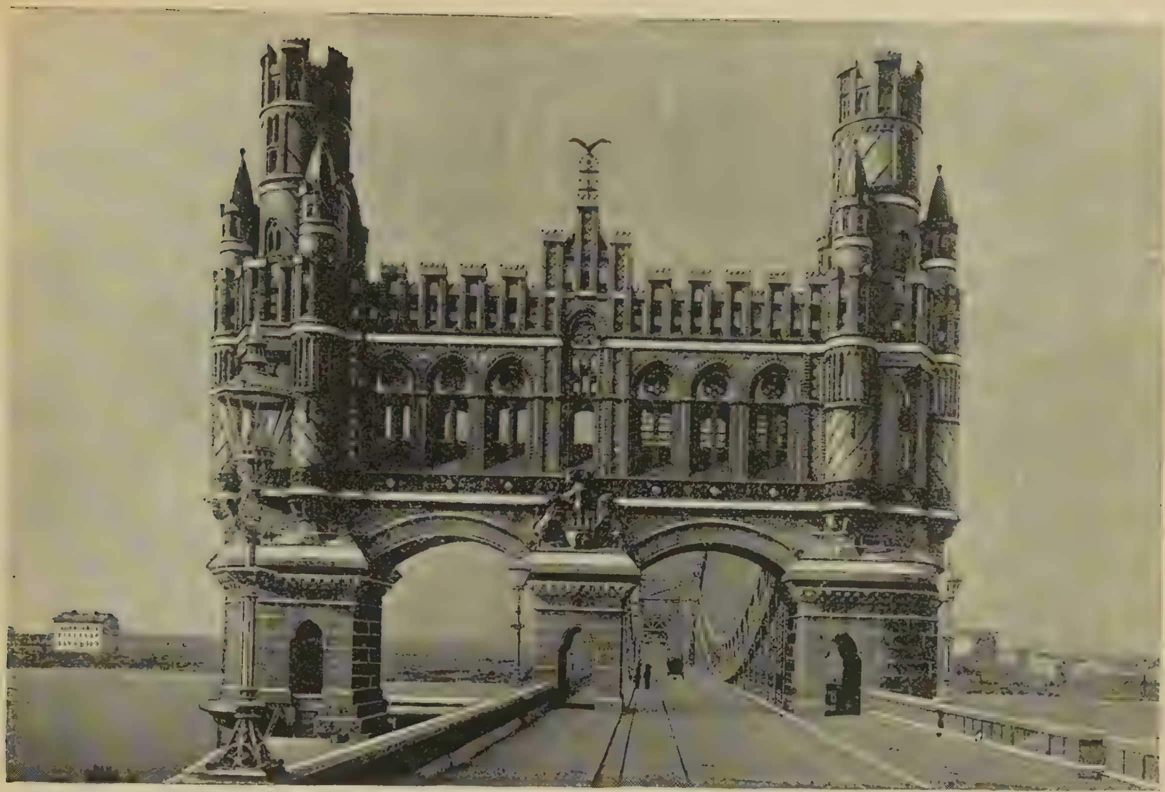


FIG. 1.—PORTAL OF NEW HAMBURG BRIDGE.

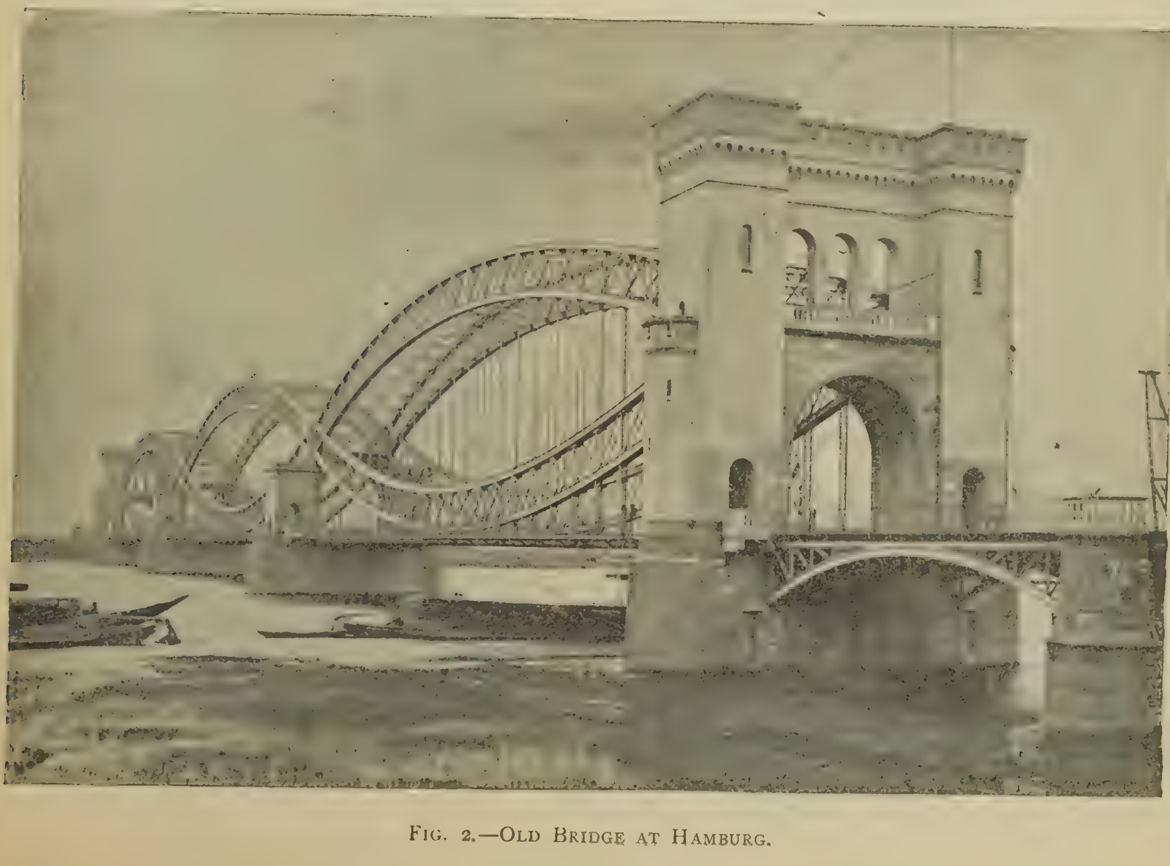


FIG. 2.—OLD BRIDGE AT HAMBURG.

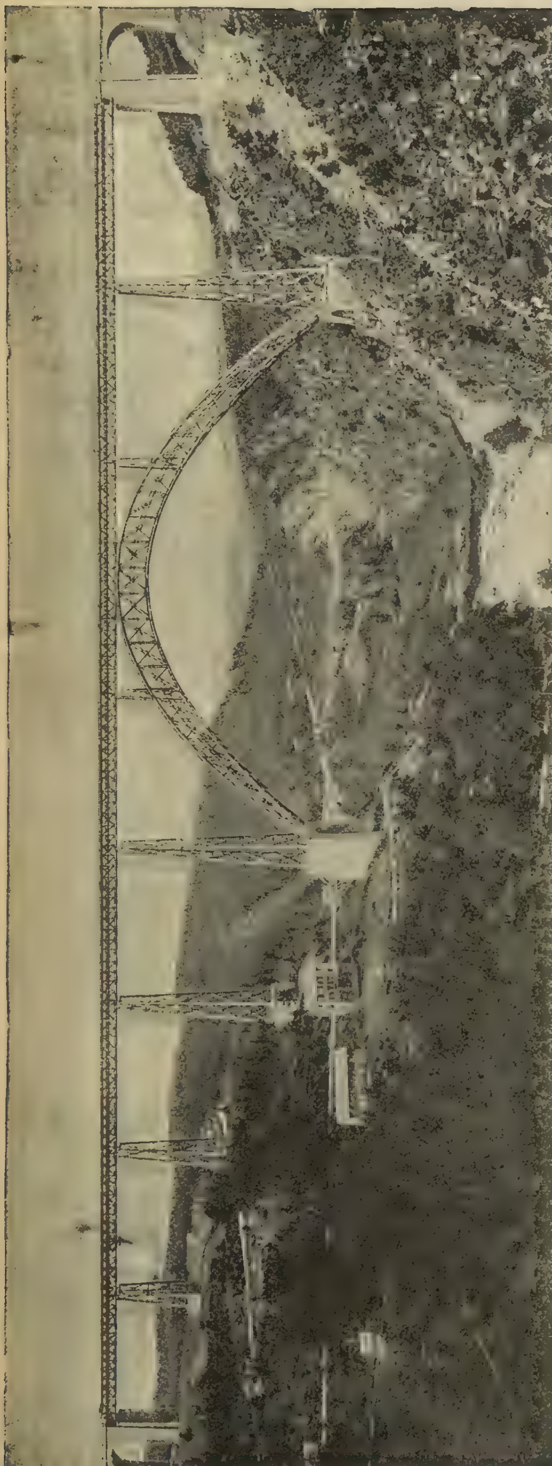


FIG. 1.—THE GARABIT VIADUCT OVER THE TRUYÈRE, CENTRAL FRANCE.

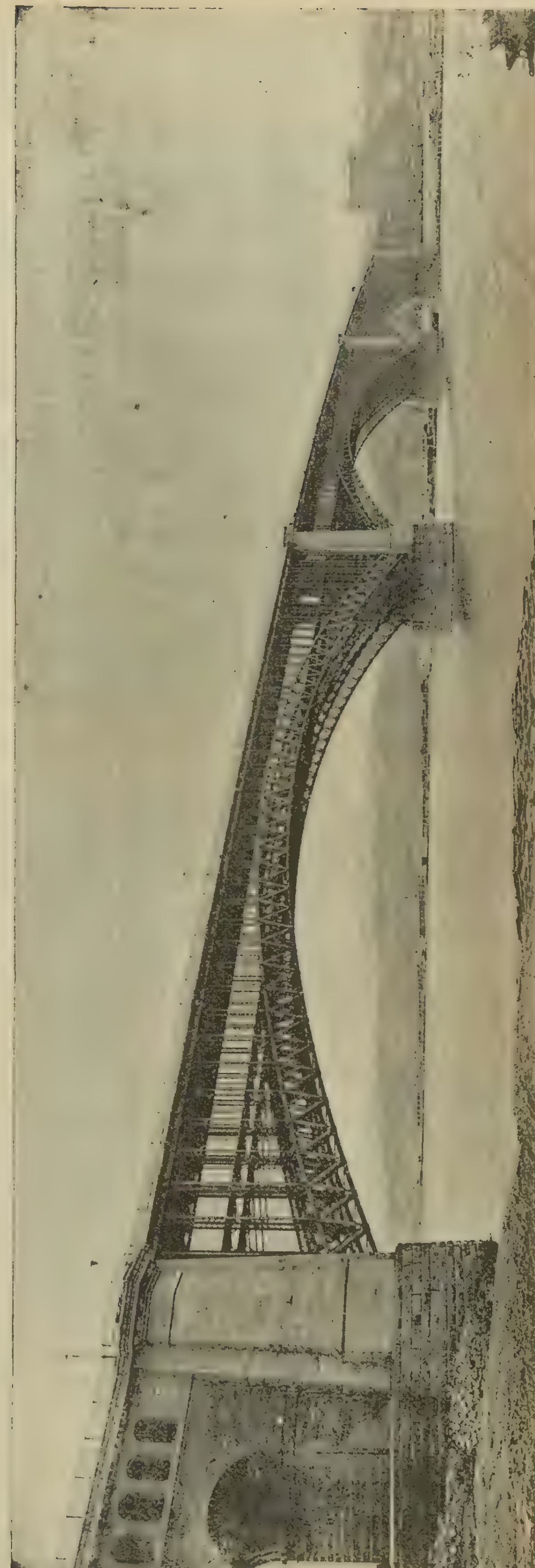


FIG. 2.—THE EADS BRIDGE OVER THE MISSISSIPPI RIVER AT ST. LOUIS.

FIG. 1.—THE TOWER BRIDGE, LONDON.

FIG. 2.—BLACKFRIARS ROAD BRIDGE, LONDON.

Fig. 1.—CHARING CROSS RAILWAY BRIDGE, LONDON.

Fig. 2.—THE VICTORIA BRIDGE, LONDON.

TOWER BRIDGE, LONDON.

BUDA-PESTH SUSPENSION BRIDGE.

CHAPTER XXVII.

STAND-PIPES AND ELEVATED TANKS.

457. **Use of Water Towers.**—Where high ground is not available for service reservoirs as a part of a city water supply system, they are replaced by the storage of a small quantity of water at a high elevation, in a steel tank or "stand-pipe." These serve to relieve the pipe system of excessive "water-rams," and to equalize the irregularities of pumping and using. These reservoirs may be large enough, in small cities and towns, to supply the night service, and also to feed three or four fire streams for an hour, or until the pumps can be started. In the case of a stand-pipe it is only the water in the upper portion which is available for fire service. It is on this account that tanks of larger diameter, elevated upon steel or masonry towers, are always of more value than stand-pipes of the same total cost. The stand-pipes are, however, much more common. From the great number of failures of these in all parts of the country, in the last few years, it is evident that they have been very poorly designed.

These two kinds of structures will be discussed separately.

STAND-PIPES.

458. **Dimensions.**—In order to decide upon the dimensions of a stand-pipe it is necessary to determine the storage capacity required above a given plane. Suppose this plane of least effective elevation of water to be 100 feet. Let it be required to compute the storage capacity needed to serve a given population through the night, when the pumps are operated only in the daytime, counting the night service as somewhat less than one half the day service. Allowing 60 gallons each per day for the entire population, we have, as *the height through which the water will be drawn down at night*, for stand-pipes of different diameters, and for towers of different sizes,

$$\left.\begin{aligned} h &= \frac{3P}{d^2} \text{ for 12 hours' pumping;} \\ h &= \frac{3.5P}{d^2} \text{ " 10 " "} \\ h &= \frac{4P}{d^2} \text{ " 8 " "} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where $h =$ height in feet through which the water is drawn down at night;
$P =$ number of population;
$d =$ diameter of stand-pipe in feet.

Thus for a town of 8000 inhabitants, with a stand-pipe 20 feet in diameter, the water would be lowered 70 feet at night for 10 hours' pumping.

Again, let it be required to find the capacity necessary to supply four fire streams, each discharging 200 gallons per minute (1-inch smooth nozzles 70 feet high, Freeman), for a period of one hour. We now have, as the number of *fire-stream hours* which can be supplied,

$$\left.\begin{array}{l} n = 0.0005hd^2 \text{ (streams of 200 gallons each per minute);} \\ n = 0.0004hd^2 \text{ (streams of 250 gallons each per minute);} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $n =$ number of fire-stream-hours;

$h =$ height through which the water is drawn down;

$d =$ diameter of stand-pipe in feet.

From equations (1) and (2) the capacity of the stand-pipe above a given plane can be found. This determines its height when the diameter has been fixed.

The height should never be more than ten times the diameter, and preferably not more than eight times the diameter.

459. **Character and Thickness of Metal.**—The material of which the plates are made should be mild steel having an ultimate strength of 55,000 to 64,000 lbs. per square inch. In the markets this is known as "sheet steel," or "boiler steel." There is a cheaper grade of steel plates on the market known as "tank steel." This is apt to be hard and brittle and *should never be allowed in any part of the structure.* Most of the failures in stand-pipes can be traced to this one cause.* And yet most of the stand-pipes in this country have been built of this material. (See Art. 468 for specification for material.)

If sufficient precaution is taken to insure obtaining the right kind of material in the plates then the thickness can be determined by the following considerations:

Allowing that the double-riveted vertical joints have an ultimate strength of 60 per cent of the gross section, and counting the tensile strength of the material at 60,000 lbs. per square inch, the actual strength of the vertical joint is 36,000 lbs. per square inch of gross section. Taking a factor of safety of four, we may allow a tensile stress of 9000 lbs. per square inch on the gross section or of 15,000 lbs. per square inch on the net section.

The tensile stress in the shell, per vertical inch, is $T = pr$, where p is the fluid pressure per square inch, and r is the radius of the cylinder in inches. But $p = \frac{62.4h}{144} = 0.434h$, where h is the head in feet. Taking $d = \frac{2r}{12}$ as the diameter in feet, we have, for the thickness of the shell required at any depth,

$$t = \frac{pr}{9000} = 0.0003hd \text{ (nearly)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where t is in inches and h and d are in feet.

The least thickness used should not be less than one fourth of an inch.

Fig. 423 shows graphically the thickness to use for different values of hd, varying by $\frac{1}{16}$ inch. It is not desirable to specify thicknesses varying by thirty-seconds of an inch. This diagram makes some allowance for imperfect workmanship and a poorer grade of material in the very thick plates. It is not possible to rivet up thick plates without considerable internal stress.

* The writer examined and tested the broken sheets from a stand-pipe which burst under a static pressure which produced a tensile stress of only one fifth of the ultimate strength of the material and found them so brittle that they could not be straightened in the rolls without breaking.

In Fig. 423, the thicknesses are shown to actual scale. It is not wise to try to use plates thicker than one inch. If this does not give the capacity required it would be better to duplicate the plant than to try to use thicker plates.

460. Wind Moment and Anchorage.—The overturning moment due to wind pressure may be taken as 40 lbs. per square foot on one half the diametral area into one half the height if a stand-pipe, or into the height of the centre of pressure if an elevated tank.

The moment of stability is the weight of the empty tank into its radius if there are no brackets, or into the perpendicular distance to a line joining two adjacent anchorage points when brakets are employed. If this is not equal to the overturning moment, the remainder must be provided for by anchorage rods extending into the masonry.

In Fig. 424, let $AB \ldots F$ represent the anchorages of the brackets of a stand-pipe whose centre is O. If we assume the wind to be in the direction OH, then the brackets A and D act with arms one half those of B and C. The pull on the rods at A and D will also be one half that at B and C, hence the moments of resistance of the brackets A and D will be one fourth those at B and C.

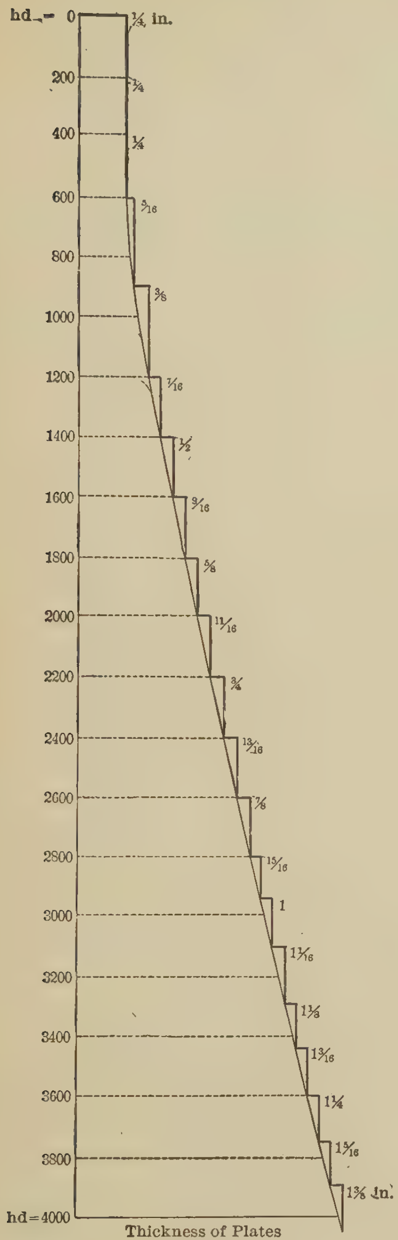


FIG. 423.

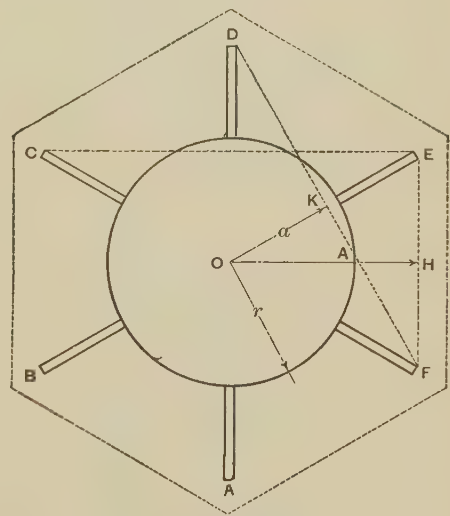


FIG. 424.

Taking moments about the line EF,

Let M = overturning moment;

M_0 = moment of resistance of dead weight = $W \times$ dist. $OH = WA$;

P = pull on anchorage rods at B and C; then we have

$$M = WA + 4PA + PA \quad \text{or} \quad P = \frac{M - WA}{5A}. \qquad (4)$$

If the wind had been in the direction BE, then the anchor rods at B would have acted

with the arm BE, and those at A and C with the arms AF and CD. Let the distance $OK = a$; then $CD = AF = 2a$, and $BE = 4a$.

The amount of the lift at A and C is to that at B as their relative distances from the diametral line mn or as a is to $2a$.

Let P_1 = pull on anchorage at B;
P_2 = " " " " C.

Then we have

$$M = 2Wa + 4P_1a + 4P_2a. \quad (5)$$

But since $P_2 = \frac{P_1}{2}$, we obtain

$$P_1 = \frac{M - 2Wa}{6a}. \quad (6)$$

Now, $\frac{a}{A} = \frac{500}{866} = 0.58$, or $a = 0.58A$.

Therefore

$$P_1 = \frac{M - 1.16WA}{3.48A}. \quad (7)$$

By comparing this with eq. (4) it will be seen that P_1 is greater than P when the overturning moment is greater than about twice the moment of stability WA, otherwise P is the greater.

Having found P or P_1, whichever is the greater, this fixes the depth of the masonry foundation, the size of the anchor rods, and the strength of the brackets. The masonry must be deep enough to supply the necessary dead weight; or that part of it which can be supposed to rest on the anchor plate, or to be lifted by this plate, must be equal to P. The weight of a cubic foot of masonry may be taken at 150 lbs.

The size of the anchor rods must be taken as the size at the base of the screw threads if they are not upset at the ends. The working stress on these may be taken at 15,000 lbs. per square inch of net section.

461. The Anchorage Brackets.—It is a very common practice, in anchoring down pedestals, to attach the anchor bolts to the outstanding legs of angle irons which are riveted to the feet of the posts. This is a very poor attachment, as at most two or three rivets are required to take the whole pull of the anchor bolts. A better arrangement for a bracket is that shown in Fig. 425.

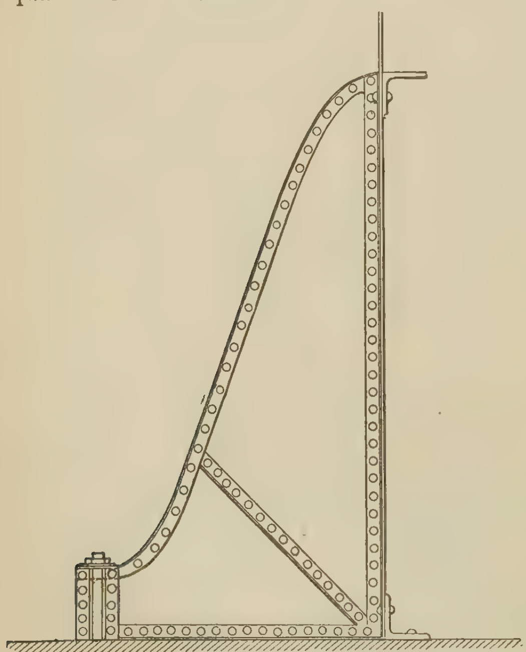


FIG. 425.

The bracket is 15 feet high and has a base of 8 feet to anchor rods. It is curved slightly for appearance, and is composed of a solid plate with double angles on all three sides. The pull from the anchor bolts is transmitted through a sufficient number of rivets to the web of the bracket, and thence by shear and bending moment to the side of the stand-pipe, which is reinforced on the inside, at the top of the bracket, by a heavy angle iron. This takes the pull or thrust coming from the outer flange of the bracket which is concentrated at the upper end.*

* This form of bracket was used on the stand-pipe at Jefferson City, Mo., which is shown in Fig. 430.

For large brackets stiffening angles should be placed diagonally across the web, as shown in Fig. 425.

In case no brackets are used the anchor bolts should be attached directly to the bottom ring of the stand-pipe by means of long angle irons, as shown in Fig. 426.

The anchor bolts must here be kept as close to the side of the stand-pipe as possible.

The brackets are preferable, however, as they serve also to distribute the dead weight over a larger area of foundation. Furthermore, they add greatly to the general appearance of the structure. (See Fig. 430.)

The use of cast-iron lugs, of short vertical dimensions, riveted to the sides of the stand-pipe, to which the anchor bolts are attached, cannot be too severely condemned.

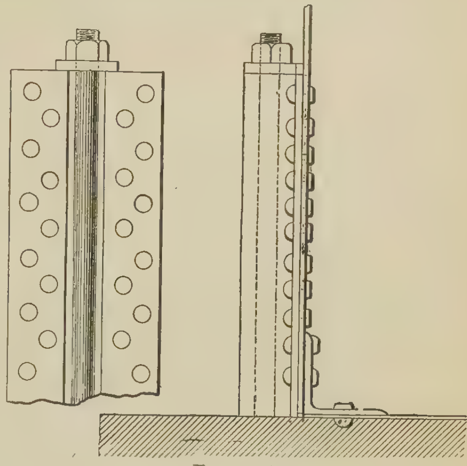


FIG. 426.

462. Details of Construction.—This not being a work on foundations or on water-works construction, the questions pertaining to character and sufficiency of the foundation; inlet and outlet pipe; cut-off valves and their operation by hand or by electricity from the pumping station; float indicators, electric and otherwise; man-holes, stairway, flushing-out pipe, etc., are here omitted; also all discussion of outer masonry structural housing and the design of the same, and whether or not it is necessary to enclose the stand-pipe in any manner. It is assumed, however, in this chapter that the tower is not enclosed.

Riveting.—The subject of riveting is discussed in Chapter XVIII. Although lap joints are almost universal in these structures, the difficulty of making a water-tight joint where three plates come together makes some other arrangement desirable. The strength of a lap

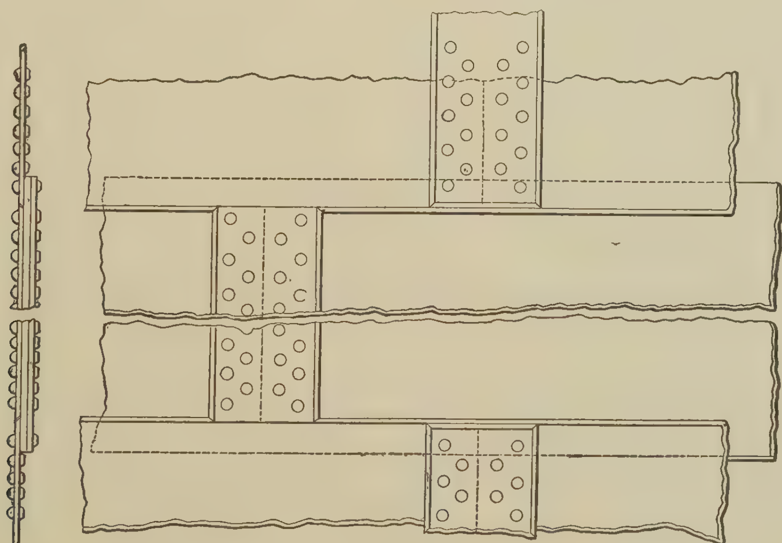


FIG. 427.

joint also is much less than that of a double-strap butt joint. Since the horizontal joints are not stressed by the water pressure, they may be lapped and single riveted. The vertical joints should be made with double butt straps, as shown in Fig. 427. Here the vertical joints have double butt straps with bevelled calking edges on all four sides of the outer strap. The inner strap is not calked. The straps should be not less than $\frac{1}{4}$ in. thick, nor thinner than one-half the thickness of the plate.

The proportions of diameter and pitch of rivets to thickness of plate are given in the table below, which has been compiled from the Watertown Arsenal experiments on riveted joints.

RIVETED JOINTS FOR STAND-PIPES.

Kind of Joint on Vertical Seams.	Thickness of Plate.	Diameter of Rivet.	Pitch of Rivets Centre to Centre.	Distance between Pitch-lines.	Distance of Pitch-line from Edge of Plate.	Percentage of Total Strength of Plate developed.	Holes.
	inch	inch	inch	inch	inch		
Single-riveted lap....	$\frac{1}{4}$	$\frac{5}{8}$	$1\frac{5}{8}$	..	1	50	Punched
" " "	$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{3}{4}$	..	$1\frac{1}{8}$		
Double- " " ...	$\frac{5}{16}$	$\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	60	
" " "	$\frac{3}{8}$	$\frac{3}{4}$	$2\frac{5}{8}$	$2\frac{1}{8}$	$1\frac{1}{4}$		
" " "	$\frac{7}{16}$	$\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{3}{8}$		
" " "	$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{7}{8}$	$2\frac{3}{8}$	$1\frac{1}{2}$		
" " butt ...	$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{7}{8}$	$2\frac{3}{8}$	$1\frac{5}{8}$		
" " "	$\frac{5}{8}$	$\frac{7}{8}$	$2\frac{7}{8}$	$2\frac{3}{8}$	$1\frac{5}{8}$		
" " "	$\frac{11}{16}$	1	$2\frac{7}{8}$	$2\frac{3}{8}$	$1\frac{3}{4}$		
" " "	$\frac{3}{4}$	1	3	$2\frac{1}{2}$	$1\frac{3}{4}$	70	
" " "	$\frac{13}{16}$	1	3	$2\frac{1}{2}$	$1\frac{7}{8}$		Drilled
" " "	$\frac{7}{8}$	$1\frac{1}{8}$	3	$2\frac{1}{2}$	2		
" " " ...	$\frac{15}{16}$	$1\frac{1}{8}$	3	$2\frac{1}{2}$	2		
" " "	1	$1\frac{1}{4}$	$3\frac{1}{8}$	$2\frac{5}{8}$	$2\frac{1}{8}$		
Triple- " "	1	$1\frac{1}{8}$	4	3	$2\frac{1}{8}$	75	

It will be observed that the butt joint is much more efficient than the lap joint, even when both are double riveted. When the plates are more than $\frac{3}{4}$ in. thick they should be drilled and not punched, as the cold flowing of the metal in punching thick plates injures it for a considerable distance around the holes.

Plates thicker than 1 in. should not be used if it can be avoided. Since all the riveting is done by hand in the field, the rivets should be of iron.

Calking.—All calking edges should be bevelled on a planer, and the calking should always be done with a round-nosed tool, as shown in Fig. 428. If a square-edged tool is used it creases the inner plate, and if this should prove to be of brittle steel it might cause a failure along this line. On all plates over $\frac{1}{2}$ in. thick the vertical joints should be made by double butt straps, as shown in Fig. 427. If single straps are used the joint is no stronger than a lap joint, which it really is. When these are not used, one of the plates, where three plates meet, must be scarfed down to a feather edge, and this should be done by heating the corner of the plate before putting under the hammer. This heating and cooling one corner of a steel plate introduces unknown internal stresses into the plate, especially if it is of a high grade of steel, which can only be removed by annealing the plate. The butt straps avoid this source of weakness also.

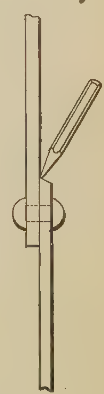


FIG. 428.

Bottom.—The bottom is attached to the sides by a heavy interior angle-iron, as shown in Fig. 425. In the case of high stand-pipes, or those over 120 feet high, there should be two of these angles to better distribute the dead weight of the sides on the masonry, as shown in Fig. 429. The bottom is usually made of $\frac{3}{8}$-in. plates, and this is attached to the first ring of the sides and thoroughly calked, and then lowered to place on the foundation. This should have been carefully levelled up by using an engineer's level, and then covered over with about an inch of soft Portland-cement mortar. The supports are then removed and the lower ring, with the bottom attached, is carefully lowered upon its cement bed. For the larger sizes the bottom must be held up at one or more points on the interior to prevent its sagging

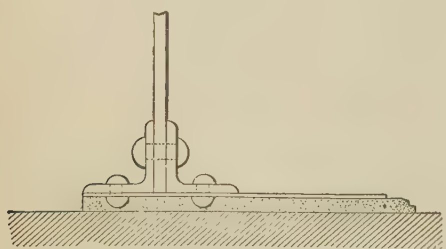


FIG. 429.

too much. This is done by attaching permanent eyes to the bottom, by riveting and fastening these to timbers across the top edge of the vertical sides. If the bottom were allowed to sag it would destroy the calking.

To the bottom is usually attached the inlet and outlet pipe, which is commonly one and the same; but the details of this will not be discussed here.

Man-holes are often placed in the lower side ring, but this is a source of weakness. It is better to arrange a blow-out pipe, with a gate-valve upon it, with many mouths uniformly distributed over the bottom on the inside, all opening downward, and properly connected with the blow-out pipe. This should be not less than ten inches in diameter for a stand-pipe fifteen feet or over in diameter, and the bottom is cleaned by simply opening the valve. The rush of water out of these various mouths cleanses the entire bottom. Even though perfectly pure ground-water is pumped into the stand-pipe, a considerable amount of sediment will collect at the bottom, which should be cleaned out occasionally. By the means here described it is not necessary to empty the stand-pipe to clean it.

Top Angle.—To prevent the top from collapsing from the force of the wind, a strong angle iron, not less than $4'' \times 4''$, should be riveted to the top, either inside or outside.

It is not wise to cover a stand-pipe where the winters are severe. A thick coating of ice forms upon the sides, which may become loosened as warm weather approaches, and if this is done suddenly, its buoyancy throws it with great force upwards. If at such a time the stand-pipe happens to be nearly full of water, this ice column may be forced many feet above the top. If a roof were provided, it would probably be torn off by such an action.

463. Ornamentation.—There is probably no homelier engineering device than a plain steel stand-pipe. They are, however, sometimes made more offensive by ill-advised attempts at ornamentation There seems to be but one class of top ornaments to a stand-pipe which are appropriate. This is a combination of a cornice and a sort of top fencing or open fret-work. The cornice is not full-surfaced, but composed of plate disks, placed near together, which in profile give the desired curves. Fig. 430 is a view of the stand-pipe at Jefferson City, Mo.,* which has such an ornamentation as here described. The photographic view here shown was taken from a point too near to show the top to the best advantage. It is considered an ornament to the city by the citizens, and yet it cost but a few hundred

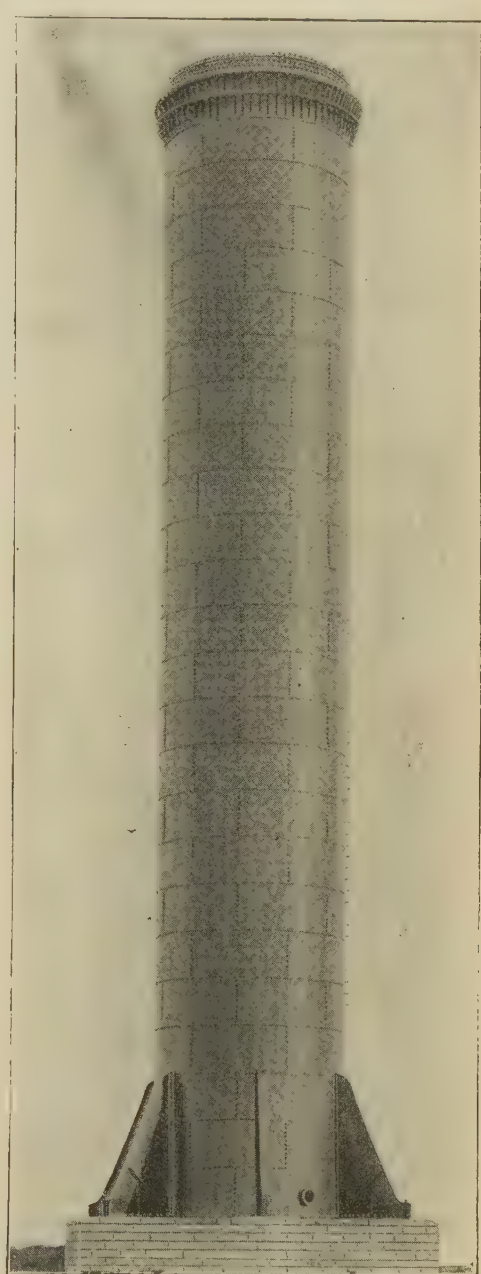


FIG. 430.

* Designed by Prof. Johnson, and erected in 1888. It stands on a prominent hill opposite the Capitol, and is visible to its base from most parts of the city. Its dimensions are 20 ft. $\times$ 125 ft. The top ornamental work is very inadequately shown in this cut which is a half-tone from a retouched photograph.

dollars to transform this from an eyesore into a pleasing monument. Since these structures must of necessity constantly meet the public gaze, it is little short of a crime to make them standing embodiments of innate ugliness.

ELEVATED TANKS.

464. General Design.—Elevated tanks should always be made of steel plates, and should rest on a wrought-iron or steel trestle-tower. Wooden tanks are not sufficiently permanent, and masonry towers over forty feet high are unreliable. It is practically impossible to make a tall masonry (brick or stone) tower act as one monolithic mass. Its strength is always problematic, and depends so largely upon the character of the workmanship that the designer can never feel assured of its safety under the heavy loads which a large water storage-tank puts upon it.

When we use a steel tank upon a trestle-tower, it is difficult to support a flat bottom and to distribute this load horizontally to the posts. It is much cheaper and more scientific to make this bottom either curved or conical.* It is more scientific to make them curved, and the most practical curve is the circle. The bottom should therefore be spherical, the segment used being somewhat less than a hemisphere. Such a bottom can be readily shaped by dishing the plates to a uniform curve under a steam-hammer. This increases the cost somewhat, but such a bottom is much cheaper than a flat bottom with its accompanying floor system.

465. Design of the Tank.—*The thickness of plates* to use in the sides is given in Fig. 423. The thickness of the plates in a hemispherical bottom need be but little more than one half that of the bottom ring on the side. The computed stress on the bottom plate is only one half that in the sides of a cylinder subjected to the same internal stress, or it is $\frac{pr}{2}$ per linear inch, where p is the internal pressure per square inch, and r is the radius of the bottom in inches. If a segmental bottom is used less than a full hemisphere, then r is larger here than for the shell, and the bottom plates must be correspondingly heavier. Allowing the same unit stress as was used for the sides, we have, *for thickness of bottom plates*,

$$t = 0.0003hr, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where t = thickness of bottom plates in inches;
h = head of water in feet;
r = radius of bottom in feet.

In any case the bottom should not be less than $\frac{5}{16}$ in. thick.

Riveting.—The side riveting is subject to the same rules as given for stand-pipes. The bottom sheets should all be single riveted, since the stress on these joints is only about one half what it is in the side plates. The attachment of the bottom to the sides may be by a single or double row of rivets as preferred. The only distortion of the bottom which can occur, due to a fluctuation of height of water in the tank, is a vertical deflection of the whole bottom. The tendency for the diametral curve to change its shape, on account of its not being at all times the curve of equilibrium for that particular pressure, is fully resisted by the circumferential stress in the bottom plates. Even though the bottom were made conical, these plates would not become curved on a diametral section, the circumferential stress holding them to true conical surfaces.

Concentration of the Load upon the Posts.—Fig. 433 illustrates the method of concentrating the load upon the columns. The bottom is here shown as segmental, and because this produces an inward pull upon the shell at its attachment, two strong angle irons are placed here to resist this compressive stress.

* In a thesis study by J. M. Raikes Univ. of Mich., 1896, the relative weights of steel in a tank 40 ft. diam. by 40 ft. high were for flat bottom, with beams and girders, 120,000 lbs.; conical bottom and circular girder, 94,200 lbs.; spherical bottom and circular girder, 77,100 lbs. (*The Technic*, 1896.)

Let $P =$ the compression produced in the shell at this circle by this action, in pounds;
$d =$ diameter of tank, in inches;
$W =$ total weight of the water in the tank plus the weight of the bottom itself;
$i =$ angle of bottom with the vertical at the attachment circle.

Then we have

$$P = \frac{W}{\pi d} \cdot \tan i \cdot \frac{d}{2} = 0.159 W \tan i. \qquad (9)$$

This is the total compression in the two angles at A. The lower sheet of the shell is continu-

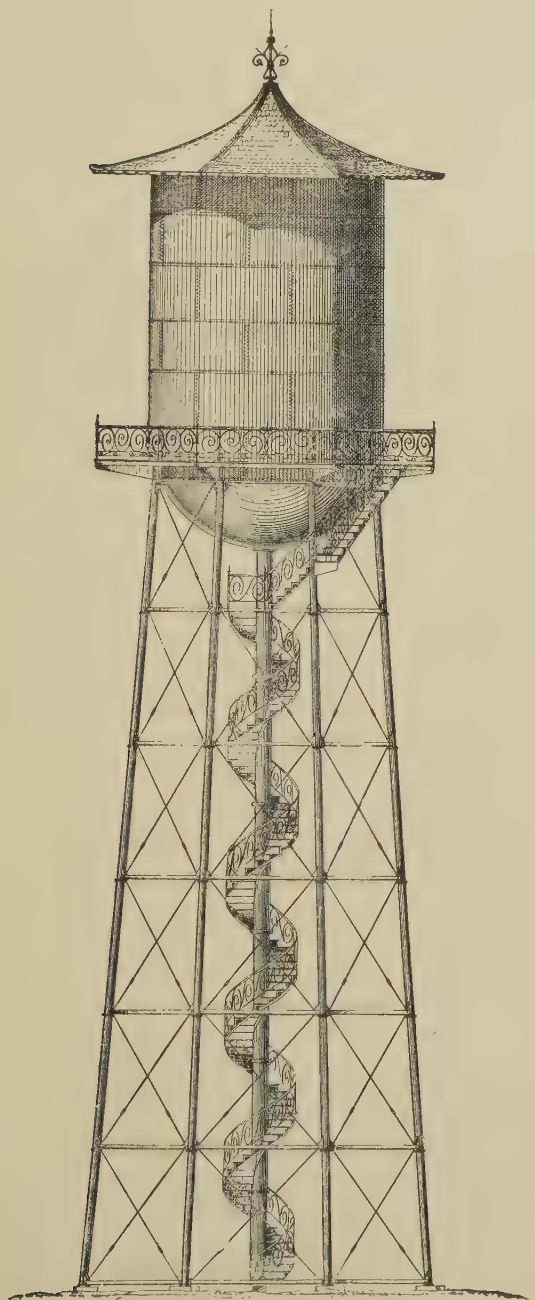


FIG. 431.

FIG. 432.

ous through the attachment joint at A to the bearings on the posts at B. Between A and B this sheet is stiffened by four angles, as shown in elevation and plan in Fig. 433. Between these angles and the top of the post a plate is inserted which reaches over to the curved bottom and is there attached as shown. The top of the column is covered by another plate which is in turn supported by the ends of the column members and angle irons in some suitable manner, so as to distribute the load equally over the members composing the column.

The Roof.—These tanks are not so high as to cause the ice to interfere with a roof, and hence they should be covered. The roof can also be made to add greatly to the appearance

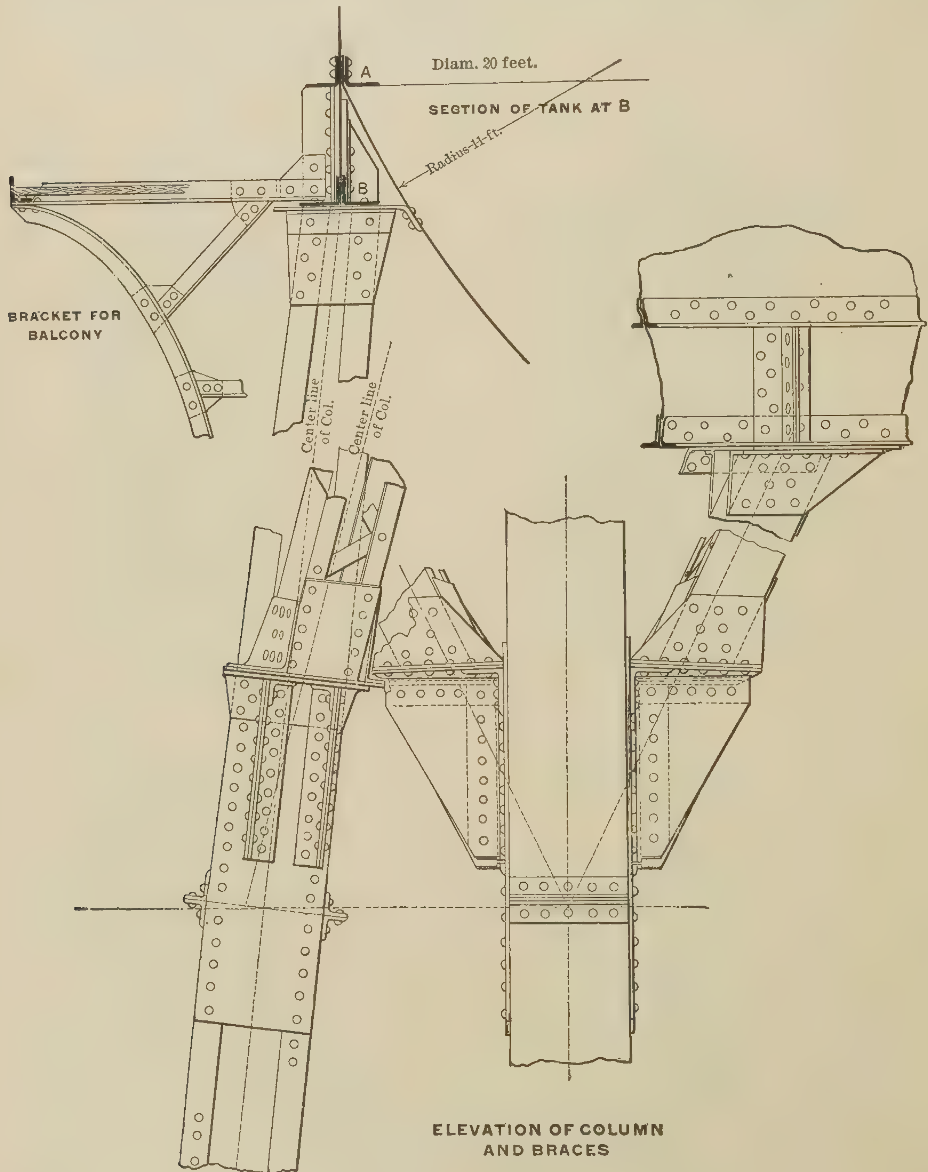


Fig. 433.

of the tank, as shown by Figs. 431 and 432.* A curved pagoda roof will always make a better appearance than a conical or pyramidal form.

* Fig. 431 is from a design prepared by Johnson and Flad, St. Louis, in 1890, for the western suburbs of that city. The tank shown in Fig. 432 was designed by Mr. Edw. Flad, C.E., M. Am. Soc. C.E., for Laredo, Tex.

Relative Dimensions.—The more nearly the tank as a whole approaches the spherical form the cheaper it will be in proportion to its volume. It makes a better appearance, however, if its height is about twice its diameter, as shown in Fig. 432. Here again the appearance should be a prominent and determining factor in preparing the design.

466. The Trestle Tower.—For economy the trestle legs should be few in number. A heavy post, or column, is relatively more economical than a light one of the same length, because $\frac{l}{r}$ is less for the large post. The material also should be medium steel, or steel of from 62,000 to 70,000 lbs. tensile strength. This material is now so cheap that there is no longer any object in using cast-iron in columns for any purpose. The forms may be either Z bars or channels, whichever is found to be best adapted to the particular details used. Probably two channels, turned with the plane sides out, and latticed on two sides, with tie-plates at the joints, as shown in Figs. 433 and 434, will be found to serve every purpose.

The smallest number of posts which is practicable is four. But four points of support under the tank are not sufficient. Mr. Edw. Flad, C.E., has probably made the best solution of this problem, and it will be here described. He uses four posts, as shown in Fig. 432, and from each post extend two braces at top, thus giving twelve points of support under the tank. This is an excellent solution, and causes the structure to present a very satisfactory appearance, but it leads to somewhat complicated details where these brackets meet the posts and the tank. The details of these connections are shown in Fig. 433. The details of the connections for struts and ties are shown in Fig. 434. In this figure are also shown the details for the bases of the posts, the anchorages, and the ladder, which is attached to the outer side of one of the posts. These details were for a tank 40 feet high and 20 feet in diameter, having a capacity of 85,000 gallons, and set on a trestle 80 feet high.

The roof has a wooden framing, set on iron brackets, as shown in Fig. 432. An air space 24 inches high was left between the top of the tank and the sheathing boards, which enables the tank to be entered.

The inlet pipe is usually provided with a stuffing-box to prevent any excessive stress coming on the pipe itself.

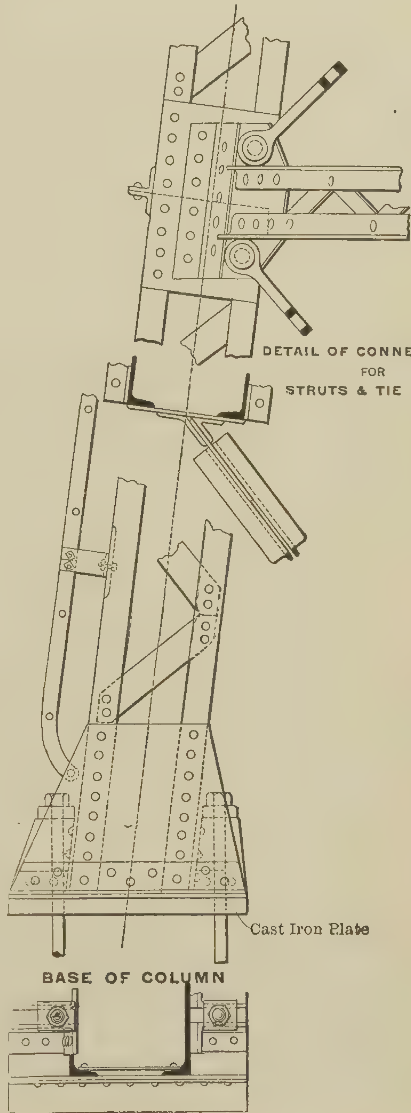


FIG. 434.

467. Relative Cost of Stand-pipes and Elevator Tanks.—When the great saving in the foundation is taken into account, the elevated steel tank will, in nearly all cases, cost less than a stand-pipe of the same efficiency, and it is usually more ornamental, or at least less offensive. The foundation of a stand-pipe will cost about four times that of an elevated tank of equal capacity for service. If we may say that only the water above the height of 75 feet

from the ground is valuable to the city, then an elevated tank should always be built in place of a stand-pipe. When large quantities of water are to be stored above the height of fifty feet, then the elevated tank is more economical for equal quantities, and the greater the height at which the storage is required the greater is the economy in the tank design. The great objection to the use of tanks has been in the flat floors with which they have usually been provided. That objection is now removed by the designs for curved bottoms here presented. Even conical bottoms may be used,* but they are not so scientific or so pleasing in appearance as the spherical forms. See Fig. 434*a*.

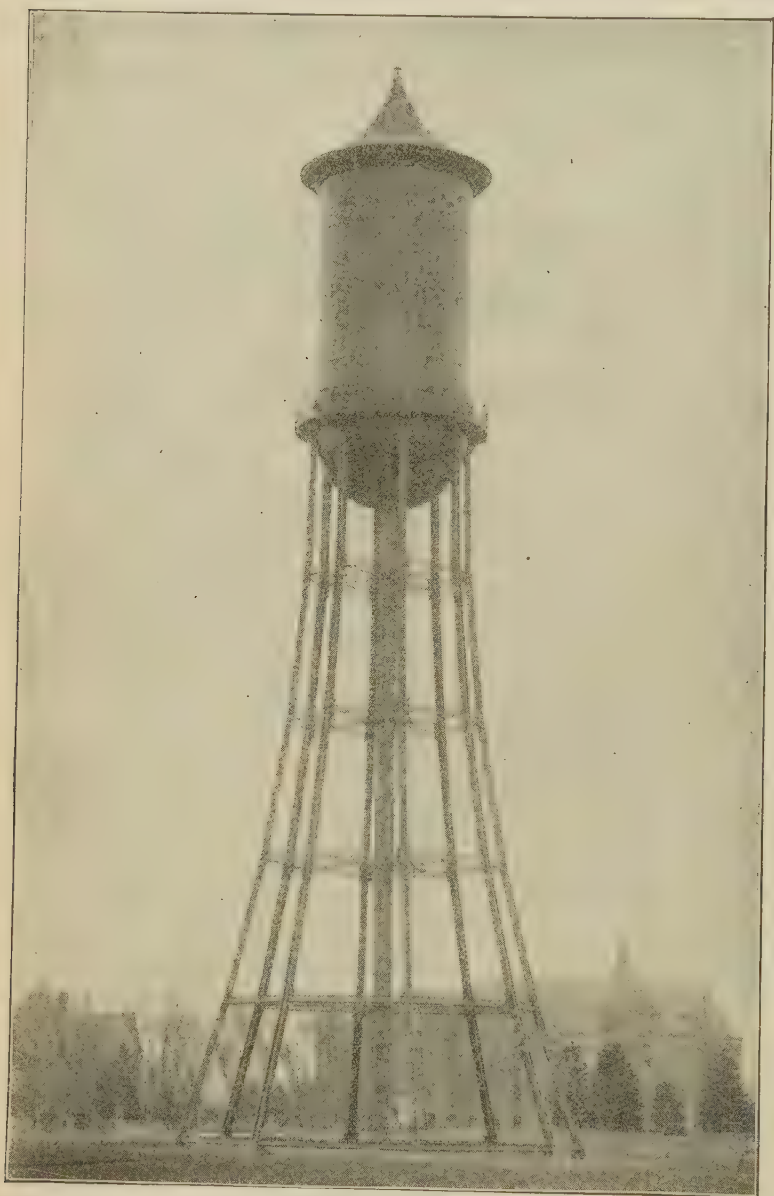


FIG. 434*a*.—WATER-TOWER AT AMES, IOWA.
Extreme height, 168 feet. Capacity, 162,000 gallons. Designed by Prof. A. Marston.

An elevated tank should be carefully designed by a competent engineer, and working drawings prepared. When so designed its safety is more secure than is possible in the case of a stand-pipe. A large proportion of the stand-pipes in this country have been designed by contractors and sales agents, and the legitimate results of such designing are being reached in the many failures of such structures which are now occurring.

468. Specification for Plate Material for Stand-pipes and Elevated Tanks.—The following clause, or its equivalent, should be inserted in all stand-pipe or elevated-tank specifications:

The material composing the plates for the sides and bottom shall be soft steel (preferably open-hearth) having a tensile strength between 55,000 and 64,000 lbs. per square inch; an elongation in eight inches of not less than twenty per cent, and a reduction of area at the broken section of not less than forty per cent. Specimens of any plate having a width not less than four times the thickness, when heated to a cherry-red and quenched in water, shall bend cold through 180° about a diameter equal to its thickness, without showing any signs of failure whatever.

This material is known amongst the dealers as "shell steel," and since it costs perhaps a quarter of a cent a pound more than "tank steel," which is made by the Bessemer process and is liable to be very hard and brittle, the contractor is pretty sure to use the cheaper grade if special care is not taken to inspect the material and prove its character by actual tests.

* See a design by Mr. Freeman C. Coffin, C.E., *Engineering News*, Mar. 16, 1893.

CHAPTER XXVIII.

IRON AND STEEL TALL BUILDING CONSTRUCTION.

469. **Modern Building Construction.**—The use of metal in the construction of large buildings has increased very rapidly in the last few years, until now the demand for structural steel for this purpose is a very considerable fraction of the whole output. A discussion of steel buildings or of steel construction in buildings becomes therefore increasingly important. It perhaps need hardly be said that an intelligent use of iron and steel in this field requires also a considerable knowledge of architecture, and more especially of the character of other building materials and of methods of using them. In general the same principles apply in this use of metal that apply in all other framed structures, but certain problems present themselves with much greater frequency and in vastly greater variety in building construction. It is the purpose of this chapter to discuss some of these, giving a few illustrations and suggestions that may assist the inexperienced to do intelligent work in this department.

The use of steel to so great an extent in large buildings is producing a new type of construction, which has been very aptly termed the "steel skeleton type of high buildings." The structural steel is used to make the frame of the building, and this frame should be strong enough to carry the loads and provide rigidity and lateral strength to the structure. The construction is typical in just the proportion that this is done and that the steel frame is relied on for strength. Several buildings have been constructed which are of this pure type. They have no supporting walls about them anywhere; the outside covering, of whatever material it may be—brick or terra cotta or stone—and all the interior partition walls, are carried on beams at each floor level and are self-supporting only one story in height; while all the weight of the building is carried on columns, arranged entirely apart from each other, but conveniently for the purpose.

Between this pure type of structure and the old construction, in which all the strength of a building was in its heavy masonry walls, there is every degree of the mixture of the two. Even some of the sixteen-story buildings of latest date have solid exterior walls of masonry, while the inside is steel construction. Oftentimes the necessities of adjoining buildings or the prejudices of owners require solid masonry party walls in what would be otherwise a steel structure of the purest type; but the same considerations govern the designing of the steel work in either case, and the use of the supporting walls only increases the variety of the problems to be solved. This characteristic feature of steel construction, that all loads, exterior and interior walls, floors, and partitions, are carried at each floor level by the frame and so taken into the columns, makes it necessary to calculate the weight of every part of the building and determine the proper dirstribution of all the loads among the columns. The columns are the most important element in the problem, and their sizes and sections must depend on these results. The foundations of the building may also depend on these loads, and especially so if the structure is to be carried on a yielding soil where the area of the footings must be proportioned to them. If the foundations are on solid rock or something equally as good, it may be sufficient that the columns simply reach their resting-place. The exact load on each bottom column is more important where piles are used to carry them, but it is most important of all when recourse must be had to a broad bearing on a compressible soil.

470. The Work of Designing should proceed somewhat as follows: (1) Arrangement of columns; (2) Arrangement of beams; (3) Calculation of loads on columns; (4) Dimensions of foundations; (5) Design of spandrel sections; (6) Calculation of the sizes of all floor-beams not already fixed; (7) Dimensions of the columns; (8) Wind bracing; (9) General details of connections.

This amount of work, with the drawings to represent it and specifications to cover it, are generally done at the expense of the architect, in his own office or by some consulting engineer. Each piece of iron in the structure should finally be drawn in complete detail, making what is known as a "shop drawing," and this work is generally done at the expense of the contractor subject to the approval of the architect or of the consulting engineer.

471. Arrangement of Columns.—The arrangement of the columns in a building depends first of all upon the plan and character of the structure. In most cases there will be certain points at which columns must be placed on account of the shape of the building, the interior arrangement of its rooms, its staircases, or its elevators, and regardless of constructional advantages or disadvantages. In planning the architectural features of a building, however, the construction ought to be kept constantly in mind and not made too subordinate, for the cost may be kept down and the strength and general good character of the framework may sometimes be greatly increased by very slight changes in construction, to permit which architectural features can be varied without detriment to the structure. A general scheme for the arrangement of the beams in the floors must also be borne in mind when the positions of the columns are fixed. The scheme should always be such that the floor loads will be taken as directly to the columns as possible. Very few rules will apply to all cases. Everything must vary with the necessities of the case. Even economy is often sacrificed to other considerations. If the building is a very high one, the bracing of the structure should also be considered at this time, for some of the columns must become a part of whatever system may be used. There is perhaps no part of the work of designing a building that is generally as little studied and yet on which so much depends regarding economy and strength and the general character of the work, as this one in particular, and none in which a large experience and a trained judgment count for more. The more completely the whole problem can be sized up in one full consideration of it, the more satisfactorily can the designer arrange his columns and at the same time plan in a general way the entire framework of the structure.

472. Arrangement of Beams.—As soon as the position of the columns is determined, the position of all the beams in the floors should be fixed as exactly as possible. To do this it will be necessary to calculate the size of the principal ones—that is, those doing the most important service.

In building lore the word "girder" has come to mean a beam, either solid, rolled, or built up of plates and angles riveted together, which is used to support joists, and the word "beam" is used to signify a joist. A "riveted girder" means a girder made of plates and angles. A "girder beam" means a girder made of a solid rolled beam. A "spandrel beam" is a common term for a beam carrying a portion of the exterior wall of a building. A "double girder" signifies the use of two rolled beams in a girder. These expressions will be used with these meanings.

It is not necessary at this time to calculate the size of the smaller or unimportant beams because the exact arrangement of the beams is needed at this juncture only to determine the true distribution of the floor load on the columns. The fewer connections between a load and the column which carries it, the better the construction; therefore the girders should always connect the columns, if possible, and multiplicity of details in arrangement should be avoided. The arrangement of elevators and stairways and pipe spaces in large office buildings, and sometimes of the machinery, makes it impossible to get the ideal plan; but a studied effort in the right direction will often greatly simplify what at first seemed a necessarily complicated

design. The arrangement of the beams may be varied to some extent with the method of fire-proofing to be employed; so it is important to have a plan for the completion of the floor also in mind at this time. There are a good many schemes for fire-proof floors, but not very many in actual service. In many large buildings the floor framing is made regardless of the arrangement of partitions, in order that these may be arranged and rearranged from time to time to suit the taste of those who use the rooms, but in some buildings built for special uses few partitions are required, and in such cases, or where partitions are to be particularly heavy, it is best to arrange the framing so that a beam or girder will come directly under the load.

In most large buildings the framing of many of the floors will be the same. A drawing should be made of this typical floor, and also drawings of those that differ from it; they should be made as simple as possible. A small square or a circle is enough to show the position of the columns, and a single line is enough to show the position of the beams. Architectural draughtsmen depend much on reference to scale to get sizes and dimensions from their drawings. All drawings of metal construction should preferably be made to a scale; dependence should be placed however, entirely on figured dimensions, and all figures should be exact. There can be no approximations in iron-work. The distance between the centres of two beams is not *about* 5 feet, it is *exactly* 5 feet $2\frac{5}{8}$ inches, and this exactness should be exercised on all *iron drawings*. These plans should show the building lines, the centre lines of all columns and their relations to the centre lines of all beams and girders. When this is done and the general construction of the building is determined, then can begin the actual work of calculating the loads on the columns.

473. Economy of Wide Spacing of Floor-beams.—In carrying the floor loads to the columns, which must always be done by the action of beams in some form, it is an evident economy to allow the floor material, whether composed of tile, concrete, or plank, to carry the distributed load as far as it can do so with safety. A stronger floor system can therefore be used with wider spacing between floor-beams. Since the strength of these beams increases in a general way with the square of their depth, it would be economical to place them farther apart, thus increasing the load on each beam, and using a greater depth. Since these beams are carried at their ends by girders resting on the columns, it is desirable also to so arrange them that the bending moment on these girders will be the least. This requires that there be an odd number of panels, or spaces between beams, on each girder. This brings two beams symmetrically on the middle portion, between which the bending moment is constant.

EXAMPLE.—Take a floor space 20 × 24 feet, to be carried on columns at the corners. Assume two arrangements of beams: *First*, seven beams (six panels) 20 feet long, resting on a plate girder 24 feet long. *Second*, four beams (three panels), supported in the same manner. In the one case the beams are 4 feet apart, and in the other case they are 8 feet apart.* Assume a total unit loading, dead and live, of 180 lbs. per square foot.

First Case.

The bending moment on the beams (counting one outside beam, and one of the girders, each fully loaded as though other similar areas of floor space surrounded this one on all sides) would be $\frac{4 \times 180 \times 20 \times 20}{8} = 36{,}000$ ft.-lbs. This would take a ten-inch beam weighing 27.75 lbs. per foot for a fibre stress of 16,000 lbs. per square inch.

The bending moment on the girder would be 259,200 ft.-lbs. If it have a total depth of 30 inches, with three eighths web and $4 \times 4 \times \frac{3}{8}''$ angles, the fibre stress will be again 16,000 lbs. per square inch, one sixth of the area of the web being added to the effective area of the angles to give total flange area. The weight of this beam would be 78 lbs. per foot, or 1872 lbs. for the 24-foot girder. Counting six beams and one girder as belonging to this elementary area, we have a total weight of 5200 lbs.

If two rolled beams be used in place of the plate girder, it would require 15-inch beams weighing 72 lbs.

* A concrete arch will readily span 8 feet.

per foot, or 3456 lbs. for the two beams to act as one girder. When these are used the total weight of iron to carry this area with four-foot spacing of beams would be 6790 lbs.

Second Case.

Here the beams are placed 8 feet apart. The bending moment in each beam would be 72,000 lbs., requiring a 15-inch beam weighing 41 lbs. per foot for a fibre stress of 16,000 lbs. per square inch.

There are now but two beams upon the girders, giving a bending moment of 230,400 ft.-lbs., which would require a plate girder 30 inches deep, $\frac{3}{8}$-inch web, and $4'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angles, the whole weighing 74 lbs. per foot, or 1776 lbs. for the girder.

If two I beams were used, as before, it would require 15-inch beams weighing 60.5 lbs. per foot, or 2904 lbs. for the two beams composing the girder.

In this case, therefore, we have three beams weighing 2460 lbs., which with the plate girder make a total of 4236 lbs., or with the double girder 5364 lbs.

Thus when the 30-inch plate girders are used, the weight of iron for the 4-foot spacing is 5200 lbs., as against 4236 lbs. for the 8-foot spacing, or a saving of 18.5 per cent of the iron used in the floor system with 4-foot spacing.

When the double 15-inch girders are used, we have a total weight of 6790 lbs. for the 4-foot spacing, as against 5364 lbs. for the 8-foot spacing, or a saving of 21 per cent of the iron in the floor system for the 4-foot spacing. These are the percentages of saving made by using 8-foot instead of 4-foot spacing. The weights of I beams have been taken exactly those required for the given bending moments in all cases, as this is necessary to obtain a fair comparison of weights from a single example.

If the flooring for the 8-foot spacing is more expensive than that for 4 feet, an allowance can be made for this and the final net saving, due to the wider spacing, computed.

474. Calculation of Column Loads.—The column loads should be divided into two classes, dead loads and live loads. The actual permanent weight of the structure itself makes the dead load; and the estimated weight of the people that will at any time enter the building, together with furniture, movables, stocks of goods, etc., make the live load. If machinery is permanently fixed, it should be counted as dead load; if it is movable, it is usually rated as live load.

Dead loads may generally be divided as follows:

Weight of
(1) Floors;
(2) Partitions;
(3) Vaults;
(4) Metal Columns;
(5) Column Coverings;
(6) Exterior Walls;
(7) Windows;
(8) Elevators;
(9) Permanent Machinery;
(10) Water Tanks;
(11) Plumbing and Heating Fixtures.

The weight of floors is usually reckoned by the square foot. The number of feet supported by each column should be determined, and this amount multiplied by the weight per foot gives the desired figure. The accuracy of the work depends very greatly upon this distribution of supported areas. If the floor plan is extremely simple, this may be done very easily and sometimes by simple observation, but in most plans there is somewhere a complication or irregularity, and the safest way is to figure the areas directly on a copy of the floor plans. There are several ways in which this can be done. The whole area may be divided into rectangles cornering at the columns. Each of these rectangles can then be subdivided and the number of square feet in each part can be noted on the drawing next the column to

which that particular part is tributary. It may be done also by computing the area carried by each individual beam, then the exact reactions of both the beams and the girders, keeping all the results in square feet. Either of these or other methods may require considerable figuring where the floor plan is very irregular, but the results can be made always definite and accurate. Supported column areas should never be estimated or guessed at.

The weight of the floor per square foot also should be determined as closely as possible Practice differs considerably in different cities, and somewhat among different architects in the same city. The most approved floor in Chicago, where this construction has perhaps reached its greatest development, is shown in Fig. 435.

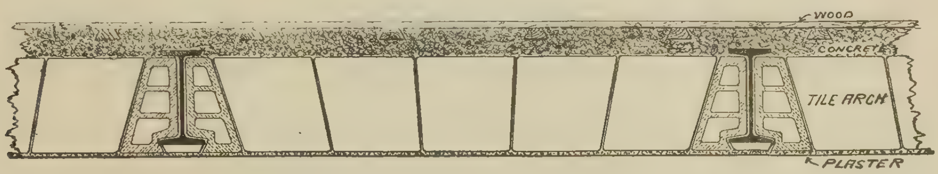


FIG. 435.

In this construction the floor weight is made up of the following items: Iron, tile arch, concrete filling, plastering, and wood floor. The iron consists of the beams, tie-rods, girders, connections, etc., which may be averaged per square foot. In ordinary floors it will be from 8 to 12 pounds. It may be easily figured from the floor plans so that it will not vary more than a fraction of a pound. The depth of the concrete should be known, also the weight of it. A concrete made of cinders and lime and a small amount of cement is one of the lightest and best for this purpose. Such a concrete weighs about 72 lbs. per cubic foot. Partitions should be of some entirely fire-proof material, light and easy to put in place. Partitions in office buildings, hotels, etc., should be built after the floor is laid and at least the first coat of plaster is on. Then they can be taken down and rebuilt in other places without leaving any permanent marks of the change. Where they are built in this way, it is best to calculate the entire weight of the partitions on a floor and find the average per square foot. In such cases this average can be included in the floor load. In most buildings it will not be greatly in error. If the partitions are sure to be permanent and the location is already fixed, a separate distribution of their weight is of course possible and preferable. The thickness of the plaster may be kept quite uniform by making it true to the woodwork about the doors, and the weight of the partition stuff, together with the weight of the plaster, will make up the whole weight of the partition. Doors can be calculated, or sample ones can be actually weighed and averaged. The common vaults in office buildings are only very small rooms with iron doors and combination locks. In such cases their weight may be lumped with that of the partitions. Bank vaults are made of solid brick walls steel lined, and they must have special treatment. The weight of the column itself will have to be estimated to some extent, but experience can make a very accurate estimate possible. The column covering consists of the fire-proofing, generally some kind of tile, and the finish, which is usually plaster, but sometimes marble or other material. The calculation of the weight of the exterior walls involves additional problems in reactions. The "curtain wall" is that part of the exterior wall extending from the line of the window cap of one story to the line of the window sill of the next story above. This part of the wall is an evenly distributed load on the spandrel beam, while the rest of it is cut up into separate concentrated loads which may or may not be symmetrical about column centres. Exterior walls are made of brick, terra cotta, stone, tile, metal, or combinations of these materials. The only way to find the correct weight, of course, is to know exactly what is to be used and just how, so that cubic contents can be figured. The floor construction sometimes projects into the curtain walls. In such cases care must be used to insure that

this portion of floor is not put in as part of the wall. The weight of windows must be doubled to allow for counter-weights. All the other dead loads are single, and most of them occur only on special floors.

The live loads used in buildings vary from almost nothing to 300 or 400 lbs. per square foot. The beams should be calculated to carry the whole of the unit taken, but in many cases it is wise to reduce the rate in figuring the girders and the loads on the columns. The live load taken on columns in office buildings vary from 20 to 50 lbs. per square foot of floor; sleeping-rooms in hotels do not require more than 10 to 20 lbs., while 60 to 80 lbs. per square foot ought to be allowed for floors where people may assemble in crowds, and warehouses sometimes require very much more. This, of course, is in addition to the dead loads. These loads must be figured for every floor, and tabulated so that the accumulated load can be determined for any particular column in any particular story. When the work is completed, the column loads are not only at hand, but the loads on the foundations can also be easily determined from the schedule.

475. **Dimensions of the Foundations.**—A general discussion of foundations is not here intended, but steel has been used so much in foundations that bear on a yielding strata, that such foundations seem almost a part of the steel construction of buildings. Most of the great buildings in Chicago have foundations resting on soft clay. In order to secure an equal settlement in those buildings, it is generally advisable to omit the live (temporary) load entirely, and then the basement column load minus the live load and plus the weight of the foundation itself will equal the permanent load on the clay.

Figs. 436 and 437 show a plan and section of such a foundation. The area covered is always in proportion, or should be, to this load on the clay. The following formula may be found useful in calculating the beams.

Let y = the width of the projecting area;
P = total load on the top,
= total reaction at the bottom;
a = the length of distribution of the load at the top;
M = maximum bending moment.

The use of a cast base under a column as shown in Fig. 436 has several advantages and is generally economical. These bases are made strong enough to carry the entire load on their perimeters, and in such cases the maximum bending moment in the beams comes at the edge of the base-plate. The length of the beam $= 2y + a$, and the total reaction of the projecting area $= \frac{Py}{2y + a}$, and this multiplied by one half the width of the projecting area is the bending moment at the edge of the plate or

$$M = \frac{Py^2}{2(2y + a)}. \quad \ldots \ldots \ldots \ldots \quad (1)$$

Either M or y may be the unknown quantity.

In the case shown in Figs. 436 and 437, in the top course

$$P = 1{,}233{,}270 \text{ lbs.};$$
$$a = 4'\ 6'';$$
$$y = 6'\ 4\tfrac{1}{2}'';$$

and these introduced into the formula make $M = 1{,}452{,}760$ ft.-lbs.

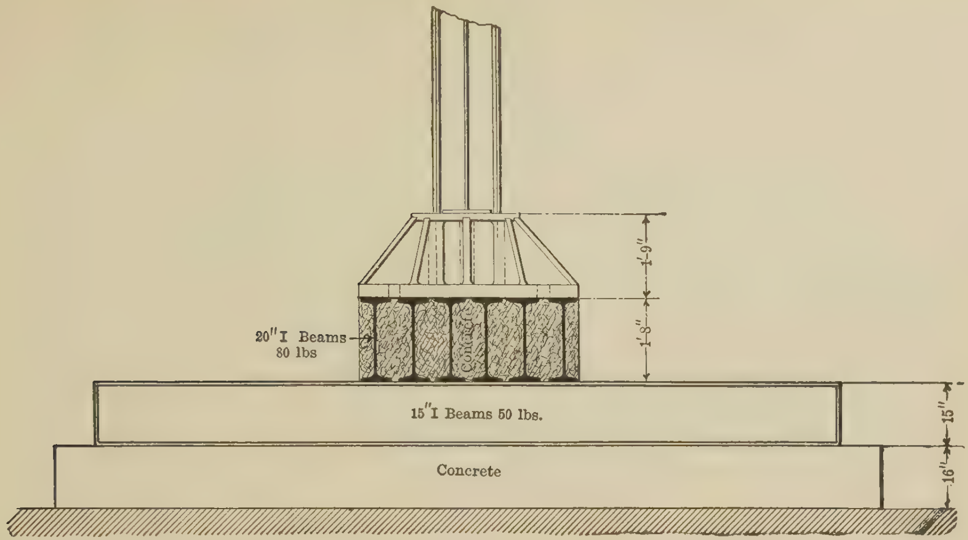


FIG. 436.

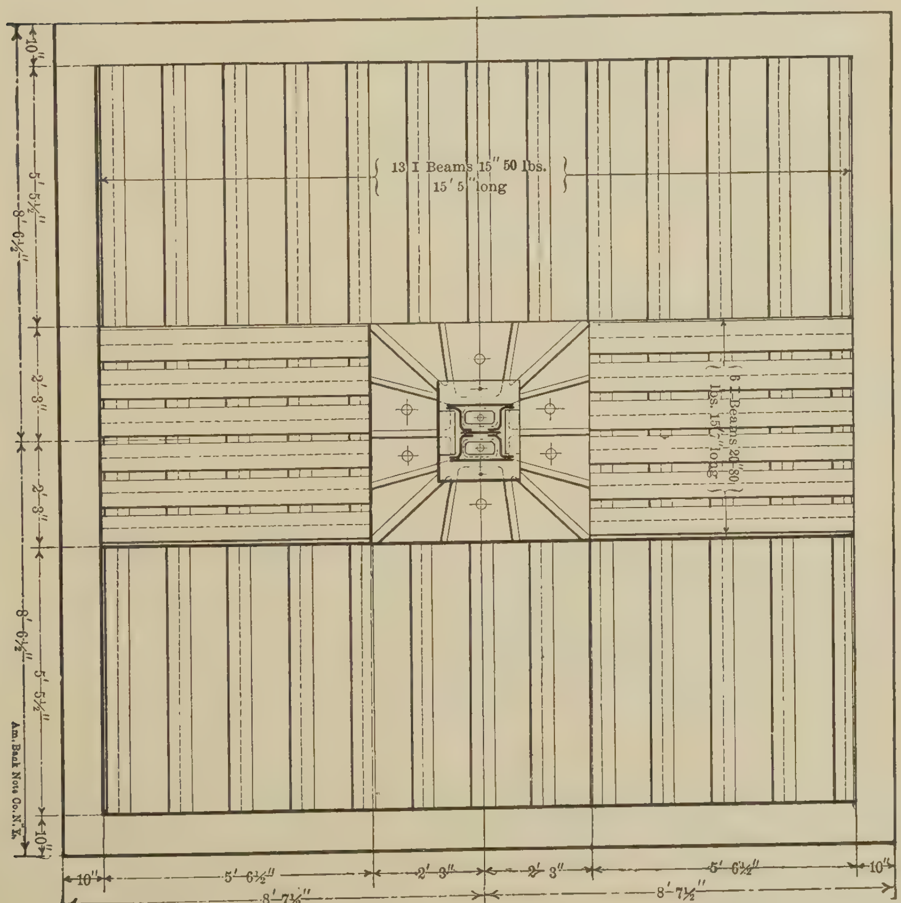


FIG. 437.

If the base-plate is only strong enough to uniformly distribute the load over the area a, the maximum bending moment in the beams is at the centre, and then if $l =$ the length of the beam,

$$M = \frac{P}{8}(l - a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It will be seen by computation and comparison that this condition will increase the weight of metal required in the top layer in the example given about 35 per cent.

The Moment of Resistance, M_0, in foot-pounds, for any given beam may be obtained as follows:

Let $f =$ ultimate allowable fibre strain per square inch;
$I =$ moment of inertia, in inch units;
$y^1 =$ one half the depth of the beam in inches.
Then from Eq. (1), Art. 121,

$$M_0 = \frac{fI}{12\, y_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

The value of I for any beam is always given by the manufacturer of it in some published book or list. The fibre stress, f, for foundation work is usually taken at 20,000 lbs. per square inch. Steel rails are sometimes used instead of beams, but are not often stressed as high. Sometimes these foundations are more than two layers of steel deep. Theoretically the sum of the moments of resistance of all the beams in one direction should equal the bending moment as given in eq. (1), irrespective of the number of layers. In practice, however, where there are more than two layers, so much steel is hardly needed. The beams are always bedded completely in Portland-cement concrete, and the friction of the beams on the cross-layers, together with the adhesion of cement and iron, tend to unite the whole into a sort of compound beam, the total moment of resistance of which is much greater than the sum of the moments of the separate beams.

Two or more of these footings are often combined into a single one where the area can only be obtained in that way, or for other reasons. Fig. 438 shows an elevation sketch of such a combined footing. The centre of gravity of such areas should coincide with the centre of gravity of the loads.

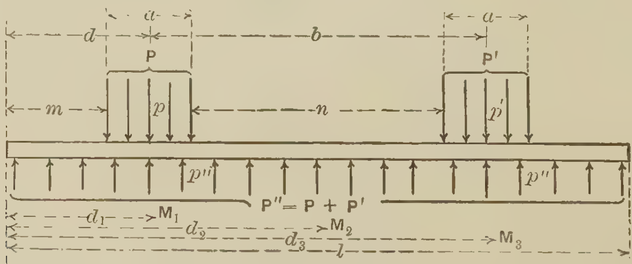


FIG. 438.

To find the maximum bending moment on the long beams we are obliged to compute three moments and compare them. In Chapter VIII it was shown that the bending moment is a maximum where the shear is zero. In this case there are three such sections, and it will be necessary to compute the moment at each one to find which is the greatest. The moments under the columns will be positive, causing convexity downwards, while that at the centre is negative.

To find the distance from the left end to the centre of gravity of the loads, we have, Fig, 438,

$$D=\frac{Pd+P'(d+b)}{P+P'}. \qquad (3)$$

If the area is rectangular, this gives the centre of it. If it is trapezoidal, its centre of gravity must be at this point.

If $p=\frac{P}{a}$, $p'=\frac{P'}{a}$, and $p''=\frac{P+P'}{l}$, as in Fig. 438, then to find the distance from the left end of the bottom beams to the sections where the shear is zero, these distances being called d_1, d_2, and d_3 respectively, we have

$$d_1p''=(d_1-m)p, \quad \text{or} \quad d_1=\frac{mp}{p-p''}; \qquad (4)$$

$$d_2p''=P, \quad \text{or} \quad d_2=\frac{P}{p''}; \qquad (5)$$

$$d_3p''=P+[d_3-(m+a+n)]p', \quad \text{or} \quad d_3=\frac{(m+a+n)p'-P}{p'-p''}. \qquad (6)$$

The bending moments at these points are readily found by taking the moments of the external forces on one side of the point about that point. Thus the bending moment at the first maximum point is

$$M_1=\frac{p''d_1^2}{2}-\frac{p(d_1-m)^2}{2}; \qquad (7)$$

$$M_2=P\left(d-\frac{d_2}{2}\right); \qquad (8)$$

$$M_3=\frac{p''d_3^2}{2}-P(d_3-d)-\frac{p'(d_3-m-a-n)^2}{2}. \qquad (9)$$

In general M_2 will be small, except where P and P' are near the ends of the beams, and the maximum moment will usually be either M_1 or M_3, whichever is the heavier load.

If the cast bases are strong enough to carry the loads on their perimeters, and the long beams are in the top course, the values of M_1 and M_3 may be reduced; M_2, however, would not be changed.

The problems of this class are sometimes exceedingly complex, as when foundations cannot extend beyond the lot line, or when a sewer or other obstruction is encountered, and the study of economy in designing offers a wide field of investigation along this line. Sometimes it happens that the clay loads are so great and the limits so narrow that it is impossible to find area enough without overloading the clay. In such cases either the dead load of the building must be lightened or the columns must be rearranged. In either case much of the work already done must be done again.

476. Design of Spandrel Sections.—The term "spandrel section" commonly means a vertical cross-section through the exterior wall of a building, showing the construction between the top of one window and the bottom of the next one above it. In most cases the spandrel beam must carry the floor and support the wall. In order to intelligently design the iron in a spandrel section, it is quite necessary to have the floor plan already arranged, and to know exactly how the wall is to be built. The architectural work proper must be practically finished. Cornice lines, reveals, projections, the dimensions of terra cotta or stone trimmings, the relative heights of window caps and floor lines, the exact position of the wall lines in the

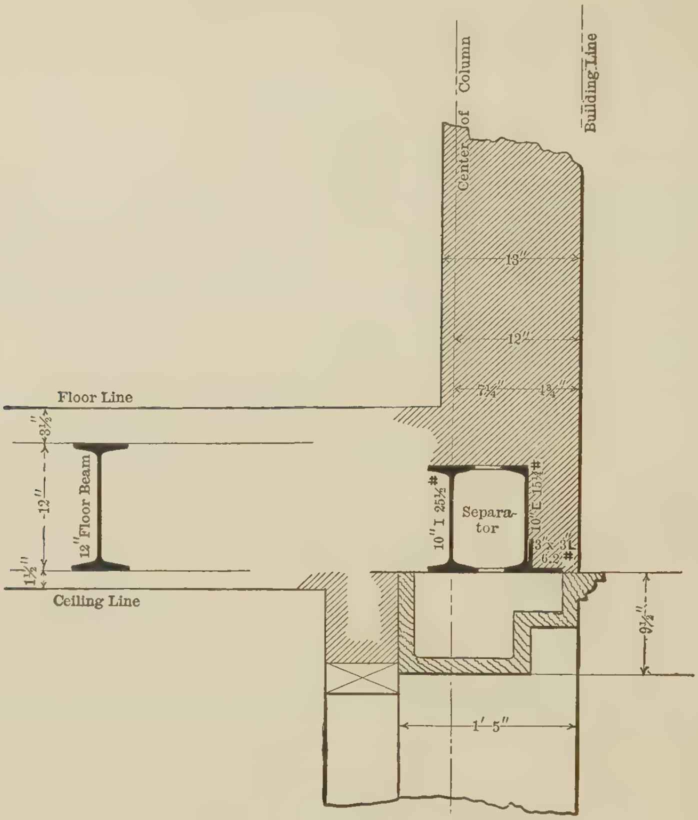


FIG. 439.

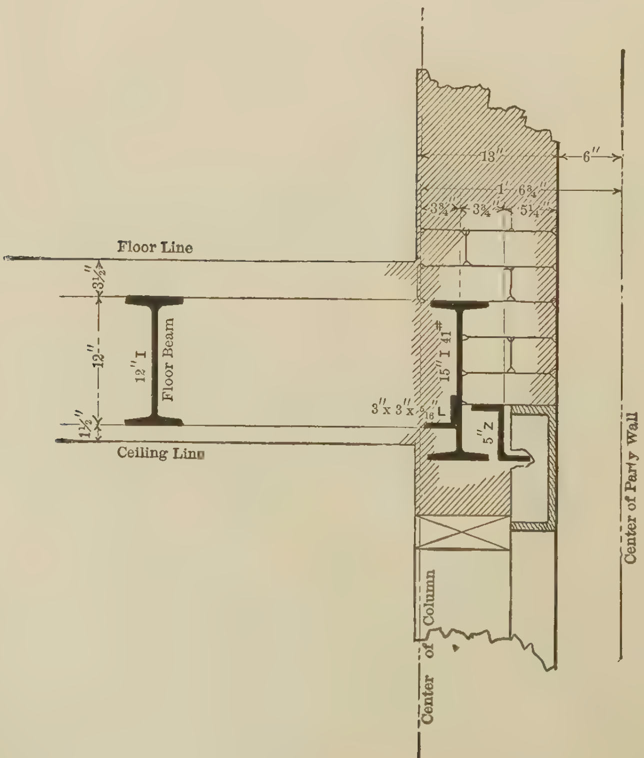


FIG. 440.

rooms both above and below, and in fact all the exact data pertaining to the construction, are needed to design the iron in the best possible manner. The iron must not only not be exposed anywhere, but it should be far enough from the lines of exposure on all sides to be thoroughly fire-proof. No part of a building is more exposed to a great heat in case of fire than this, for window openings become draft-holes, through which the flames can play with greatest fury. The iron must also not only be strong enough to carry the load, but it must be so arranged that both the wall and the floor shall be fully supported.

If the floor arch rests directly on the spandrel, either the bottom flange of the beam must be on the same level as the floor-beams, or an angle must be riveted to the web of the beam, as shown in Figs. 439 and 440. When the spandrel beam serves as a girder, it can be placed as high or as low as desired. This is illustrated in Figs. 441 and 442. Terra cotta is used to a

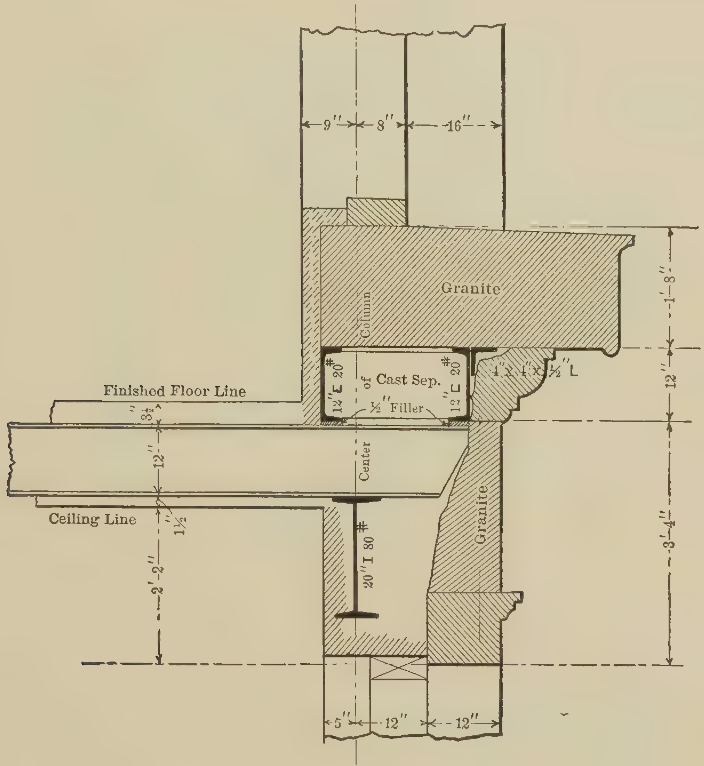


FIG. 441.

very great extent outside as a finishing material in this type of construction. It is used for the entire outside finish, or it is used for window caps and sills and in various other ways in connection with pressed brick. The proper idea is to get the iron as well under the terra cotta as possible, and where that cannot be done, to arrange the iron so that the bottom course in each spandrel can be readily suspended. Fig. 439 shows such construction. The terra-cotta blocks making the window cap are drawn into position close against the iron-work with hook bolts, so that each has its own support. The next piece is anchored with slight iron rods in position with one edge resting on the horizontal flange of an angle, so that none of its weight comes on the suspended blocks. Fig. 440 shows another construction where the reveal is less and the window cap is notched so as to ride directly on the flange of the Z bar. Care should be exercised in all cases to make the connections in some way that will prevent these angles and

Z bars, or any other iron that may be used for the purpose, from deflecting or twisting so as to crack the wall after all the work is done. Such cracking may not endanger the construction in any way, but it will be sure to mar the appearance of the building, and it can be entirely avoided. Brick can be used without terra cotta by turning a flat arch for a window cap and taking all the load above it directly on the iron or, indeed, by suspending the brick directly over the window, in which case special brick are required. When stone window caps are used, they can also be carried directly on the iron, but more generally the window cap itself is made self-supporting, and the load above is taken on the iron, as shown in Fig. 441. Where the distance between columns is very great the beams should not be strained as high as usual on account of deflection, and for the same reason deep beams are preferable to shallow ones. Deflection should be kept at a minimum in all spandrel work.

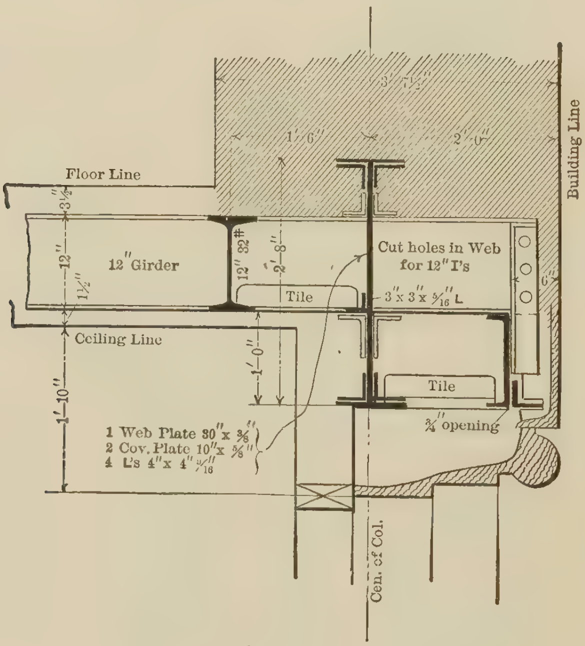


FIG. 442.

In calculating the loads, that part of the wall between the lower line of the window cap and the top line of the window sill next above is almost always evenly distributed. The mullions between the windows and the windows themselves sometimes must be treated as separate loads, while the floor loads coming on these beams may come under either class.

477. **Calculation of Beams.**—The best method of determining the size of beams is that of moments. To use this method readily it is necessary to have a table showing the values of M_0 for each section of beam which there is any possibility of using. Such a table can easily be made, using formula (2) for the calculations. The value of p is usually taken at 16,000 lbs.

Determine the bending moment of each beam, taking all distances in feet, then select a beam from the table whose moment of resistance is not less than the calculated bending moment. This method of work would be understood anywhere, and is applicable to all cases that can possibly occur.

Dimensions of Columns.—In actual practice the treatment of columns varies greatly. This is mostly due to the following circumstances: The formulæ for the strength of columns do not agree. The underlying principles seem to be sufficiently established, but every

authority has his own treatment of them, his own form of expression and his own nomenclature. To some extent, also, they are empirical, containing factors entirely dependent upon the results of actual tests, and these have given rise to further differences. All common formulæ are based on a condition of ideal loading which cannot always be obtained in building construction; indeed, it would be nearer the truth to say it is rarely obtained. There is also a lack of full-sized tests right along the line of these irregularities of loading. The tests that have been made are not full enough to properly show the relative value of the different sections in use, and are not conveniently available to the profession at large. All this helps to explain the lack of uniformity in the estimate of column sections and methods of calculating them.

In the treatment of columns given in Chapter IX three kinds of compressive stresses are pointed out to which the concave side of the bent column is subjected: that uniformly distributed over the section, that due to eccentric loading, and that due to the flexure. In the derived formulæ the second of these elements is omitted because there can be no eccentricity in ideal loading, and so it is in Gordon's formula, and all the others that have been derived from it or based upon it. In building construction this second element must not be omitted. The metal of one column should be directly over the metal of the column below, continuously through the entire height of the building, and this necessitates the application of the loads on the sides of the columns. If the loads are equal and are on opposite sides of the column, the effect of the eccentricity is neutralized, otherwise it increases the stress on the side of the column on which the greater load is applied. Owing to the short length of most of the columns used in this construction, and to the fact that the ends are flat bearing, the value of $\left(\frac{l}{r}\right)^2$ is so small that it gives the third of these elements the least importance. In the basement columns of a sixteen-story building the value of the term $\frac{f-p}{10E}\left(\frac{l}{r}\right)^2$ in equation (1) of Chapter IX is about .022, while the value of the second term, $\frac{vy_1}{r^2}$, is quite commonly as much as .07, and often considerably more. In the smallest columns at the top of the building the value of the term $\frac{f-p}{10E}\left(\frac{l}{r}\right)^2$, owing to the reduced section, is about 0.220, while 0.6 or 0.7 would not be an unusual value for the term $\frac{vy_1}{r^2}$ which in these smallest columns occasionally doubles the section. These figures are taken from examples at hand. They show first that the important effects of eccentricity of loading increase rapidly as the section of the column decreases, and that the importance of this element in columns thus eccentrically loaded is three or more times as great as that of the element dependent upon the flexure of the column. These effects are entirely independent of the character of the column, varying of course in values with different kinds of columns, but always true when the loading is as irregular and eccentric as the architecture of modern sixteen- and twenty-story buildings necessitates.

Mr. James Christie in his report of tests made at Pencoyd Iron Works, in a paper read before the American Society of Civil Engineers in 1883, says: "Very minute changes in the position of the centre of pressure produces greater differences in the resistance of the bars than was anticipated." And in another place he says: "For reasons not always evident, occasional results were obtained either abnormally high or low, as will be found illustrated on the diagrams; but there is little doubt that the principal cause of low resistance was eccentricity of axes, or non-coincidence between the centre of pressure and the axis of greatest resistance of the specimen." These tests were made on small bars, but the results would

hold equally good on sections and conditions found in practice. All that we have in actual experiment bears out this theory that seems too well founded for question.

There are two other factors having a very practical bearing on the strength of columns, neither of which are accounted for in this discussion and application of equation (1) of Chapter IX. One of these is contained in the form of the sections used, and in the manner in which they are fastened together; the other concerns imperfections in workmanship and material. Both of these factors are alike empirical, and no function expressing these conditions enters into any of the column formulæ arranged for possible practical use. They are, however, unlike in this important feature. The imperfections of workmanship and materials do not differ greatly with different kinds of columns, but rather with different shops and mills, and the only way to guard against them is to employ the best service and in close and careful inspection. On the other hand, the form of the sections used and the manner in which they are put together is a factor of strength or weakness peculiar to each kind of column manufactured, and is an important feature in any comparison of the strength of different kinds of columns.

By "the form of the section" is meant, not its capacity to produce in the finished column a large moment of inertia for the actual area, but the possible assistance that the different parts of the section may afford each other in general stiffness. We have no scientific discussion of this feature as it applies to the strength of columns, no function of the form of the sections independent of their position, nor of effects of multiplied punching and riveting in any column formula, and no adequate comparative tests that can establish the relative merits or demerits of different sections in this respect even empirically. There is testimony to the fact that it is an important consideration, though sometimes authorities do not agree as to the facts involved. For example, in a book published by the Phœnix Iron Company they criticise the Z-bar columns because "their thin unsupported flanges, flaring out at extreme points, are much to be deprecated, owing to their inherent tendency to buckling," while Mr. Strobel in his discussion of the same column in a paper before the American Society of Civil Engineers says: "It will be seen that the Z-iron columns compare favorably with other columns in ultimate resistance. The values obtained are near approximations to the Watertown results with Phœnix columns, and exceed those heretofore obtained with other types of columns. This favorable showing for the Z-iron columns should probably be attributed to the fact that the material in the outer periphery of the cross-section, on which dependence must be placed to hold the column in line, is not weakened by rivet-holes, but is left solid and unbroken, and is therefore in best shape to do its work effectively."* Each of these arguments, one for the strength of the column and one for its weakness, is based on the conditions concerned in this consideration of the column.

FIG. 443.

Fig. 443 shows the sections of the columns in common use in building construction. In practice it often occurs that columns must be calculated rapidly, and it is important to curtail the work as much as possible. Some of these column types are manufactured as a specialty. In such cases the manufacturers have a working formula for the strength of the columns, and a table showing the safe concentric loads for columns of different sizes. These formulæ and

* These columns were made of iron which showed very high elastic limits in the specimen tests.—J. B. J.

tables may be relied upon as conservative. They are made for the use of unprofessional men, and it must be remembered that they are for concentric loads only, that is to say, they represent only the first and third elements of equation (1) before referred to, with possibly an empirical factor supposed to cover the conditions of form and riveting, etc., peculiar to the column. If a proper allowance is made for the bending moment arising from the eccentric loading, and moderately heavy sections are used with skilful detailing, there seems to be no good reason why columns for most buildings should not be subjected to a higher unit stress than is usually given by these formulæ and tables. The moment due to the eccentric loading may be provided for as follows: Multiply the eccentric load by the distance from the point of its application to the centre of the column; the result is the bending moment due to the eccentric load. Then from the formula $M_0 = \frac{fI}{y_1} = \frac{fAr^2}{y_1}$, or $A = \frac{M_0 y_1}{fr^2}$, we find the area of section required to resist the bending moment.

In designing short columns, as used for buildings, select a maximum working stress on the extreme fibres of the column, on the side of the eccentric load, and neglect all bending of the column for such working loads. We then have, for both concentric and eccentric loads,

$$A = \frac{P}{p} + \frac{My_1}{pr^2}, \quad \ldots \ldots \ldots \ldots \quad (10)$$

where A = total area of column; P = total load on column, both eccentric and concentric; p = maximum working stress in lbs. per sq. in.; M = bending moment from eccentric load = $P_e v$ (where P_e = eccentric load, and v = distance of eccentric load from axis of column); y_1 = distance of extreme fibre on loaded side from neutral plane of column, and $k = \frac{v}{y_1}$; r = radius of gyration of cross-section of column in direction of eccentric load. But for Z-bar columns and for most of the iron and steel forms $\frac{y_1}{r} = 1.73$. If v be found in terms of y_1 we may write $v = ky_1$; hence $\frac{My_1}{pr^2} = \frac{P_e v y_1}{pr^2} = \frac{kP_e}{p}\left(\frac{y_1}{r}\right)^2$. But since $\frac{y_1}{r} = 1.73, \left(\frac{y_1}{r}\right)^2 = 3.0$, hence our equation becomes

$$A = \frac{1}{p}(P + 3kP_e)^*. \quad \ldots \ldots \ldots \ldots \quad (11)$$

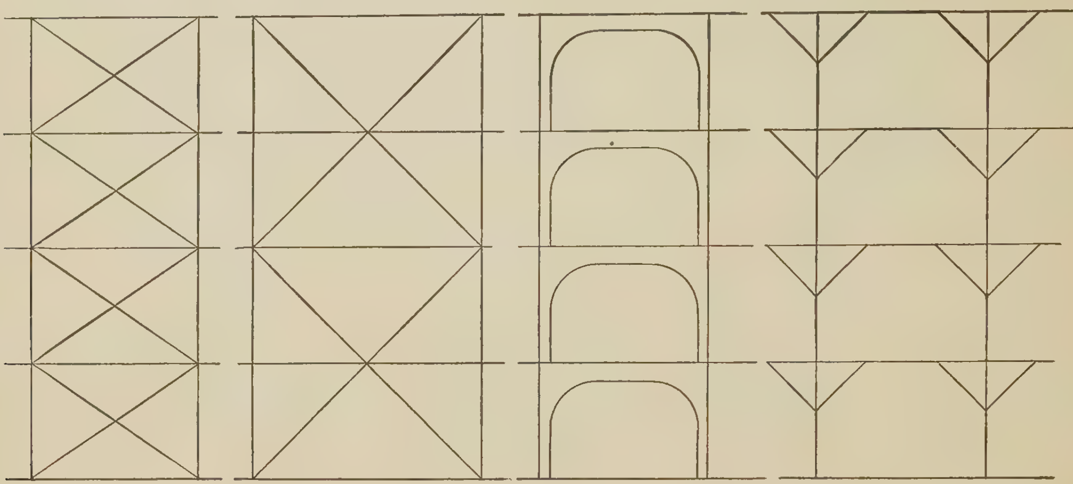
This is an exceedingly simple formula and readily applied.

479. Wind Bracing.—Buildings are always subject to lateral strains from wind forces. But little attention has been paid to this fact heretofore, and really there has been little need. If a building of unusual height was proposed, an extra wall was put into it, but otherwise the lateral strength of the exterior walls and the ordinary partitions have almost always been quite sufficient to resist these forces. Where buildings have been blown down it has generally been shown that there was a reckless want of care in the construction where only a little care was needed. Steel buildings, however, are built to such great heights, and are so destitute of these ordinary means of resisting wind forces, that it is necessary to give the subject much more serious consideration and to brace the steel frames so that the strength of the buildings in this respect shall be assured. This can be done in a variety of ways; but the arrangement of the rooms, the architectural features, and other requirements prescribe so greatly that the designer will probably be left with but one way, and be very glad that he has that one.

The bracing, whatever it is, must of course be vertical, reaching down to some solid connection at the ground. It should also be arranged in some regular symmetrical relation to the outlines of the building. For example, if the building is narrow and is braced crosswise with one system of bracing, that system should be midway between the ends of the buildings, and if two systems are used they should be equidistant from the ends, the exact distance

* This formula was added by Prof. Johnson in the fourth edition of this work.

being unimportant, because the floors, when finished, are extremely rigid. The symmetrical arrangement is necessary to secure an equal service of the systems and prevent any tendency to twist.



a *b* *c* *d*

FIG. 444.

Fig. 444 shows in outline several ways in which such a system of bracing may be constructed. Horizontal lines indicate floors, and vertical lines indicate columns. It is obvious that both the horizontal iron-work and the columns must do a good part of the work, and that each arrangement must have its own treatment and must create stresses unlike those created by the other systems. The loads, however, will be the same. If one system is used, the length of the building, that is, the width of the side perpendicular to the direction of the bracing, multiplied by the distance between floors half-way below and half-way above, will equal the exposed area tributary to each panel point, and this multiplied by the force per square foot will equal the horizontal external force applied at each panel point. Then the total shear at any point will equal the sum of all the external forces at and above the point taken. These shears may be reduced in actual practice on several accounts, and if such reduction is made it is well to make it at this point in the computations. The weight of the building affords some resistance, and in most cases is worth taking into account. Most buildings are filled with tile or some other sort of partitions, and when these are really constructed and their continuance is assured, there is no good reason why we should not rely also on them to some extent. There is also some resistance to lateral strains in the connection of the beams to the columns where they are well riveted. Some of these considerations will admit of calculation; but in using them much must depend on the experience and judgment of the engineer.

The simplest form of bracing is that marked *a* in Fig. 444. In Fig. 445, which is the same thing, $B =$ the load or shear directly tributary to that panel point; $A =$ the sum of the loads or shears tributary to all the points above, or, in other words, the horizontal component of the stress in rod r; $E =$ vertical component of the stress in rod r; $D =$ accumulated vertical wind loads in the column next above column 2.

Then $A + B =$ the horizontal load on rod s;

$$\frac{(A + B)b}{e} = \text{vertical component of the stress in rod } s.$$

The compressive stress in any horizontal strut must equal $A + B$. The load on any

column 2 must equal $D+\frac{(A+B)b}{e}$, and this wind load must be added to all the other regular loads on the column. It must also be noted that $\frac{(A+B)b}{e}$ is an eccentric load, the length of arm being the distance from the bearing or point of attachment at the end of the horizontal

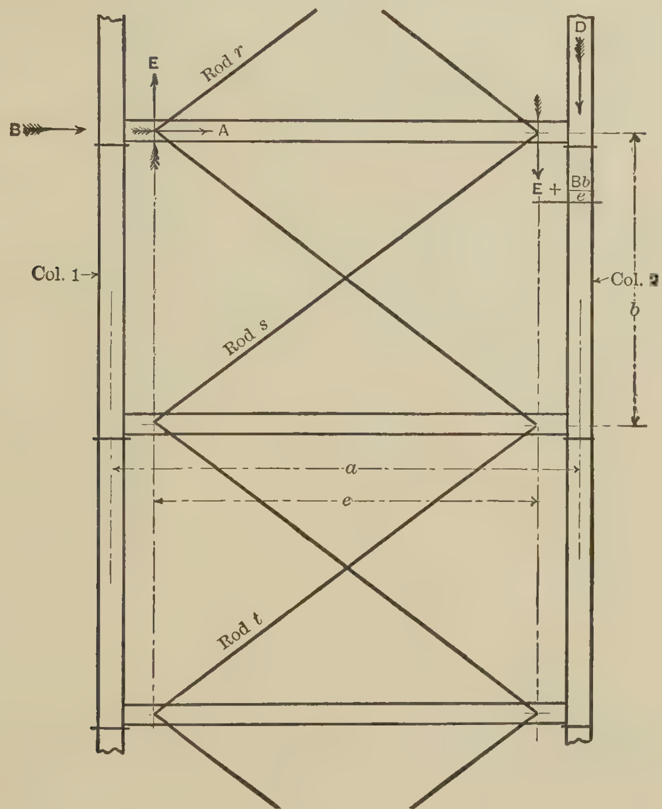


FIG. 445.

strut to the axis of the column. If this connection is to the axis of the column itself, or if the rods connect directly to the centre of the column, the eccentricity is reduced to zero and the eccentric load becomes a direct load the same as D.

The regular load carried by column 1 resists the upward vertical component of the stress in rods, connected at the bottom of the column, and the same is true at every other connection to this tier of columns. The dead load in column 1 is reduced the full amount of the total compression for wind in column 2, that is, $D+\frac{(A+B)b}{e}$; and when this amount exceeds the dead load in column 1 there must be tension in the connection of the column to the next one below, a condition which is not provided for and which in any ordinary case should not be allowed to occur. The horizontal shears multiplied by the secants will give the stresses in the rods the same as in a truss.

The arrangement marked *b* in Fig. 444 is a slight variation of that marked *a*. The arrangement marked *c* consists of a system of portals one above the other. This is shown more definitely in Fig. 446.

A = accumulated force or horizontal shear from wind at the floor next above floor M, applied one half on one side and one half on the other;

B = the force of the wind or shear directly tributary to floor M;

$D =$ the accumulated vertical wind load in the column next above column 2;

A, B, $D =$ the total exterior forces acting on the portal, then

$$(Ab + Bb - Bc)\frac{1}{a} = \text{vertical resistance due to } A \text{ and } B;$$

$$\frac{A + B}{2} = \text{horizontal reactions due to } A \text{ and } B.$$

In column 2 the vertical column load due to the wind must be added to the regular load of the column the same as in the arrangement shown in Fig. 445. The load D and its equal reaction, being directly applied along the same straight line, may be omitted from considera-

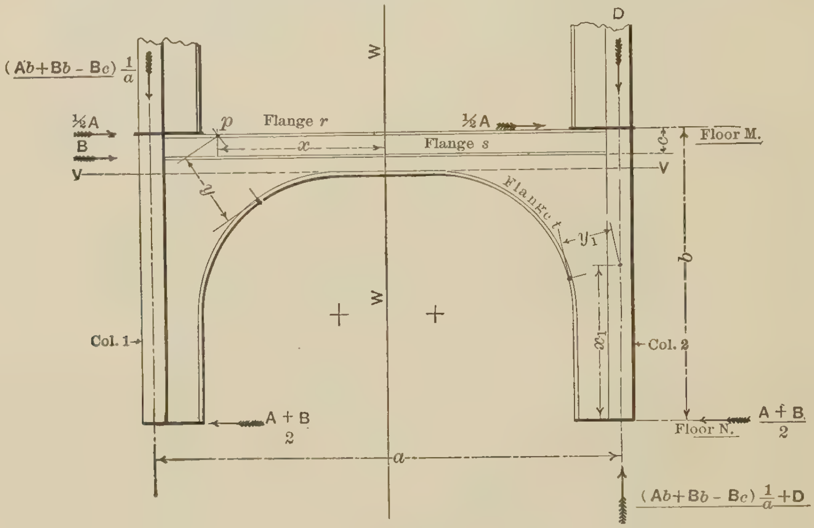


FIG. 446.

tion in discussing the strength required in the bracing, as may also the negative effects equal to D which occur in column 1, the same as in the case shown in Fig. 445.

The horizontal shear along the line $vv = A + B$.

The horizontal shear in either leg below the line $vv = \frac{1}{2}(A + B)$.

The vertical shear on all vertical planes $= \dfrac{(Ab + Bb - Bc)}{a}$.

The thickness of the web plates must be determined by these shears. It will be noted that the connection to the columns must be equal to the whole vertical shear. The direct compression in the flange $S = \frac{1}{2}B$. Taking moments about the point of intersection of flange r with the line ww, it will be found that the sum of the moments equals zero, that is, that there is no bending moment in the portal on the line ww, and that flange t is not strained at this point. For maximum stress in flange t take a point p in flange r, distant x from the line ww and at right angles to any given section of the flange t; then x times the vertical shear divided by $y =$ the stress at the section taken, and this is maximum when $\dfrac{x}{y}$ has its greatest value. The leg of the portal including column 2 might be also taken as a cantilever with two forces acting on it, $\dfrac{A + B}{2}$ and $\dfrac{(Ab + Bb - Bc)}{a}$, with flange t in compression and the column itself acting as a tension chord. Take a point in the centre of the column, distant x_1 from the bottom of the leg and at right angles to any given section in flange t; then $\dfrac{A + B}{2} \cdot \dfrac{x_1}{y_1} =$ the strain in flange t, and this is maximum when $\dfrac{x_1}{y_1}$ has its greatest value. There is a slight error

in this treatment, but it is on the side of safety. If flange t has a section proportioned to these maximum stresses, the requirements will be fulfilled.

The stress and area required in flange r can be obtained in a similar manner. The connections of the portal above this flange to the portal and column above must be equal to $\frac{1}{2}A$ at each leg.

The arrangement marked d in Fig. 444, if used at all, would probably be made to include more than two columns, and the stresses would vary greatly with the number of columns included in the system. It is not an economical method of stiffening a structure, as it produces heavy bending moments in both the horizontal struts and in the columns themselves. Methods a and b, on the other hand, if connections are properly made, do not cause any bending in the columns or in the lateral struts.

480. Details.—In many buildings, owing to irregular lines or to an elaborate exterior, the details of the construction are difficult and complicated. Many architects also have but little knowledge of the practical ways of connecting and working metal in the shop, and the result is that otherwise good frames are often decidedly weak in connections and details. It is therefore very important that the general drawings and specifications should cover all these points.

Beams fitting into beams should have an eighth of an inch clearance at each end. Standard connections, when the manufacturers of the beams have any, should be used

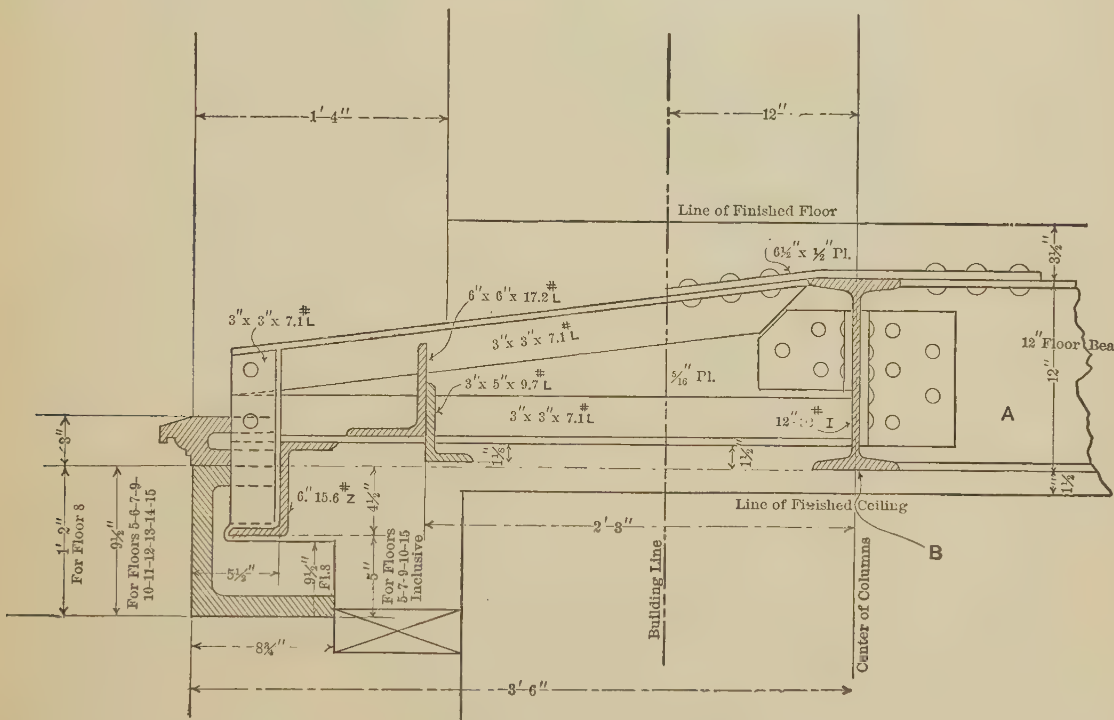


FIG. 447.

wherever they can be conveniently. Care should always be exercised to see that the flanges are not weakened by rivet-holes. The common connection of a beam to a column requires two rivets in each flange. These rivets should attach to a lug directly connected to the column itself by an equal number of rivets. As good a connection, and in some ways a better one, is to put all four rivets in the bottom flange and fill the clearance space between the beam and the column at the top of the beam with iron wedges tightly driven. When the columns are cut off under the beams so that the latter can rest on the cap-plate of the column below, this cap-plate should not be used as a lug. The supports of brackets in bay-windows

and under a heavy cornice needs especial care. These should be made as parts of beams wherever possible. Fig. 447 shows such a connection. When attached as shown in this case, the beam marked *A* may be calculated as though it were one continuous beam to the end of the bracket. If the bracket were attached only to the beam marked *B*, the latter would twist to some extent, and a very little yielding on the part of the beam in this way would make the vertical deflection of the bracket relatively large. This would certainly be an injury to the building. All beams carrying floor arches should be provided with tie-rods to counteract the

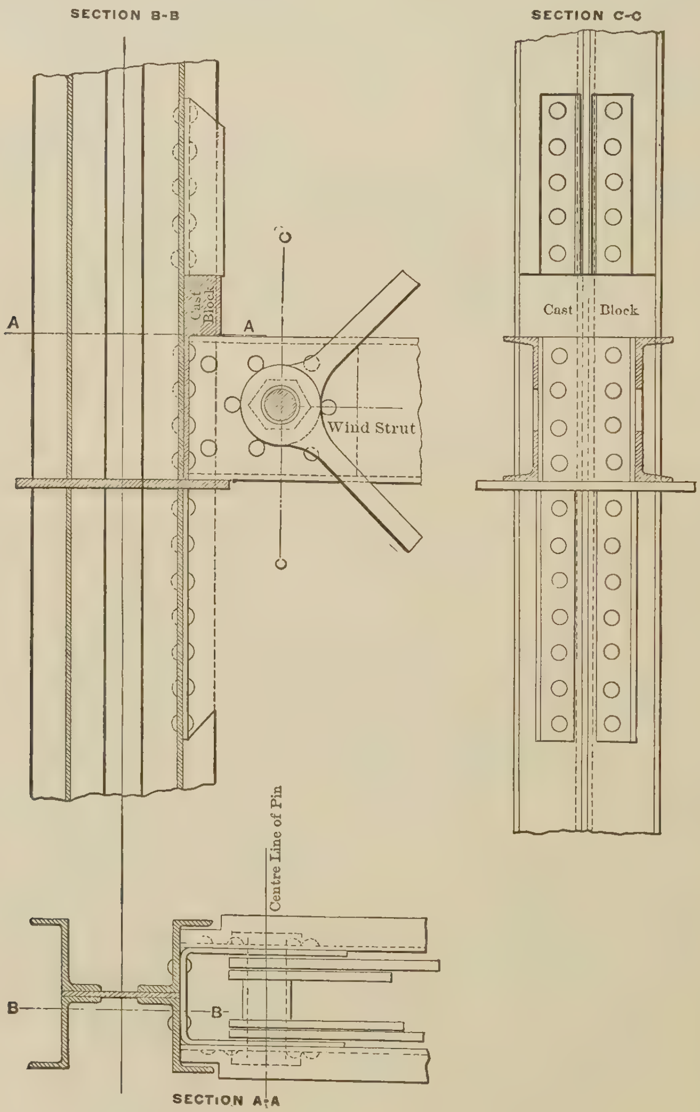


Fig. 448.

thrust of the arch. Double beams should always be provided with separators to equalize the load. Connections in high buildings, wherever possible, should be riveted.

Columns should be connected to each other through the cap-plates by four or more rivets. This should be independent of the attachment of the columns through the beam connections. Columns should be milled at each end, smooth, and at right angles to their axes, and the use of sheets of lead or of any other material under one side of a column, to make it

plumb, is exceedingly bad. The shop-work should be dône so well that the need for such adjustments should not be felt.

Details of connections in wind bracing are exceedingly important, as it is very easy to destroy the efficiency of any system by a single faulty connection. It is difficult to describe exactly what these details should be, for the requirements vary greatly in different buildings. To properly detail the work it is as necessary to have a perfect understanding of the outward forces and the manner in which they must be resisted, as it is to calculate the sections required. This may be illustrated in the connection of the struts to the columns in that system of bracing marked *a* in Fig. 444. The strut need not be connected to the column to resist horizontal forces, for there is no force tending to tear the strut away from the column in this direction. The force to be resisted here is vertical. Fig. 448 shows such a connection; the strut is made to butt the column squarely instead of fastening to the sides of the column by rivets passing through the two members, or indirectly through connection plates, because the forces producing stresses in the bracing at this point must come into the strut by compression from without and not through any possible tensile stress. When the strut butts the column these forces are introduced into the strut without the aid of rivets, and the full value of all the rivets can be used to resist the vertical component of the rod stress. It serves also to keep the arm at the end of the strut, or the distance from the centre of pin to the bearing at the end, as short as possible, all of which is important. The top angles may be placed several inches above the strut, and a cast filler-block introduced between them. Such an arrangement has several advantages. It generally happens that these angles cannot be riveted to the column directly under the channels of the strut, as shown in Fig. 448. The consequence is that whatever intervenes must carry a cross-strain. The cast-block will do this well. It is also important that there should be absolutely no clearance; otherwise the whole system would lack in stiffness and efficiency. The block can be cast a little large, and if necessary it can be chipped at the building in order to crowd it into position. The block also has the further advantage of cheapness, and is always easily obtained. Every detail in wind bracing should receive the most careful consideration.

CHAPTER XXIX.

IRON AND STEEL MILL-BUILDING CONSTRUCTION.*

481. General Types of Buildings.—There are three general types in common use. The first has a rigid iron frame throughout. Each transverse bent is made up as follows: An iron roof truss is supported at its ends by iron columns firmly anchored to masonry piers; the bent is made rigid by transverse bracing, consisting of knee-braces in intermediate bents and vertical bracing in the end bents between columns. In the sides of the building vertical bracing is provided between the columns, and lateral bracing in the planes of the top and bottom chords of the roof trusses. When long panels are desired, the alternate roof trusses can be supported on longitudinal girders, or trusses, running between the columns. At the ends of the building one or more gable columns should be introduced to shorten the unsupported length of transverse vertical bracing. The sides and ends may be covered with corrugated iron sheeting supported on suitable framework.

The second type differs from the first in having thin curtain walls of brick built up to the tops of the columns on all sides of the building. These walls carry no vertical loads, but must do the work of the vertical bracing between the columns in the sides and ends. This bracing is, therefore, omitted; but there should be in the sides of the building a top strut running from column to column, and connecting to the inner flanges, in order to clear the brick wall. If desired, the top strut may connect on the centre line of columns, in which case the wall must be built around it.

Buildings of the first type of construction are suitable to withstand the action of the heaviest jib and travelling cranes, since they are able to resist large horizontal forces, as well as the usual vertical loads. The second type, while suitable for buildings of heavy construction, and especially adapted for machine-shops, are not so well fitted to endure the action of cranes of large capacity, particularly if the building be high and narrow. If for any reason the brick walls are considered essential and it is desired to secure the greatest strength and rigidity possible, regardless of expense, a combination of these two types would give a result which could not be surpassed.

The last type of construction to be mentioned is well adapted to buildings intended for heavy machinery and light jib-cranes, and may be used for suspended travelling-cranes. In this third type there are no iron columns. The walls of the building are brick, and the roof trusses rest directly upon them. Additional rigidity can be secured by the introduction of iron columns at the four corners of the building. These columns should be well anchored to the foundations. The roof trusses should be braced in the planes of the top and bottom chords. If there are no jib-cranes to be provided for the bottom chord bracing may be put in alternate panels only; in which case, however, the top chord bracing should be in every panel to keep the walls in line.

482. Vertical Loads or Forces.—Under this head come the weight of the iron frame and its covering; the weight of snow; the vertical component of the wind pressure; travelling cranes and their lifted loads; the weight of pipes, machinery, and shafting; the action of

* This chapter has been adapted from a paper contributed to the Engineers' Society of Western Pennsylvania, October, 1892.

driving-belts; and such additional loads as may arise in individual cases. The importance of considering the action of stresses due to cranes is evident when it is remembered that there are now in use jib-cranes of varying capacity up to fifty tons and travelling cranes up to one hundred tons and even one hundred and fifty tons lifting power. These cranes produce not only direct stresses in main members and bracing, but also heavy bending moments in the columns and alternating stresses in various members throughout the building. The action of long lines of shafting, with their driving-pulleys and belts, and of hydraulic and steam machinery, frequently contributes a large share to the sum of uncertain and, too often, unconsidered stresses to which a mill building is subject. While it is impossible to determine these stresses with accuracy, it is, nevertheless, necessary to make allowance for them; and the proper way to deal with them is to assume, as nearly as may be, an equivalent uniform load and a single concentrated load which may be applied at will to any member of the structure. These vertical loads vary so in amount, depending on the span and type of building, that it is impossible to give an equivalent load per square foot which could be used indiscriminately. It must be ascertained independently for individual cases. The action of the vertical loads on the structure, when once they or their equivalents have been computed, is readily determined, and need not be further discussed.

483. **Horizontal Forces.**—Under this head come the horizontal component of the wind pressure and the thrusts from the travelling-cranes, jib-cranes, belts, etc. The horizontal action due to the cranes and wind is of considerable importance, and is an element that does not receive the attention in merits. Fortunately, however, the maximum wind and crane loads occur but rarely, and the probability of their occurring simultaneously is so small that, if we proportion each member for the larger stress, we may disregard the smaller. Horizontal forces act on a building, tending to move the structure horizontally and to overturn it as a whole. As a result, stresses are produced in the individual members throughout the building. Friction and the weight of the structure are usually sufficient to counteract the tendency to movement, and we need consider the action on the individual members alone. This action can be best shown by means of a stress diagram (see Plate XXXII, Fig. 1).

484. **Method of Analysis.**—The following conditions will be assumed to illustrate the method of analysis:

	Ft.	In.
Height to centre line of bottom chord of roof truss.... ...	42	6
Rise of roof truss..	16	0
Height of ventilator......................................	7	6
Total height of building..................................	66	0
Span of building, centre to centre of columns............	64	0

The building consists of a rigid iron framework throughout.

The wind force will be taken at the low value of 20 lbs. per square foot. The wind will be assumed to blow in a horizontal direction in all cases. The vertical dead load acting on roof is taken at 25 lbs. per square foot, horizontal projection; with an additional vertical loading, acting on columns, of 25 lbs. per square foot, horizontal projection. These two loads cover the weight of the iron framework of the structure, the roof load being the weight of the trusses and their covering, and the additional column load being the weight of the columns and their bracing. The permanent loads only have been selected for analysis, so that the liability to alternating wind stresses in the roof trusses may be fully shown. While the estimate for dead weights is made to more than cover their amount in general practice, no extreme data have been used.

Were we designing the building for construction, it would be necessary to assume, in addition to those already mentioned, vertical loads something like the following:

For snow, 10 lbs. per square foot.

For equivalent uniform load, 10 to 15 lbs. per square foot.

For concentrated load, 10,000 lbs.

The equivalent uniform and concentrated loads above mentioned are those referred to in Article 482, and cover the action of shafting or other indeterminate loads which would affect a building according to the particular purposes for which it is intended. If cranes were to be put in the building, their action would also have to be considered. For the present discussion these loads are not included in the analysis.

The diagrams on Plate I show:

CASE I. The stresses in roof trusses, columns, and knee-braces, for wind acting on one entire side of the building and roof, for columns hinged at base.

CASE II. The same conditions of loading, for columns rigidly fixed at base by anchor bolts.

CASE III. The stresses in roof trusses and columns for permanent dead load.

CASE IV. The stresses in roof trusses for wind on roof only. The roof trusses rest on walls, and are anchored at both ends.

For Cases I, II, and IV the entire horizontal force of 20 lbs. per square foot is considered on the vertical projection of all surfaces acted on by the wind. The vertical component is disregarded.

CASE V. The stresses in roof trusses, columns, and knee-braces, for wind acting on one entire side of building and roof, for columns rigidly fixed at base by anchor bolts.

For Case V only, the analysis is made for the resultant normal pressure on the several surfaces acted on by the wind.

It is customary to consider the resultant wind pressure as acting normally to the roof surface. We have, however, in the present instance, for the purpose of comparison, found the stresses resulting from the normal pressure, and also from the full horizontal force of the wind acting on the vertical projection of the entire building. In the latter case the vertical component has been neglected. When the resultant normal pressure is taken, as in Case V, the wind stresses may be found in one diagram for this normal pressure, or the stresses for the horizontal and vertical components may be found separately and combined for total wind stresses.

Cases II and V show a comparison of the effects of the full horizontal force of the wind and of the resultant normal pressure. All the conditions in these two cases are the same, except that of the direction of the resultant wind pressure.

In high buildings, especially with ventilators, on account of the small proportion of exposed roof area, the difference in the resulting stresses is not very marked; the normal pressure giving in most members the smaller results. For a discussion of wind pressures, see Chap. III, Art. 52.

The intensity of the wind stresses throughout the roof truss is dependent, other conditions remaining constant, upon the ratio of the total length of the column to the length of the portion above the foot of the knee-brace. The greater this ratio, the greater will be the stresses. Although the knee-braces have been assumed of greater depth than will ordinarily occur in practice, an examination of the diagrams will show for Cases I and II, and even for Case V, that the wind stresses in some of the principal members on the windward side of the roof trusses are equal to the dead load stresses, while on the leeward side the stresses are of opposite character and of even greater intensity; note especially the stresses in the top and bottom chords of the roof trusses, in the two main diagonal ties running to the ridge, in the knee-braces and in the adjoining diagonals.

The horizontal reaction or shear at the base of the columns is a known quantity; it remains the same whether the columns are hinged or rigidly fixed at the piers. For the two

columns of a bent it is equal to the horizontal component of the wind on one panel of the building and roof. This total shear or reaction is assumed to be equally distributed between the two columns. The wind loads are considered concentrated, and the vertical reactions at the feet of the columns are disregarded in the analysis of stresses in trusses and columns, but are considered in the calculation for anchor bolts, masonry, etc.

External Forces.—The columns, in deflecting from the wind loads, have a leverage action producing certain horizontal reactions at the feet of the knee-braces, and at the bottom chords of roof trusses, which must be considered as external forces in finding the vertical reactions, and in the analysis for stresses in the knee-braces and roof truss.

Vertical Reactions from Horizontal Forces.—Taking the centre of moments at any point in the axis of either column, the vertical reaction at the opposite column, in any case, is equal to the algebraic sum of the moments of all the external forces above this point, divided by the span of the building. The centre of moments may be taken at any point in the axis of the column; but the most convenient points are at the bases of the columns, at the points of contraflexure, and at the feet of the knee-braces.

Vertical Reactions from Vertical Forces.—The vertical reaction at either column resulting from the vertical components of the horizontal wind force is equal to the sum of the moments of these vertical forces about a point in the axis of the other column divided by the span of the building.

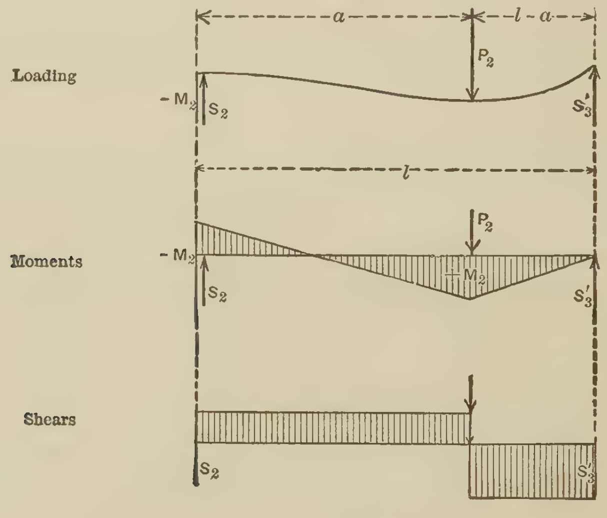
For analyses in which only the horizontal component of the wind force is dealt with, as in Cases I and II, the vertical reactions resulting from the horizontal component only need be considered. When, however, as in Case V, the analysis is made for the horizontal and vertical components of the resultant normal wind pressure, the algebraic sum of the corresponding vertical reactions must be considered. In all cases the wind and dead load stresses have been determined separately, for the purpose of comparison and to locate the alternating stresses. To determine the net stresses in the members and the resultant overturning action on the building, the stresses and reactions for dead and wind loads must be combined algebraically.

485. **Action of Horizontal Forces on the Columns.**—The column in resisting horizontal forces acts as a beam, and when rigidly anchored it may be considered as fixed in direction at the base, but when not properly anchored it must be considered as hinged at the base, or, in other words, free to rock on the pier. Theoretically we might treat the column as fixed in direction at the top also, when rigidly connected to the roof truss; but as we are not at all sure of realizing a sufficient degree of rigidity at this point, it will be safer to consider the column as simply supported at the top in all cases. In reality there would be but little gain in fixing the column in direction at the top, as it must be done at the expense of resulting bending moment in the roof truss. The column, then, in resisting horizontal forces will, when rigidly anchored at the base, act as a beam fixed at one end and supported at the other, but when not properly anchored it will act as a simple beam supported at both ends. The anchor bolts which fix the column have to resist the bending moment at the base; or, in other words, they must counteract the tendency of the column to rock on the pier. They must also resist the horizontal thrusts not overcome by friction. The rocking tendency is influenced by the width of column base and by the depth of vertical bracing. The lower the bracing extends, the smaller will be the bending moment at the foot of the column, and the less the pull on the anchor bolts. If the bracing were to extend the entire length of the column, the bending moment at the base and the pull on the anchor bolts would be reduced to zero. In practice the bracing can run down the column a short distance only, and bending moments will occur at the foot of the bracing and at the base of the column. As a result, the column, acting as a beam, will be distorted, and a point of contraflexure will occur in the unsupported portion. If the anchor bolts be removed, the case will be a beam supported at

both ends. The point of contraflexure will then disappear and the bending will be largely increased.

From this discussion we observe the following important facts: Deep bracing and strong anchorage reduce the stresses in the columns, roof trusses, and the bracing itself. In designing the column the moments at the base and at the connection of the bracing should be provided for in the section. The anchor bolts should be made of sufficient section to thoroughly fix the column at its base.

The analysis for Case I is the same as that for similar cases in Art. 115, of Chap. VII, so far as the reactions at the base of the columns are concerned. The analysis of Case II is given in Art. 151, Chap. X. In this case the horizontal and vertical reactions act at the point of inflection, as found in Art. 151, and the remaining analysis is the same as that in Art. 115. In the analysis for these reactions as shown in Plate XXXII, the tops of the posts were assumed to remain in the same vertical with the bottom, and the ordinary continuous girder formulæ were used, as is graphically shown in Fig. 449. This puts the point of inflection too



Case II.

FIG. 449.

low, and increases the stresses in the knee-braces and roof truss. The more rigid analysis given in Art. 151 should be used.

After the horizontal and vertical reactions have been determined, the stresses in the roof trusses and the knee-braces can be easily found graphically. The graphical treatment is shown in full on Plate XXXII, Fig. 2. Taking the foot of the windward knee-brace as a convenient starting-point, the force polygon is constructed as follows: from a to l to c-d-e-f-x-y-t to a to close at point of starting; the polygon being made up of the wind loads and the horizontal and vertical reactions mentioned above. The shear at the base of the column does not directly appear in the force polygon or in the stress diagram, but its equivalent has been considered in the load at the foot of the knee-brace and in the shear at the top of the column.

Notice that at the foot of the windward knee-brace the concentrated wind load at that point must be deducted from the load P_1 or P_2 to find the net horizontal reaction which is to be used in the force polygon and which is the horizontal component of the stress in the windward knee-brace.

The analysis for the horizontal action of crane loads is similar to that given for wind, except that it is simpler, as there are fewer loads to deal with; therefore it is not necessary

to give the analysis for this class of forces, but in practice the maximum stresses arising from either wind or crane loads should be provided for, considering such part of the crane load as may be deemed proper for the bent under consideration.

486. **Systems of Bracing.**—Mill buildings should be thoroughly braced to resist the action of horizontal forces and to keep them properly lined up. For this purpose longitudinal and transverse bracing in a vertical plane and lateral bracing in a horizontal plane are required.

Ventilator Bracing.—There should be a strut running from the top of each ventilator post to the ridge of the main roof truss, to prevent distortion of the ventilator frame. Vertical longitudinal bracing is needed in the sides, and at the centre line of ventilators, to keep the frames upright and parallel. When swinging windows are placed in sides of ventilators the bracing there must be omitted, and replaced by stiff bracing fastened to the ventilator purlins.

Rafter Bracing.—The purpose of this bracing is to keep the trusses from overturning, and it should be strong enough to resist the action of the wind on the gable end of the roof. This bracing may be omitted in alternate bays.

Bottom Chord Bracing.—The purpose of this bracing is to keep the columns in line and to distribute concentrated loads.

Vertical Bracing between Columns should be provided to transmit to the foundations all stresses due to horizontal forces. The greater the depth of this bracing for given length of column, the less will be the resulting stresses in the columns, trusses, and in the bracing itself. Usually it may come down to a point eight to ten feet from the ground. The bracing in the sides takes the longitudinal thrust of the wind and crane loads, and the bracing in the ends resists the part of the transverse action of those loads received through the bottom chord bracing.

It will be well to consider, in a general way, the action of the wind and crane loads on these systems of bracing in the sides and ends of a building. Both of these classes of forces, if allowed, would act on the structure in the same general manner; but the different conditions under which they are applied warrant the provision of separate systems of bracing to transmit them to the foundations. It is impracticable, in long buildings, to make the bottom chord bracing heavy enough to carry the cumulative wind loads from the successive panels to the ends of buildings. It is, therefore, necessary that each intermediate bent should resist its own wind load and a portion of the distributed crane loads. To do this knee-braces are provided at each bent, running from bottom chord to column. The conditions of head room allow the knee-brace to extend down the column a short distance only, and the resulting stresses from the wind, as has been shown in the diagram, are large in the trusses and columns and in the knee-braces themselves. These members should, therefore, be made abundantly strong to resist these stresses. The uniform wind loads having been provided for, the crane loads and other local loading only remain to be considered. The bottom chord bracing must take these loads and distribute them to the adjacent bents, transmitting the residue, if any, to the vertical bracing in the ends of the building. This is an economical arrangement, since the maximum crane stresses occur simultaneously in two or three panels only, and, consequently, are not seriously cumulative. Crane stresses are occurring constantly, and their repeated action would be a severe test upon the rigidity of the structure if each bent were required to resist the stresses from loads in the adjacent panels. On the other hand, large wind loads occur rarely, and the individual bent can withstand them without injury. We do not mean to say that in practice the wind will be taken by the individual bents and the crane loads by the bottom chord bracing, because there is, necessarily, some uncertainty as to the distribution of these loads; but the bottom chord bracing will have served its purpose if it relieve individual bents of the racking effect of concentrated loads.

487. Types of Roof Trusses.—*Sections and Details.*—While there are several types of roof trusses well adapted to mill buildings, probably the one in most common use is the "French" roof truss. This truss is simple in design and suitable for either light or heavy construction. Whatever the type, it is always best to make the bottom chord of the truss straight, except when the conditions of head room require it to be raised.

Irrespective of the type of roof truss, there are two radically different styles of construction which may be used: one is pin-connected and the other is riveted work throughout. While there is a large variety of sections from which to choose in the riveted style, the T-shaped section is most commonly used. This is composed of two angles placed back to back, with or without a single web plate between. This form of section may be used for both tension and compression members. A single flat bar may be used for the smaller tension members. Roof trusses built in this manner lack somewhat in rigidity and lateral stiffness.

In the all-iron type of building—that is, where the roof trusses rest directly upon the columns—rigidity and stiffness are important factors, and the construction shown in Fig. 1. Plate XXXIII, is frequently used with satisfactory results. In this case the compression members are made up of two channel-bars turned back to back and latticed, forming a box section, For members subject to tension only loop-eye rods are used. The truss is pin-connected throughout. Upon examination of the figure, it will be observed that all the connections are readily made, and that the details are simple in design. But little riveting is required, the shop-work can be easily and quickly turned out, and the building rapidly erected in the field. The bottom chords of the trusses will stand heavy loads in bending, and afford especially good connections for jib-cranes and lateral bracing. The roof trusses have such strength and rigidity as to impart stiffness to the entire building. This style of construction is well adapted to both light and heavy buildings.

There is another style of construction suitable for heavy work only, which is quite as rigid as the one just mentioned. In this the top and bottom chords are of box sections, and made up of plates and angles instead of channels. The web members have an I shape, and are made up of four angles placed back to back in pairs and latticed. All the truss members are stiff and may have either pin or riveted connections.

When these built sections are used with riveted connections, the shop-work is somewhat less expensive than for the pin-connected channel construction. When, however, pin connections are to be used for both styles of construction, the channel sections require less shop-work and afford simpler details than the built sections.

Roof columns and light crane columns may be constructed of Z bars, channels, or plates and angles, but for simplicity of details the channel columns are to be preferred to all others. For heavy crane columns, however, it may be necessary to use plates and angles to secure a column of sufficient width.

Referring again to roof trusses, whatever style of construction is used, the following points should be well considered: The trusses must be designed to withstand the maximum stresses produced by any possible combination of vertical and horizontal loads or by the vertical loads alone. The bottom chords of trusses should be made stiff enough to take both the tension and compression to which they are subjected as well as the bending from local loads. They must afford a convenient connection for the longitudinal struts in the bottom chord bracing; in fact, they are themselves important struts in that system of bracing.

488. Columns and Girders for Travelling-cranes.—Track girders for light travelling-cranes may be supported on brackets on the roof columns; but as this arrangement gives an eccentric loading, it is better, for heavy cranes, to provide a separate column to carry this load. To prevent unequal settlement, the crane columns and roof columns should stand on the same base. The connection between these columns should be rigid, so that, as far as possible, they

may act as one member. The girders must be thoroughly connected to the crane columns and braced laterally to the roof columns. Knee-braces from crane columns to girders give longitudinal stiffness. For track girders I beams or plate girders are preferable to lattice girders. Long, heavy girders should have the box section, with diaphragms at intervals between the webs. (See Plate XXXIII, Fig. 5.)

Rails for Track.—In order that the alignment of the track may be true, it is best to drill the holes for rail-fastening with girders in position and the rails lined up. Oak packing is frequently put under the rails and at the girder seats to insure smoothness in the running of the cranes. Several rail sections, with method of fastening, are shown on Plate XXXII. Details of crane columns and track girders are shown on Plate XXXIII, Fig. 5.

489. **Details of Construction.**—*Splices and Connections.*—Members subject to tension and compression should have their splices and end connections made to resist the maximum stresses of both kinds. When bending stresses occur, they should be considered in designing the splices; this necessitates both flange and web splices. As an example, the splices in the bottom chord of the roof trusses should be able to develop the full strength of that member in tension, compression, and bending. The splice at the ridge of the roof truss has to resist the horizontal thrust from the rafters and any vertical shear which may arise from unsymmetrical loading. It is well to remember that the wind is an important case of unsymmetrical loading. In addition, provision must be made to introduce into the rafters the stresses from the web members which meet at this point.

The connection of roof trusses to columns has to resist large horizontal and vertical stresses. In the analysis given for wind in Case I, shown on Plate XXXII, the horizontal shear on the leeward side is 40,000 pounds and the upward pull from the roof truss is 22,000 pounds, which gives a resultant of about 46,000 pounds to be resisted by the connection. This resultant is net, that is, exclusive of the 25,000 pounds dead load from the rafter. On the windward side there occurs a resultant of 55,000 pounds from wind alone, or 70,000 pounds from wind and dead loads combined. The style of connection determines whether, on the windward side, the larger or the smaller resultant should be provided for.

Column Bases.—The gusset plates and bracket angles at the foot of the column have several important duties to perform, which require them to be thoroughly connected thereto. They assist in distributing the column loads to the base-plate; they give stability to the column by broadening its base, and furnish a connection for the anchor bolts. The base-plate should be large and strong enough to distribute to the pier the loads received from the enlarged column base. The anchor bolts should pass through the bracket angle, and at the same time be placed as far from the centre line of the column as practicable.* They should be anchored in the masonry sufficiently to develop the full strength of the bolts. Large anchor bolts should be built deeply into the masonry; small anchor bolts, however, may be roughened or split and wedged and set in drilled holes, with hydraulic cement, lead, or sulphur. (See Plate XXXII, Fig. 1.)

Piers.—The masonry piers for supporting the columns should be of sufficient dimensions, weight, and strength to resist the vertical loads, the horizontal thrust, and the overturning action from the column. They should extend well below the frost line and reach a firm bottom. If necessary, concrete or piles may be used to secure a suitable foundation. To secure a uniform distribution of the column loads, the piers should be provided with a cap consisting of either a single stone or a cast-iron plate. If the stone be used, it should be large enough to give a margin all around the column base equal to one third of its thickness, which latter should be at least one third its largest dimension.

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* This connection cannot possibly develop the full bending resistance of the column. Vertical angles riveted to the base of the column, with a top plate to receive the nut of the bolt, is to be preferred if the column is assumed to be fixed in position at bottom. (See Figs. 426 and 434.)—J. B. J.

Purlins.—Angles, Z bars, or I beams may be used for roof purlins, with or without trussing. Single channels do not have sufficient lateral stiffness to make good purlins. Several styles of trussed purlins are shown on Plate XXXVII, Figs. 1, 2, and 3. Fig. 2 shows a simple and desirable style of trussing, using star shapes and flat bars. Fig. 1 shows a good section for a long and heavily loaded purlin, made up of two channels, forming a box section and held together by tie-plates. Loop-eye rods are used in the trussing, which is pin-connected. Care should always be taken to turn the shape used as a purlin so as to secure the greatest vertical depth. Figs. 5 and 6 show respectively the correct and incorrect method of placing the purlin upon the rafter. Fig. 5 also shows the method of bracing I-beam purlins to rafter by the use of bent plates. Purlins act as longitudinal struts in the system of rafter bracing. At the ridge, under ventilators, no purlins are required. It is, therefore, necessary to have a ridge strut to complete this system of bracing. Long purlins are liable to sag in the plane of the roof. Fig. 4 shows a method of holding them in place by tie-rods between the purlins and running from ridge to eaves.

Expansion.—Roof trusses resting on brick walls should have one end free for expansion. A wall plate, for a sliding surface, should be used for spans to about 75 feet, and rollers should be provided for larger spans. Roof trusses resting on columns must be attached rigidly thereto, no provision being made for expansion. It is customary to introduce an expansion panel in long buildings, at points 100 to 150 feet apart, to provide for longitudinal expansion.

Corrugated Iron Sheeting.—The use of corrugated sheeting to close the sides and ends of buildings, where brick walls are not present, has already been mentioned. It is used as well for a roof-covering, and has the advantage of being cheap, light in weight, and incombustible. Furthermore, it is quickly and easily put on and readily renewed. Its most objectionable quality is its liability to rust. This can be retarded if the sheeting is kept well painted on both sides, and still further by using galvanized sheeting. Common weights of sheeting for roofs are Nos. 18 and 20, and for sides of buildings Nos. 20 and 22. For No. 20 sheeting, purlins should be spaced not over 6 feet centres, and preferably less than this.

Lighting and Ventilation.—Ample provision should be made for lighting and ventilation. Windows with swinging, sliding, or fixed sash may be introduced in the sides and ends of the building and in the sides of ventilators. Louvres for ventilation may replace the windows where desired. Additional lighting surface can be obtained by the introduction of skylights on the main and ventilator roofs. In the sides of a building which has brick walls it is a simple matter to provide openings for windows and doors. In buildings with no brick walls, framework between the columns is necessary to support the doors, windows, and sheeting. This framework may be all timber, all iron, or a combination of iron and timber. The framework entirely of timber is suitable for light buildings only. The framework entirely of iron is required in buildings where combustible material is prohibited. In the latter the bracing and framework supporting the windows are of structural iron, and the window frames and sash are galvanized iron. An all-iron frame for a gable is shown on Plate XXXVII, Fig. 7. The combination timber and iron framework is suitable for all buildings where absolute fire-proof construction is not needed. In this case we have a complete system of iron bracing between the columns. The main girts are of iron, but the girts and posts supporting the windows and corrugated iron are of timber and are introduced between the main girts wherever needed. This style of construction is shown on Plate XXXIII. In the all-iron type of construction the bracing between the columns is frequently stopped 8 to 12 feet above the ground. The space below the bracing may be left entirely open or closed with corrugated sheeting and swinging, sliding, or lifting doors.

General Conclusions.—In the design of mill buildings of the present day, it seems to the writer that there are certain stresses incident to the every-day operation of the mill which do not receive proper recognition. We can with profit make a comparison of the design of the

mill building with that of the railroad bridge, since in general the same principles apply to both. In the latter, provision is made for certain secondary stresses, such as impact, wind, and centrifugal force. In addition, the probable increase in loading during the life of a structure is considered. In like manner, we have in the mill certain secondary stresses due to the action of cranes and wind, of equal importance. The question of the future increase in loading of the mill building should be given the same attention as it receives in bridge construction. We have already considered the action of these secondary forces upon the building, and have shown that they must be provided for by the use of liberal sections and suitable bracing in order to secure rigidity. The uncertainty respecting the increase in loading which future conditions may dictate for the structure is especially marked in the case of extensive plants, in which, on account of rapid development, radical changes are not unusual. While the writer would not be understood to advocate the introduction of material in members throughout the structure regardless of their present or probable future requirements, he does hold that in the long-run it is economy, in the case of permanent structures, to provide not only for loads which it is known will occur, but for those which experience teaches are within a reasonable range of probability. The objection to all this is that it costs money, but small first cost is not always true economy. A cheap building will in time cost enough for repairs and remodelling to make it an expensive investment. Furthermore, delays in the operations of the mill, which are liable to result from weak and faulty construction, are at best expensive drawbacks, and frequently are far-reaching in their consequences.

A principle well worth noting, and one which argues strongly in favor of designing for liberal loads, may be stated in this connection: After the material has been provided to make a structure strong enough to carry a moderate loading, the introduction of a reasonable amount of extra material will give an increase in carrying capacity which is entirely out of proportion to the expense incurred for such increase.

In conclusion, we may say that the three most essential factors in the design and construction of mill buildings, named in the order of their importance, are strength, simplicity, and economy.

The accompanying plates (XXXII to XXXVII) represent several types of structures recently erected in the vicinity of Pittsburg.* They are largely self-explanatory and need no further description in the text.

ANALYTICAL TREATMENT.

490. **Analysis for Wind.**—The following is a brief statement of the conditions assumed and the methods employed in the analysis for wind stresses. The reactions, shears, and moments are determined analytically; the resulting stresses in the knee-braces and roof trusses are determined graphically, as shown on Plate XXXII.

Case I in the accompanying sketch represents the loads, shears, and moments for column hinged at base and acting as a simple beam supported at both ends.

Case II represents the loads, shears, and moments for column rigidly fixed in direction at base by anchor bolts and acting as a beam fixed at one end and supported at the other.

NOTATION.

Known Terms:

b = span of roof;

l = length of beam or column;

a = distance from base of column to foot of knee-brace or to the point of application of the load P_1 or P_2;

* The authors are indebted to the courtesy of the management of the Keystone Bridge Works for the drawings of work, constructed by them, from which these details were selected.

$k = \frac{a}{l}$

S_1 = the horizontal component of reaction, or shear, at base of column when hinged at base. Case I;

S_2 = horizontal component of reaction or shear at base of column when fixed at base. Case II;

$S_1 = S_2$ = one half external horizontal force.

S_2' = shear at top of column when hinged at base $= S_1 \frac{a}{l - a}$.

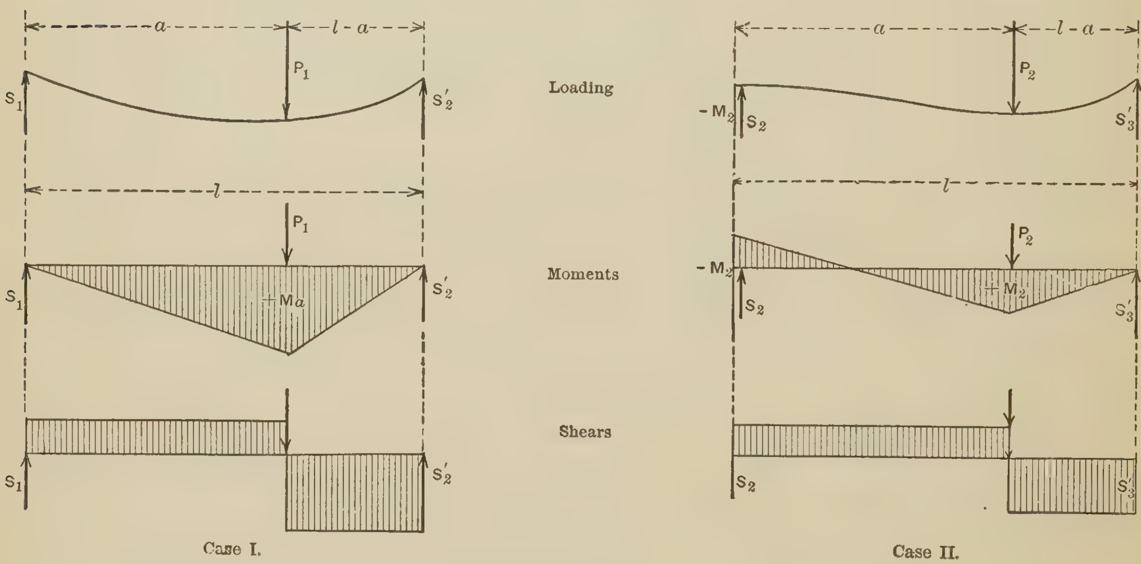

FIG. 450.

Unknown Terms:

S_3' = shear at top of column when fixed at base;

P_1 = horizontal thrust or load at foot of knee-brace due to the leverage action of the column when hinged at base;

P_2 = horizontal thrust or load at foot of knee-brace due to the leverage action of the column when fixed at base;

M_2 = bending moment at the foot of column when fixed at base;

M_x' = moment at any section of column distant x from the base, for column hinged at base;

M_x'' = moment at any section of column distant x from the base, for column fixed at base;

M_a' = moment in column at foot of knee-brace = maximum bending moment for column hinged at base;

M_a'' = moment in column at foot of knee-brace = maximum bending moment for column fixed at base;

M = the sum of the moments of the horizontal wind loads above any point in the axis of either column distant x above the base, which is taken as the centre of moments; note especially that this is a variable quantity, its value depending upon the height of the point taken as the centre of moments;

V_1 = the vertical reaction at either column due to the overturning action of the wind on one entire side of building and roof for columns hinged at the base;

V_2 = the vertical reaction at either column due to the overturning action of the wind on one entire side of building and roof for columns fixed at base.

FORMULÆ.

491. Case I.*—For columns hinged at the base and considered as a simple beam supported at both ends:

$$S_2' = \frac{S_1 a}{l - a}; \quad \ldots \ldots \ldots \quad (1)$$

$$P_1 = \frac{S_1 l}{l = a}; \quad \ldots \ldots \ldots \quad (2)$$

$$P_1 = \frac{S_2' l}{a}; \quad \ldots \ldots \ldots \quad (2a)$$

$$M'_{x \leqq a} = S_1 x; \quad \ldots \ldots \ldots \quad (3)$$

$$M'_{x > a} = S_1 x - P_1(x - a); \quad \ldots \ldots \ldots \quad (3a)$$

$$M'_{x > a} = S_2'(l - x); \quad \ldots \ldots \ldots \quad (3b)$$

$$M_a' = S_1 a; \quad \ldots \ldots \ldots \quad (4)$$

$$M_a' = S_2'(l - a). \quad \ldots \ldots \ldots \quad (4a)$$

$$V_1 = \frac{m}{b} = \frac{\text{hor. comp. of wind force on one panel of building and roof} \times \frac{1}{2} \text{ total height}}{b}. \quad (5)$$

This vertical reaction is constant throughout the column up to the knee-brace. Above that point it is increased or diminished by the vertical component of the stress in the brace.

492. Case II.†—For columns rigidly fixed at the base and considered as beams fixed at one end (the base) and supported at the other (the top):

$$M_2 = \frac{-P_2 l}{2}[2k - 3k^2 + k^3]; \quad \ldots \ldots \ldots \quad (6)$$

---

* In the analysis for this case the column has been considered as a beam with unyielding supports; strictly this condition will not be realized, for as the top of the column deflects to the leeward, the support at that end will yield an equal amount, and the resulting stresses will be somewhat larger than found by the analysis. However, this increase is in a measure counteracted by the condition of partial fixedness at the top of the column, which has been disregarded in the analysis.

† The following analysis is proximate. It assumes that the top of the building does not deflect laterally. The rigid analysis for the general case is found in Art. 412, and for the case here assumed in Art. 151.—J. B. J.

or taking moments about the base of column, we may write, in terms of P_2 and S_3',

$$M_2 = -P_2 a + S_3' l; \quad \ldots \ldots \quad (6a)$$

$$M''_{x<a} = -M_2 + S_2 x; \quad \ldots \ldots \quad (7)$$

$$M''_{x>a} = -M_2 + S_2 x - P_2(x-a); \quad \ldots \ldots \quad (7a)$$

at point of contraflexure $M_{2x} = -M_2 + S_2 x = 0$, and

$$x = \frac{M_2}{S_2} = \text{distance from base of column to point of contraflexure}; \quad \ldots \quad (7b)$$

or independently in terms of S_3' we may write

$$M''_{x>a} = S_3'(l-x); \quad \ldots \ldots \quad (7c)$$

$$M''_{x<a} = S_3'(l-x) - P^2(a-x); \quad \ldots \ldots \quad (7d)$$

$$M_a'' = -M_2 + S_2 a; \quad \ldots \ldots \quad (8)$$

$$M_a'' = S_3'(l-a); \quad \ldots \ldots \quad (8a)$$

$$S_2 = \frac{P_2}{2}[2k - 3k^2 + k^3] + P_2(1-k). \quad \ldots \ldots \quad (9)^*$$

Solving eq. (9) for P_2,

$$P_2 = \frac{2S_2}{(2k - 3k^2 + k^3) + 2(1-k)}; \quad \ldots \ldots \quad (9a)$$

$$S_3' = -\frac{P_2}{2}[2k - 3k^2 + k^3] + P_2 k.^* \quad \ldots \ldots \quad (10)$$

The vertical reaction V_2 is best found for the point of contraflexure, and this is then the vertical stress in the post, from wind, from the base to the foot of the knee-brace. While the vertical reaction could be found by passing a horizontal section through both columns at any elevation, and summing the moments of the external forces on either side of this section, we would, in general, have to include in these moments the bending moments at this section in the two columns cut. Taking the section through the points of contraflexure, however, we obtain

$$V_2 = \frac{m}{b} \text{ at point of contraflexure or when } x = \frac{M_2}{S_2}, \quad \ldots \ldots \quad (11)$$

or, so far as the overturning action of the wind on the building is concerned, the columns have virtually been shortened by the amount $x = \dfrac{M_2}{S_2}$, which is the distance from the base of

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* Equations (6), (9), and (10) are expressions for a beam fixed at one end and supported at the other, referred to the origin of co-ordinates at a point over the support at the fixed end, and can be found in text-books on the subject, and can be derived from the formulæ given in Art. 131. Valuable tables for the easy solution of these equations are given in Howe's "The Continuous Girder." (*Engineering News* Pub. Co., 1889.) The other equations under Case II and all the equations for Case I can be written directly from a consideration of the external forces, and require no extended demonstration.

column to the point of contraflexure. In fact, an examination of the several equations for moments, shears, and horizontal reactions will show that Case II becomes in all respects Case I, with the base of column moved up to the point of contraflexure.

This can also be shown by an inspection of the moment and shears diagrams on Plate I. This is true not only for the moments and shears for the part of the column above the point of contraflexure, but also for the stresses in trusses and knee-braces. There will occur, however, below the point of contraflexure, shears, and a negative bending moment, of which the action on the column and pier must be considered.

In the above formulæ, several expressions have been given for the values of V_1 and V_2.

For a given analysis, however, only one of these expressions need be used; the position taken for the centre of moments determining which formula shall be chosen.

For an analysis under Case I, for columns hinged at the base, formulæ (1), (2), (4), and (5) only, need be used; and for an analysis under Case II, formulæ (6), (8), (9*a*), (10), and (11*a*) only, are required. The supplementary formulæ may be used as substitutes for those first named when so desired.

The horizontal reaction or shear at the base of a column is a known quantity; it remains the same whether the columns are hinged or rigidly fixed at the piers. For the two columns of a bent it is equal to the horizontal component of the wind on one panel of the building and roof. This total shear or reaction is assumed to be equally distributed between the two columns. The wind loads are considered concentrated, and the concentration at the foot of the column is disregarded in the analysis of stresses in trusses and columns, but is considered in the calculation for anchor-bolts, masonry, etc.

493. External Forces.—The columns in deflecting from the wind-loads have a leverage action producing certain horizontal reactions at the foot of the knee-braces and at bottom chord of roof truss, which must be considered as external forces, in finding the vertical reactions and in the analysis for stresses in the knee-braces and roof truss.

494. Vertical Reactions from Horizontal Forces.—Taking the centre of moments at any point in the axis of either column the vertical reaction at the opposite column, in any case is equal to the algebraic sum of the moments of all the external forces above or below this point plus the sum of the moments in the two columns themselves at this horizontal plane, divided by the span of the building. The centre of moments may be taken at any point in the axis of the column, but the most convenient points are at the base of the column, at the point of contraflexure, and at the foot of the knee-brace. When taken at this point of contraflexure the moments in the columns themselves disappear.

495. Vertical Reactions from Vertical Forces.—The vertical reaction at either column resulting from the vertical components of the horizontal wind-force is equal to the sum of the moments of these vertical forces about a point in the axis of the other column, divided by the span of the building.

For analysis in which only the horizontal component of the wind-force is dealt with, as in Cases I and II, the vertical reactions resulting from the horizontal component only need be considered. When, however, as in Case V, the analysis is made for the horizontal and vertical components of the resultant normal wind pressure, the algebraic sum of the corresponding vertical reactions must be considered. In all cases the wind and dead load stresses have been determined separately for the purpose of comparison and to locate the alternating stresses. To determine the net stresses in the members and the resultant overturning action on the building, the stresses and reactions for dead and wind loads must be combined algebraically.

496. Analytical Process for Case I.—Find S_1, the shear at the base of the column, and substitute its value in equations (1), (2), and (4). Solve (1) for S_2', the shear at the top of the column. Solve (2) for P_1, the horizontal load at the foot of the knee brace. Solve (4) for M'_a,

the maximum positive bending moment in the column. This bending moment occurs at the foot of the knee-brace.

Find m, the sum of the moments of the horizontal wind loads, taking the foot of one of the columns as the centre of moments (see nomenclature), and divide by b, the span of the building; this gives V_1, the vertical reaction at the foot of either column due to the horizontal component of the wind force, as shown in equation (5).

497. Analytical Process for Case II.—Find S_2 the shear at the base of the column, and substitute its value in equation ($9a$), and solve for P_2 the load at the foot of the knee-brace.

Substitute value of P_2 in (6) and (10), and solve (6) for M_2, the negative bending moment at the foot of the column; and solve (10) for S_3', the shear at the top of the column. Substitute values of M_2 and S_2 in (8), and solve for M_a'', the maximum positive bending moment in the column. This bending moment occurs at the foot of the knee-brace. Finally find m, the sum of the moments of the horizontal wind loads (see nomenclature), and substitute the values of m and S_3' in ($11b$), and solve for V_2, the vertical reaction at foot of either column due to the horizontal component of the wind force. (See paragraph relating to "Vertical Reactions from Vertical Forces.")

498. Graphical Process for Cases I and II.—The action of the horizontal forces on the columns has now been fully determined. The horizontal and vertical reactions have also been determined, and the stresses in roof trusses and knee-braces can be easily found graphically. The graphical treatment is shown in full on Plate XXXII, Fig. 2. Taking the foot of the windward knee-brace as a convenient starting-point, the force polygon is constructed as follows: From a to b to c-d-e-f-x-y-t to a to close at the point of starting, the polygon being made up of the wind loads and the horizontal and vertical reactions mentioned above.

The shear at the base of the columns does not directly appear in the force polygon, or in the stress diagram, but its equivalent has been considered in the load at the foot of the knee-brace and in the shear at the top of the columns.

Notice that, at the foot of the windward knee-brace, the concentrated wind load at that point must be deducted from the load P_1 or P_2 to find the net horizontal reaction which is to be used in the force polygon, and which is the horizontal component of the stress in the windward knee-brace.

Plate 37

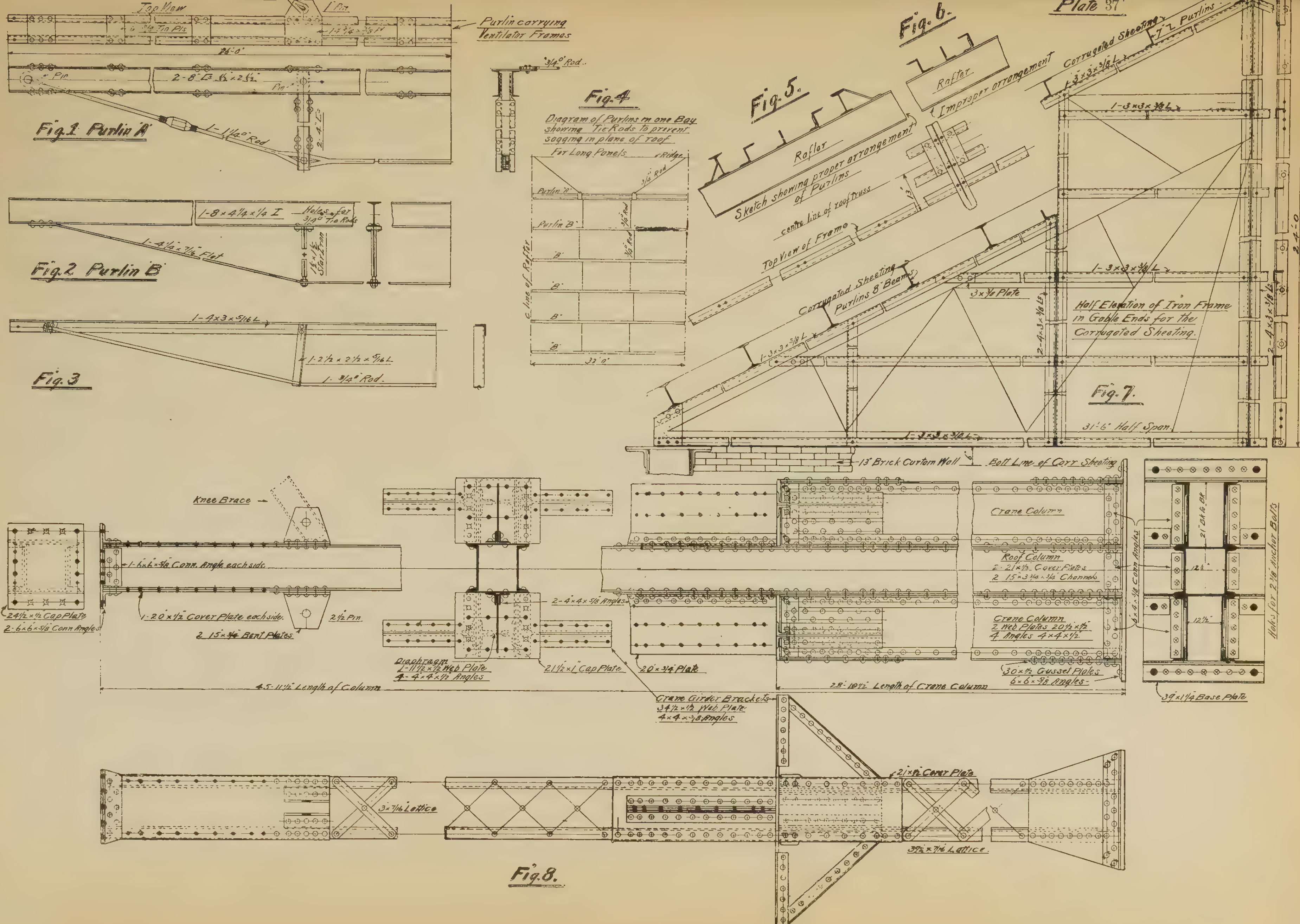

Fig. 8.

APPENDIX A.

STRUCTURAL STEEL AND GENERAL SPECIFICATIONS.*

SOFT STEEL IN BRIDGES.†

IT is not to be expected that a bridge specification, prepared at this time, will contain much that is new or novel. It will, no doubt, contain something of the individual tastes and preferences of the party who expects to use it. If the writer is fortunate enough to include the approved features of existing specifications, a new idea or two will quite suffice for novelty. The accompanying specification is, in fact, a revision of an older one, which in turn derived its salient features from the well-known specification of the Keystone Bridge Company. It is not the writer's purpose, therefore, to enlarge upon the subject-matter in any general or extended way, but to confine his remarks chiefly to one feature. It will be evident to any one who examines the unit stresses that they are expressly intended to put soft steel on at least an equal footing with wrought-iron and medium steel; and the idea in laying the specifications before the Club at this time was to have this feature fully discussed for mutual benefit. While, therefore, any part of the text is open for criticism or suggestion, the steel question is the primary one; and to that alone the writer proposes to address himself.

There are, at this time, three grades of material offered to engineers for structural work—viz., wrought-iron, soft steel, and medium steel. Of these we are using wrought-iron for short spans and medium steel for long spans, while the soft steel, which is in many respects the best grade of material of the three, we are using very little; and there seems to be no immediate prospect, under existing specifications, of extending its use. It has been the writer's good fortune to have an intimate acquaintance, at first hand, with the character of the material now being turned out by our manufacturers, and to have been much impressed, in consequence, with the idea that in neglecting to use soft steel we are doing so to the disadvantage of our bridge structures, and that it is really important to give it a standing in specifications which will allow it to be used on at least equal terms with the other grades of material. Our wrought-iron is certainly not improving in quality, and in certain directions it is distinctively poorer than it formerly was. In the manufacture of steel, however, there has been a notable improvement in quality and uniformity, and the best qualities of soft steel now leave very little further to be desired. As compared with wrought-iron, we have a material which, with 15 per cent higher ultimate strength, has from 20 to 25 per cent higher elastic limit, and from 50 to 60 per cent greater ductility. We have a material which has equal strength, ductility, and bending qualities, lengthwise, and crosswise of the material, in comparison with wrought-iron, which has less strength or ductility and will not bend across its grain. We have, again, a material which has no fibre, and which, consequently, has less disposition to tear or pull out; and, lastly, we have the fine finish and sound, clean metal characteristic of steel, in comparison with the rougher finish and frequently doubtful welding of wrought-iron.

Now, in certain well-known specifications, and with the tacit consent of many engineers, our bridge builders have been privileged, for several years past, to use soft steel on the same basis as wrought-iron. At no time, however, have they taken advantage of this privilege to any great extent, and it seems quite uncertain whether they will do so. We get a little soft steel occasionally when there are long bars to be rolled, which can be gotten out easier in steel, or when there is a large order of plates that can be rolled more economically, and we get occasionally pieces of steel in our structures for other purposes; but anything like a general use of it has not come to pass, and the very simple reason is that it costs more. The writer has endeavored to learn from our manufacturers whether the relative difference between wrought-iron and soft steel would be likely to decrease soon, but, so far as he has been able to ascertain, none of them are prepared.

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* This appendix is adapted from a paper which was read before the Engineers' Club of Philadelphia, by M. F. H. Lewis, October 17, 1891. See also paper by Mr. Lewis before Engrs. Soc. W. Penn., in *Engineering News*, April 25, 1895 (Vol. XXXIII, p. 276).

† The reader is referred to Prof. Johnson's *Materials of Construction* (1897) for a full working knowledge of all kinds of steel and other kinds of building material.

to commit themselves on this subject. It seems likely, if there is a considerable market for it, under standard specifications, that the cost will be reduced to that of wrought-iron, or lower; but it also seems likely that this will not transpire until we do use it and cheapen it by making a market for it. Within the last few years quite a number of engineers have evidently formed favorable opinions of this grade of material and have essayed to use it, but under conditions which do not seem to be entirely warranted. In one or two cases they are reaming it, just as they would medium steel; but this can hardly be regarded as an advantageous procedure, because, first, the metal is not of a character adapted to reaming, being of a soft, waxy texture, which drags on a tool and clogs it, making more expensive work than medium steel; and, second, we are not warranted in figuring it at high enough unit stresses to cover this expense. We might, perhaps, rate the unit stresses at 15 per cent higher than wrought-iron; but such a percentage will certainly not pay the cost of reaming, added to the extra cost of the material. Again, several engineers have been using soft steel under the same conditions as wrought-iron, but with the higher stresses which have been adopted for medium steel. It is an open question whether such a course as this is warranted by the facts of the case. It is not necessary, in the first place, to figure it so high to place it on a par in cost with wrought-iron; and then the material is certainly not of equal value with reamed medium steel, for the reasons, first, that its elastic limit and ultimate strength are both lower, and, second, because reaming undoubtedly adds to the value of any grade of steel, or of wrought-iron, for that matter.

The correct solution of the matter would appear to be to strike a balance between the cost of soft steel in bridge structures, as compared with wrought-iron on the one hand, and medium steel on the other, and to rate it accordingly if we are fully warranted to do so, at unit stresses high enough to overcome the difference in cost. The present difference in cost between wrought-iron and soft steel may be reckoned about as follows:

Cost of wrought-iron, say	100
Increased weight of steel 2 per cent	2
Add increased cost of soft steel 5 per cent (not well maintained)	5.1
Total	107.1

In other words, the cost of soft steel is nominally a little over 7 per cent higher than wrought-iron. In the shop there would be a slight saving in using soft steel due to a rivet saved here and there, and other small savings, provided we can use it just like wrought-iron. The writer takes it for granted that the propriety of using soft steel, having an average ultimate strength of 58,000 pounds, at unit stresses in compression 10 to 15 per cent higher than wrought-iron, will not be seriously questioned; that is to say, under the actual conditions in which it is used in our bridges, or with values of $\frac{l}{r}$ not exceeding 150. The propriety of using soft steel under higher unit stresses than wrought-iron hinges then upon its qualities in tension in flanges of girders and in the tension members of trusses; and this phase of the subject will now be discussed somewhat at length:

Whether Soft Steel, after Punching, constitutes a Good Tension Member.

If left to our practical shopmen, there would probably be no hesitation whatever on this score. It is, perhaps, no exaggeration to say that for exacting conditions of all sorts it is becoming more and more the practice to substitute soft steel. It is being used for latticing, because it will not split; it is being used for the hitch angles of stringers and floor-beams, because it will not crack; it is being used for pin plates on suspenders, and in various other ways where its superior qualities have been practically shown to practical men. It is necessary to get more conclusive evidence than this, however, and the presentation of this evidence, as I have been able to collect it, follows below. The writer wishes it understood at the outset that in nothing which follows is there any attempt to show that reaming is not a good thing *per se.* On the contrary, the tests given below show that it generally benefits soft steel whenever the metal is over $\frac{3}{8}''$ thick. It is well to state this very clearly, because otherwise the discussion might hinge on the relative merits of punching and reaming, which is not now at issue. We cannot ream soft steel and use it economically.

The question is not, therefore, whether punching is better than reaming, but *whether we are as fully warranted to punch soft steel as we are to punch wrought-iron.* We are punching iron for all conditions of work; and *if soft steel holds its value after punching as well as or better than iron, then we are clearly free to use it in the same way.* To test the question it is only necessary to take tested samples of iron and steel,

and after punching them under the same conditions, again test them, and by simple rule of three ascertain which holds its value best. This has been done in the tables which follow. Except where noted in the tables, the values obtained in specimen tests are always rated at 100, and the values after punching are rated by percentage from this basis.

Table I.

U. S. GOVERNMENT TESTS OF RIVETED LAP-JOINTS AT WATERTOWN ARSENAL.

Reports of 1882–83.

PUNCHED HOLES.

Specimen.	Kind of Joint.	Grooved Iron. Specimen Test. Ultimate.	Grooved Iron. Net Section of Joint. Ultimate.	Percentage.	Grooved Steel. Specimen Test. Ultimate.	Grooved Steel. Net Section of Joint. Ultimate.	Percentage.
10″ × ¼″ plate	Single lap	47,925	43,230	90	55,765	60,250*	
		"	45,520	95	"	59,240	106
		"	52,160	109			
		"	54,930	115			
	Double lap	"	50,592	106	"	61,510	110
		"	49,950	104	"	60,300	108
			Mean =	103		Mean =	108
10″ × ⅜″ plate	Single lap	47,180	36,130	77	53,330	55,215*	
		"	41,750	89	"	65,460	123
		"	41,290	87	"	65,210	122
		"	61,700	131	"	73,394	138
		"	58,510	124	"	73,970	139
	Double lap	"	48,450	103	"	62,800	118
		"	50,730	108	"	64,720	121
		"	46,255	98	"	63,210	119
		"	46,110	98	"	54,930	103
			Mean =	102		Mean =	121
10″ × ½″ plate	Single lap	44,615	31,100	70	57,215	60,210	105
		"	31,395	70	"	49,590	87
		"	35,650	80	"	47,530*	
	Double lap	"	44,320	99	"	64,602	113
		"	42,920	96	"	64,519	113
		"	46,400	104	"	68,920	120
		"	46,140	104	"	66,710	118
			Mean =	89		Mean =	109
10″ × ⅝″ plate	Single lap	44,630	34,680	78	52,445	52,770*	
		"	34,230	77	"	49,610*	
	Double lap	"	43,580	98	"	69,680	133
		"	45,850	103	"	67,100	128
			Mean =	89		Mean =	131
10″ × ¾″ plate	Single lap	46,590	29,290	63	51,545	39,970	78
		"	30,730	66	"	47,370	92
	Double lap	"	42,000	90	"	48,970	95
		"	43,950	94	"	47,510	92
			Mean =	78		Mean =	89

* Did not rupture the plate.

This table is derived from a series of tests of riveted joints which appear in the Government Reports of tests made at the Watertown Arsenal. In this series of tests, with which many members are, no doubt, familiar, steel and wrought-iron were compared under practically the same conditions through a long series

of tests of both punched and reamed material, and the ultimate strengths of the samples were very carefully determined by tests on grooved specimens. The conditions were not always exactly identical, as the rivets were sometimes of wrought-iron and sometimes of steel, and the joints failed accordingly, sometimes by shearing the rivets and sometimes by tearing the plate. I have endeavored, however, in this table to compare, on essentially similar terms in each case, the behavior of the wrought-iron and of the steel. The table includes every punched test in the series which failed by tearing across the sheet, and also includes a few tests which failed by shearing, but which probably developed nearly the full value of the material. These last tests are indicated by stars. Unfortunately, the elastic limit was not determined in these tests, and no comparison can therefore be made between the elastic limit of the specimens and the elastic limit of the riveted joints. The ultimates, therefore, only are compared, and the relative ultimate strength developed in net sections at the joints is figured out in the columns headed "Percentage" throughout the table. It will be found that not only does the steel show less loss in ultimate strength as an effect of punching than the wrought-iron, from an average of these tests, but in every identical test, with one exception, this fact is also true, and as the thickness of the pieces ranges froms $\frac{1}{4}''$ up to $\frac{3}{4}''$, the series is very complete. It is true some of this steel is a little softer than the soft steel which it is proposed to use for bridge structures; but, on the other hand, the iron is also of good, soft quality, probably boiler-plate iron, as the tests indicate it to have an unusually large percentage of strength across the grain. It would, therefore, probably give better results than ordinary wrought-iron would, and the comparison is therefore a fair one.

[NOTE.—In Table I*a* are given the results of tests on iron and steel single-riveted butt-joints with punched holes, as compared with the strength of the grooved plates. This is the kind of riveted joints used in structures, and hence indicates more fully the relative effect of punching iron and steel in structural work than the tests in lap-joints given in Table I.—J. B. J.]

TABLE I*a*.

RELATIVE STRENGTH OF IRON AND STEEL SINGLE-RIVETED BUTT-JOINTS WITH PUNCHED HOLES.*

Thickness of Plate.	Diameter of Rivet-holes.	Pitch of Rivets.	No. of Tests.	Iron Plates.				Steel Plates.			
				Strength of Grooved Plate.	Strength of Net Section of Riveted Joint.	Excess Strength of Joint.	Per cent of Excess.	Strength of Grooved Plate.	Strength of Net Section of Riveted Joint.	Excess Strength of Joint.	Per cent of Excess.
				Pounds per sq. in.	Pounds per sq. in.	Pounds per sq. in.					
$\frac{3}{8}$ in.	$\frac{3}{4}$ in.	2 in.	2	47,180	46,620	− 560	− 1.2	53,330	62,000	+ 8,670	+ 16.2
$\frac{1}{2}$ in.	$\frac{13}{16}$ in.	2 in.	2	44,615	46,270	+ 1,655	+ 3.6	57,215	67,820	+ 10,605	+ 18.5
$\frac{5}{8}$ in.	$1\frac{1}{16}$ in.	$2\frac{1}{8}$ in.	2	44,635	43,300	− 1,335	− 3.1	52,445	62,380	+ 9,835	+ 18.8
$\frac{3}{4}$ in.	$1\frac{3}{16}$ in.	$2\frac{3}{4}$ in.	2	46,590	42,020	− 4,570	− 10.9	51,545	54,425	+ 2,875	+ 5.6

[This table shows that whereas wrought-iron riveted plates are weaker than the grooved specimen tests, the corresponding steel-riveted plates are much stronger.—J. B. J.]

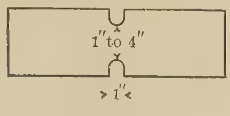
Table II is also from the Government tests at Watertown, and consists of comparative results obtained from grooved specimens alternately drilled and punched. The percentages of this table are made up by rating the ultimates of the drilled specimens at 100, and in this table also it will be found that the steel is less impaired by punching than the wrought-iron. The steel and iron used in these tests are the same material as that tested in Table I.

* From Watertown Arsenal *Reports of Tests of Metals, etc.*, for 1882 and 1883, given also in Lanza's "Mechanics."

TABLE II.

U. S. GOVERNMENT TESTS OF GROOVED SPECIMENS AT WATERTOWN ARSENAL

REPORT OF 1892.

Shape of specimen : 

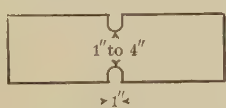
Iron.			Specimen.	Steel.		
Drilled.	Punched.	Percentage.		Drilled.	Punched.	Percentage.
44,980	38,950	86	½″ × 3″ wide	66,310	60,320	93
47,030	37,200	79	" "	66,190	62,430	94
45,330	35,730			64,470	48,010	
45,000	36,690	81	½″ × 4″ "	64,810	55,190	80
46,100	37,000			64,690	55,780	
	37,420			64,140	46,250	
		82				89
47,220	49,770	105	⅝″ × 1″ wide	60,290	66,720	110
48,350	52,960	110	" "	63,610	64,800	102
47,170	46,320	98	" "	63,450	64,400	102
46,350	46,750	101	" "			
48,220	40,140	83	⅝″ × 3″ "	59,270	57,180	96
44,840	37,310	83	⅝″ × 4″ "	60,330	54,450	90
45,100			" "	61,120	57,380	94
		97				99
47,500	50,840	107	¾″ × 1″ wide	58,480	67,930	116
52,780	47,590	88	" "	58,790	67,630	115
48,470	45,970	95	¾″ × 1½″ "	59,290	62,890	106
47,750	40,350	84	¾″ × 3½″ "	58,700	56,730	96
46,350	39,380	85		59,180	54,220	92
		92				105

[NOTE.—Table II might have been greatly extended by including the tests reported in 1883. The following table gives the summaries by averages of all such tests made at the Watertown Arsenal.—J. B. J.]

TABLE II*a*.

RESULTS OF TESTS* SHOWING EFFECTS OF PUNCHING WROUGHT-IRON AND MILD-STEEL PLATES.

SUMMARY OF WATERTOWN ARSENAL TESTS.

Shape of specimen : 

Thickness of Plate.	Width at Bottom of Groove.	Iron Plates.					Steel Plates.				
		No. of Tests.	Average Strength Drilled.	Average Strength Punched.	Loss from Punching.	Percentage of Loss.	No. of Tests.	Average Strength Drilled.	Average Strength Punched.	Loss from Punching.	Percentage of Loss.
			Pounds per sq. in.	Pounds per sq. in.	Pounds per sq. in.			Pounds per sq. in.	Pounds per sq. in.	Pounds per sq. in.	
¼ in.	½ in. to 3 in.	42	50,100	43,500	6,600	13.2	31	64,200	61,800	2,400	3.7
⅜ in.	1 in. to 3½ in.	9	48,500	40,000	8,500	17.5	1	63,620	61,890	1,730	2.7
½ in.	1 in. to 4 in.	15	47,100	40,300	6,800	14.5	17	66,030	58,160	7,870	12.0
⅝ in.	1 in. to 4 in.	8	46,670	43,460	3,210	6.9	10	61,050	59,740	1,310 (Gain)	2.1 Gain of
¾ in.	1 in to 3½ in.	5	48,570	44,650	3,920	8.1	5	58,890	61,880	2,990	5.1

* From Watertown *Reports of Tests of Metals, etc.*, for 1882 and 1883; given also in Lanza's "Mechanics."

[NOTE.—By comparing Tables II*a* and I*a* we see that wrought-iron punched plate is, say, 12 per cent weaker than the same drilled, and that the riveted sheet is, say, 4 per cent weaker than the punched plate, or the total weakening of wrought-iron has been some 16 per cent, while there has been a corresponding loss in strength of the punched steel plates of, say, 3 per cent, but a *gain* in strength of the riveted steel plate over the punched sheet of, say, 15 per cent, or a *net gain of some 12 per cent*. This is the average gain in strength of a punched and riveted steel plate as compared with the drilled grooved specimen.—J. B. J.]

In these Government Reports will also be found some interesting tests on the tearing out of rivet-holes in steel and iron. Unfortunately the series is limited to $\frac{1}{4}''$ metal, and is consequently of limited value. In $\frac{1}{4}''$ metal the difference between punched and drilled holes is very small indeed. So far as these tests go, however, they show that *steel with punched holes and sheared edges gives better relative results than iron with drilled holes and planed edges.*

Not long since, it was the writer's good fortune to find a very interesting series of comparative tests of iron and steel on record at one of the larger iron-works of this district, and through the courtesy of the proprietors he has been able to examine it to determine the same ratios given in the tables above. As the complete results will probably be published, the details will not here be given, but only the general results. The tests are important, because punched, reamed, and drilled specimens of iron and steel are compared under exactly similar conditions, and the series comprises some fifty or sixty tests. The bars used for the purpose were $6 \times \frac{1}{2}$, $6 \times \frac{5}{8}$, and $6 \times \frac{3}{4}$ respectively in both iron and steel, the steel having an ultimate strength of about 64,000 pounds, and the iron an ultimate strength of about 52,000 pounds, and both grades of material were of excellent quality. The writer has checked up the entire series and finds that *the steel is very uniformly less injured by punching than iron;* the average difference being about 10 per cent in favor of steel, and this difference is essentially true of all tests save, perhaps, one in the series. Thus, if we rate the average ultimate strength of the steel in specimen tests at 100, the average ultimate of the iron in specimen tests would rate at 83; but rating the average ultimate of the punched steel on net sections at 100 per cent, the average ultimate of punched iron on net sections would rate at $71\frac{1}{2}$ per cent. More than this, a comparison of the punched specimens with the reamed ones in both cases shows the iron to have been quite as much benefited by reaming as the steel.

In Table III are given some tests of wrought-iron, soft steel, and medium steel, which have been made recently under the auspices of the Pittsburgh Testing Laboratory. Some of the specimens were tested with open holes and some with rivets in the holes. In these tests the elastic limit was taken in all cases, and it will be observed that with punched holes the material retains its full value at the elastic limit, and indeed generally records a gain in this respect, showing the punching to have made the metal harder and denser. In the series of tests with rivets driven in the holes, it is evident that generally the rivet acted as a splice, and the elastic limit is really that of the gross section. The tests on their face hardly indicate as high values as this, but the fact seemed to be as stated. The heating in riveting no doubt softened the metal and reduced the elastic limit.

In Table IV a number of tests of punched medium steel are given, and also two of hard steel. With the exception of the latter grade of metal, a comparison of results decidedly favors the steel.

It will be evident to any one who examines the facts here presented that there are several points suggested by these tests which are not proved, and which can only be demonstrated by more extended tests. The field for such experiments is evidently an ample one. But on the definite question raised in this paper may it not be said that the tests presented, whether considered as a whole or in detail, are *positive evidence that the tensile strength of structural steel is injured rather less by punching than is the tensile strength of wrought-iron?* It is not the writer's intention to claim that this demonstration is conclusive, much less exclusive of testimony to the contrary. But it is entirely true so far as the evidence goes; and this evidence, as presented, is entirely full and candid, including everything that the writer was able to find bearing on this subject, and the entire series of ninety special tests made under his own direction. He has no idea that it can be successfully gainsaid. As conclusions suggested, but not demonstrated, he would state the following:

(1) *The thicker the metal the greater the injury in punching steel.*

(2) *The softer the steel the less the injury in punching; but,*

(3) *The first conclusion is the major factor; the grade of the steel making much less difference than the thickness of the material.*

(4) *The effect of riveting is advantageous, increasing the strength of net sections.*

Conclusions 1 and 2 have ample support from other sources and may be considered as facts. Thus, Kirkaldy's recent book, in commenting on his well-known series of tests of Fagersta plates, speaks as follows: "Punching is less detrimental the softer the plate; the great difference produced by punching is due to its *hardening* effect upon the plates, which becomes more severely felt as the thickness increases." In proof of this he then gives the elongation of holes at the fracture, in a series of tests of $\frac{1}{8}''$, $\frac{1}{4}''$, $\frac{3}{8}''$, $\frac{1}{2}''$, and

Table III.

TENSILE TESTS OF SPECIMENS OF SOFT STEEL AND WROUGHT-IRON HAVING PUNCHED HOLES.

ALSO, COMPARATIVE TESTS WITH REAMED HOLES.

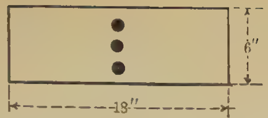


Fig. 1.

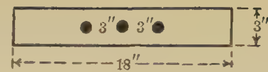


Fig. 2.

	Test cut from.	Grooved Specimen Test. Elastic Limit.	Grooved Specimen Test. Ultimate Strength.	Grooved Specimen Test. Elong.	Punched Piece. Elastic Limit.	Punched Piece. Per Cent.	Punched Piece. Ultimate Strength.	Punched Piece. Per Cent.	Reamed Piece. Elastic Limit.	Reamed Piece. Per Cent.	Reamed Piece. Ultimate Strength.	Reamed Piece. Per Cent.
Soft Steel, Riveted Holes	7" × 1/4" flat	Assumed	60,000		58,840		64,230	107	60,890		64,900	108
	8" × 5/16" flat	34,250	54,480	22.50	50,780	148	59,600	109	53,100	156	62,100	114
	15" × 3/8" plate	39,410	55,540	31.00	51,620	131	61,220	110	52,220	133	66,900	120
							Mean	108.7			Mean	114
					Av. elong. in 8" = 8.00 per cent.				Av. elong. in 8" = 7.92 per cent.			
Soft Steel, Open Holes		38,200	58,300	26.50	46,200	121	62,200	107				
	*6" × 1/2" flat	39,800	62,400	25.30	45,300	114	65,900	106				
		39,700	63,500	25.00	45,500	115	68,300	108				
		39,500	62,600	27.60	43,800	111	62,100	99				
	*6" × 3/8" flat	40,100	64,200	28.20	48,200	120	59,900	93				
		40,100	64,000	27.50	41,800	104	63,100	99				
	4" × 3" × 3/8" angle	39,180	58,910	26.25	38,170	98	50,330	85	40,350	103	61,320	103
	12" × 1/2" flat	41,370	55,900	32.25	47,450	114	48,960	88	42,870	103	64,820	116
	19½" × 9/16" plate	35,820	56,460	32.25	38,950	109	52,400	93	41,570	116	61,450	109
	12" × 9/16" flat	35,650	62,210	30.00	35,310	99	58,890	95			63,270	102
	18" × 19/32" plate	37,540	55,540	31.50			45,080	81	39,440	108	63,450	114
	14½" × 5/8" plate	34,710	57,520	30.75			43,820	76	38,260	120	64,660	112
	8" × 11/16" flat	40,690	60,540	31.00	44,170	109	45,500	75	44,140	109	63,030	104
	18" × 3/4" plate	35,240	55,350	33.75	40,070	114	41,180	74				
							Mean	91.4			Mean	108.6
					Av. elong. in 8" = 4.19 per cent.				Av. elong. in 8" = 10.20 per cent.			
Wrought-iron, Open Holes	7" × 3/8" U. M. plate	30,140	45,000	7.75	31,740	105	36,600	81	33,510	111	46,870	104
	6½" × 3/8" U. M. plate	34,790	51,390	15.00	36,150	104	40,570	79	33,320	95	44,240	86
	6" × 4" × 3/8" angle	34,080	50,560	18.00	34,300	101	36,450	72	34,360	101	42,360	84
	6" × 6" × 7/16" angle	33,050	49,870	17.25	35,720	108	36,670	74	34,520	105	43,580	87
	6" × 6" × 1/2" angle	33,680	48,830	25.00	36,160	97	40,210	82	35,180	104	46,660	96
	6" × 6" × 1/2" angle	32,690	47,990	16.00	33,620	103	36,510	76	34,040	104	44,970	94
	12" × 1/2" plate	32,750	49,800	24.00	36,250	111	38,460	77	34,540	105	47,490	95
	14" × 5/8" plate	30,170	49,780	24.25	34,800	115	37,140	75	35,280	117	52,060	105
	16" × 3/4" plate	29,690	48,800	24.75	34,800	117	37,550	77	33,080	111	43,680	89
	14" × 3/4" plate	30,430	49,160	28.00	34,260	112	37,500	76	32,970	105	42,630	87
							Mean	76.9			Mean	92.7
									Av. elong. in 8" = 5.83 per cent.			

* These six tests were 6 inches wide and had 3 holes across the section, as in Fig. 1, the pitch varying in the different tests. All the other tests above were of the form shown in Fig. 2, and nearly all of them were 3 inches wide with 15/16-inch holes. In punched pieces holes were punched 15/16 inch; in reamed pieces punched 13/16 inch, and reamed to 15/16 inch.

Table IV.

TESTS OF MEDIUM STEEL AND HARD STEEL.

	Test cut from.	Specimen Test.			Punched Piece.				Reamed Piece.			
		E. L.	Ult.	Elong.	E. L.	P. Ct.	Ult.	P. Ct.	E. L.	P. Ct.	Ult.	P. Ct.
Medium Steel, Riveted Holes	8" × $\frac{5}{16}$" plate		69,280	20.25	51,080		70,460	102	57,010		72,700	105
	14" × $\frac{1}{2}$" plate	41,750	67,600	20.50	45,550	109	66,190	98	53,460	128	72,180	107
	10$\frac{1}{4}$" × $\frac{21}{32}$" plate	39,460	68,540	29.38	48,900	124	56,690	83	41,720	106	71,630	105
	6" × 6" $\frac{3}{4}$" angle	40,090	66,340	25.38	45,970	115	23,300	80	46,990	118	68,550	103
	" "	38,400	67,070	23.00	45,100	118	52,300	78	47,450	124	66,840	99
							Mean	88.2			Mean	103.8
					Av. elong. in 8"=4.70 p. ct.				Av. elong. in 8" = 8.75 p. ct.			
Medium Steel, Open Holes	7" × $\frac{3}{8}$" plate	41,190	61,080	19.50	40,610	99	54,260	89	45,420	110	58,980	97
	6" × 6" × $\frac{1}{2}$" angle	33,010	66,790	23.75	33,950	103	63,540	95	35,710	108	66,740	104
							Mean	92			Mean	100.5
					Av. elong. in 8"=8.50 p. ct.				Av. elong. in 8" = 12 p. ct.			
Hard Steel, Open Holes	15" × $\frac{3}{4}$" plate	48,380	81,120	22.25	55,430	115	57,420	71	67,500	139		
	*7" × $\frac{7}{16}$" plate	44,890	73,040	22.75	46,000	102	55,200	69	43,770	98	71,210	98
							Mean	70				

* This specimen had a flaw in it. Form of specimens same as in Table III.

$\frac{5}{8}$" plates, as follows: With drilled holes 5.7, 14.1, 17.2, 18.7, and 19.0 per cent, with fractures all silky; with punched holes 3.2, 9.6, 13.5, 3.2, and 1.9 per cent, with the last two fractures showing 95 and 100 per cent granular respectively. Then he anneals the specimens in a similar series, to remove the hardening effect, and gets elongations as follows: With drilled holes "extensions of 11.5, 16.3, 19.4, 21.0, and 21.9;" with punched holes "10.3, 14.6, 18.1, 19.3, and 20.7 per cent, all fractures being wholly silky." This makes a very pretty demonstration.

Kirkaldy compares the ultimate strength of drilled and punched specimens in these same tests, as follows: When not annealed, the strength of punched pieces as compared with drilled ones showed losses of 7.92, 8.10, 7.53, 23.45, and 26.22 per cent respectively. When he anneals the specimens, however, the punched pieces show losses of but 5.85, 6.70, 6.74, 7.08, and 7.18 per cent respectively. "This manipulation or treatment rectifies the injuriousness of punching to a great extent," he says. This is argument for conclusion No. 4, because, evidently, so far as the rivets anneal and soften the metal they benefit it and lessen the injury in punching; and, evidently also, this advantage accrues more to thick metal than to thin because the original injury was greater. The tables in this paper appear to offer evidence that this beneficial effect of riveting is an actual fact; certainly, the riveted pieces show higher percentages than the pieces with open holes.* Regardless of any such effect, however, *the fact that punching is more injurious in thick material makes it desirable to use moderate sections in tension members and in floor-beams, stringers, and girders subject to impact.*

In concluding the discussion of the tables, attention is called to the elongation of the punched steel pieces in a length of 8 inches. The stretch is, of course, necessarily confined chiefly to the three holes; but the percentages are given in 8-inch length, to show what it might be expected to average in any long riveted member.

Having thus reasonably demonstrated more than an equal standing for our punched steel, we could logically claim for it unit stresses from 12 to 15 per cent higher than for wrought-iron. In the specifications, however, it was thought well to be conservative on this point, especially as it is not necessary to figure soft steel so much above wrought-iron in order to make the cost practically the same. The unit stresses, therefore, for soft steel in tension are placed at 8 per cent higher than wrought-iron, and in compression 10 per cent higher. These figures are conservative, and the remainder is so much advantage to the bridge structure.

* This is probably due more to the frictional resistance to movement under the rivet heads than to the annealing action of the hot rivet.—J. B. J.

It will be observed that web-plates are required to be of steel in all cases. It is, of course, impracticable to dimension web-plates for unit stresses. It would, however, be absurd to use steel angles in connection with an iron web, since the equal strength of steel plates in all directions exactly fills the condition of a web-sheet, and makes them particularly desirable for such purposes. There is, besides, good reason for throwing iron web-plates out altogether, because of poor quality. It is doubtless no longer profitable to make a first-class iron web-plate; hence many of those now sold are of very doubtful quality, and owe whatever virtues they possess to the percentage of steel which they contain. It will be noted that both iron eye-bars and soft steel eye-bars are ruled out of the specifications. Excepting with the most conservative, the iron bar has already been practically superseded, because steel bars are now cheaper and have besides abundantly demonstrated their superiority. As regards soft steel bars, there is no reason, on the face of it, why soft steel should not make a good eye-bar: it doubtless would; but medium steel has been found to answer the requirements with entire satisfaction, and, after annealing, is considerably softer than when tested in its natural state in small specimens.

There is a practical reason also for not using soft steel in the fact that it is more apt to have piping and blow-holes in it than medium steel. This is not objectionable in angles or plates where these defects close up and disappear; but it is objectionable in large masses, like pins and forged bars. It was not thought worth while, therefore, to introduce a soft steel eye-bar in the specifications. The result is that the soft steel is limited to compression members, to girders, and to riveted tension members, and the idea in fixing the unit stress has been to place it for these purposes on a par with other material; possibly to give it a little advantage to overcome the inertia of manufacturers who have been accustomed to using wrought-iron only. It is possible that it is not necessary to make as much allowance as has been made, or perhaps it should be increased to suit different cases.

Screw Threads on Steel Bars.

There is one point more that may require special mention—that is, the provision of the specifications permitting steel bars to be used with upset screws on their ends. This is probably the last point that engineers are ordinarily willing to concede in regard to steel; that is, the possibility of putting a screw-thread on it; nevertheless this may be done with entire success, and in evidence thereof the following tests are submitted:

TABLE V.

TESTS AT EDGE MOOR BRIDGE WORKS OF STEEL EYE-BARS WITH UPSET SCREW ENDS.*

Eye-bars.	Size.	Length.	Elastic Limit.	Ultimate Strength.	Elongation, per cent.	Reduction, per cent.	Remarks.
No. 1	$5'' \times \frac{7}{8}''$	$37'\ 1''$	39,370	61,620	12.66 (33′)	48.08	Broke in long end
No. 2	$4'' \times \frac{7}{8}''$	$31'\ 6''$	37,700	59,240	15.63 (27′)	51.42	Broke in short end
No. 3	$5'' \times \frac{7}{8}''$	$37'\ 0\frac{1}{2}''$	38,170	62,470	11.71 (33′)	43.48	Broke in long end

These tests are certainly crucial tests, as it would be impossible to imagine a more difficult upset than to put a round screw-end on a $5 \times \frac{7}{8}$ bar.

Not long ago one of the leading Southwestern railroads desired to renew a large number of hangers, which were necessarily limited in size, because of the place they had to fit. In this connection Mr. Henry G. Morse, and President of the Edge Moor Bridge Works, had some experimental tests of steel hangers made. The results of these tests are given in Table VI.

The material that was put in the hangers was taken from stock, being ordinary steel rounds which had been ordered for cotter pins. Attention is called to the fact that the excess in areas at the root of the thread on these hangers was but 7 per cent, and it may well be doubted whether iron hangers could have stood as severe a test.

The tests which have been presented show that the steel held its value, after punching, better than iron, up to say $\frac{3}{4}''$ thickness, when the percentages in steel and in iron were just about equal. But these tests also show that the value of the steel decreased quite rapidly as the thickness increased; the percentages running down from about 100 at $\frac{3}{8}''$ thickness to 75 at $\frac{3}{4}''$ thickness.

In iron the percentages seemed to be much more constant, the range being only from 82 to 72 per cent. (See Tables III and IV.) In order to make this fact more clearly apparent, the results are plotted as a diagram,

* This table is of little value without some knowledge of the relative net areas on screw and bar portions.

TABLE VI.

TESTS AT EDGE MOOR BRIDGE WORKS OF STEEL STIRRUP HANGERS AND UPSET RODS.

Stirrup Hangers.	Diameter of Bar.	Diam. at Root of Thread.	Area. Orig.	Area. Fract.	Elastic Limit.	Ultimate Strength.	Elong. in 12″, per cent.	Reduction of Area, per cent.	Remarks.
No. 1	1.75	1.81	2.41	1.23	38,540	59,780	26.2	48.96	Broke in one leg. 1′ 3½″ from end.
No. 2	1.75	1.81	2.41	1.09	37,760	58,990	20.33	54.72	Broke in bend.
No. 3	1.75	1.81	2.41	.95	37,760	58	15.17	60.58	Broke in bend.
Upset Rods									
No. 1	1.75	1.81	2.41	1.91	34,610	55	16.67	20.75	Broke 2′ 9 11/16″ from end.
No. 2	1.75	1.81	2.41	2.06	40,900	56	11.67	14.52	Broke 2′ 5¼″ from end.
No. 3	1.75	1.81	2.41	.99	37,760	5	33.33	58.92	Broke.

in Fig. 451, which shows the percentage values of the punched iron, the punched steel with open holes, and the punched steel with riveted holes for different thicknesses of metal. It also shows a mean line for steel.

In addition to this decline in percentage value, there was a gradual change in the character of the fracture from fine silky in ⅜″ material to 100 per cent granular in the ¾″ material. This gradual change is shown by the photographs in Plate XXXVIII, which indicate how the granular structure first appears at the edge of the holes and radiates from them in larger percentages as the thickness increases. In order to show the latter characteristics more fully, Table VII, is inserted, which gives the details for most of the tests of soft

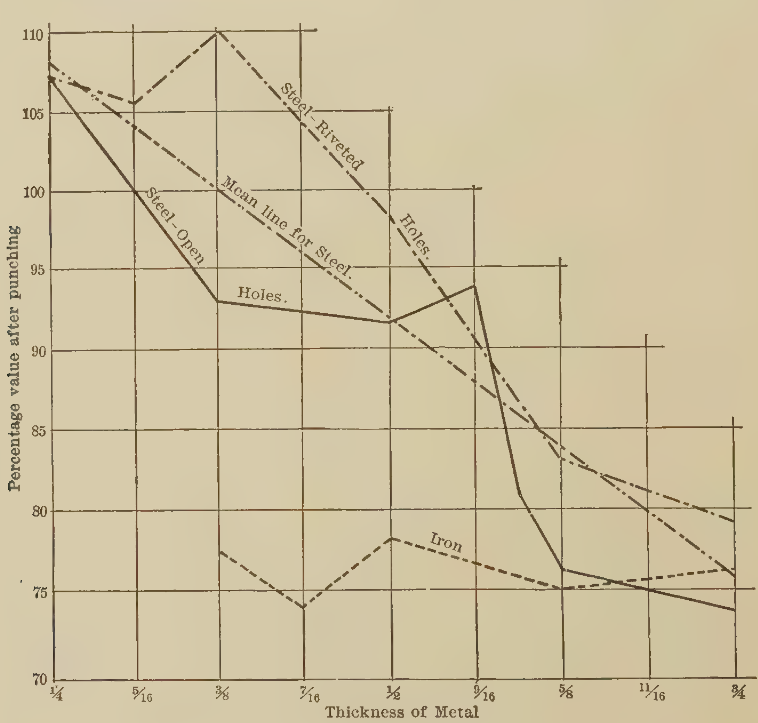
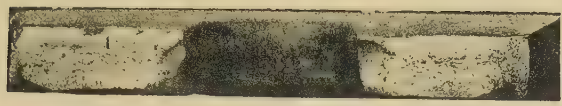
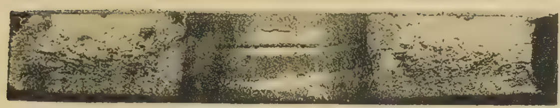


FIG. 451.

and medium steel which appear in Tables III and IV. The tests of iron did not, of course, show granular fractures, since the normal structure of iron is not granular, as that of steel is. But it was apparently very dead, and it would be difficult to say which of the two metals was more reliable at ¾″ thickness. The writer is disposed to think, however, that this decrease in value and increase in granular fracture in thick steel should be considered, regardless of a satisfactory comparison in ultimate strength with wrought-iron of the same gauges. He has decided, therefore, to limit the thickness of punched material in the specifications,

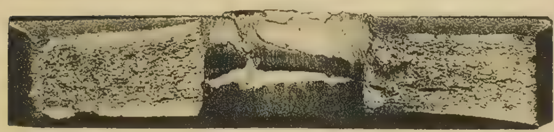


All Fibrous.

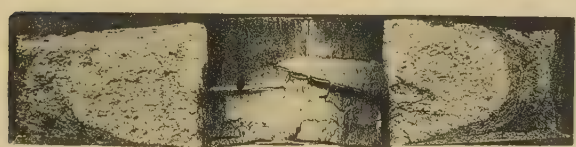


All Fibrous.

Partly Granular.



All Fibrous.



Largely Granular.

All Granular.

NOTE.—See also cuts showing effects of shearing and punching soft-steel plates in Johnson's *Materials of Construction*, pp. 531 and 532.

Showing the Effects of Punching Mild-Steel Plates of Different Thicknesses.

SCALE FULL SIZE.

and to do this for most members by drawing the line at the gauge in which the crystalline fracture becomes evident and the stretching qualities decline. From Table VII it is apparent that all the tests are satisfactory until we pass $\frac{9}{16}''$ thickness. Above that gauge there is but one test which would pass muster on the basis here proposed. A third condition to the use of soft steel has therefore been added, limiting to $\frac{1}{2}$ inch the thickness of material subjected to punching, excepting that in girders over 50 feet long it may be $\frac{9}{16}''$, in top chords and end posts it may be $\frac{5}{8}''$, and in shoes, pedestals, and bed-plates it may be $\frac{3}{4}''$.

TABLE VII.

DETAILS OF PUNCHED TESTS GIVEN IN TABLES III AND IV FOR SOFT AND MEDIUM STEEL.

Thickness.	Specimen cut from	Elastic Limit, lbs. per sq. in.	Ultimate Strength, lbs. per sq. in.	Stretch in 8 in., per cent.	Stretch of Hole, per cent.	Character of Fracture.	Class of Steel.	
$\frac{1}{4}''$	$7'' \times \frac{1}{4}''$ Bess.	58,840	64,230	8.50	46.9	Silky	Soft steel	Riveted holes
$\frac{5}{16}''$	$8'' \times \frac{5}{16}''$ "	50,780	59,600	8.00	40.8	"	" "	" "
$\frac{5}{16}''$	$8'' \times \frac{5}{16}''$ "	51,080	70,460	5.75	29.6	"	Medium steel	" "
$\frac{3}{8}''$	$15'' \times \frac{3}{8}''$ "	51,620	61,220	7.50	36.1	"	Soft steel	" "
$\frac{3}{8}''$	$4'' \times 3'' \times \frac{3}{8}''$ "	38,170	50,330	7.25			" "	Open "
$\frac{3}{8}''$	$7'' \times \frac{3}{8}''$ "	40,610	54,260	9.50			Medium steel	" "
$\frac{1}{2}''$	$12'' \times \frac{1}{2}''$ "	47,450	48,960	4.75	34.0	20 per ct. gran.	Soft steel	" "
$\frac{1}{2}''$	$14'' \times \frac{1}{2}''$ "	45,550	66,190	7.00	32.6	Silky	Medium steel	Riveted "
$\frac{1}{2}''$	$6'' \times 6'' \times \frac{1}{2}''$ O. H.	33,950	63,540	7.50			" "	Open "
$\frac{9}{16}''$	$19\frac{1}{2}'' \times \frac{9}{16}''$ Bess.	38,950	52,400	7.50	48.9	Silky	Soft steel	" "
$\frac{9}{16}''$	$12'' \times \frac{9}{16}''$ O. H.	35,310	58,890	8.75			" "	" "
$\frac{19}{32}''$	$18'' \times \frac{19}{32}''$ Bess.		45,080	.38	3.09	Granular	" "	" "
$\frac{5}{8}''$	$14\frac{1}{2}'' \times \frac{5}{8}''$ "		43,820	.62	4.25	"	" "	" "
$\frac{21}{32}''$	$10\frac{1}{4}'' \times \frac{21}{32}''$ O. H.	48,900	56,690	1.75	7.22	"	Medium steel	Riveted "
$\frac{11}{16}''$	$8'' \times \frac{11}{16}''$ Bess.	44,170	45,500	3.50	23.4	70 per ct. gran.	Soft steel	Open "
$\frac{3}{4}''$	$18'' \times \frac{3}{4}''$ "	40,070	41,180	.75	6.38	Granular	" "	" "
$\frac{3}{4}''$	$6'' \times 6'' \times \frac{3}{4}''$ O. H.	45,100	52,300	1.50	7.22	"	Medium steel	Riveted "
$\frac{3}{4}''$	" " "	45,970	53,300	1.35	5.15	"	" "	" "

SPECIFICATIONS FOR FIRST-CLASS BRIDGE SUPERSTRUCTURE.*

GENERAL DESCRIPTION.

Materials. 1. All parts of the structure, except ties and guard-rails, and bed-plates of stringers, shall be of wrought-iron or steel. Stringer beds will be cast-iron. Cast-iron or steel for other purposes will only be used for special conditions at the discretion of the Chief Engineer.

Ties and guard-rails. 2. Ties and guard-rails shall be of wood. They will be supplied by the railway company, but must be put in place by the contractor, who will also furnish all bolts, nuts, washers, etc., (except rail spikes), for this purpose.

Ties on tangents will be 8 inches by 10 inches, laid on 8-inch face and spaced 14 inches between centres; guard-rails will be 7 inches by 8 inches, spaced 8 feet between centres.

On curves the outer rail will be elevated $\frac{1}{2}$ inch per degree, and this elevation will be framed in the ties, as no shims will be allowed.

Ties will be notched $\frac{3}{4}$ inch over stringers, and guard-rails $\frac{3}{4}$ inch over ties.

Guard-rails will be bolted to each end of every other tie; and ties and guard-rails will be secured to stringers by hook bolts at each end of every fourth tie.

Superstructure. 3. For spans of 16 feet or less, rolled beams will be used, and from 16 feet to 100 feet, riveted plate girders. All spans over 100 feet will be pin-connected trusses.†

Stringer-spacing. 4. Beams or deck girders on masonry will be spaced 7 feet 0 inches centre to centre (ties 10 feet long). Riveted stringers on through single-track bridges and on trestles will be spaced 9 feet centre to centre (ties 12 feet long), beam stringers 8 feet 6 inches on double-track through bridges 7 feet centre to centre, symmetrically disposed under the rails.

Tracks will be 13 feet apart centre to centre.

* Compiled by F. H. Lewis, C.E. † See Chap. XVI.

Clearance for through spans.

5. Through bridges on tangents shall not be less than 14 feet in width in the clear between trusses for single track, and 27 feet for double track, nor less than 20 feet in height in the clear, measuring from base of rail to the lowest point of portals.

Trusses on curves.

6. On curves the truss on the convex side will be 7 feet from the centre line at the middle of the span; at the ends of the span the truss on the inner side of curve will be spaced 7 feet $+ 0.2d$ feet from the centre line, d being the degree of curve.

The case of a curve near enough to a bridge to require elevation of the rail will also be considered and provided for.

Deck trusses.

7. Deck truss bridges on tangents will be spaced at least 10 feet centre to centre of trusses.

Through girders.

8. Through plate girders will be spaced at least 13 feet 5 inches centre to centre on tangents.

Girders on curves.

9. Stringers, deck girders, and deck truss spans on curves will be spaced as may be necessary to suit the circumstances and to satisfy the Engineer.

Design.

10. All structures will be simple in design, and admit of accurate calculation of the stresses in each member.

11. Pin trusses will be in all ordinary cases of the single-intersection type with leaning end-posts.

Collision strut.

12. All end-posts of through-pin spans will be braced by collision struts, and the end-panels of the bottom chord and the vertical suspenders will be stiff members.

Proposals.

13. Proposals for bridge-work will be submitted on invitation of the Chief Engineer, and must conform with these specifications, with general plans or descriptions furnished by the Engineer, and with other conditions provided for in the letter of invitation.

Plans, etc.

14. Complete strain sheets, general plans of structure, and detail drawings, shall be furnished to the Chief Engineer of the railway company without charge.

Stress sheets.

The stress sheets must show for each member the maximum stress or stresses caused by the dead load, the live load, and the wind separately; the unit stress and the dimensions and areas of cross-sections. Also the dead weight assumed in the calculation, which must not be less than the actual weight of the structue as built.

Grade of material used.

15. It will be noted that the specifications below provide as follows:

(1) That all eye-bars and pins shall be of medium steel (see §§ 133 and 146).

(2) That all web-plates shall be of steel (see § 76).

(3) That loop rods and all other devices which are welded shall be of wrought-iron.

These requirements, as defined below, are common to all bridges, whether built of wrought-iron, soft steel, or medium steel. The other parts of bridges, however, may be built of such grades of material as the contractor may elect, provided only that each member and each set of members performing similar functions must be of the same grade of material throughout.

Approval of plans and details.

16. Complete detail drawings must be submitted for approval to the Chief Engineer, and work will not be commenced until the stresses and details have been approved. The Chief Engineer or such assistants as he may appoint shall have free access at all times to the working drawings and shops of the contractor for the purpose of examining the plans and inspecting the material used and the mode of construction.

17. The contractor shall furnish to the Chief Engineer of the railway company, free of cost, such detail drawings of each structure as he may require.

Loading.

Live Loads.

18. Live loads will be as per diagram furnished by the Chief Engineer.

19. The structure will be proportioned to carry the live loads as per diagram, and the live load stresses will be the maximum stresses produced by the rolling load considered as stationary or as moving in either direction. In double-track structures, one track or both will be considered loaded, whichever may produce the greater stresses, and the trains will be supposed to move either in the same or in opposite directions.

Dead Load.

20. The dead load shall consist of the entire structure, including the floor system and rails and fastenings. The weight of the ties, guard timbers, rails, spikes, etc., shall be taken at 400 lbs. per lineal foot for each track.

The load of the structure when complete shall not exceed the dead load used in calculating the stresses.

21. In through bridges, two thirds ($\frac{2}{3}$) of the dead load shall be assumed as concentrated at the joints of the bottom chord, and one third ($\frac{1}{3}$) at the joints of the upper chord.

In deck bridges, two thirds ($\frac{2}{3}$) of the dead load shall be assumed as concentrated at the joints of the upper chord, and one third ($\frac{1}{3}$) at the joints of the bottom chord.

WIND IN TRUSSES.

22. The bottom lateral bracing in deck bridges and the top lateral bracing in through bridges must be proportioned to resist a uniformly distributed lateral force of 150 lbs. per lineal foot of bridge for all spans of 200 feet and under, and an additional force of 10 lbs. per lineal foot for every 25 feet increase in length of span ove 200 feet.

23. The bottom lateral bracing in through bridges and the top lateral bracing in deck bridges must be proportioned to resist a uniformly distributed force the same as above, and an additional force of 300 lbs. per lineal foot of bridge, which will be treated as a moving load.

WIND IN TRESTLES.

24. Trestles shall be so proportioned, and the trestle bents shall have such a spread of base, that no tension * may occur in the windward trestle-leg when the structure is loaded with a light train weighing 600 lbs. per lineal foot of track, and when the wind pressure on this train is 300 lbs. per lineal foot, acting 9 feet above the rail. In addition, the wind pressure on the structure itself shall be assumed at not less than 150 pounds for each longitudinal foot of structure, and not less than 100 pounds for each vertical foot of height of each trestle bent, and more if the exposed wind surface of track and structure exceeds 5 square feet for each foot in length, and the exposed wind surface of each trestle bent exceeds $3\frac{1}{3}$ square feet for each vertical foot of height.

In all cases the projected surface of both sides of the towers and of one train is to be taken as the surface acted upon by the wind.

CENTRIFUGAL FORCE.

25. When the bridge is on a curve, add to the maximum wind stresses a moving lateral stress equal to 3 per cent of the live load on all tracks (acting in the direction of centrifugal force) for each degree of curvature.

26. The effects of wind and centrifugal force in the lateral system of structures must be fully provided for at unit stresses given below.

LONGITUDINAL BRACING AND ANCHORAGE.

27. Longitudinally the bracing of trestle towers and the attachments of the fixed ends of all trusses shall be capable of resisting the greatest tractive force of the engines or any force induced by suddenly stopping the assumed maximum trains, the coefficient of friction of the wheels upon the rails being assumed to be 0.20.

Double-track structures will be braced to provide for trains moving either in the same or in opposite directions.

TEMPERATURE.

28. Variations in length from change of temperature to the amount of 1 inch in 100 feet shall be provided for.

CALCULATIONS AND UNIT STRESSES.

29. All parts of the structure will be proportioned to sustain the maximum stresses produced by the live and dead loads specified above, and by the wind and centrifugal forces under special conditions provided in paragraphs 26, 32, and 39.

30. In calculating stresses, conventional assumptions will be used throughout. The

Assumed spans. lengths of spans will be the distance between centres of end-pins of trusses, and between centres of bearing plates of beams and girders.

[* An unnecessary limitation in high trestles.—J. B. J.]

Assumed lengths, etc.

The length of stringers will be the distance between centres of floor-beams, and the length of floor-beams the distance between centres of trusses.

The depth for calculation of girders will be the distance between centres of gravity of flange sections, provided it does not exceed the distance out to out of angles, in which case the latter amount shall be considered the depth.

The length of posts will be the centre-to-centre length between pins, except in trestle-posts, where the length will be from cap or base-plate to the centres of intermediate struts.

In estimating the section of the end-posts, the collision-strut connection will not be considered.

Any modification of this will be at the discretion of the Engineer.

Bending in pins and rivets will be estimated between centres of bearings.

Formulæ for Unit Stresses.

31. The following formulæ for unit stresses in pounds per square inch of net sectional area shall be used in determining the allowable working stress in each member of the structure:

TENSION MEMBERS.

	Wrought-iron.	Soft Steel.	Medium Steel.
(*a*) Floor-beam hangers or suspenders, forged bars	Will not be used	Will not be used	7,000
Counter-ties	6,000	" " " "	7,000
Suspenders, hangers and counters, riveted members, net section (see § 140)	5,000	5,500	7,000
(*b*) Solid rolled beams (by moments of inertia)	8,000	8,000	Will not be used
(*c*) Riveted truss members and tension* flanges of girders, net section (see § 82)	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)$	8 % greater than iron	$9,000\left(1+\frac{\text{min.}}{\text{max.}}\right)$
Forged eye-bars	Will not be used	Will not be used	$9,000\left(1+\frac{\text{min.}}{\text{max.}}\right)$
(*d*) Lateral or cross-section rods	15,000	16,000	(For eye-bars only) 17,000

COMPRESSION MEMBERS.

	Wrought-iron.	Soft Steel.	Medium Steel.
(*e*) Chord sections: Flat ends	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)-30\frac{l}{r}$	† 10 % greater than iron	† 20 % greater than iron
One flat and one pin end	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)-35\frac{l}{r}$	" " " "	" " " "
Chords with pin-ends and all end posts	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)-40\frac{l}{r}$	" " " "	" " " "
(*f*) All trestle-posts	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)-35\frac{l}{r}$	" " " "	" " " "
(*g*) Intermediate posts	$7,500-40\frac{l}{r}$	" " " "	" " " "
(*h*) Lateral struts, and compression in collision struts, stiff suspenders and stiff chords	$10,500-50\frac{l}{r}$	" " " "	" " " "

In these formulæ, l = length of compression member in inches, and r = least radius of gyration of member in inches. If the allowable stresses given by formula (*c*) are less than those given by (*e*), (*f*), and (*g*), the former will be used.

* The compression flanges of beams and plate girders will have the same cross-section as the tension flange.

† Prof. Burr recommends 15 % and 22 % here in place of 10 % and 20 %.—J. B. J.

MEMBERS SUBJECT TO ALTERNATE TENSION AND COMPRESSION.

	Wrought-iron.	Soft Steel.	Medium Steel.
(*i*) For compression only.	Use the formulæ above		
For the greatest stress. . . . . Use the one giving the greatest area of section	$7{,}000\left(1 - \frac{\text{max. lesser.}}{\text{2 max. greater}}\right)$	8 % greater than iron	20 % greater than iron

COMBINED STRESSES.

(*k*) When the cross-ties rest directly on the top chords, the latter will be considered as beams of one panel length, subject to the maximum bending that will result from the wheel loads and floor system; the beam to be considered as supported at the ends for section in centre of panel, and fixed at the ends for section at the ends of panel. The chords will be proportioned to sustain the algebraic sum of the stresses resulting from the direct compression or tension and the transverse loading given above, in which the allowed stress per square inch shall not exceed:

	Wrought-iron.	Soft Steel.	Medium Steel.
At centre of panel.	8,000	8,800	9,600
At ends of panel.	10,000	11,000	12,000

SHEARING.

	Wrought-iron.	Soft Steel.	Medium Steel.
(*l*) On pins and shop rivets.	6,000	6,600	7,200
On field rivets.	4,800	5,200	Will not be used
In webs of girders.	Will not be used	5,000	6,000

BEARING.

	Wrought-iron.	Soft Steel.	Medium Steel.
(*m*) On projected semi-intrados of main pin-holes.	12,000	13,200	14,500
On projected semi-intrados of rivet-holes.	12,000	"	"
Excepting that in pin-connected members taking alternate stresses, the bearing stress must not exceed 9,000 *pounds for iron or steel*			
On lateral pins.	15,000	16,500	18,000
Of bed-plates on masonry.	250 lbs. per square inch		

BENDING.

	Wrought-iron.	Soft Steel.	Medium Steel.
(*n*) On extreme fibres of pins when centres of bearings are considered as points of application of strains	15,000 *	16,000 *	17,000 *

* Prof. Burr would use the Launhardt Formula here, the same as for the tension and compression members.—J. B. J.

(*o*) Coefficients of Friction will be used as follows:

Wrought-iron or steel on itself	.15
" " cast-iron	.20
" " masonry	.25
Masonry on itself	.50

Effects of wind in truss members.

32. In case the maximum stresses in chords, girder flanges, trestle-posts, or the bending effects on posts due to wind or centrifugal force shall exceed 25 per cent of stresses due to dead and live load, the section will be increased until the total stress per square inch will not exceed by more than 25 per cent the maximum fixed for live and dead load only.

Collision struts.

33. Collision struts will be proportioned to carry a thrust of 50,000 pounds acting in a direction at right angles to the end-posts.

Stiff suspenders.

Stiff suspenders must be able to carry a compressive stress equal to six tenths ($\frac{6}{10}$) the maximum tensile stress.

Stiff chords.

Stiffened chords will be proportioned to take compression equal to $60\frac{T}{L}$, in which T is the maximum tension in pounds in the chord, and L is the span in feet. Use formula (*h*) for these members.

Bending due to weight of member.

34. The effects of the weights of horizontal or inclined members in reducing their strength as columns must be provided for. It will also be considered in fixing the position of pin centres.

Flange areas of girders.

35. Plate girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing or web strains are resisted entirely by the web plate.

Value of rivets.

36. The effective diameter of the driven rivet shall be considered the same as the diameter before driving.

Deducting rivet-holes.

In deducting for rivet-holes, the diameter of the hole will be considered $\frac{1}{8}$ inch greater than the rivet for full-headed rivets, and $\frac{1}{4}$ inch larger for countersunk rivets.

Minimum compression members.

37. No compression member shall have a length exceeding 45 times its least width, and no post will be used in which $\frac{l}{r}$ exceeds 125.

Camber.

38. All bridges with parallel chords shall be given a camber by making the panel lengths of the top chord longer than those of the bottom chord in the proportion of $\frac{1}{8}$ inch to every 10 feet.

Initial stress.

39. An addition of 10,000 pounds * must be made to the stress obtained in all lateral rods to provide for initial tension, and the proper component of this total stress is to be used in the calculation of all lateral struts.

Details of Construction and Workmanship.

General.

40. All details must be of approved forms and satisfactory to the Chief Engineer.

41. Preference will be had for such details as will be most accessible for inspection, cleaning, and painting.

Minimum sections.

42. No shape iron weighing less than six pounds per lineal foot will be used, nor any iron less than $\frac{3}{8}$ inch thick, nor any bar of less than one square inch section.

No angle smaller than 3 inches by 3 inches will be used in girders or truss members, or in any member having $\frac{7}{8}$ inch rivets.

No angle smaller than $2\frac{1}{2}$ inches by $2\frac{1}{2}$ inches will be used in any part of bridge structures.

End angles carrying stringers and floor-beams will be at least $\frac{1}{2}$ inch thick (see §§ 60 and 101).

43. All bed-plates will be at least $\frac{3}{4}$ inch thick.

Tension bars.

44. No tension bar will be less than 1 inch thick, and all tension bars will be of rectangular section.

[* Whenever the working stress in these members exceeds twice the initial stress, the initial stress in the counter rod has disappeared, and hence does not need to be added. See footnote, p. 228.—J. B. J.]

No eye-bars over 2 inches thick will be used; the maximum size of eye-bars will be 8 inches by 2 inches.

Pins. 45. No main pins will be less than 3½ inches diameter, nor less in diameter than three quarters of the width of the widest bar attaching to them.

46. Angles, cover-plates and web sheets shall be as long as practicable to avoid splicing.

Universal plates. Plates for all purposes shall be universal-rolled when the width does not exceed 30 inches (see §§ 76, 78, 124).

Pitch of rivets. 47. The pitch of rivets will not be less than 3 diameters, nor more than 6 inches (see § 92), nor more than 16 times the thickness of the thinnest outside plate. No rivet will have a longer grip than 5 times its diameter, nor be nearer the edge of the metal through which it passes than 1¼ inches when the edges are machine- or roll-finished, *and will not be nearer than 1½ inches to a sheared edge.*

48. The unsupported width of any plate subjected to compression shall never exceed 30 times its thickness, excepting cover-plates of top chords and end-posts, where it may be 40 times its thickness.

49. Lattice-bars will be as described in § 122.

50. All workmanship must be first class; the several pieces forming a built-up member must fit snugly together without open joints. This will be specially insisted upon with reference to pin-bearing chords and end-posts.

All members must be straight and out of wind; holes accurately bored both in position and direction; lengths correct within $\frac{1}{32}$ inch in all cases and within $\frac{1}{50}$ inch for all pieces of the same set.

51. Work will be designed in clean, handsome lines. In addition to the value and usefulness of members in the structure, they will be neatly finished.

52. All material must be straightened before being laid off, and also after punching, if necessary.

I-Beam Spans.

53. I-beam stringers on masonry will preferably be double under each rail, spaced 7 feet 0 inches between centres of bed-plates.

54. They will have planed sole plates riveted to the flanges, and bolted through bed-plates to the masonry at one end, and free to slide longitudinally at the other. They will have rigid cross-struts and transverse bracing riveted to the webs.

55. When in pairs, they will have wrought-metal separators.

56. Bed-plates will be cast-iron, 3″ high.

57. All holes in flanges will be drilled. Holes in webs may be punched.

I-beam stringers. 58. I-beam stringers between floor-beams will preferably be in single lines under each rail, and will have lateral bracing between webs for all spans over 12 feet.

59. They will be spaced 8 feet 6 inches between centres, and be riveted to webs of floor-beams.

Hitch angles. 60. The angles carrying them at the ends will be either 4 inches by 4 inches by ½ inch, or 6 inches by 6 inches by ½ inch.

61. If resting on top of floor-beams, they will be riveted to the floor beams, and knee-braces will support both ends of each line of beams.

62. On the abutments, they will have cross-struts, and be stayed to the main shoes of girders or trusses. Sole-plates and bed-plates (see §§ 54 and 56).

Milled ends. 63. Beams fitting between floor-beams must be milled to length and have the hitch angles accurately set, square to the stringer and flush with its end.

64. Beams which are not required to be of exact length may vary ¼ inch from dimensions called for, and may have the ends cut by cold saw.

65. Heating beams to cut them will not be allowed, and no patched beams will be received.

Rivets.

66. Rivets will be ¾ inch or ⅞ inch diameter in the rod before upsetting.

67. For main members ⅞-inch rivets will preferably be used, with ¾-inch rivets for lateral members.

68. Rivets will be power-driven wherever practicable, and must have full round heads concentric over the shank of the rivet.

69. Whenever the grip length of rivet exceeds 2½ inches, power-driven rivets will be insisted upon.

70. All rivet-holes will be accurately laid off and punched.

71. The dies will not exceed the diameter of rivet by more than $\frac{1}{16}$ inch.

72. When the several pieces forming a member are bolted up, the holes must match accurately throughout, or the material may then be condemned.

73. No drifting will be allowed; holes requiring it must be reamed; hot rivets must enter the holes without the use of a hammer.

74. Countersinking will be neatly done; all holes of the same size, and countersunk rivets must completely fill the holes.

75. Rivets must completely fill the holes, and no loose or badly formed rivets will be allowed, nor any calking. Field riveting must be reduced to a minimum.

Plate Girders.

Steel webs.

76. The webs of all plate girders will be of steel (see Quality of Material, §§ 171–184).

Universal plates will be used for widths up to 30 inches, and sheared plates for greater widths.

77. No rivet-holes will be pitched *nearer than* 1¾ *inches to a sheared edge*, or nearer than 1¼ to a roll-finished or machined edge of the webs.

78. Web splices will be made by two universal steel plates of the same thickness as the web plate.

Stiffeners will be used at all web splices.

The width of the splice-plates shall be sufficient to admit the requisite number of rivets and to receive the stiffeners.

79. Wherever the unsupported distance between the flange angles exceeds 50 times the thickness of the web sheet, vertical stiffeners of angle iron shall be placed on each side of the girder.

Stiffeners.

Stiffeners will be symmetrically spaced from the centre of the girder, and the distance between them, centre to centre of rivets, will not be greater than the distance between centres of flange angles. If unequally spaced, the distance between them will decrease toward the ends.

Fillers.

80. There will be a pair of stiffeners at each end of all bed-plates.

81. All stiffeners will have fillers under them of the same thickness as flange angles and as wide as stiffener angles.

82. The net section of tension flanges will be reckoned as the minimum section square across the flange, and the net section on any diagonal or broken line through two or more rivet-holes must have 25 per cent excess.

83. In calculating shearing and bearing stresses on web rivets of plate girders, the maximum shear acting on the outer side *MM* of any panel will be considered to be transferred to the flange angles in a distance *MO* (equals *MM*), and the number of rivets in the stiffener *MM* will follow the same rule.*

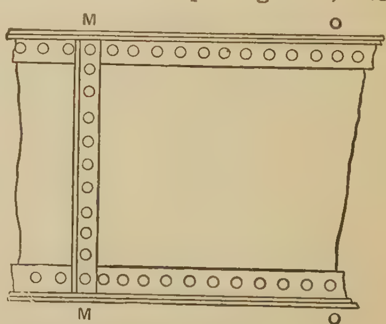


Fig. 452.

Fit of stiffeners, etc.

84. All stiffeners, fillers, and splice plates on the webs of girders must fit at their ends to the flange angles sufficiently close to be sealed, when painted, against admission of water, but need not be tool-finished.

85. Web plates of all girders must be arranged so as not to project beyond the faces of the flange angles, nor on the top be more than $\frac{1}{16}$ inch below the face of these angles at any point.

Rivet spacing for local shear.

86. To provide for local shear of heavy wheel loads, the rivet spacing in top flanges of deck-plate girders and stringers will not exceed 3 inches pitch when there are no cover plates, or 4 inches with cover plates.

[* See Art. 279 for the true theory and practice.—J. B. J.]

Lateral bracing.

87. The compression flanges of girders will be stayed at intervals not exceeding 15 times their width.

88. In through spans, stiffened gussets will run from top flange to each floor-beam.

89. In deck spans the lateral bracing will extend from end to end, and no brace will make an angle less than 40 degrees with the girders, excepting end braces of skew spans.

One top flange plate, etc.

90. All girders having flange plates will have one plate in each flange extending from end to end; and, with the exception of floor-beams, girders will preferably have at least one flange plate.

91. When two or more plates are used on the flanges they shall either be of equal thickness or shall decrease in thickness outward from the angles, and shall be of such lengths as to allow of at least two rows of rivets of the regular pitch being placed at each end of the plate beyond the theoretical point required.

92. When two or more cover-plates over 12 inches wide are used in the flanges of plate girders, an extra line of rivets shall be driven along each edge to draw the plates together and to prevent the entrance of water.

Plates over 17 inches wide will have three rows of rivets with 9 inches pitch for the outer row.

Flange splices.

93. All joints in flanges, whether in tension or compression members, must be fully spliced, as no reliance will be placed upon abutting joints. The ends, however, must be dressed straight and true, so that there shall be no open joints.

94. Flange angles must be spliced with angle-covers whenever cut within the length of the girder.

95. Splices must break joints with each other, one piece only being spliced at any point.

Cross-frames.

96. Cross-frames will be used at the masonry ends of all girders and at intermediate points when wind or centrifugal force makes it desirable.

97. All cross-frames will be made of angles and plates, and will be stiff rectangles of four members, viz., top, bottom, and two diagonals.

Finish of girders.

98. Girders will be neatly finished at the ends. They will have a plate corresponding in width with the cover-plates riveted to end stiffeners, and a corner cover at the top riveted to both top and end plates.

Stringers and Floor-beams.

Steel webs.

99. Webs of stringers and floor-beams will be steel (see § 76).

100. Stringers and floor-beams will preferably be carried by rivets at their ends, and all such ends will be milled to exact length.

101. The end angles carrying floor-beams and stringers will be either $4 \times 4 \times \frac{1}{2}$, or $6 \times 4 \times \frac{1}{2}$, or $6 \times 6 \times \frac{1}{2}$ inch angles, as one or two rows of rivets are required, and they will have fillers under them 7 inches and 9 inches wide, respectively, of same thickness as flange angles, and having a row of rivets outside the hitch angles.

Hitch angles.

102. The hitch angles of stringers will have a full complement of rivets, and, in addition stringers may have a bracket and stiffeners under them unless they practically cover the web of floor-beam between flanges.

Reamed holes for field rivets.

103. All holes for field rivets in stringer and floor-beam connections will be punched $\frac{3}{4}$ inch diameter, and afterward reamed out to $\frac{15}{16}$ inch, using cast-iron templates. The wire edge on reamed holes must be removed.

104. Floor-beams having stringers resting on their top flanges will have stiffeners under the points of support.

105. Stringers will preferably have one cover-plate from end to end (see § 90).

The rivet pitch through the web in top flanges of stringers will not exceed 4 inches throughout (see § 86).

Hangers.

106. Hangers will be either solid forged eye-bars riveted to floor-beams, or riveted plate hangers. Stirrup-hangers or bent hangers will not be used.

107. Eye-bar hangers will have the usual excess of material across the eye, and all rivet-holes in them will be bored, and the edges of holes will be chamfered to make a fillet under rivet-head.

108. All riveted hangers will have excess sections at the pin-holes, as described in § 139

Long Plate Girders.

109. Plate girders of lengths from 75 ft. to 100 ft. may be built of medium steel, under special conditions, as follows:

110. All rivet-holes in the angles and plates in both flanges will be drilled in the solid. Occasional small holes may be punched for purposes of bolting up and reamed afterward.

111. All other rivet-holes in web plates, stiffeners, lateral braces, and fillers may be punched of full-size for riveting, provided the metal does not exceed $\frac{9}{16}$ inch thickness.

112. The spliced ends of tension flange plates or angles will be milled off $\frac{1}{4}''$ to remove sheared edges; and the ends of all splicing pieces in tension flange will be similarly milled. No other milling of sheared edges will be required, excepting such as is specified for iron girders.

113. All web splices will have four rows of rivets, the middle rows being pitched five inches between centres.

114. Rivets will be of soft steel and power-driven whenever practicable.

115. The general requirements given under Plate Girders will apply to these also, except as modified above. (See §§ 77 to 98 inclusive.)

Pin-connected Spans.

116. See §§ 10, 11, 12, 31, 33, and 47.

117. No web member, except collision struts, shall make a less angle with the horizontal than 45 degrees.

Compression Members.

118. All parts working together as one member shall be uniformly stressed.

Centre of gravity.

119. All eccentricity of stress shall be avoided. Pin-centres will be in the centre of gravity of the members, less the eccentricity required to provide for their own weight; and in continuous chords, pin-centres must be in the same plane.

120. See §§ 46 and 69.

Batten plates.

121. The open sides of all compression members shall be stayed by batten plates at the ends, and diagonal lattice-work at intermediate points. The batten plates must be placed as near the ends as practicable, and shall have a length of $1\frac{1}{2}$ times the width of the member.

Lattice.

122. Lattice bars will preferably be of soft steel, and be at least $2\frac{1}{2}$ inches wide for $\frac{3}{4}$-inch or $\frac{7}{8}$-inch rivets, and increase with the width of the member. They will make an angle of 60 degrees with the axis of the member, and the same angle with each other.

123. Double lattice will be used in all members having a clear width between webs of more than 20 inches.

Pin plates.

124. When necessary, pin-holes shall be re-enforced by plates, so that the allowed pressure on the pins will not be exceeded. These re-enforcing plates must contain enough rivets to transfer proportion of the bearing pressure, and at least one plate on each side shall extend not less than 6 inches beyond the edge of the furthest batten plate.

Pin, splice, and batten plates will be universal-rolled plates.

Forked ends.

125. When the ends of compression members are forked to connect to the pins, the aggregate compressive strength of these forked ends must equal the compressive strength of the body of the members; in order to insure this result, the aggregate sectional area of the forked ends at any point between the inside edge of the pin-hole and 6 inches beyond the edge of the batten plate, shall be about double that of the body of the member.

126. In compression chord sections, the material must mostly be concentrated at the sides, in the angles and vertical webs.

Pin-holes.

127. Pin-holes shall be bored exactly perpendicular to a vertical plane passing through the centre line of each member when placed in a position similar to that which it is to occupy in the finished structure.

Milled ends.

The ends of all members which make contact joints shall be planed smooth, to exact lengths and to exact angles, with the axis of the member.

Reamed splices.

128. Abutting members must be brought into close and forcible contact when fitted with splice plates, and the rivet-holes reamed in position before leaving the works, all pieces being match-marked, so as to fit in the same position in erecting.

Pitch of rivets at ends.

129. In all compression members the pitch of rivets at the ends shall not be over four times the diameter of the rivets for a length equal to twice the width of the member.

Couplers.

130. The couplers on chords and end-posts will be at least $\frac{1}{2}$ inch thick.

131. In intermediate posts, both web and pin plate will extend beyond the pin for a length about equal to the diameter of pin-hole.

Collision struts.

132. Collision struts will be disposed to best advantage to take a shock of derailed train striking $3\frac{1}{2}$ feet above base of rail. They will preferably make an angle of 80 degrees or more with the end-post, and may be placed 3 feet away from the point indicated above in order to increase the angle.

Tension Members.

Eye-bars.

133. All eye-bars will be solid forged-steel bars of approved quality and rolled by mills having established reputations for the manufacture of eye-bar steel.

134. No work on the bars will be done at a blue heat, and all bars must be thoroughly annealed after forging.

135. The heads of eye-bars shall be so proportioned that the bar will break in the body instead of in the eye. The form of the head and the mode of manufacture shall be subject to the approval of the Chief Engineer of the railway company before the contract is made.

136. The bars must be free from flaws and of full thickness in the necks. They shall be perfectly straight before boring. The holes shall be in the centre of the head and on the centre line of the bar.

137. The bars must be bored of exact lengths and the pin-holes $\frac{1}{50}$ inch larger than the diameter of the pin.

All bars on the same item must be bored at one setting of the drills and at the same temperature.

Bars may vary $\frac{1}{32}$ inch from ordered length, but bars on the same item must not vary $\frac{1}{50}$ inch in length.

Rivet-holes in eye-bars will be drilled, and have the wire edges cut off the edges of the holes.

Upset screw ends

138. Upset screw ends may be used on steel bars if fully guaranteed and tested in full-sized bars. The area at base of thread and at all parts of upset ends will be 15 per cent in excess of the area of the bar.

Riveted Tension Members.

Excess at pin-holes.

139. Riveted tension members must be designed with special care. Members with pin connections will be required to have net areas across the pin-hole, and back of the pin-hole, respectively of 150 per cent and 80 per cent of the net area required in the body of the member, and there will be a corresponding excess of rivets to make this material effective.

The length from back of eye to end of member must be greater than the radius of the pin.

140. In members with riveted connections special care will be used to have the rivets symmetrically arranged from the centre line and to avoid eccentricity of stress.

If rivets are staggered they will all be deducted as though they occurred at the same cross-section.

Rods.

141. Rods for counters or laterals will, preferably, be either simple loops or clevis rods.

142. Loop rods will be of wrought-iron, and must be upset and welded in a thoroughly efficient and workmanlike manner.

143. The eyes of all loop rods must be bored to fit the pins.

Upset screw ends must have a net section at base of thread 15 per cent greater than the body of the bar.

144. Steel rods may be used for upset screw ends, but must be fully guaranteed and tested in full-sized section. They must be annealed after forging.

145. Clevises, turnbuckles, and sleeve nuts must be of approved pattern, and fully guaranteed.

Pins.

146. All main pins will be made of medium steel.

The diameter of the pin will not be less than three-fourths ($\frac{3}{4}$) the largest dimension of any tension member attached to it.

147. The several members attached to the pin shall be packed close together, and all vacant spaces between the chords and posts must be filled with wrought-iron filling rings.

The pins shall be turned straight and smooth, and shall fit the pin-holes within $\frac{1}{50}$ inch. They shall be turned down to a smaller diameter at the ends for the thread, and driven in place with a pilot-nut when necessary to save the thread.

148. Nuts will be of wrought-iron or wrought-steel. There will be a washer under each nut, or else Lomas nuts will be used.

Lateral Bracing.

149. The attachment of the lateral system to the chords shall be thoroughly efficient. If connected to suspended floor beams, the latter shall be stayed against all motion.

150. Preference will be given to lateral bracing in the floor system, which is capable of resisting both compression and tension.

Portals and knee-braces.

151. Portals and intermediate knee-braces shall be used in all through bridges, so designed as to form efficient and rigid connection between top lateral struts and web members. Where the height of the truss makes it practicable, the knee-braces shall be replaced by suitable cross-section bracing at each pair of intermediate posts.

152. When brackets only can be used for portal braces they will have plate webs.

Portals will be as deep as the head-room will allow.

Deck spans.

153. In all deck bridges transverse bracing shall be provided at each panel; this bracing shall be proportioned to resist the unequal loading of the trusses, and the wind and centrifugal forces; the transverse bracing at the ends shall be of the same equivalent strength as the end top lateral bracing.

154. The hitches for lateral braces must be thoroughly efficient; shear on field rivets will be reckoned as per formula (*l*).

Shoes, Bed-plates, etc.

155. There must be a pier box or plate of approved form under pedestal shoes at both ends of sufficient depth to distribute the weight properly on masonry. These boxes or plates must be at least $\frac{3}{4}$ inch thick, must have planed surfaces, and be of such dimensions that the greatest pressure upon the masonry will not exceed 250 pounds per square inch, and sheet lead * not less than $\frac{1}{8}$ inch thick shall be interposed between them and the masonry.

Two spans.

156. Where two spans rest upon the same masonry a continuous plate not less than $\frac{3}{4}$ inch thick shall extend under the two adjacent bearings.

Bolts.

157. All the bed-plates and bearings under fixed and roller ends must be fox-bolted to the masonry; for trusses, these bolts must not be less than $1\frac{1}{4}$ inches diameter; for plate and other girders, not less than $\frac{7}{8}$ inch diameter. The contractor must furnish all bolts, drill all holes, and set bolts to place with sulphur.

Rollers.

158. All bridges over 75 feet span shall have at one end nests of turned friction rollers, formed of wrought steel, running between planed surfaces. The rollers shall not be less than $2\frac{1}{2}$ inches diameter, and shall be so proportioned that the pressure per lineal inch of roller shall not exceed the product of the square root of the diameter of the roller in inches multiplied by 500 pound ($500\sqrt{d}$).† Bridges less than 75 feet span will be secured at one end to the masonry, and the other end shall be free to move by sliding upon planed surfaces.

No lateral movement.

159. Friction rollers must be so arranged as to be readily cleaned and to retain no water.

160. While the roller ends of all trusses must be free to move longitudinally under change of temperature, they shall be anchored against lifting or moving sideways.

Workmanship on Medium Steel.

161. Medium steel will be subject to the general conditions given under "Workmanship" above, and in addition to the following requirements:

[* Sheet aluminum is now used extensively for this purpose.—J. B. J.]

† See Arts. 254 and 255.

(*a*) All sheared and hot-cut edges shall have not less than ¼ inch of metal removed by planing. (Lattice bars only will be exempted from this.)

(*b*) All punched holes will be reamed to a diameter ¼ inch larger, so as to remove all the sheared surface of the metal.

(*c*) No sharp or unfilleted re-entrant corners will be allowed.

(*d*) All rivets will be steel.

(*e*) Any piece which has been partially heated or bent cold will be afterward wholly annealed.

Quality of Material. *

Wrought Iron.

162. All wrought-iron must be tough, ductile, fibrous, and of uniform quality for each class, straight, smooth, free from cinder-pockets, flaws, buckles, blisters, and injurious cracks along the edges, and must have a workmanlike finish. No specific process or provision of manufacture will be demanded, provided the material fulfils the requirements of these specifications.

163. The tensile strength, limit of elasticity, and ductility shall be determined from a standard test piece not less than ¼ inch thick, cut from the full-sized bar, and planed or turned parallel. The area of cross-section shall not be less than ½ square inch. The elongation shall be measured after breaking on an original length of 8 inches.

164. The tests shall show not less than the following results:

	Ultimate Strength.	Limit of Elasticity.	Elongation in 8 inches.
	Pounds per square inch.	Pounds per sq. inch.	Per cent.
For bar iron in tension	50,000	26,000	18
" shape iron	48,000	26,000	15
" plates under 36 inches wide	48,000	26,000	12
" " over " " "	46,000	25,000	10

165. When full-sized tension members are tested to prove the strength of their connections, a reduction in their ultimate strength of (500 × width of bar) pounds per square inch will be allowed.

166. All iron shall bend, cold, 180 degrees around a curve whose diameter is twice the thickness of piece for bar iron, and three times the thickness for plates and shapes.

167. Iron which is to be worked hot in the manufacture must be capable of bending sharply to a right angle at a working heat without sign of fracture.

168. Specimens of tensile iron upon being nicked on one side and bent shall show a fracture nearly all fibrous.

169. All rivet iron must be tough and soft, and be capable of bending cold until the sides are in close contact without sign of fracture on the convex side of the curve.

170. Samples from each rolling will be tested, and also widely differing gauges of the same section.

Steel.

171. All steel will be uniform in quality, low in phosphorus, and from works of established reputation.

Phosphorus. 172. The phosphorus in all melts of acid open-hearth steel must be less than 0.10 per cent, and in all Bessemer or basic steel must be less than 0.08 per cent. Certified analyses of all melts will be furnished the Engineer free of charge.

Tests from finished material. 173. The material will be tested in specimens of at least one-half square inch section, cut from the finished material. Each melt of steel will be tested, and each section rolled, and also widely differing gauges of the same section.

* See also *Specifications for Structural Steel for Modern Railroad Bridges* (1894), by Geo. H. Thomson, Consulting Engineer, Grand Central Station, New York; also *Specifications for Structural Steel*, by H. H. Campbell, Trans. Am. Soc. C. E., Vol. XXXIII (1895).

174. If several different sections are rolled from the same melt of steel, all of them may be tested at the discretion of the inspector.

Melt numbers. 175. All finished material will be plainly and distinctly marked with correct melt numbers.

176. If the melt number is lacking, or is illegible, or has been changed, the material may be condemned at the discretion of the inspector.

177. If first tests are unsatisfactory, the material will not be accepted unless—

Re-tests. (1) A majority of the tests fill the specifications.

(2) All the tests show good material of reasonable uniformity.

Soft Steel.

178. Soft steel may be used under the same conditions as wrought-iron except—

(1) It must be used consistently. The occasional use of pieces of steel will not be permitted.

(2) It must not be welded.

(3) The thickness of material subjected to punching will be limited to $\frac{1}{2}$ inch, excepting (*a*) in plate girders over 50 ft. long, in which it may be $\frac{9}{16}$ inch; (*b*) in top chords and end-posts, in which it may be $\frac{5}{8}$ inch; and (*c*) in shoes, pedestals, and bed-plates, in which it may be $\frac{3}{4}$ inch thick.

179. Soft steel when tested as described above must meet the following requirements:

An elastic limit of at least 32,000 lbs. per square inch.

An ultimate strength of 54,000 to 62,000 lbs. per square inch.

An elongation in 8 inches of at least 25 per cent.

A reduction of area of at least 45 per cent.

Web plates. For web plates over 36 inches wide the elongation will be reduced to 20 per cent and the reduction of area to 40 per cent.

180. It must bend cold 180 degrees and close down on itself without cracking on the outside.

181. When $\frac{7}{8}$-inch holes pitched $\frac{3}{4}$ inch from a roll-finished or machined edge and 2 inches between centres are punched the metal must not crack; and when $\frac{7}{8}$-inch holes pitched $1\frac{1}{8}$ inch between centres and $1\frac{1}{2}$ inches from the edge are punched, the metal between the holes must not split.

Rivets. All rivets will be soft steel.

Medium Steel.

182. Medium steel only will be used for eye-bars and main pins, and it may be used for other members under conditions given in § 161.

183. Medium steel when tested as described above must meet the following requirements:

An elastic limit of at least 35,000 lbs. per square inch.

An ultimate strength of from 60,000 to 70,000 lbs. per square inch.

An elongation in 8 inches of at least 20 per cent.

A reduction of area of at least 40 per cent.

It must bend 180 degrees on itself around a $1\frac{1}{2}$-inch round.

Eye-bar tests. 184. Full-sized eye bars, when tested to destruction, must show an ultimate strength of at least 56,000 pounds, and stretch at least 10 per cent in a gauged length of 10 feet.

There will be two bars tested from each span.

Inspection.

185. Ample facilities for inspection and testing of material and workmanship must be furnished to the Chief Engineer of the railway company or his assistants. Tests on small specimens to determine the quality of materials will be made free of charge to the railway company before any work is done on the material.

Full-sized tests. 186. Full-sized parts of the structure may be tested at the option of the Engineer of the railway company, and shall be paid for at cost less their scrap value, if they prove satisfactory. If the test is not satisfactory the contractors will receive no recompense.

Painting.

187. All surfaces in contact with each other must receive one coat of red oxide-of-iron paint or other metallic paint mixed in pure linseed oil, approved on sample by the Engineer of the railway company.

188. All work before leaving the shops must be thoroughly cleansed from all loose scale and rust, and be given one good coat of pure raw linseed oil, well worked into all joints and open spaces.

189. All surfaces that will be inaccessible after erection must receive one coat of the approved paint during erection, the iron to be perfectly cleaned before painting.

190. The railway company will paint the structure after erection.

191. All planed or turned surfaces must be coated with white lead, mixed with tallow, before shipment.

192. No painting will be allowed during wet or freezing weather.

False-work.

193. The contractor will furnish all necessary false-work and remove the same after the completion of the bridges, leaving the several streams and rivers unobstructed, except the actual space occupied by the masonry.

Risks.

194. The contractor shall assume all risks from floods and storms, and also casualties of every description, and must furnish all material and labor incidental to or in any way connected with the manufacture, erection, and maintenance of the structure until its final acceptance.

195. If any patented parts be used, the contractor shall protect the railway company against any and all claims on account of such patents.

Preservation of Old Material.

196. In the erection of bridges, the contractor will be required to remove all old material in such a manner as will not impair its future use in structures similar to that from which it was taken, unless otherwise agreed.

Maintaining Tracks.

197. The contractor will be required to maintain the track in proper condition for the passage of all schedule trains without delay, except when previously arranged for by the Division Superintendent.

Final Test.

198. Before the acceptance the Chief Engineer of the company may make a thorough test by passing over each structure the specified trains or their equivalent at a speed not exceeding thirty miles an hour, and bringing them to a stop at any point by means of the air or other brakes, or by resting the maximum train load upon the structure for such period of time as he may deem proper.

Specifications for Second-class Bridge Superstructure.

For Divisions and Branches Carrying Light Traffic.

1. The loading will be the same as for first-class bridges, and the bridges fully conform to the specifications for first-class bridge superstructures, excepting as follows:

(1) Outside of the lateral system, the stresses due to wind and centrifugal force need not be provided for unless they exceed 40 per cent of the stresses due to dead and live load.

(2) The unit stresses will be modified as follows:

TENSION.

	Wrought Iron.	Soft Steel.	Medium Steel.
I-beams (by moments of inertia)......	8,500	8,500	Will not be used

COMPRESSION.

	Wrought Iron.	Soft Steel.	Medium Steel.
Trestle posts........................	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)-30\frac{l}{r}$	10% greater than iron	20% greater than iron
Intermediate posts..................	$7,000\left(1+\frac{\text{min.}}{\text{max.}}\right)-40\frac{l}{r}$	" " " "	" " " "

Stresses not specified will be the same as for first-class bridge superstructures.

(3) One sixth ($\frac{1}{6}$) of webs of girders will be considered available section in each flange, except at web splices, where the full section shall be provided for by extending the flange plate or by the addition of cover-plates.

APPENDIX B

PROCESSES IN THE MANUFACTURE AND IN THE INSPECTION OF IRON AND STEEL STRUCTURES.*

Of the four natural divisions into which the construction of a metallic framed structure is divided, i.e., Design, Mill-work, Shop-work, and Erection, the first has been treated in the body of this work, and of the remainder, the mill-work and shop-work alone will be considered here.

MILL-WORK.

Processes in Manufacture.—Iron.—The order of the manufacturing processes are as follows:

Mill-work in Iron:
1. Making the Pig.
2. Puddling.
3. Rolling the Muckbars.
4. Cutting and Piling.
5. Rolling.
6. Straightening.
7. Working and Shearing.
8. Inspection.

For what is called double-refined iron, after item No. 5, it would be necessary to insert two more items, i.e., Cutting and Piling, and Rerolling.

The *Pig* is made by melting the ore with a flux, in a blast-furnace, the product being run out into a series of small "pens" to cool.

This is then puddled in a square box furnace, also called a reverberatory furnace, where it is remelted, much of the combined carbon and other impurities burned out of it, when the iron is gradually gathered by the puddler into spongy masses called puddle-balls, weighing about seventy-five pounds each. These are then removed from the furnace and taken to either a hammer or a cam-squeezer, and worked down into a shape suitable for rolling into muck-bar. This process also removes most of the slag contained in the ball. The material is next sent through the muck-rolls and reduced to muck-bar. These bars are usually about four or five inches wide and one inch thick, and are very rough in appearance.

This muck-bar, together with waste scrap-iron, is then cut up into lengths, usually not over six or seven feet, depending upon the size of the piece to be rolled, and reheated in an oven and then passed through the rolls. As has been said, for double-refined iron this material would again be cut up, piled, re-heated, and again rolled into flat bars, and these repiled and rolled to the final shapes. This material is much stronger and more homogeneous than the first product, which is called Refined Iron or Merchant Bar. After the material has been rolled to its final forms it is run out on a series of skids called the hot-bed, where it is allowed to cool. From here it goes through the straightening machine. This may be either a gag-press or a train of rolls, three below and two above. The latter is much the better, producing straighter bars with less injury to the material.

After coming from the straightening machine the material is marked and sheared to length, and then inspected. Each piece is marked in white-lead with its true dimensions, and also, in the case of steel, with its heat and bloom number. The material is now ready to be shipped to the bridge manufacturer.

Steel-work.—Steel is made by various processes, of which the best, to date, are the Open-hearth and the Bessemer. For a complete discussion of these processes see standard works on the metallurgy of iron and steel.† The Open-hearth processes (for there are several), though slower (requiring from seven to ten hours for one heat, while the Bessemer blow can be made in half an hour), are considered by many engineers to be more thorough, producing a more homogeneous material than the Bessemer product. With the same grade of ore in each case this is undoubtedly true. A cheaper grade of ore is employed usually in the Open-hearth process. When the melt is finished it is run off into ingot-moulds. The ingots are about eighteen inches square at one end and twenty inches square at the other, and about six feet long. One

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* See also *Specifications for Structural Steel for Modern Railroad Bridges*, and Instructions to Inspectors, by Geo. H. Thomson, Consulting Engineer, Grand Central Station, New York.

† Especially Howe's *Metallurgy of Steel;* also a paper on Specifications for Structural Steel, by H. H. Campbell, in Trans. Am. Soc. C E., Vol. XXXIII.

blow from the Bessemer converter makes three such ingots, while a melt or heat from the open-hearth furnace will make six.

From each heat or blow there is also cast a small billet about four inches square, which is rolled down into a three-quarter-inch round, from which the heat or blow-tests are made for determining the quality of the steel. A chemical analysis is also made of each heat. The large ingots are next reheated, and taken to the blooming-mill, where they are rolled down and cut into blooms. The size of these will vary according to the order in hand. Each bloom is supposed to make a certain number of ordered pieces.

The remaining processes in the mill are the same for steel as for iron, with the exception noted.

INSPECTION.—In the inspection of iron, no tests can be made before the material is rolled. Specimens are then selected, two or three or more, depending upon the size of the order, from each size and shape, and from these test specimens are cut, about 18″ in length, 1″ in width in the reduced portion, the thickness being left the same as that from which the specimen was taken.

In iron each pile may differ from all the others. This difference is in general very slight, the same kind of muck-bar and the same kind of scrap being used. Nevertheless, for this reason more tests are made of iron than of steel.

In steel, tests are made on the ¾″ round specimens before the material is rolled. These tests will usually run a little below the final finished material tests in elastic limit and ultimate strength, and a little above them in elongation and reduction. Allowance should be made for this variation in the acceptance of the heat.

After the material is rolled test specimens are taken, one from each size of each heat for tension tests. Specifications usually require also bending, nicking and bending, punching, and tempering tests. One test of each kind for each heat is all that would be required.

These represent the physical tests of the material. For each of them the inspector can get numerical values. For the superficial inspection, however, he cannot do this, and here he is largely left to his own judgment. Rejections at this point are for various reasons, such as unwelded or ragged edges, burns, cinder spots, blisters, etc., not easily described, but learned by experience.

In inspecting the material the inspector should have a copy of the mill order and check off such as he accepts, so that he as well as the mill people may know how much remains to be furnished.

With his hammer he should also stamp every accepted piece, putting a ring of white-lead around the mark. This will save much trouble at the shop, and should never be neglected. When the material is shipped he should compare the invoices with his mill order, and see that no more than he has accepted has been shipped. In iron, superficial inspection of the muck-bar is sometimes required, and also of the piling. If, however, the finished material is to be thoroughly tested these restrictions are not necessary.

It is not just to demand a certain grade of article and also specify how it shall be obtained. The same is true with regard to the limitations placed upon the chemical composition of steel. The physical tests should be sufficient to develop its capacity for performing the work it has to do.

SHOP-WORK.

PROCESSES IN MANUFACTURE.—The various processes in the shop are as follows:

Order of processes in shop:

1. Straightening (when necessary).
2. Marking-off and punching.
3. Straightening.
4. Reaming.
5. Assembling.
6. Reaming.
7. Riveting.
8. Facing.
9. Boring.
10. Finishing.
11. Fitting up.
12. Inspecting.
13. Oiling and Painting.
14. Shipping.

This table applies to large members as a whole, such as posts, chords, girders, etc. On any given contract there will be more or less machine and blacksmith shop-work not provided for in the table.

STRAIGHTENING.—This is often necessary, either because it has not been done properly at the mill or has been abused in handling. The work cannot be laid off well if the material is very crooked.

MARKING-OFF AND PUNCHING.—Templets are made for the proper gauge and pitch of rivets, and from these the material is marked off. Upon this work single punches are used. For complementary chord angles no templets are necessary. They are clamped together and put through a rack punch. Multiple punches are used on web plates. With these the entire row of stiffener rivet-holes can be punched at one setting. The rack and multiple punches save a great deal of time, but they require more skill and care to achieve as good results as are obtained from the templet, single-punch work. A broken gauge-line on chord angles makes very unsightly work.

All material is stretched slightly by the process of punching. This is very noticeable in chord angles, especially if both legs are punched, in which case it may amount to as much as $\frac{1}{8}$ to $\frac{1}{4}$ inch in 20 feet. It will vary according to the size of the angle and size and pitch of the rivets. The effect upon web plates is to stretch a small strip of the plate next to the punched edge. This results in a continuous series of short buckles along this edge, impossible to take out in bolting up the member.

STRAIGHTENING.—On account of this buckling of the material in punching, the material should always be sent to the rolls and straightened before assembling. This is a very important item, but one which is sometimes entirely disregarded. If it is not done, the angles and web plate cannot be made to come together. Then in the ordinary process of rapid riveting, the rivet will be sufficiently hot after the pressure is removed to allow the spring between the angles and plate to distort or draw the rivet slightly. When the next rivet goes in it draws the material up again, and this leaves the first rivet, if not loose, at any rate not cupped down. This will be true of every rivet driven unless the pressure is held on an unusual length of time. In cases like this it is not unusual for as high as 20 per cent of the rivets in a girder to be cut out. The finished member also never looks as well as if the material had been straightened.

REAMING.—This may be done by hand or by flexible tube-reamers attached to the shafting. The latter method is of course much the better. Nearly all specifications require that the material shall be punched to a diameter $\frac{1}{16}$ in. less and reamed to a diameter $\frac{1}{16}$ in. greater than that of the rivet. Some specifications require the punched holes to be reamed to a diameter $\frac{1}{8}$ in. larger than the rivet. This is for the purpose of removing the material affected by the punch. It has not been established, however, that in soft steel the material around the hole has been materially injured in punching plates less than $\frac{5}{8}$ in. in thickness.*

On the punch side of the plate the material remains in its normal condition and uninjured, but on the die side the effect is different and the material is injured from the cold flowing produced here. This injury will vary directly with the thickness of the plate, and with the bluntness of the edges of punch and die.

ASSEMBLING.—The next process is to collect all the various pieces forming a member of the structure and bolt them up preparatory to riveting.

REAMING.—Before going to the riveter, however, a reamer is passed through all the holes to make sure that the rivet will enter. It is thus seen that we have two reamings. Owing to the stretching of the material in the punching, the angles stretching more than the web, and to accidental causes, it is impossible to make all the holes fit. In a 30-ft. girder the holes near the ends may not match by $\frac{3}{16}$ in. The reamer is put through and takes off not more than $\frac{1}{16}$ in. from the angles and the remaining $\frac{1}{8}$ in. from the web. This leaves a pocket $\frac{1}{8}$ in. deep at the web into which the rivet must back up in order to fill the hole and be able to transmit stress. It takes a good riveting-machine to do this well. Usually it would not be done. For this reason the writer is in favor of punching the material to a diameter $\frac{1}{8}$ in. less than the diameter of the rivet, and of having but one reaming, that being done after the pieces are assembled. Some would make the objection to this method, that all of the injured material around the hole would not be removed. But as it would be better to have the vacant space filled with anything than with nothing at all, and as what proof there is available upon the subject seems to indicate that no real injury has been done to the working strength of the member, the method would seem to be worthy of adoption.

RIVETING.—Whenever it is practicable, rivets are driven by machine riveters. These are of many kinds. Compressed air, steam, and water furnish three different kinds of power, and each of these are applied to several different styles of machines. They may be divided into two general classes, *direct* and *indirect* acting. By a direct-acting riveting-machine is meant one in which the ram moves in the line of final pressure throughout the stroke. The indirect-acting machines, instead of a ram acting directly from the piston, have two jaws pivoted in the middle, the power being applied at one end and the pressure on the rivet at the other. The jaw therefore moves in the arc of a circle. These machines act well enough where the thickness of material through which the riveting is being done is equal to the distance between the cups on the fixed and movable jaws when the faces of these cups are parallel. If these distances are not equal, lop-sided rivets will be the result. To make good rivet-heads with such a machine, therefore, requires for each diameter of rivet a set of cups, one for each length of rivet. Some of these cups may be dispensed with by having a set of washers to go under the cup.

* See Appendix A on this subject.

The indirect-acting riveter is not as satisfactory as the direct-acting, and is very often forbidden on large contracts.

The hydraulic, direct-acting machine is the most satisfactory of all. Steam and compressed air work well on the average, but when it happens that all the machines in a shop are working to their full capacity at the same time, the pressure usually runs low.

FACING.—All abutting ends of members need to be planed off. This is done by means of a rotary planer or facer. This machine has a face-plate revolving in a vertical plane, into and at right angles to which are set a series of cutting tools, from three to six inches apart, forming a circle around the centre of the face. The piece to be planed is bolted to a bed in front of this face, and the latter is, by means of a screw, fed across the end of the piece, each tool taking off a slight chip. The frame holding the face-plate can also be revolved around a vertical axis any required number of degrees so as to plane the end on a mitre if so desired, as, for example, the ends of batter-posts.

BORING.—For boring pin-holes, etc., two kinds of machines are used, vertical and horizontal. The latter is generally preferable, allowing a more accurate adjustment of the member.

FINISHING.—This includes all hand work necessary to finish the piece, such as putting on of brackets, driving such rivets as could not be driven by machine, chipping, and making whatever changes the inspector may require.

OILING AND PAINTING.—Before shipping, the material should be cleansed of loose scale and rust, and oiled with a coat of pure raw linseed oil. After this it should be painted with some good quality of paint, but this is not usually done at the shop.

INSPECTION.—On a large contract of from 5000 to 10,000 tons of material, the best shop inspection is secured by the employment of special inspectors who give the work their personal supervision and report directly to the Chief Engineer. The mill work, however, is more economically done by the agents of some regular Inspection Bureau.

Such a contract as the one named above requires, in the shops, two men, one being the Chief Inspector and the other his Assistant. The Chief Inspector does all of the actual inspection, receives reports from mill inspectors, looks out for possible causes of delay, and has the general supervision of all the work. He reports weekly to the Chief Engineer. His assistant helps in taking measurements, keeps up invoice reports (checking weights of same if the contract is by the pound), keeps track of material for the daily progress report, makes tests of material, and does many other things of a similar nature.

THE RECORDS.—If there is a time-penalty clause in the specifications, as there usually is, another object of the inspection is to be able to testify as to the cause and character of any delay in the prosecution of the work. To keep account of the material in such a way that this can be done is not a simple matter. On such a contract as has been named it would require the inspector to know, at any time, the exact condition of from 12,000 to 24,000 pieces of material. This is not as difficult a task as it would seem at first. Of these 12,000 to 24,000 pieces not all are changing their condition each day. Probably half of them are. Then, too, they go in groups, so that we can greatly reduce the number.

To keep this record requires a day-book ruled as below:

Left-hand page. *Right-hand page.*

General Remarks.	Punched.	Reamed.	Bolted.	Riveted.	Milled.	Bored.	Turned.

In these columns, each day, the inspector enters the designation of the piece which has received that particular treatment. It will not always be possible for him to get these independently of the contractor, but that is not necessary. If he goes about it in the right way the contractor will allow him access to the daily reports of the shop, which of necessity are as correct as may be. From these he can readily get the last five columns and enter them directly into his day-book. Punching and reaming are more difficult to get. But these also in most shops he can get from the shop records. But they will probably give the record for the component pieces and not for the whole member. In this case he cannot use the day-book at first, but

must take the shop list on which all of the members with their component pieces are given, and a copy of which he can get.

On these sheets, using red ink entirely, he rules two columns, one for punching and one for reaming, and then enters up these records for each of the essential component pieces with date. Then when these have all been treated for that member he enters it in his day-book for the day on which the last really necessary piece of the member was finished.

For keeping account of the mill material he checks off on the mill order, a copy of which he has, the pieces from invoices as they come in, entering date of receipt of car. Thus the assistant has a record of every piece from the time it left the mill until ready for finishing and inspecting. The rest is obtained by the Chief Inspector himself.

When a large number of pieces of similar but slightly-varying construction are under contract, the inspection is sometimes rendered difficult by the large number of sheets of drawings involved. This difficulty is best obviated by the preparation of tables giving all of the important descriptive features of each member. These tables are made out in a note-book which the Chief Inspector always has with him.

The table for chords and posts is given below:

CHORDS AND POSTS.

No. of Drawing.	Name of Piece.	Length over all.	Length between Pin-centres.	Size of Pin-hole.		Size of Web or Bar.	Size of Chord Angles.		Thickness of Pin-bearing.	Clearance.		Cover-plates.	Splice-plates.	Remarks.
				N.	S.		Top.	Bot.		Inside.	Outside.			

The floor-beam and stringer table would be as follows:

FLOOR-BEAMS AND STRINGERS.

No. of Drawing.	Name of Piece.	Length.	Size of Chord Angles.		Size of End Stiffeners.	No. of Rivets in End Connections.	Size of Web.	Remarks.
			Top.	Bottom.				

In case the floor-beams have hangers or a direct connection to the pin a slightly different form would be required.

In elevated or viaduct work, also, the stringer table would require some extra columns to provide for the bevels at the ends on grades and curves, thus:

Bevels.			
Vertical.		Horizontal.	
Fixed End.	Expansion End.	Fixed End.	Expansion End.

A very simple table suffices for eye-bars. Other things such, as pins, rollers, bracing rods, lateral plates, pedestals, etc., are all easily tabulated. Often it is neither necessary nor advisable to do this. They may be more easily inspected either from the drawings or the shop lists. But this advantage of tabulation should be taken into consideration. It enables the inspector to have always with him, without any inconvenience, a complete record of *all* the pieces. There is no trusting to memory. If in passing through the shop at any time he notices something wrong with a certain piece, he enters it in the column for remarks against that piece, and then notifies the foreman of that department. When this piece comes up for final inspection the note is found and that point re-examined.

The inspector should so arrange his work as to inconvenience the contractor as little as possible. He cannot leave his inspection until the material is ready to ship. Neither can he inspect and accept the piece in each of its successive steps through the shop. The proper adjustment of the work will vary somewhat for different shops and for the kind of work. But he should always be on hand when wanted, and should keep a watchful eye on all of the departments.

One objection to the subdivision of the inspection is that the inspector finds it difficult to keep account of what has been inspected and what has not. By the method of tabulation spoken of above this is rendered perfectly easy and reliable. If certain things on certain members have been inspected before the completion of the member, some sign or letter indicating the same is entered opposite each of these members in his note-book, and the same signs put upon the members themselves in soapstone. When the member is finished the remaining requirements are checked.

Details of the Work of Inspection.—The inspector should see that the material is not so crooked when it starts into the shop as to prevent its being properly laid off or gauged and punched. After being punched the material should be straightened.

The punch-dies should be examined occasionally to see that the edges are sharp and unbroken, and that the difference in diameter between the upper and lower does not exceed $\frac{1}{16}$ inch.

After being straightened and reamed, the various pieces forming a single member are assembled. This requires a good deal of care. Web splices and all abutting sections should be made to close tightly and the splice-plates fitted on and reamed while in this position. Excessive drifting, or drifting for any purpose other than bringing the pieces to the proper position, should not be allowed.

The inspector should see that a sufficient number of bolts are used to hold the material snugly together while being riveted. The inspector should see that all stiffeners fit tight against the chord angles, and that all surfaces to be riveted together are painted before being bolted up. After being assembled all the rivet-holes are to be reamed.

Ordinarily not much trouble is experienced with power riveting-machines. Sometimes, however, the power gets low and many loose rivets are found. One cause for this is the removal of the bolts too far ahead of the riveting-machine. In testing rivets they should be struck two short, sharp blows, one on each side of the head, with a hammer weighing about one pound, the handle to which is quite small in the shank, allowing the absorption, at this point, of some of the spring of the hammer. When the handle is held at the proper point and the rivets are solid, no jarring effect is felt in the hand. A little experience enables one to detect loose rivets by means of the action of this handle where no rattling sound could be heard, and where no movement could be detected by the finger placed at the angle between rivet-head and girder.

Very often there is trouble with countersunk rivets driven by a machine. The reason is this: The rivets are a trifle too long. This excess material spreads out under the die and overlaps the hole. Being thin this edge hardens quickly, and then no amount of pressure will upset the body of the rivet any further. It will appear tight until chipped, when it is often found to be loose. Drawings often require flat-head rivets in certain places where there is not enough clearance for the hemispherical head and yet where all the space obtained by countersinking is not necessary. On account of the difficulty mentioned above, such rivet-heads, less than $\frac{1}{4}$ inch in thickness, should not be allowed. If left unchipped it cannot be known whether the rivet fills the hole or not.

In hand riveting, spring dollies, for holding up the rivet, should be used where possible, especially for heavy pieces. These dollies consist of a long bar of wood or iron used as a lever, the short end of which is bent up and contains a cup that fits on the head of the rivet. In this way the effective weight of the dollie, for the resistance of the impact of the sledge, is equal to whatever weight is used on the long end, multiplied by the ratio of the lever-arms. With two men on the dollie this can be made quite large. This is assuming that the piece being riveted is sufficiently heavy to hold them up.

Spring dollies cannot be used on light work. For this, simple hand dollies, weighing from fifteen to twenty-five pounds, are used, and give good results, since, in light work, small rivets are, or ought to be, used. The use of $\frac{7}{8}$-inch steel rivets in $\frac{3}{8}$-inch and even $\frac{5}{16}$-inch metal is, however, not uncommon. It is very difficult to replace loose rivets in such cases, since the material is apt to split before the rivet-heads will shear. Then in backing the rivet out, unless the holes match well, the material will be badly bent.

Material lighter than $\frac{5}{16}$ inch does not work up well in the shop.

In marking rivets to be cut out, the inspector should use a centre punch or the stamping end of his hammer with which to cut the head of the rivet, which should then be painted with white lead. Some mark should also be made on the material near the rivet so that he may be able to find and test the new rivets.

In facing and boring, care should be taken that the ends of girders are planed to the proper length and bevel, and that the pin-holes are of the proper size and distance apart centre to centre. These are all subject to careful measurements, which should be taken with a steel tape after being compared with the company's standard, the correction for every ten feet being determined.

The inspector should supervise the laying out of the sections that are to be fitted up in the shop, and see that everything goes together so that no unnecessary work has to be done in the field.

When he receives invoices of shipments he should be able to check off all of these pieces from those marked "Accepted" in his note-book.

Such a rigid record of the work as outlined above requires some office work which has to be done at odd times or in the evening. But it pays for itself many times over in the sense of absolute security which the inspector is thereby enabled to enjoy.

APPENDIX C.

AMERICAN METHODS OF BRIDGE ERECTION.*

BRIDGE ERECTION, in a broad sense, includes the assembling in place, connection and adjustment of almost all framed and trussed structures, chiefly bridges and roofs, either permanent or temporary, primary or auxiliary; but in this part of this country, and as the most developed art, it refers chiefly to large structures composed of iron or steel members, with which we may properly deal from the time they leave the manufactory until the final inspection and acceptance by the purchaser's engineer.

The subject has three principal divisions: first, Primary Structures, usually permanent; second, Auxiliary Structures, usually temporary; third, Working Plant.

Bridges may be assumed to include all structures designed to transmit strains of flexure to relatively solid seats, and thus embrace roofs, girders, highway and railroad bridges, viaducts, aqueducts, towers, columns, and wind and crane bracings that form most of the first division. Primary structures may be either simple or compound; a simple structure practically consisting of a single piece, as a column or plate girder; simple structures may be either directly placed or temporarily supported.

Compound structures may be very elaborate, like the complicated trusses of a long-span railroad bridge, and are essentially structures formed by assembling several members or parts delivered separately at the site. Compound structures may be, during erection, naturally self-supporting, artificially self-supporting, or non-self-supporting. Non-self-supporting structures may be erected on the ground or on falsework.

Auxiliary structures are chiefly designed solely for the erection of permanent constructions, of which they may serve the whole or portions; they may be fixed or movable. Fixed structures include trestling, towers, piles, framed trusses, and suspended platforms. Movable structures include shear-legs, gin-poles, derricks, rolling towers and platforms, and boats.

The present American practice is notably superior to the foreign in the completion of members by power tools in the manufactories, their design with special reference to rapidity and accuracy of field assembling and completion of joints, and for the liberal use of special engines and steam and hydraulic power in the field that was promoted by the magnitude and economy of American work.

In the admirable monograph on American Bridges presented by Theodore Cooper to the Am. Soc. C. E., the devolopment of long spans and consequently of heavy members and difficult erection problems is traced, and it is shown that, except some moderately long timber spans, no great and heavy trusses existed until recent years, so that their erection is the art of this quarter century and its most able masters are of the present generation, who have created methods and appliances at least as fast as the designing engineers and manufacturers have furnished them with structures of increasing proportions to handle.

The first wooden bridges were doubtless built on continuous timber scaffolds, each moderate-sized piece being framed on the spot and readily placed in position by hand tackle, levers, and skids; and as the light highway iron work of twenty or thirty years ago was introduced, old methods were modified to suit. When railroad bridges became important, the erection of almost every structure of magnitude was a problem requiring special solution, and new methods and tools have been constantly devised, modified, and perfected until the mechanical and constructive skill, ability, and facilities now acquired are probably unparalleled in the world's development of physical undertakings of magnitude.

The erection of simple structures considers chiefly girders, roof trusses, and columns. Girders vary from the dimensions of rolled I beams to those of solid plate girders more than 120 feet long and weighing over 100,000 lbs. each, or of lattice girders of 150 feet or more in length, the girders up to 120 feet long having been shipped from the shops in single rigid pieces. Such long and heavy pieces must be loaded skilfully to ride the railroad curves, and each requires from three to five flat-cars for its transportation. The girder is supported at each end on a transverse beam that has an iron bar or old rail on top on which the girder rests, and can easily slip to conform to the chords of the curves. This transverse beam is supported by the centres of two or more longitudinal ones whose ends rest on transverse beams placed on the car floor and

* Adapted from an illustrated lecture delivered by Mr. Frank W. Skinner, M. Am. Soc. C. E. and of the editorial staff of *The Engineering Record*, before the College of Civil Engineering of Cornell University, April, 1893. Most of the cuts were taken from the *Journal of the Association of Engineering Societies*, Mar. 1898, article by Mr. Frank P. McKibben, Boston. Plates XLVIII, XLIX, and L are from *The Engineering Record* of July, 1898.

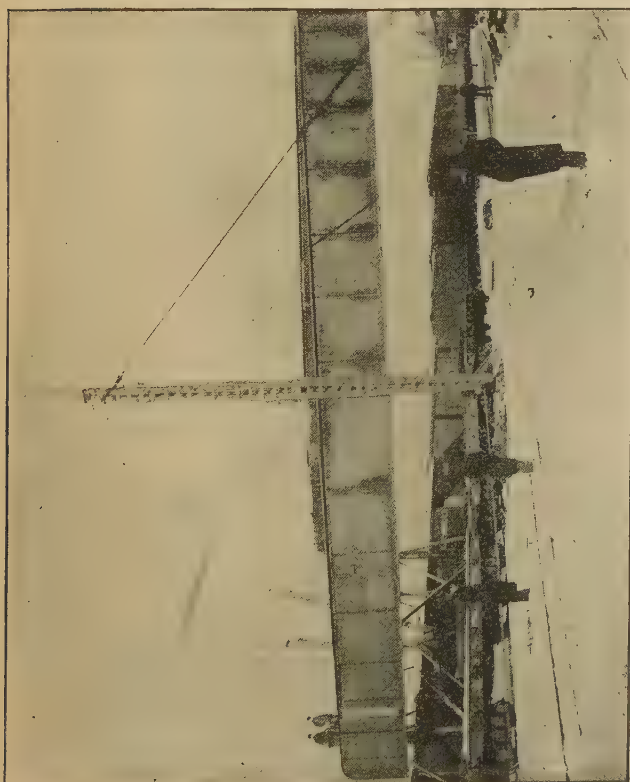

FIG. 1.—HOISTING A 25-TON GIRDER WITH GIN-POLE AND LOCOMOTIVE, MINNEAPOLIS, MINN., 1896.

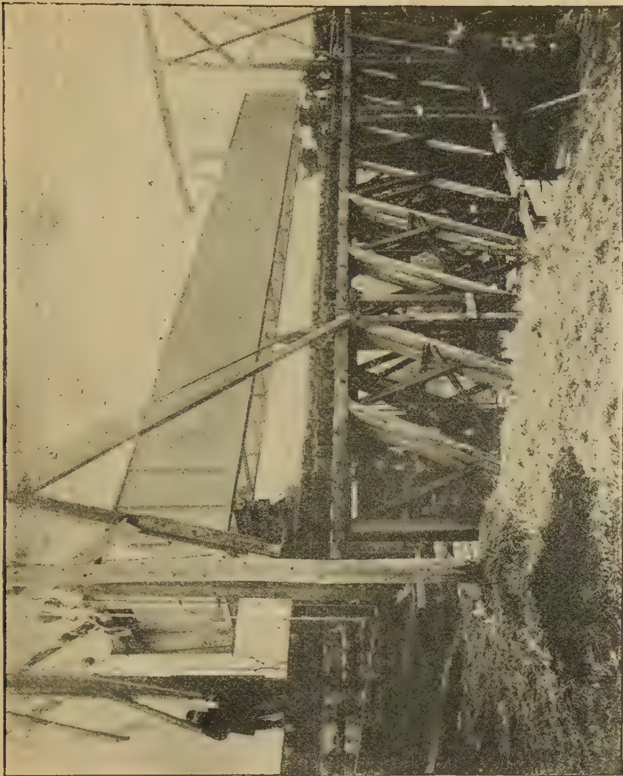

FIG. 2.—SUNBURY BRIDGE. C. A. & C. RY. YOUNGSTOWN BRIDGE CO.

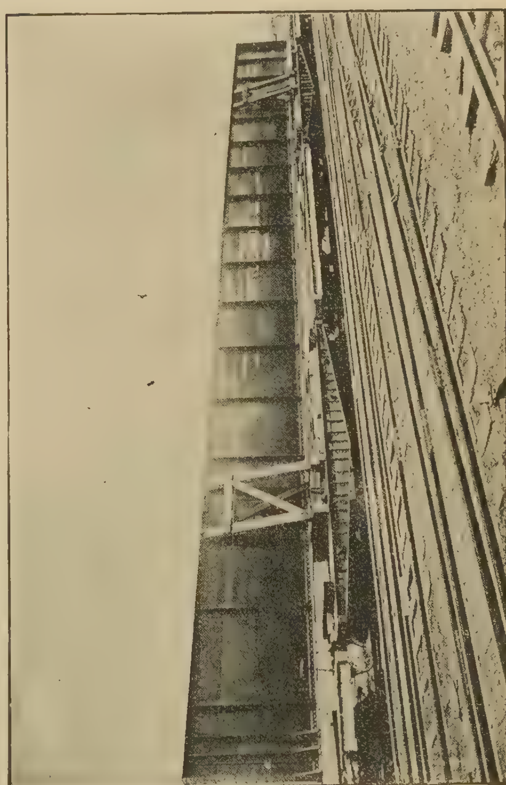

FIG. 3.—123-FOOT GIRDER, PHILADELPHIA.

FIG. 4.—SHIFFLER BRIDGE COMPANY, 123-FOOT SPAN, PHILADELPHIA.

To face page 508.

PLATE XL.

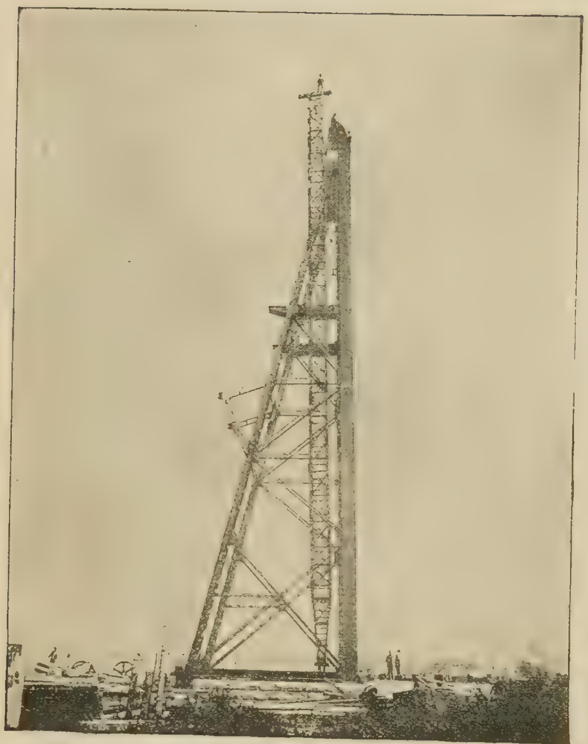

FIG. 1.—HIGHEST GIN-POLE ON RECORD.

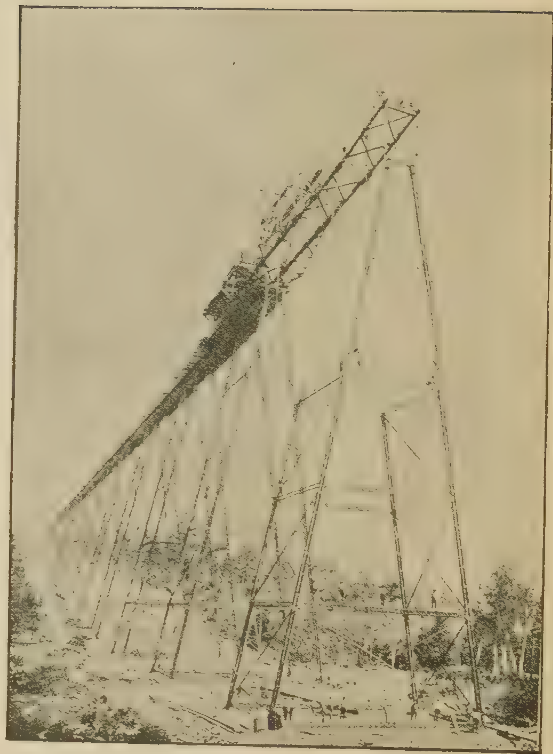

FIG. 2.—PANTHER CREEK VIADUCT.

FIG. 3.—SIMPLEST FORM OF TRAVELLER.

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thus distributing the load on two lines, one at each end of the car, or else rest on another set of longitudinals that are set on four transverse beams, one of which would thus be directly over each axle and sustain one eighth of the total load, half of which is carried by each end car, the intermediate ones acting only as spacers. See Fig. 3, Plate XXXIX, for methods of shipment and the other figures on this plate for methods of handling plate girders. Whenever practicable, they are not unloaded until brought across the openings they are intended to span and parallel to their final positions from which they do not vary longitudinally and not more than is necessary transversely. They are then usually raised a little by hydraulic jacks and supported by timber blocking till the cars are run out from beneath them and then jacked down and skidded to their seats, or, less frequently, are lifted from gallows frames and turned if necessary and lowered by tackle. These gallows frames, one at each end, ordinarily consist of single bents of, say, 12 × 12 posts and single or reinforced caps that just span one or two tracks and are guyed both ways. When no old or temporary track exists across the opening, the girders have been unloaded at one end of it and placed in the required position by protrusion, i.e., pushed out cantilever-wise over a stationary roller on the abutment until the forward end reached its seat on the opposite side. This method requires either a pilot extension, a rear counterweight, overhead guys or intermediate supporting rollers; after it is half way across a pilot extension would generally be used and would be a long beam lashed firmly to the girder so as to engage a roller on the further side before the centre of gravity of the girder passed the first abutment.

A remarkable example of this method of erection is that of the Souleuvre Viaduct in France; its spans, of riveted lattice-girder type, were completely assembled about 1600 feet from one abutment, connected together as continuous spans, and rolled out on fixed rollers; the spans weighed nearly 100,000 lbs. each and had in front a trussed pilot 66 feet long that weighed 40,000 lbs. The bridge was protended by the revolution of the fixed rollers at each pier. These rollers were turned by ratchets operated by long levers, one on each side of the span, connected by a cross-bar over the top of the bridge. Men walking back and forth on the top of the bridge pushed this cross-bar before them and thus turned the rollers, but considerable difficulty was experienced in securing uniformity of action between different gangs. This example is notable in that it was entirely successful, and for its striking difference from American practice.

Roof trusses up to about 100 feet span are generally lifted and set in place as one complete finished truss, whether with rigid or flexible joints. If with riveted joints, they have been shipped from the shops in one, two, three, or four sections each, that are riveted together on the ground at the site; or, if pin-connected, they are assembled there, and in either case raised and set by a gin-pole or derrick that moves backward with each successive truss.

These trusses often depend largely upon the roof sheathing boards for lateral bracing, without which they have little transverse stiffness. They are also likely to be set on slender isolated columns, and require special care in guying until permanently braced after being released from the derricks. When supported on columns the trusses may be assembled between them exactly parallel, and each with its lower chord nearly in the vertical plane of its final position, but if supported on masonry walls they must be assembled with their lower chords sufficiently oblique to clear and be adjusted after rising above the tops of the walls. They may also be assembled on the same platform or blocking at one end of the building and raised there without moving the derrick, and skidded along on top of the walls to their respective positions; but this method will usually be more difficult, tedious, and hazardous than moving the derrick to raise them in position. This method was employed at one of the mills in the famous Homstead Plant, and either one end of a certain truss was advanced beyond the other, or else the flexibility was so great that it was pulled out of a plane and the lower chord became curved horizontally so that it fell off and tumbled to the ground.

Two gin-poles are often used together to raise a roof truss, each gripping it about one fourth or one fifth of its length from the centre, and, of course, always above its centre of gravity. The writer once used this method of erecting a rolling-mill roof of over 140 feet span whose very light trusses had slender gas-pipe struts and deck-beam top chords, and were about limber enough to be considered funicular frames, but were handled without difficulty by the renforcement of planks judiciously lashed on.

A gin-pole is simply a timber mast with four guys and a sheave at the top over which the hoist line leads to a crab bolted three or four feet from the bottom. In use it should always be inclined a little from the vertical so that it overhangs its burden and gives a positive strain on the back guys and on them only, the front guys coming into service when the pole is moved. By taking up or slacking the guys the truss may be very quickly swung backwards, forwards, or transversely and adjusted to position, and for heavy work a tackle is advantageously used to operate at least the back guys. See Fig. 1, Plate XXXIX.

The foot of a gin-pole is generally supported by and shifted upon a plank or timber along which it is pinched with bars or pushed upon rollers. Gin-poles are ordinarily from 40 to 60 feet long and up to 16 inches square at the butt. In erecting a lofty dome recently Horace E. Horton, of the Chicago Bridge Works, used a trussed gin-pole 120 feet long. Gin-poles are often rigged with ¾-inch wire guys and 1½-inch

manilla line that would, according to the load, be operated directly by the crab or be rove over a fixed and loose single block or a two-three pair of blocks. See Fig. 1, Plate XL. This gin-pole is 146 ft. high.

An A derrick is two inclined masts braced together and united at the top; needs but one guy and is very often preferable to a gin-pole.

A gin-pole must always be carefully handled and may be easily raised or lowered by a boom or an A derrick that is likely to be found at any large building, provided the height of the pole is not more than twice that of the derrick, which can then pick it up just above its centre of gravity, and swing it into its vertical position. If the gin-pole is only a little too long for this it can still be handled, as by counterweighing the butt. A short pole can be raised by fastening the foot and blocking the other up beyond the centre of gravity until the angle is great enough to enable it to be revolved by a rope leading to the ground, but much the easiest and best way is to provide a secure resistance for the foot, making a virtual hinge there, and hoist it from the ground by a line led over the top of a shears, which need only be any convenient pair of timbers lashed together at the top. When the pole rises high enough to carry the rope off from the shears they will be no longer needed, and if the hoisting crab has been properly set it will continuously pull the pole up to its vertical position. The foot of the pole must be carefully watched, and reliable men stationed at the guys which always, whether moving or raising the pole, should be kept free from slack.

When the span is very great, or when the ground underneath must be preserved free from obstruction, the roof trusses are either assembled and erected from a strident traveller, or a tower, or upon a movable platform whose surface conforms to its lower chord or intrados, is as long as the span, and usually is a little wider than the distance from out to out of the two most distant adjacent trusses, so that each pair may be simultaneously erected in position, braced together, and left in stability while it moves forward two panels' lengths to the next pair.

Lofty viaducts have been erected with the utmost simplicity by booms carried on the structure itself as its towers were built up section by section from the ground,

The famous Kinzua viaduct, over 300 feet high, was erected in this manner several years ago, but the method is evidently best adapted to locations where the iron work is most readily delivered in the bottom of the chasm, which is not usually the case, and probably would not now be used except for very short structures or where the spans between towers were extremely long, the material being chiefly handled from overhead in the present practice.

By far the most usual and generally economical way of erecting viaducts, including metropolitan elevated-railroad structures, is by means of an overhanging derrick that moves on top of the completed portion and reaches far enough beyond it to set all members in the next one or two panels in advance of its own support, setting and maintaining each piece until it is braced and self-supporting; the connections usually being quickly and temporarily bolted up to enable the derrick to move forward to the new panel and commence erecting the next one before the main joints are drifted and riveted and secondary connections completed. These derricks are called "Erecting Travellers," and comprise essentially a base moving on the finished work, and carrying the hoisting engine, coal, etc., that partly counterbalance the overhang and its burden. A reach of 60 feet usually suffices for elevated railroads whose travellers may consist simply of long, single beams, mounted on central wheels and set with the front end slightly elevated and the rear or trailing end lashed or otherwise secured to the longitudinal girders as was done on some of the earlier Brooklyn work. Generally, however, a braced platform, of the full width of the structure, carries a vertical head frame in which are set masts of two or three boom derricks, whose booms are usually trussed and swing nearly around a semi-circle so as to be able to pick up the iron from the side of the street and swing it into position. Often these booms are arranged so that the two side ones can set and hold the columns of the next panel while the longer centre boom puts the transverse girder in position upon them and after it is connected to them maintains the whole bent until the released side booms set the longitudinal girders and make the whole panel stable.

Railroad viaducts are generally considered to be most economically proportioned when the towers are 30 feet wide and 60 feet apart, and many have been built approximately to these dimensions, thus requiring an overhang to support a transverse beam or pair of column sections at a distance of 90 feet, a short longitudinal girder at 75 feet, and a long one at 30 feet. These travellers always consist of two parallel trusses, usually combination, always firmly braced together horizontally and anchored to the structure when in service. Sometimes the overhangs are single heavy beams with iron guys from the end and intermediate points to the top of a mast placed over the end of the supported part, and guyed back to the rear of the platform. But they are more often square Howe or Pratt trusses not unlike bridge spans, overhanging about half their length, and having cross-beams and eye-bolts in the lower chords from which to suspend the tackles required to lift and support the pieces in the different positions. See Fig. 2, Plate XL.

Viaduct travellers are designed according to special conditions so as to receive the members for erection

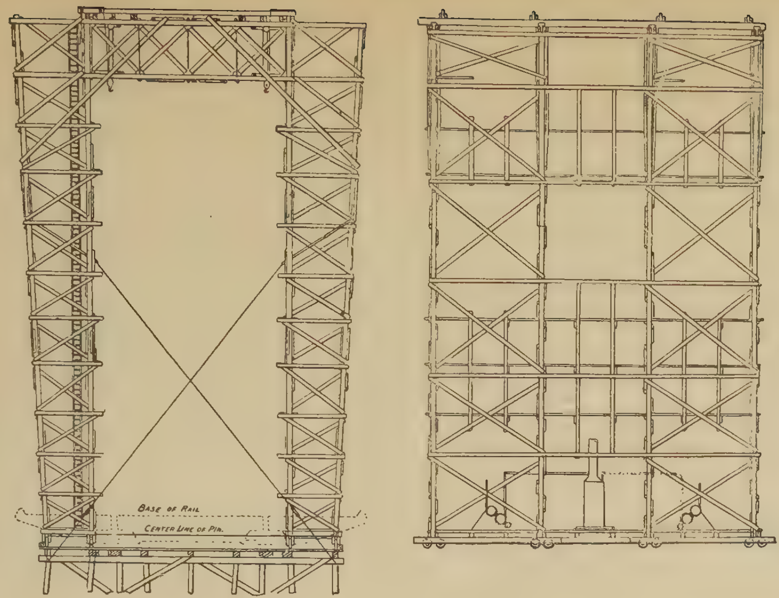

FIG. 1.*—TRAVELLER FOR LARGE SPANS.

FIG. 2.—OHIO BRIDGE AT CINCINNATI

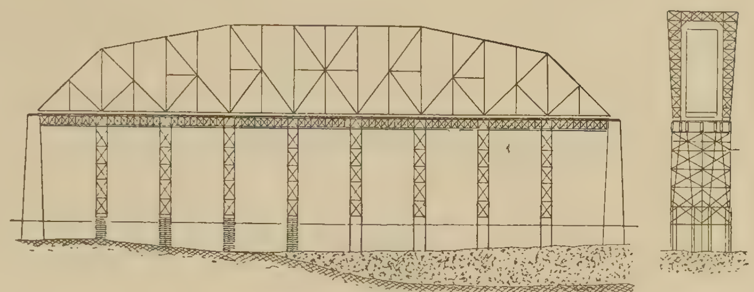

FIG. 3.*—FALSEWORK OF TOWERS AND HOWE TRUSSES.

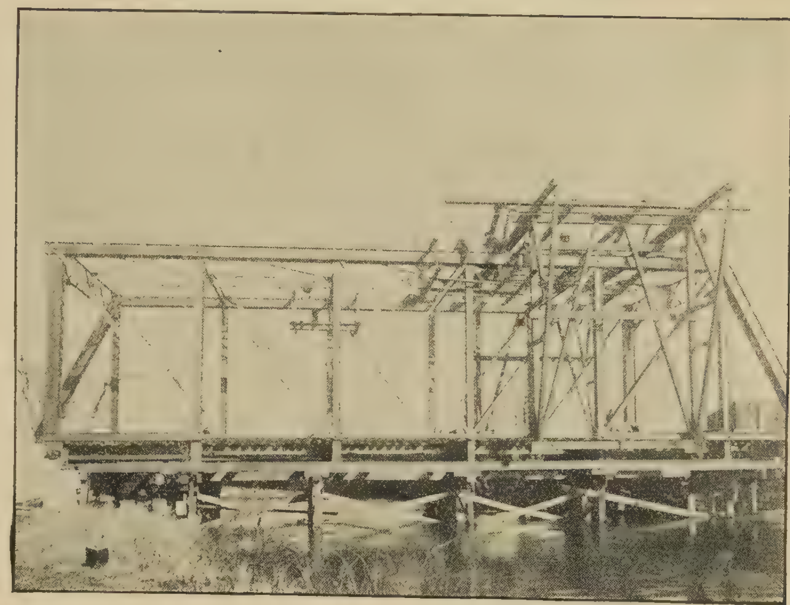

FIG. 4.—MILLBURY BRIDGE DURING ERECTION.

* These cuts appeared originally in Dubois' *Strains in Framed Structures.*

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PLATE XLII.

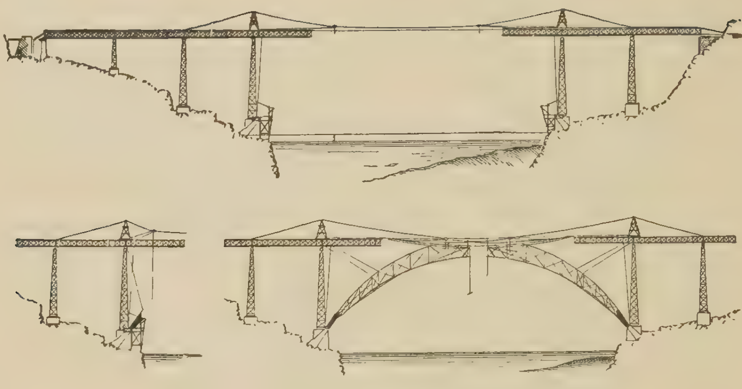

FIG. 1.—GARABIT VIADUCT IN FRANCE.

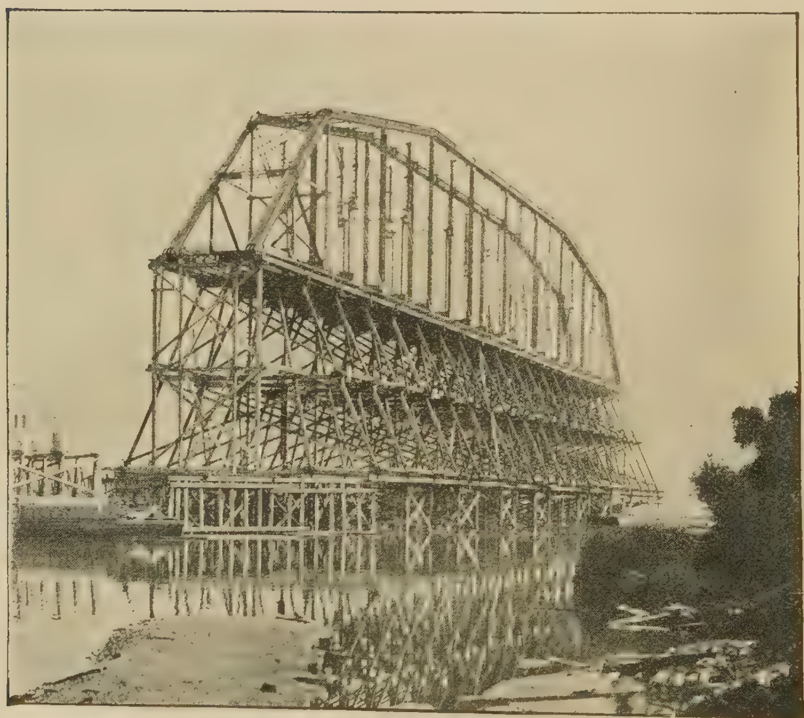

FIG. 2.—OHIO CONNECTING RAILWAY BRIDGE NEAR PITTSBURG.
KEYSTONE BRIDGE COMPANY.

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directly underneath the overhang, or from cars that run on its own track level and come from behind up to or underneath the main platform and deliver to trolleys that carry the hoisting tackle out on the overhang, or to booms that swing it around, or to falls that slack it off to position. In building the St. Paul High Bridge across the Mississippi River, Horace E. Horton erected the long and lofty viaduct by a huge travelling tower that was 150 feet high by 68 feet square, and straddled the 125 feet high trestle bents, with a clear spring of more than 135 feet high. It was built chiefly of 5″ × 10″ and smaller sizes of timber with iron main tension diagonals; ran on eight double-flanged wheels, and was probably the largest and tallest traveller ever constructed.

As crane bracing and horizontal trusses must be permanently supported by columns, walls or vertical trusses, the supports greatly facilitate their convenient erection, which is almost invariably accomplished by simple tackle directly supported and by stationary crabs or engines.

The only primary structures remaining to be considered are Long Span Bridges, i.e., say above 150 feet long. They may have either pin or riveted connections, or be suspension bridges or arches. Most of them in this country are the former, although there is a growing tendency to design arches for locations where the geological formation affords good seats that in receiving the thrust save the metal required for a tension chord.

Cantilevers and suspension bridges are the only types that are self-supporting during erection. The former may include draw bridges when they are erected symmetrically with the panels simultaneously added each side of the centre so as to balance each other upon the pivot pier, but they are generally erected upon the fender piling in the axis of the river each side of the pier.

In cantilevers the anchor arm is first built on falsework and counterweighted so as to enable the channel arm to be built as an overhang, its members being self-sustaining as soon as each panel is connected. See Plate XLVI.

American cantilevers are almost invariably connected by a suspended centre span of from $\frac{1}{3}$ to $\frac{1}{5}$ of the total opening, and this is usually erected as an extension of the cantilever arms from each side, special temporary or permanent stock being provided in the truss members if necessary to meet the erection strains, which are usually allowed a high unit value.

Wire Suspension Bridges are commonly erected from their own cables, which, when twisted rope is used, are drawn across the river in strands and then lifted to the tops of the towers.

The large cables are merely bundles of parallel straight wires that are carried across singly by special machines, looped over the end pins, and spliced at the end of each coil so as to form one practically continuous filament, the different individual catenaries of which must be carefully adjusted and secured to uniform tension and then compacted and encased. After the cables are completed the members of the floor and stiffening trusses and working derricks and platforms are supported readily from them.

Arches are generally assembled on falsework, but may be sustained without it by commencing at the skew-backs and supporting each section by overhead guys, as was notably done in Eads's St. Louis Bridge. Such a method, or that of temporary reinforcements to enable an ordinary truss to be erected cantilever-wise, may be termed artificial self-support. See Plates XLIV and XLV, and Fig. 1, Plate XLII.

Tubular Bridges are happily obsolete in this country, the only important one in America being the Victoria Bridge across the St. Lawrence at Montreal. It has many long spans, over deep and rapid water, the bottom is too rocky for pile driving, and the writer has been informed that the superstructure was built *in situ* in the winter, upon falsework erected on the ice which forms and packs and freezes there to remarkable thicknesses so that higher up on the river it is not uncommon to run ordinary locomotives across on it. Stevenson's other great tubular bridges at Conway and the Straits of Menai, England, were built at the water's edge, floated complete to the unfinished pier, and raised many feet to final position by hydraulic jacks, their masonry seats being continually built up close under them as they rose.

Sometimes ordinary fixed spans are erected as cantilevers by the use of temporary anchorages, as shown in Figs. 2 and 4, Plate XLV, and Fig. 1, Plate XLIX. In this last the stone towers were already on the ground.

With the above exceptions, long-span arches and trusses are supported on falsework substructures until self-sustaining.

Fixed Falsework.

This may consist of a simple temporary suspension bridge with a more or less stiffened floor for comparatively light work or where the height or cost of falsework would be prohibitory, or where it could not be safely maintained. Excellent examples are afforded by the South American work of L. L. Buck, who erected some of the first railroad bridges in the Andes mountains, with the simplest equipment and unskilled labor; the structures spanned deep chasms traversed by mountain torrents that were liable to sudden floods that would destroy trestle falsework, and the trusses could not be erected cantilever-wise. A suspension bridge was therefore made across the river, and on its floor the trusses were erected by a light special derrick

that commenced at the centre and erected to one end by balanced overhangs that were lowered to allow it to return inside the erected structure to the centre and thence erect the remainder of the truss.

In an adjacent viaduct crossing, a cable was stretched from side to side, and upon it a trolley hoist travelled by gravity, receiving the iron members on top of the bank and lowering them as it carried them out to position so that the resultant motion carried them in a nearly straight line to their required locations in the towers. See a similar use of cables in Fig. 1, Plate XLII.

Reverse conditions existed at the Elkhorn viaduct erected recently by Coffrode & Evans, Philadelphia. The timbers for a 600-foot permanent viaduct 140 feet high were delivered, framed and connected up in sections of trestle bents, at the centre of the bottom of the chasm where a stationary hoisting engine raised them, traversed them along the line of the structure, and set them in position by means of a trolley travelling on a suspension cable and operated by hoist and traverse lines leading from it to snatch-blocks at the foot of the opposite towers.

Trussed-span falsework has been used chiefly where there was great danger from floods, drift or scour, or where it was imperative to offer the least obstruction to navigation.

Howe trusses or combination trusses have generally been used; and can be quickly erected on trestles to seat on the bridge piers and become self-sustaining, and permit the removal of the trestles and allow plenty of time for the removal of old and assembling of new structures.

The long spans of one of the first Missouri River bridges were erected on one temporary Howe truss deck span that was assembled on trestle work of the required height, then transferred to two wooden towers built on the decks of pontoons which were floated with their burden to position between the piers; water was then admitted to the pontoons and they sunk sufficiently to deposit the span on the piers and clear it and be removed, leaving a platform safe from floods, for the assembling and connecting at leisure of the permanent span. After it was swung the pontoons and towers were brought back underneath, pumped out till their buoyancy lifted the Howe span, which was then transferred to the next opening and seated as before, for the erection of another span, and so on. See Plates XLII and XLIII for similar method.

This erection was notable for the length of the wooden span, the height of the towers, the swiftness of the current, and the success and economy of the operation.

Several years after this erection was that of the bridge of the Canada Atlantic Railroad across the St. Lawrence River at the head of the Coteau rapids, with a 355-ft., swing span and one 139-ft., two 175-ft., four 223-ft., and ten 217-ft. fixed spans. The water was from 20 to 30 feet deep, with a current of 5 to 7 miles an hour, making it impossible to erect falsework on the rocky bottom, and very difficult to build the masonry piers, which were constructed in bottomless wooden caissons that were floated into position, loaded and sunk with inside canvas bottom flaps on which the concrete was laid.

Three miles above the site, in a sheltered bay, falsework was built parallel to the shore, and at its ends transverse tracks were built out to deeper water. Between these tracks and parallel to the falsework were moored a pair of timber pontoons each 90 × 26 × 6 feet deep, and braced together 70 feet apart. On the deck of each pontoon a trestle a little higher than the bridge piers was built. The permanent spans were assembled and connected complete on the falsework, skidded down the transverse tracks above the towers, which rose and lifted them free when the water was pumped out of the pontoons, and supporting them at two panels' distance from each end were slacked off down the river and sunk between piers enough to deposit the span on its seats, after which they returned for another span, and so on.

In the erection of the Harvard Bridge at Boston, the long plate-girder spans were built on shore, lifted by a traveller that overreached their ends and at high tide carried out by it, deposited on a special pontoon that was towed to position, and with the falling tide deposited the girders on their pier seats.

The Brunot's Island Bridge near Pittsburgh, and the Hawkesbury Bridge in Australia erected by C. L. Strobel and Charles McDonald, are notable illustrations of moving very large spans at a great height upon pontoons.

TRESTLES.—These are essentially sets of columns in vertical planes transverse to the axis of the structure; they are in rows or bents of four or more with transverse and longitudinal bracing, the latter either continuous or connecting alternate pairs so as to form towers. When the height is considerable the trestles are built in stories so as to give convenient sections. The simplest framed trestling has bents each composed of two plumb posts and two batter posts, a cap, a sill, and two diagonal braces, and the bents are braced longitudinally by a horizontal ledger piece at the top and a diagonal from the top of one to the foot of the next, on each side. Usually the caps are connected by two or more lines of stringers which should rest directly above the adjacent tops of the plumb and batter posts, the latter having a run of 1 in 12 to 1 in 4 horizontal to vertical according to circumstances. The dimensions of the timber should be proportioned to the load carefully in high or exposed structures or for very heavy burdens, but must never be made very light, for the connections never develop all the efficiency of the sections, and weight and stiffness are gener-

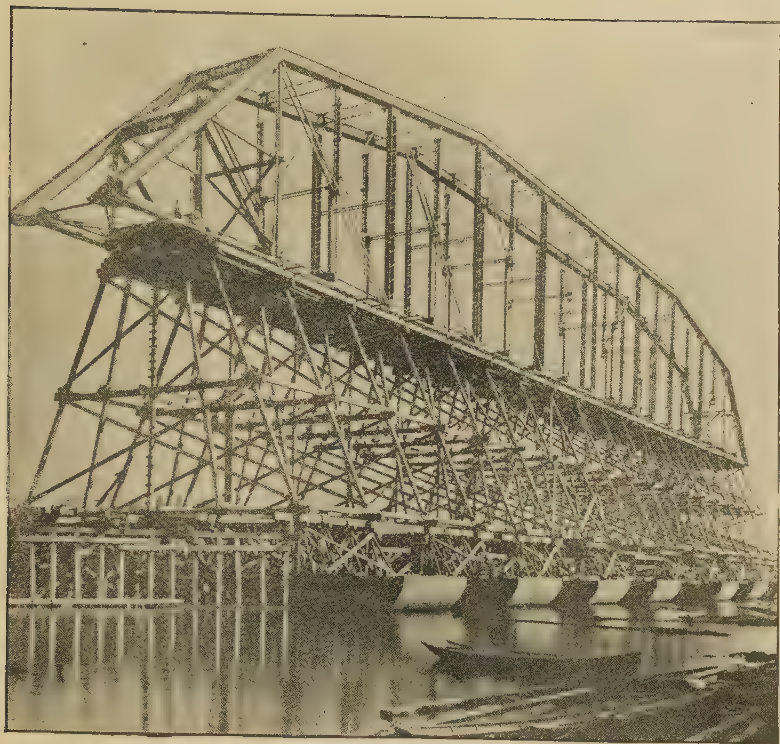

FIG. 1.—OHIO CONNECTING RAILWAY BRIDGE.

FIG. 2.—OHIO CONNECTING RAILWAY BRIDGE.

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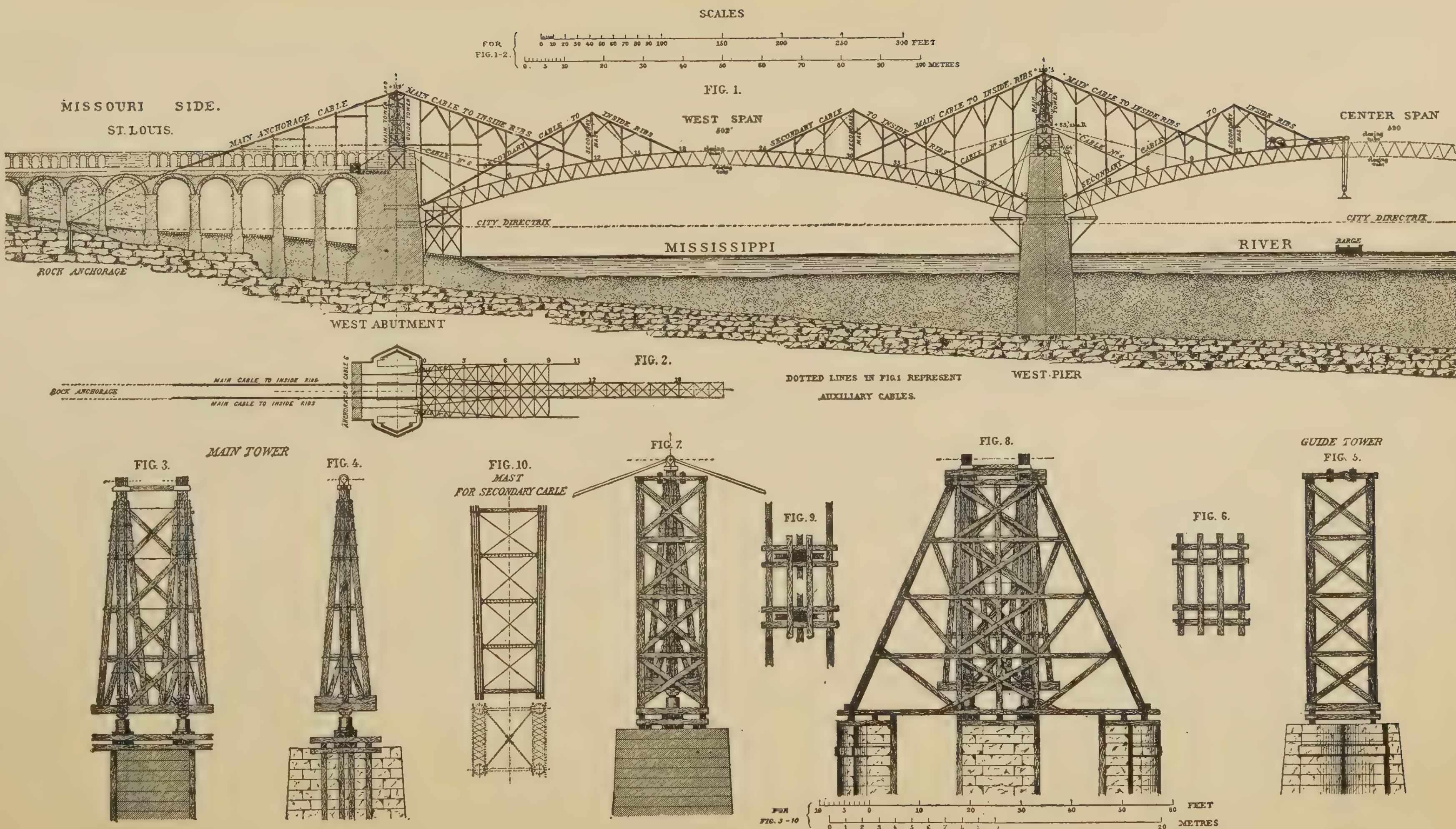

METHOD OF ERECTING THE ST LOUIS STEEL ARCH BRIDGE.

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ally more important than direct theoretical strength, so that careful judgment and experience are much more reliable and necessary than elaborate calculation of strains.

For ordinary simple trestles 10 × 10 and 12 × 12 posts and caps, 3 × 10 or 3 × 12 braces, and 8 × 16 stringers are much used, and for lofty and elaborate structures the variation is more in the number and arrangement of pieces than in their sections, though some 6 × 6 and 8 × 8 are used for secondary braces, and larger ones for important members if obtainable. But the few sizes mentioned with 2-inch platform plank and plenty of ¾ and ⅞ bolts, large washers, wood screws, and steel spikes comprise most of the bill of materials, except occasionally screw-ended iron tension rods with cast angle blocks. Mortise and tenon joints are seldom used; sometimes a batter post is toed in, but most joints are butted and covered with a splice or "batten" piece each side; usually a 2- or 3-inch plank as wide as the timber, and from 4 to 6 feet long, secured with four or more bolts.

Sometimes double caps are used, one piece on each side of the posts, but usually single pieces of the same thickness as the posts are put in their vertical plane, and between the tops of the lower and the bottoms of the upper sections, and all are bound together by batten pieces across the caps.

Trestle bents may be from 5 to 20 feet apart and of any height from 4 to 150 feet. Smaller heights are blocked up solid, and greater ones are usually avoided or otherwise provided for. Stories do not ordinarily exceed 20 feet in height. Ledger pieces and diagonals are sometimes spiked on, especially to promote rapidity of erection, but the latter at least should generaly be bolted eventually.

When framed trestles are to be set with sills on a river bottom, careful soundings must first be taken preferably with a pole, and a profile made to determine the heights of the posts. The bents are laid out with uniform tops and bottom widths varying to suit the different heights and bottoms. They are bolted together on shore, floated to position, swung up and set vertically from a derrick or pile driver boom or from the cap of the last bent if it is high enough, and stayed by ledgers, braces, or stringers.

If the water is deep, the lower end of the longitudinal diagonal brace is bolted to the foot of the batter post before the bent is set, so as to easily bring it eventually at the desired position under water.

Sometimes the bottom is explored ahead, or the bents raised from a "balance beam," which is simply a long heavy timber projecting half its length beyond the last trestle set, upon whose cap its middle is supported, while its rear end is under the cap of the preceding bent.

Ordinarily the "mud sill" is a sufficient footing when framed trestles are admissible, but sometimes special provisions are requisite on account of severe burdens or very soft bottom. In trestling over a very soft mud John Devin secured very cheap and efficient footings by spiking old railroad ties transversely to the bottom of the sills of his trestles, and thus constructing what was substantially a grillage under each bent.

PILE TRESTLES.—Wherever piles can penetrate they form the best foundations for falsework. They are driven to a moderate refusal and loaded much more heavily than in permanent structures. They are used in single lengths up to about 60 feet, beyond which they must usually be spliced, the joint being generally a square butt with comparatively narrow batten splice pieces bolted or spiked on all around it. At the Poughkeepsie Bridge piles were thus driven 120 feet through the mud and water.

Piles are driven either by a floating driver or by one running along on top of the completed bents. The batten posts are driven by inclining the ram guides to correspond to the required inclination.

If the top of the trestle is within 10 or 15 feet of the surface of the water the piles are usually sawed off and capped just below that height, but if the elevation is much greater they are usually capped near the water level, and one or more stories of framed trestle built upon them.

Sometimes, to prevent brooming, piles are driven with a temporary iron ring or ferrule on top; sometimes they have iron points or cast shoes at the bottom.

Great judgment is required in driving piles, which should be controlled far more by skilled experience than by theoretical considerations; sometimes, when the penetration is excessive, if they are allowed to rest a few hours they will resist heavier blows of the ram than previously drove them, and be amply safe for severe static loads. At other times the pile will refuse to penetrate and will bound out at every blow, but will sink steadily if loaded with a dead weight and tapped or twisted.

Some remarkable and apparently extravagant pile work was done at the Gour-Noir Railroad viaduct over the La Vézère River, France, where the 213-ft. main stone arch span 26 feet wide between parapets and 5′ 7″ thick at the crown, was built on centring carried by seven continuous wooden trusses 14′ 6″ deep that each rested on sand boxes on the tops of oak piles 12 inches or more in diameter; their bottom ends cut square and shod with plate iron. The river is subject to floods and has a granite bed into which holes 3 feet deep were drilled and the piles wedged and cemented in them. Quartz veins were encountered in which only one tenth of an inch per hour could be drilled until cofferdams were built. The cost of setting 126 piles was about $6800.

Timber dimensions for Eastern trestling seldom exceed 12 and 16 or at the utmost 20 inches, but a recent letter from a bridge man on the Pacific coast says his regular bills call for sizes like 11″ × 30″ × 42′, 20″ × 20″ × 48′, and 7″ × 18″ × 64′, while he saw some logs being sawed into timber 48″ × 48″ × 50′ and 20″ × 20″ × 100′.

Sometimes, to afford passageway for navigation, a few bents will be omitted at the bottom of a line of falsework, and the opening will be bridged by a high temporary span to carry the continuous top bents. Such an opening may be as much as 75 feet wide at the bottom and narrowed at the top by inclined posts so as to permit a sort of queen-post span of comparatively short pieces or the use of stringers borrowed from the permanent structure to close the opening at the top.

In the early days of iron bridge building, it was frequently customary to build "two-story falsework," i.e., to provide two complete platforms the whole length of the bridge, to support top and bottom chords throughout; but now it is almost universal to support only the bottom chord directly, and to make the rest of the truss self-supporting when assembled upon it, stability being of course assured as soon as any two opposite panels of the two trusses are assembled and connected transversely to each other.

Wooden derricks or towers usually traverse the falsework to lift and place various members, or are fitted to follow out at the extremity of a cantilever arm and overhang it so as to assemble and connect the successive panels upon which it continually advances. These structures are called travellers and properly belong to the

WORKING PLANT,

and form the most important item of erection equipment. A traveller in its simplest form consists of one transverse bent double guyed or two bents braced together longitudinally to form a tower, each bent consisting of a post each side and a cap and knee braces at the top. Commonly, however, there are three or four bents on large work, leaving at each side a vertical post and a batter post braced together, the latter flaring out at the top and their feet united on a heavy double longitudinal stringer, underneath which are three or more double-flanged wheels that run on special tracks laid on the ends of the trestle caps; the caps of the traveller bents are developed into Howe or Warren girder trusses, and usually carry longitudinal straining beams from which the hoisting tackle is hung. The bents are braced together with iron or wooden diagonals, and the structure carries working platforms and hoisting-engine and perhaps other machinery, besides sometimes tracks for material cars. Generally the traveller goes astride the completed structure and clears every portion of it, and is so required to have a large free opening from end to end with no transverse connection between the bottoms of the posts; but it sometimes is designed to go inside the trusses, or backwards in advance of them from a starting point, or upon the top chords, in which cases it may have bottom and interior cross and diagonal bracing. See Fig. 3, Pl. XL, for an interior, and Pl. XLI for an exterior, traveller.

Traveller Bents, as at Poughkeepsie, Wheeling, Memphis, etc , have been made more than 100 feet high, assembled and connected in a horizontal plane and the first one erected by simply lashing down the heel and revolving it into a vertical plane by two or three carefully adjusted lines on each side, attached at different points, and leading over shear poles to the hoisting engine. Of course the lofty timber frame was securely held by guys front and back that were kept constantly taut. After the first bent is erected the second one is easily hoisted from it, both are braced together, and the structure immediately becomes stable and rigid. See Pl. XLVIII and Fig. 2, Pl. XLIX, for an adjustable steel traveller.*

In erecting an ordinary simple truss span the members of the floor system and lower chords are distributed on the falsework in position before the traveller lifts and assembles the other members, commencing either at one end and going straight across or beginning at the centre and working to one end and then returning and working out to the other end.

The trusses are carefully placed in alignment and vertical plane, but are blocked up at the joints, where they rest on wedges, to a much greater camber curve than exists in the completed structure, so as to allow the adjustments for the final connections being made by driving out the wedges.

In suspended spans between cantilevers, the end piers rest in expansion slots in the cantilever arms, thus permitting longitudinal motion that would allow the portions of the centre trusses to hang down and make the final connection impossible unless special provision is made for adjusting their length and controlling their position and inclination. This usually consists of, at each of the eight slots, a fixed and movable roller separated by a wedge which is commanded by a powerful screw so that by entering or withdrawing one or both of its wedges the extremity of any truss segment may be raised, lowered, protruded, or withdrawn. See Fig. 2, Pl. XLVII, for the toggle joint used for closing the Niagara Arch, shown in Fig. 1.

Usually they are set in the beginning so as to allow for the deflection of dead load and traveller, and still be inclined above the required cambered position, so that the adjustment requires only the slacking off of the wedges.

* Described in *The Engineering Record*, Vol. XXXVIII, p. 180.

PLATE XLV.

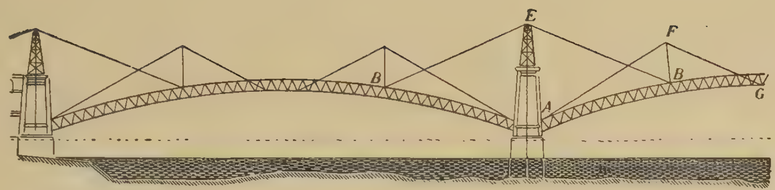


FIG. 1.—ERECTION OF THE EADS BRIDGE.

FIG. 2.—EADS BRIDGE AT ST. LOUIS.

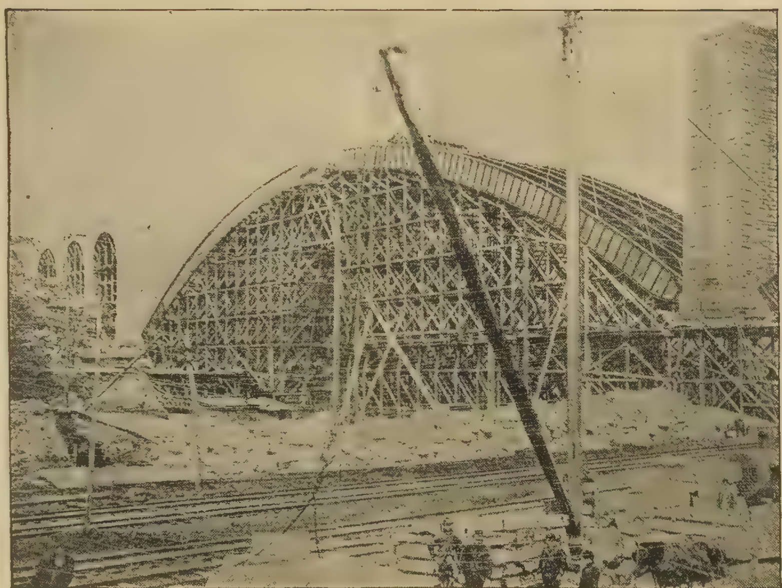


FIG. 3.—WASHINGTON BRIDGE OVER THE HARLEM RIVER.

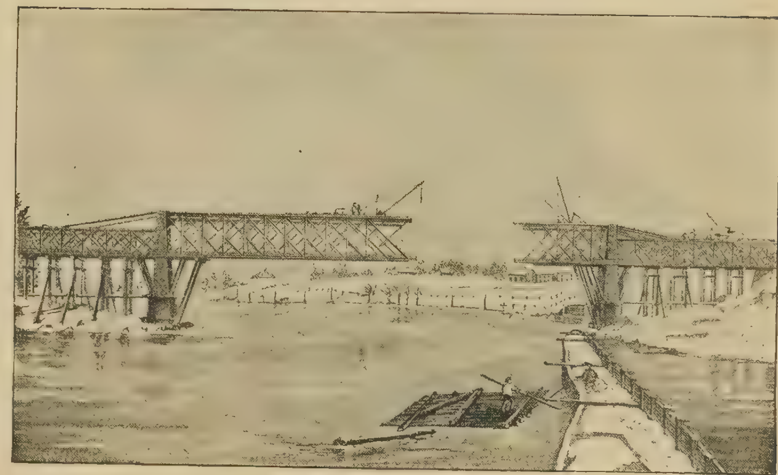


FIG. 4.—BRIDGE OVER RIVER DAL IN SWEDEN.

To face page 514.

PLATE XLVI.

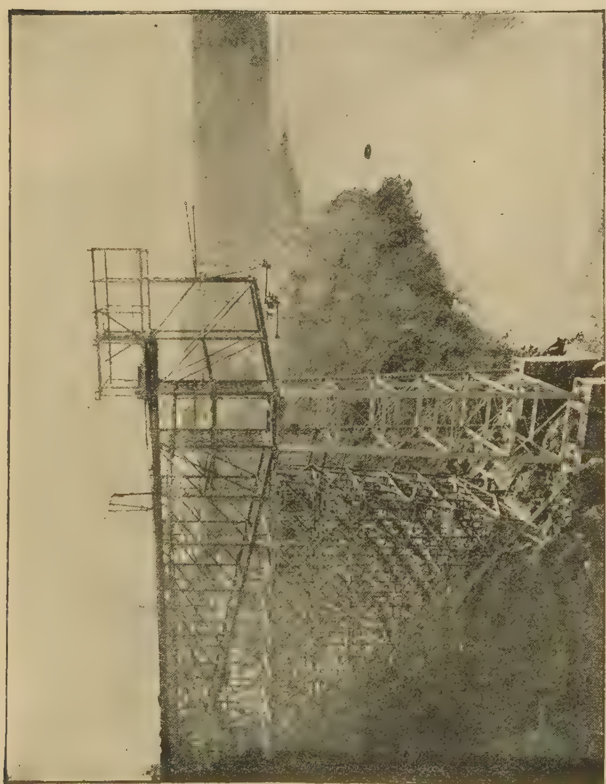


FIG. 1.—CANTILEVER AT NIAGARA.

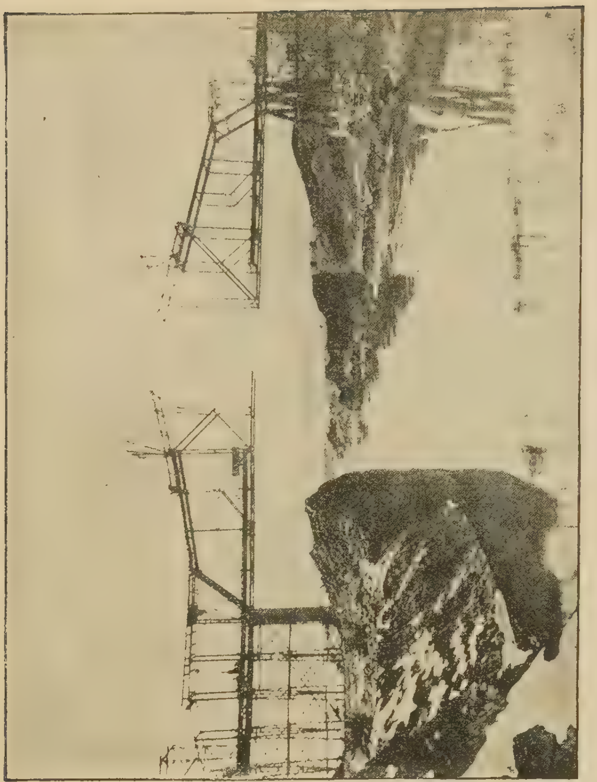


FIG. 2. COLUMBIA RIVER BRIDGE. EDGE MOOR BRIDGE WORKS

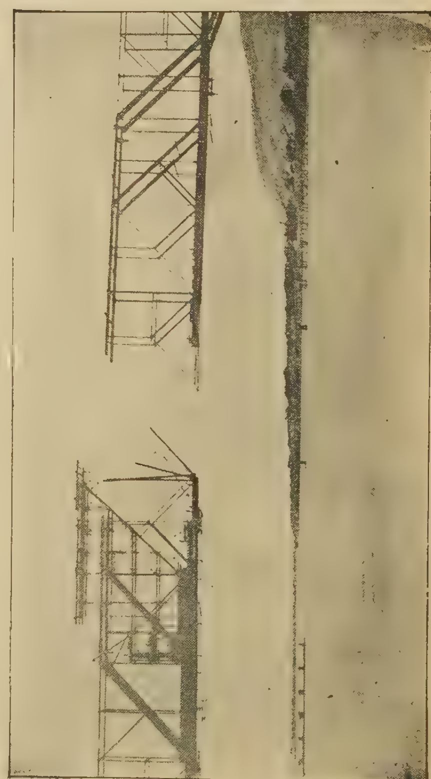


FIG. 3.—RED ROCK CANTILEVER OVER COLORADO RIVER.
PHŒNIX BRIDGE COMPANY.

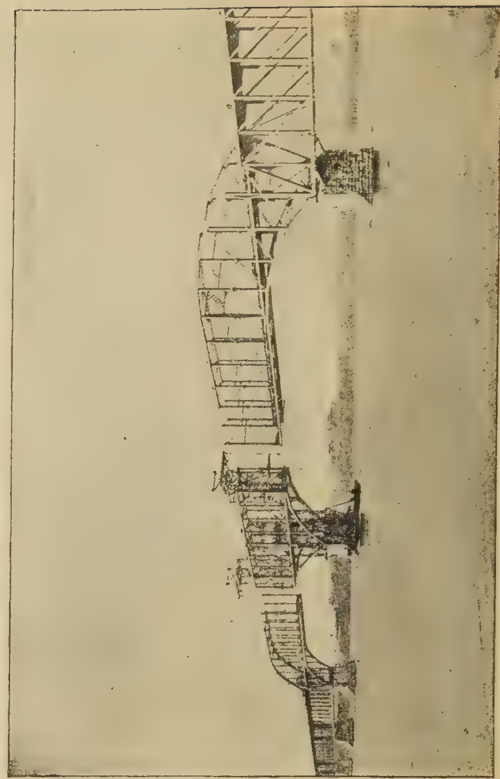


FIG. 4.—CANTILEVER AT LACHINE, CANADA.

To face page 515.

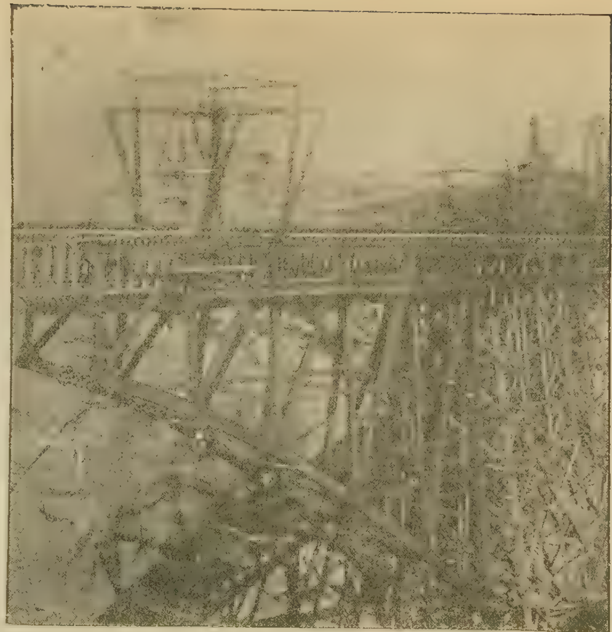


FIG. 1.—NIAGARA ARCH. PENNSYLVANIA STEEL COMPANY.

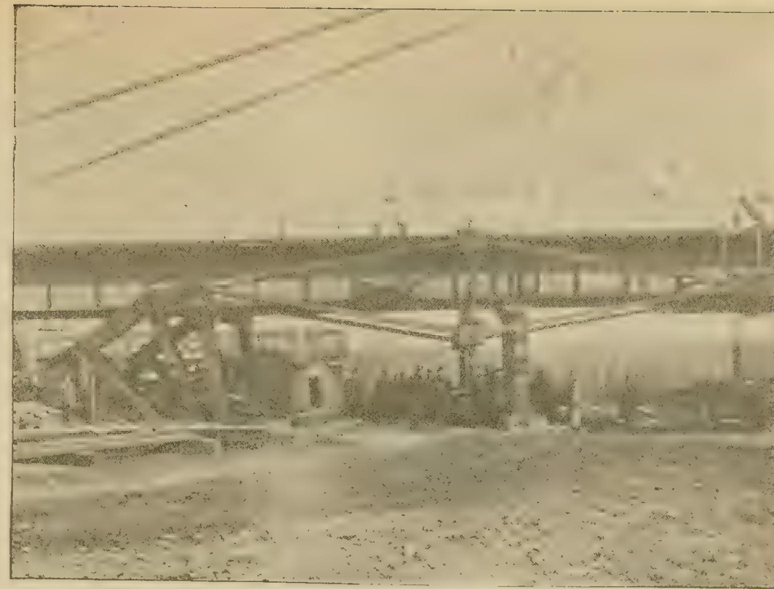


FIG. 2.—TOGGLE OF NIAGARA ARCH.

FIG. 3.—BOYLSTON STREET, BOSTON.

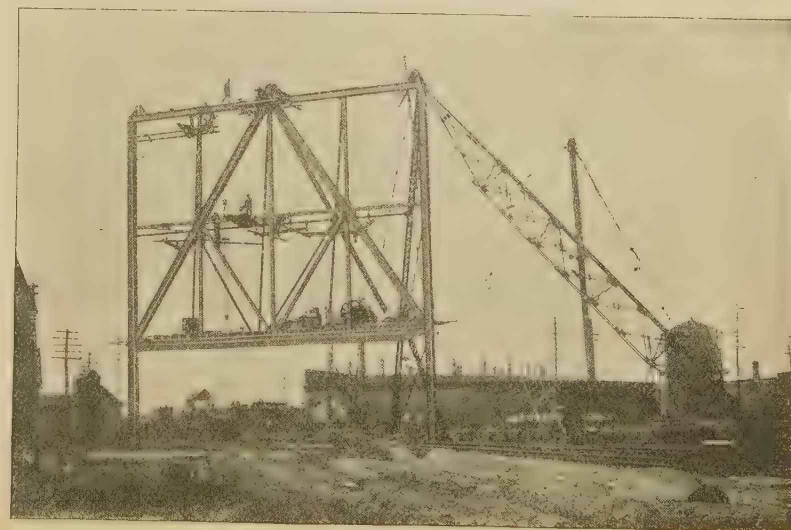
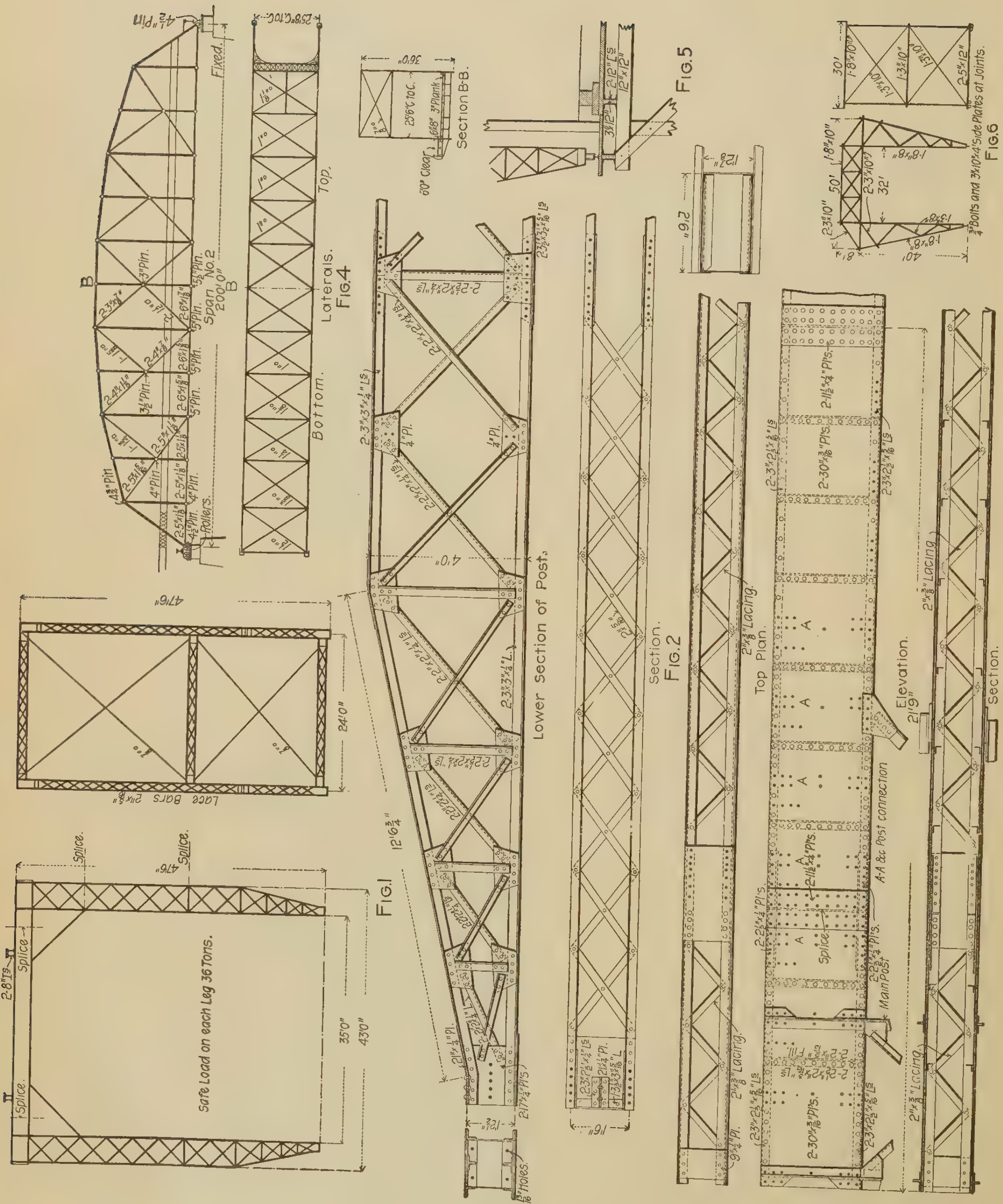


FIG. 4.—MILLER'S RIVER DRAWBRIDGE DURING CONSTRUCTION, B. & M. R. R.



ADJUSTABLE STEEL TRAVELLER OF BERLIN IRON BRIDGE COMPANY.

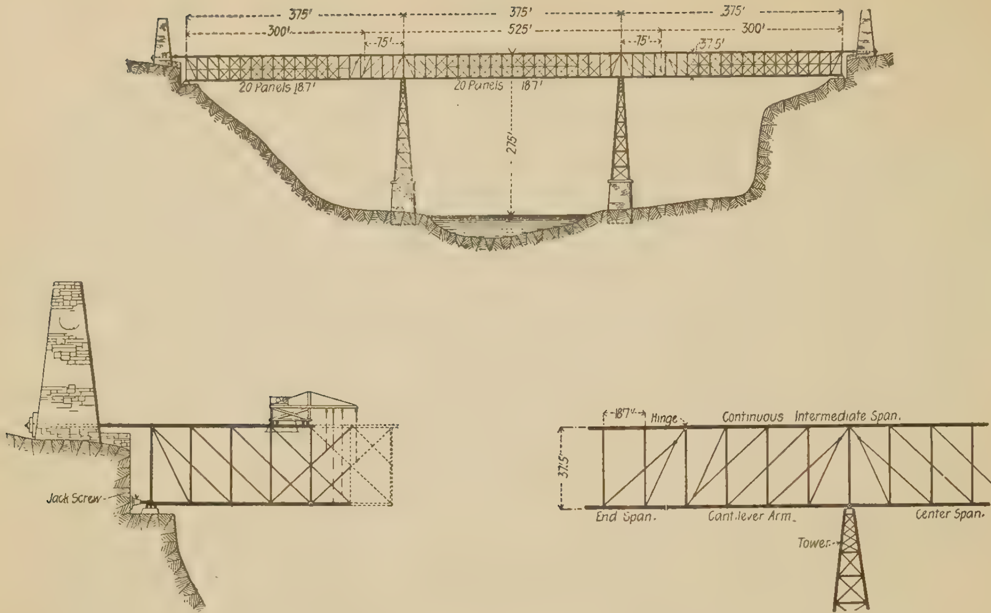


FIG. 1.—THE KENTUCKY RIVER BRIDGE. THE FIRST CANTILEVER BUILT IN 1876.

FIG. 2.—VIEW OF ADJUSTABLE STEEL TRAVELLER.

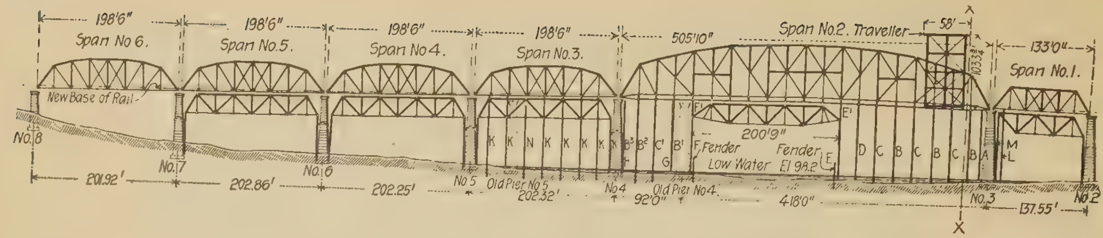


Fig. 1.—Erection of the Newport and Cincinnati Bridge.

Fig. 2.—198-foot Span with Falsework Truss, Newport and Cincinnati Bridge.

Fig. 3.—Channel Span with Falsework, Newport and Cincinnati Bridge.

Before this device was well known the speaker was required to provide erection adjustment for the second large cantilever bridge ever built in this country, at St. Johns, N. B., and designed simple stirrup irons or U bolts that engaged the movable top chord pins and had screw ends passing through fixed bearing plates against which their nuts rested. This afforded a tension adjustment for drawing or letting out the top chord, while large bolts screwd through solid boxes in the ends of the lower chords and having rounded ends bearing in hemispherical cups in a casting bolted to the end of the cantilever formed a compression adjustment and could be easily set up or slacked off so as to lengthen or shorten the lower chord. This method proved simple, economical, and satisfactory.

In the erection of the steel arch bridge spanning the Mississippi River at St. Louis (see Plates XLIV and XXXI) an effort was made to insert the closing twelve-foot sections of the tubular ribs of the arches by cooling them down with ice. The ribs were covered with gunny sacks and ice was placed upon their upper sides, but the temperature of the tubes was lowered by this means only 28° F. in the day and 5° at night (from 95° and 62° respectively). These effects were not sufficient to allow tubes of the normal length to be inserted. Other sections were used, which were provided with adjustable sleeve-nuts with right and left threads, allowing an extension of 1½ in. These were worked by means of a powerful wrench operating upon a 1½-in. steel bar inserted in holes through the nut. The upper and lower nuts were turned alternately as the arch rose and fell with rising and falling temperatures, till the normal length was obtained.

WORKING PLANT comprises travellers, derricks, hoisting engines, tackle, pile drivers, pumps, locomotives, cars, differential hoists, hydraulic and screw jacks, dynamometers, steam, pneumatic, hydraulic, and electric appliances and hand tools. Stationary derricks are much used for unloading and handling material and for hoisting secondary members. They are ordinarily of the familiar mast and boom pattern, with hollow vertical mast pivot through which the fall lines lead to the hoisting drums, and are generally rigged with manilla running gear and wire-rope standing guys. Sometimes they are stiff legged, and sometimes have also bases of timber sills, enabling them to be easily moved. Balanced derricks are sometimes used. At the Washington Bridge (see Fig. 3, Pl. XLV) all the material for one of the 510 ft. arches was lifted to a distributing platform by a balanced derrick that had twin overhangs, a bearing on top and a friction collar and rollers at the bottom of the trussed arms. Small four-wheeled trucks or "lauries" are usually used for distributing the iron work on the span and bringing it from the yards, but these often are provided with special lifting devices for expediting loading and unloading, especially if there is no yard derrick. A car having two braced vertical posts in the middle, capped by an overhanging beam with a tackle suspended from each end, is very convenient for picking up a pair of stringers and bringing them and unloading, one on each side of the track, nearly in the required position. A very effective arrangement was devised by a young erector, that simply consisted of a long trussed beam with its forward end somewhat elevated and carrying a tackle that was mounted close to the front end on a small car; it could be run out in the yard and the fall belayed and the lever tipped down to hook on to a heavy piece; then several men, mounting the long arm of the lever, would raise the load and push the car to any required place, when its burden would be lowered by slacking off the fall line. This device was especially convenient for handling floor beams.

The many varieties of hoisting apparatus in use are generally some form of multiple spooled engine, with capstan heads and drums; some of them have eight spools, each driven by independent gearing and commanded by clutches and brakes, so that any one may haul up or slack off independently, the rope only taking two or three turns around the spool and then being tailed off so as to maintain the same efficiency always. These engines usually have a locomotive gearing to enable them to propel themselves on standard-gauge track. An ingenious method has been employed to hoist them to the top of high falsework. At the Poughkeepsie Bridge a six-spool engine was delivered by a boat underneath the traveller, from whose top beams, 250 feet above, four sets of tackle were hung and their lower blocks were hooked on to the engine-bed, one on each corner; the fall line of each was wrapped around a capstan head and kept taut by a man mounted on the frame. Another man started the engine, and as the fall lines were wound up and tailed off, the engine pulled itself and its five men up to the top of the falsework and was let down on beams slipped under to receive it. Sometimes a locomotive can be advantageously employed to hoist heavy members, as at the Niagara Railroad Suspension Bridge, where the heavy saddles were hoisted to the tops of the 80-ft. towers by a line rove through upper and lower snatch-blocks, and led from the latter to a loco motive that simply steamed off with it and drew the load up after it. The running tackle used should be best manilla rope; 1¼, 1½, and 1¾ are the common sizes, generally rove through double and treble blocks with lignum vitæ or steel shelves 16″, 18″, or 20″ in diameter, and steel hooks and cases. Members are generally lifted with chain slings which must be properly fastened to avoid cross-straining the links, which can be very easily snapped. Special hook clamps are often provided to fit the flanges of girders. For extra-heavy

strains a luff tackle may be used, consisting merely of a second purchase attached to the fall line of the first. When this is commanded by a heavily geared windlass or a hoisting engine great power is developed, but is slow and troublesome in its application.

Very heavy pieces and large masses or assembled structures may be moved horizontally on greased skids by hydraulic jacks that are made of from 5 to 100 tons capacity, and can also raise or lower it, but should then be closely followed by solid blocking.

In commencing to drive a connecting pin, the holes in the different members often do not match, and a square-ended pin could not be entered. Therefore, the end which is shouldered and threaded for a nut receives a pilot of the same diameter as the body of the pin, up to which it is screwed to fit tightly, while the front end is made conical and can enter a half-hole into which it is driven, drawing the pieces into position and allowing the pin to follow easily. The pin should always be driven by a wooden maul or ram to avoid battering its threads. An iron-hooped beam or log suspended at the pin, by its centre of gravity held from a considerable height and swung against the pin, does excellent service.

The specifications for most large recent bridges have required machine field riveting which, in this country, has been done by pneumatic or hydraulic tools similar to those used in the shops. At the Memphis Bridge the riveters for the floor system were simply swung by long ropes; at the Poughkeepsie Bridge they were swung from a trolley that ran on a longitudinal track, moving in a transverse arc and all revolving about the centre of a small traveller that cleared the inside braces of the main traveller. At the Washington Bridge the arch rib splices were riveted up by a machine that hung from two differential hoists carried by a trolley whose deeply grooved wheels rolled on 4-inch round bars that themselves rolled freely on the horizontal braces. Electric field riveters are being much used abroad. A recent pattern has a small motor that by reducing gear operates a screw piston which develops hydraulic pressure for driving each rivet.

Adjustments of tension members sometimes are required to be made accurately and verified; this can be done by interposing an ordinary spring dynamometer so as to form a temporary link in the connection. But this often is a needless refinement, since the proportionate, if not the actual, tensions of members can generally be closely estimated after some experience by striking them with a hammer. In adjusting over 600 floor-beam suspenders of the Niagara Bridge the writer was able to estimate the strain by feeling of the ropes almost within the limits of graduation of the dynamometer.

In the same bridge L. L. Buck reinforced the original anchor chains by additional new links, and accurately adjusted the load taken up by the latter by the elongation produced.

When only a slight discrepancy at first exists between tension bars of the same members they will usually adjust themselves by proportionate elongations; but if the variation is too great it may be sometimes eliminated by heating both bars (as by wrapping them with oily waste and igniting it), and allowing them to set themselves in cooling. The attempt to shorten the steel ribs of the St. Louis arches by packing them in ice proved unsuccessful as described above.

Thus it is seen that in bridge erection unforeseen and perplexing contingencies continually arise, and must be met by all the resources of science and mechanics.

The equipment of a complete general erection outfit should comprise full kits of carpenters' and blacksmiths' and masons' tools, portable forges (these may be improvised with half barrels filled with clay, and a large bellows pumping into another large barrel with tuyere pipes to three or four of them will furnish very satisfactory and economical blast, and may be very convenient for such work as riveting up buckle-plate flooring), riveters' outfits, tool steel, plenty of steel spikes, fitting bolts, long bolts and washers, hand screws, hand chisels and gouges, ratchets, reamers, screw and set wrenches, large key wrenches with rings, pinch bars, crowbars, hand hammers, sledges, a 50-ft. and a 100-ft. steel tape, rubber clothing, lights, and the more important tools, etc., mentioned above.,

American bridge engineers design their structures with careful consideration for the erection requirements, and of the combination bridges lately built in the far West, some, if not many, have been specially constructed to afford facility for replacing the wooden compression members with iron without disconnecting them or impairing the integrity of the structures.

A large and increasing proportion of the bridge erection here to-day consists in the renewal of existing structures where it is almost invariably demanded that the traffic shall not be interrupted. When permissible the road is usually moved to a temporary crossing on one side of the old structure, which is then demolished and replaced unrestrictedly, but this method is often not practicable and numerous other expedients are resorted to; most often, probably, trestle work is put up under the span and the track transferred to it, as well as the old structure after it is disconnected. As soon as it is removed the new structure is assembled upon the trestles, everything being scrupulously made to clear the trains or only used in fixed intervals between them. Sometimes the old bridge is made to support the new one till the latter is swung and self-supporting.

when it in turn supports the old one until completely removed. Sometimes the old structure is taken out piecemeal and replaced by the new, or clamped to it for certain periods.

Some very remarkable achievements have been accomplished by L. L. Buck, whose work on the Railroad Suspension Bridge at Niagara Falls is a masterpiece of skill and ability. He, at first, opened the anchor pits, disconnected the main cables, replaced numerous corroded wires, removed the anchors, replaced them and added new links and pins to their chains. Afterwards he replaced the whole suspended wooden superstructure with steel and iron floors and trusses. At another time he removed portions of the masonry towers, and put in new stones, and finally he replaced the massive towers with steel structures standing on substantially the same foundations, and accomplished all the difficult operations quickly, cheaply, and without loss of life or any serious interruption of traffic.

The rapidity with which erection work can be executed is illustrated by the bringing of the material for a 200-ft. railroad bridge from a storage yard 1000 feet away, and erecting it in 16 working hours after falsework was ready.

The 518-ft. span of the Cairo Bridge was erected by Baird Brothers in six days; two spans were erected and the falsework and traveller twice put up and moved in one month and three days, inclusive of five days of idle time.*

Bridge erection is subject to many dangers, and serious accidents appear to be inevitable. Those occurring to single individuals are often not the fault of the victim, who is frequently injured by an article dropped by some one else or knocked off from the work by some carelessness. An experienced bridgeman seldom falls from a great height through dizziness or missteps, but may do so by carelessly stepping on a loose plank. Some terrible accidents have occurred by the collapse of falsework, trestling, and travellers, some of them perhaps due to derailments or breakages while hoisting heavy pieces, or possibly to outright general weakness, but comparatively few are ever exactly determined, except where, as is too often the case, trestles are destroyed by floods scouring out the bottom underneath them or piling vast quantities of drift against them, as was the case at Wheeling, W. Va., and has been frequent in the Ohio River bridges.†

No other calling demands and receives the experience, courage, good judgment, and personal endurance displayed by the leaders and skilled workmen in bridge erection. They must construct the loftiest and most difficult scaffolding solely by their own resources, often in remote and dangerous positions, and upon them must handle and perfectly adjust heavy girders and huge chords, etc., weighing perhaps 100,000 lbs., while subject to constant peril of destruction by storm and flood, or they must build great trusses in the very path of frequent express trains without impeding their progress or prejudicing their safety. Under such trying circumstances their work is accomplished with a rapidity and accuracy exceeding that in some comfortable and well equipped shops and mechanical plants, and the great address and faithfulness, general integrity and reliability that they exhibit in their difficult tasks brings them into deserved prominence among constructive workmen. These men are characteristic of our grand nation. Keeping pace with the unparalleled creations of this generation of bridge designers, they have applied no ordinary engineering skill to the devising and execution of erection methods whose success is attested by scores of monumental constructions, and the absence of many great disasters.

---

* Perhaps the most remarkable achievement on record (to March, 1896) is found in the erection of three Pegram-truss spans (see p. 67), of 217 feet each, on the Union Pacific R. R. These three spans were erected, floor system put in, and ties and guard-rails finished on Feb. 8, 1896, the bridge gang having arrived on the ground on the evening of Jan. 24. Thus in twelve working days the false-work was put in, the traveller erected, the old Howe-truss spans removed, and 660 feet of new bridge erected and completed. The last span, not including the floor system, was erected in five hours and twenty minutes, the material being brought from the storage yard, several hundred feet away. On an average 70 men were employed, 28 of them being on the traveller. Only one hoisting-engine was used.

† To avoid a recurrence of such accidents the Edge Moor Bridge Co. employed in 1897 an auxiliary truss, 200 feet long, for the erection of the new Newport and Cincinnati bridge, as shown in Plate L. This was made compulsory for the channel span, which was not allowed to be wholly obstructed during erection, and its length was then reduced so as to employ it on the other (198 foot) spans.

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